Triple-Regge phenomenology. A new approach

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The amplitude for the inclusive reaction $A+B\rightarrow C+$anything is evaluated in terms of known quantities, overcoming the difficulty of the triple-Regge vertex by expressing the three-Reggeon residue $\beta^P_{\alpha_i\alpha_i}$ in terms of the residue $\beta^P_{R_iR_i}$, where R_i is a known resonance on the α_i trajectory. An application to $pp\rightarrow\pi^\pm X$ is made.

The amplitude for the inclusive reaction $A+B\rightarrow C+$anything is usually evaluated from the "two-body" process $A+B\rightarrow C+X$ where the "particle" X has mass M. By taking the limit $s/M^2\rightarrow\infty$, $M^2\rightarrow\infty$, one obtains the following expression for the differential cross section:[1-3]

$$\frac{d^2\sigma^C_{AB}}{dt\,dM^2}\underset{\substack{s/M^2\rightarrow\infty\\ M^2\rightarrow\infty}}{\longrightarrow}\frac{1}{16\pi s^2}\left[\frac{d}{dt}(A+C\rightarrow C+A)\right]^{1/2}|\xi(\alpha_i(t))|\left[\frac{s}{M^2}\right]^{\alpha_i(t)}\beta^P_{BB}(0)\beta^P_{\alpha_i\alpha_i}(0)(M^2)^{\alpha_P(0)}\,, \tag{1}$$

where $\alpha_i(t)$ is a Regge trajectory in the $A\bar{C}$ channel, ξ is the signature factor, and β^a_{bc} denote the coupling of a to b and c.

Thus we see that in order to evaluate $d^2\sigma^C_{AB}/dt\,dM^2$, we have to calculate the cross-channel high-energy scattering amplitudes (i) $A+\bar{C}\rightarrow A+\bar{C}$ at c.m. energy $s(\cos\theta_s\sim s/M^2)$ and momentum transfer $t_1=t$, and (ii) $\alpha_i(t)+\bar{\alpha}_i(t)\rightarrow B+\bar{B}$ at c.m. energy M^2 and momentum transfer $t_2=0$. The first amplitude can be easily calculated within the framework of the Regge-pole theory, whereas the second involves Reggeon scattering. This is rather unfortunate, since it is well known that Reggeons are not free objects to interact between themselves or with other particles and so one can say little theoretically about the triple-Regge vertex $\beta^P_{\alpha_i\alpha_i}$. Due to this, formula (1) does not have much predictive power and it is usually used to evaluate the form of the exchanged trajectory $\alpha_i(t)$ (Ref. 4).

For this reason it is quite interesting to see whether one could be able to relate the triple-Regge residue $\beta^P_{\alpha_i\alpha_i}$ to the residue $\beta^P_{R_iR_i}$, where R_i is a resonance on the α_i trajectory. Since $\beta^P_{R_iR_i}$ can be easily calculated from the high-energy data of the total cross section $\sigma_T(R_iB)$, this implies that formula (1) is simplified a lot being expressed in terms of known quantities. We know, of course, that one can always analytically continue a Regge-pole formula from the scattering region to the resonance region when the Reggeon is the exchanged object. Here, we play the extrapolation game once more with the Reggeon now being the interacting particle.

As we know, Regge-pole theory does not predict the exact form of residue functions, apart from the fact that they should be smooth functions of their variables. Thus for the trajectory

$$\alpha_i(m^2)=\alpha_{0i}+\alpha_i'm^2\,,$$

we expect to have

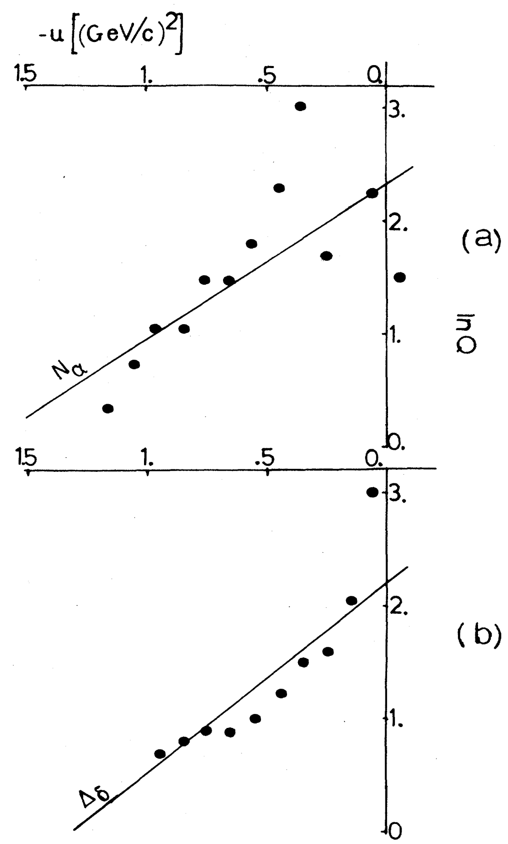

FIG. 1. $\ln Q$ vs u for (a) $pp\rightarrow\pi^+X$ at $M_X{}^2=825$ GeV^2 and $\sqrt{s}=45$ GeV, and (b) $pp\rightarrow\pi^-X$ at $M_X{}^2=875$ GeV^2 and $\sqrt{s}=45$ GeV. Both data are taken from Ref. 5. The differential cross section for $\pi^+p\rightarrow\pi^+p$ has been evaluated from the $p_{\rm lab}=7$ GeV data of Baker *et al.* (Ref. 9), whereas that for $\pi^-p\rightarrow\pi^-p$ has been evaluated from the $p_{\rm lab}=5.91$ GeV data of Owen *et al.* (Ref. 10). The straight lines represent the N_α and Δ_δ trajectories found in Ref. 5.

$$\beta^P_{\alpha_i\alpha_i}/\beta^P_{R_iR_i}=f_1(m^2)\ , \qquad (2a)$$

where $f_1(m^2)$ is a dimensionless smooth function of m^2 satisfying $f(m_{R_i}{}^2)=1$. So we put

$$f_1(m^2)=f_2\left[x=\frac{m_{R_i}{}^2-i\epsilon}{m^2-i\epsilon}\right],\ f_2(1)=0 \qquad (2b)$$

in order to avoid any trouble at $m^2=0$. ϵ is a parameter. For large values of $-u$ the data prefer $f_1(m^2)$ to be of the form $f_1(m^2)\propto 1/m^2$. So we parametrize $f_2(x)$ in the form

$$\begin{aligned}f_2(x)&=A_0+A_1x+A_2x^2\\&=(1-A_1-A_2)+A_1x+A_2x^2\end{aligned} \qquad (3)$$

and expect that ϵ is a small number.

The final expression for $f_2(x)$ is a consequence of the constraint $f_2(1)=1$. Of course, we do not expect the above parametrization to be valid for all values of x. We expect it, however, to describe in a good approximation the variation of $f_2(x)$ in the small region $t\le x\le m_{R_i}{}^2$. It is interesting to notice that the signature factor ξ_i in (1) avoids any coupling with the α_i trajectory at $\alpha_i=-\frac{1}{2}, -1, -\frac{3}{2},\ldots$.

Substituting (2b) into (1) we get

$$\frac{d^2\sigma^C_{AB}}{dt\,dm^2}\underset{\substack{s/M^2\to\infty\\ M^2\to\infty}}{\longrightarrow}\frac{1}{16\pi s^2}\left[\frac{d}{dt}(A+C\to C+A)\right]^{1/2}|\xi(\alpha_i(t))|\left[\frac{s}{M^2}\right]^{\alpha_i(t)}\beta^P_{BB}(0)f_2(x)\beta^P_{R_iR_i}(0)(M^2)^{\alpha_P(0)}\ . \qquad (4)$$

We shall apply now (4) for the inclusive process $pp\to\pi^\pm X$ using the data of Singh *et al.*,[5] since we have good experimental information for the related processes $\pi^\pm p\to\pi^\pm p$ in the backward direction, whereas by taking $R=p$ we need the amplitude for elastic pp scattering at zero momentum transfer. $\pi^+p\to\pi^+p$ and $pp\to\pi^+X$ are dominated by N_α exchange, whereas $\pi^-p\to\pi^-p$ and $pp\to\pi^-X$ are dominated by Δ_δ exchange.[5,6]

According to (4) the quantity $\ln Q$, where

$$Q=\left|\frac{d^2\sigma^{\pi^{+(-)}}_{pp}}{du\,dM^2}16\pi s^2\left[\frac{d\sigma}{du}(\pi^{+(-)}p\to\pi^{+(-)}p)\right]^{-1/2}|\xi_{N_\alpha(\Delta_\delta)}|^{-1}f_2(x)\beta^P_{pp}(0)^{-2}(M^2)^{-\alpha_P(0)}\right|$$

is proportional to $\alpha_{N_\alpha(\Delta_\delta)}\ln(s/M^2)$. So we can determine the values of A_1 and A_2 by demanding that by plotting $\ln Q$ versus u at fixed s and M^2 we get a straight line of slope equal to that of the N_α (Δ_δ) trajectory.

The best fit to the data gives $A_1=1$, $A_2=0$, $\epsilon\approx0.05$. In Fig. 1 we have plotted $\ln Q$ versus u using formula (4) and the values for $f_2(x)$ found above for both $pp\to\pi^+X$ and $pp\to\pi^-X$, and also the N_α and Δ_δ trajectories exchanged in these reactions found in Ref. 5.

Is it now reasonable to assume that (4) with $f_2(x)$ given above holds for any exchanged trajectory α_i and not only for $\alpha_i=N_\alpha,\Delta_\delta$? Indeed, if we believe in QCD this seems to be a reasonable hypothesis. Thus, as we slip on a Regge trajectory α_i, $\beta^P_{\alpha_i\alpha_i}$ should be a function of the mechanism which brings us from one point of the trajectory to another. However, both the bag model[7] and the string model[8] show that the mechanism for the formation of Regge trajectories is the same for all trajectories independently of being baryon of meson trajectories (this explains why all the trajectories have the same slope), implying that the ratio $\beta^P_{\alpha_i\alpha_i}/\beta^P_{R_iR_i}$ is described by the same function $f_2(x)$. Thus formula (4) describes the differential cross section for the inclusive reaction $A+B\to C+$anything for any particles A, B, and C in terms of known quantities, overcoming the difficulty of the triple-Regge vertex.

It is interesting to notice that according to our results, the total cross sections $\sigma_T(R_1B)$ and $\sigma_T(R_2B)$, where R_1 and R_2 are two resonances lying on the same Regge trajectory, are related by means of

$$m_{R_1}{}^2\sigma_T(R_1B)=m_{R_2}{}^2\sigma_T(R_2B)\ .$$

The verification of the above formula by the experiment, would be the most decisive test for the validity of (4). Unfortunately, existing experiments, up to now, do not give us the information we need in order to reach a definite conclusion.

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