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Dirac equation in a six-dimensional spacetime: Temporal polarization for subluminal interactions

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We examine the effect of the introduction of a six-dimensional extension of the Dirac equation on the problem of the physical meaning of the additional temporal coordinates of a (3+3) spacetime. We demonstrate that the electromagnetic field introduces a polarization of the temporal axes and that this polarization effect divides the (3+3) spacetime into six (3+1) Lorentzian subspaces. Subluminal interactions which involve fields and particles within a specified (3+1) subspace do not introduce multitemporal motion. We have therefore shown that a (3+3) formulation of the problems of subluminal and superluminal motion along with extended relativity is consistent with ordinary interactions predicted by the Dirac equation.

I. INTRODUCTION

Efforts to formulate a four-dimensional approach to the problem of superluminal motion have consistently led to the introduction of imaginary, or complex, numbers in situations where the physical system requires real observables.[1–17] Further, a recent paper by Marchildon, Antippa, and Everett[18] appears to have demonstrated that there is no possibility of an extended theory of relativity within the context of a purely (3+1) framework.

One alternative is a (3+3) spacetime, but this six-dimensional approach is not without problems. The most complete of the classical (3+3) systems, that of Cole,[19–26] provides an immediate insight into one of the more significant of these difficulties. If we postulate a symmetric six-dimensional metric of the Cole form

$$g_{\mu\nu}=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

and therefore admit time on an equal basis, then we must be prepared to deal with the possibility of multitemporal motion. In turn, this presents us with the problem of the meaning and physical observability of these additional time axes. In that we have no *a priori* knowledge of the nature of superluminal matter, this situation lacks any major significance with respect to the $v>c$ case. We are not so fortunate when dealing with $v<c$. The postulate of a multitemporal nature for spacetime also yields conflicts with experimental data. These have been discussed by Strnad,[27] who notes that these problems do not exist if interacting subluminal objects are forbidden nonparallel time vectors. The case of common spatial coordinates only is also admissible: two objects in this state cannot undergo an extended interaction. Patty[28] has attempted to deal with this problem by assuming that matter, although contained within a (3+3) spacetime, is intrinsically of a (3+1) nature. While this has some appeal, it constitutes an additional assumption. It would be far better if a reasonable extension of one of the laws governing subluminal matter were to impose the conditions necessary to remove the difficulty.

We report the results of an examination of the validity and effects of extending the well-known four-dimensional Dirac equation to six dimensions.[29] It is important to note we assume the metric given by Eq. (1.1) and the energy relation of the associated classical relativity theory. For the sake of clarity, we take the viewpoint of an observer of the type K. This K type designates any observer that considers the axes x^s, $s=1,2,3$ to be spacelike and the axes x^t, $t=4,5,6$ to be timelike. We designate an observer who takes a viewpoint inverted with respect to that of a K type as a J-type observer. We note that these viewpoints are equivalent: our selection of one does not result in a loss of generality. We let $\hbar=c=1$. The direct extension of the energy operator is the energy vector operator.

$$\begin{aligned} \mathbf{E}\psi &= i(\hat{\mathbf{e}}^4\partial_4+\hat{\mathbf{e}}^5\partial_5+\hat{\mathbf{e}}^6\partial_6)\psi \\ &= (\hat{\mathbf{e}}^4E_4+\hat{\mathbf{e}}^5E_5+\hat{\mathbf{e}}^6E_6)\psi \\ &= (\hat{\mathbf{e}}^4\eta_4+\hat{\mathbf{e}}^5\eta_5+\hat{\mathbf{e}}^6\eta_6)E\psi \;, \end{aligned} \tag{1.2}$$

where the η_t are the direction cosines of the time vector $\mathbf{t}$ and E is the magnitude of the energy vector.

We restrict our primary derivation to matter viewed as subliminal by a K-type observer. We designate this K-type matter and note that it must obey

$$(p^\mu p_\mu - m^2)\psi=0 \;, \tag{1.3}$$

where p^μ is the six-momentum operator. (Greek indices run from 1 through 6.) Again, this operator is a direct extension of the four-momentum operator.

In Sec. II we develop the covariant form of the six-dimensional Dirac equation and introduce the Hamiltonian form in Sec. III. We discuss the nature of our generalization in Sec. IV and give our conclusions in Sec. V.

II. THE COVARIANT FORM OF THE EXTENDED WAVE EQUATION

The extended wave equation my be written as either

$$(i\gamma^{\mu}\partial_{\mu}-m)\psi=0 \tag{2.1}$$

or

$$(\gamma^{\mu}p_{\mu}-m)\psi=0\ . \tag{2.2}$$

The γ^{μ} are expected to be matrices of the form $n\times n$. It is clear that Eq. (1.3) may be satisfied only if

$$\{\gamma^{\mu},\gamma^{\nu}\}=2g^{\mu\nu}I_n\ , \tag{2.3}$$

where I_n is an $n\times n$ unit matrix. ($\{a,b\}$ is the anticommutator of a and b. Square brackets denote ordinary commutators.) A representation of the six γ^{μ}, in terms of 8×8 matrices, is not difficult to find. This representation may be expressed in compact form by introducing

$$\sigma^1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},$$

$$\sigma^2=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},$$

$$\sigma^3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},$$

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},$$

$$0=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},$$

where we note that the first three 2×2 matrices are the standard form of the Pauli matrices. Our γ^{μ} representation is written as

$$\gamma^s=\begin{bmatrix}0 & 0 & 0 & \sigma^s\\ 0 & 0 & \sigma^s & 0\\ 0 & -\sigma^s & 0 & 0\\ -\sigma^s & 0 & 0 & 0\end{bmatrix}$$

for $s=1,2,3$ with the remaining three γ^{μ} given by

$$\gamma^4=\begin{bmatrix}I & 0 & 0 & 0\\ 0 & I & 0 & 0\\ 0 & 0 & -I & 0\\ 0 & 0 & 0 & -I\end{bmatrix},$$

$$\gamma^5=\begin{bmatrix}0 & 0 & -iI & 0\\ 0 & 0 & 0 & iI\\ iI & 0 & 0 & 0\\ 0 & -iI & 0 & 0\end{bmatrix},$$

$$\gamma^6=\begin{bmatrix}0 & 0 & 0 & I\\ 0 & 0 & -I & 0\\ 0 & -I & 0 & 0\\ I & 0 & 0 & 0\end{bmatrix}.$$

It is not difficult to demonstrate that the γ^{μ} and I_8 are generators of 64 Γ_j which divide into seven sets. The first set has one element, I_8, and is a zero-index object. The six γ^{μ} form the one-index objects of the second set. Fifteen $\sigma^{\mu\nu}=[\gamma^{\mu},\gamma^{\nu}]/2i$ form the third set of two-index objects. Twenty $(i/3!)\epsilon_{\alpha\mu\nu\sigma\rho\theta}\gamma^{\sigma}\gamma^{\rho}\gamma^{\theta}$ form the three-index object set. Fifteen $(i/4!)\epsilon_{\alpha\mu\nu\sigma\rho\theta}\gamma^{\nu}\gamma^{\sigma}\gamma^{\rho}\gamma^{\theta}$ provide an additional two-index object, $(i/5!)\epsilon_{\alpha\mu\nu\sigma\rho\theta}\gamma^{\mu}\gamma^{\nu}\gamma^{\sigma}\gamma^{\rho}\gamma^{\theta}$ provides the six components of an additional one-index object, and $(i/6!)\epsilon_{\alpha\mu\nu\sigma\rho\theta}\gamma^{\alpha}\gamma^{\mu}\gamma^{\nu}\gamma^{\sigma}\gamma^{\rho}\gamma^{\theta}$ gives another zero-index object. We label these the scalar, vector, antisymmetric rank-2 tensor, fully antisymmetric rank-3 tensor, pseudotensor, pseudovector, and pseudoscalar, respectively.

In the standard manner, explicit construction of the spinor transformation gives

$$\psi'=S\psi=\left[\exp\left[-\frac{i}{4}\omega\sigma_{\mu\nu}I^{\mu\nu}\right]\right]\psi\ , \tag{2.4}$$

where, for Lorentz boosts with $v<c$,

$$\tanh\omega=\frac{v}{c}=\beta,\quad \cosh\omega=(1-\beta^2)^{-1/2}\ ,$$

and

$$\sigma_{\mu\nu}=\frac{[\gamma_{\mu},\gamma_{\nu}]}{2i}\ .$$

$I^{\mu}{}_{\nu}$ gives the usual unit boosts extended to a $(3+3)$ space-time. Using Eq. (2.4), one can demonstrate the covariance of Eq. (2.1) or (2.2) under a proper Lorentz transformation from one K-type observer to a second K-type observer.

It is also straightforward to demonstrate that the 64 Γ_j are linearly independent and may be used as the basis for 8×8 Clifford numbers. These obey the algebra associated with the six-dimensional Dirac ring. As a direct consequence, one finds that

$$\det|G_{ij}|=(\mathrm{tr}I_8)^{64}\ ,$$

where

$$G_{ij}=\mathrm{tr}\Gamma_i\Gamma_j$$

and thus, up to a similarity transformation, our representation of the Γ_j and γ^{μ} is the only representation. We note that a second Γ_j representation may be obtained by taking all possible double direct matrix products of σ^1, σ^2, σ^3, and I. This presents an interesting comparison to the basis elements of the Dirac ring for which the same four 2×2 matrices may be combined under all possible single direct matrix products to yield the sixteen 4×4 Γ_j of the $(3+1)$ theory.

The above indicates that the $(3+3)$ theory duplicates the formal properties of the $(3+1)$ theory. The actual connection between $(3+1)$ and $(3+3)$ spacetimes is discussed in Sec. IV.

III. THE HAMILTONIAN FORM OF THE WAVE EQUATION

We now seek to place Eq. (2.1) in the form

$$H\psi=E\psi \tag{3.1}$$

and thus discover the form of H. Before proceeding we add a real electromagnetic six-vector potential A^μ to Eq. (2.2), such that

$$[\gamma^\mu(p_\mu-eA_\mu)-m]\psi=0 \tag{3.2}$$

or

$$\gamma^\mu p_\mu\psi-e\gamma^\mu A_\mu\psi-m\psi=0\ . \tag{3.3}$$

The first term on the left may be written as

$$(-\gamma_1\cdot\mathbf{p}+\gamma_2\cdot\mathbf{E})\psi=\gamma^\mu p_\mu\psi\ , \tag{3.4}$$

where

$$\gamma_1=\gamma^1\hat{\mathbf{e}}_1+\gamma^2\hat{\mathbf{e}}_2+\gamma^3\hat{\mathbf{e}}_3\ , \tag{3.5}$$

$$\gamma_2=\gamma^4\hat{\mathbf{e}}_4+\gamma^5\hat{\mathbf{e}}_5+\gamma^6\hat{\mathbf{e}}_6\ . \tag{3.6}$$

$\mathbf{p}$ is the usual three-momentum operator and $\mathbf{E}$ is defined by Eq. (1.2). Using Eqs. (3.5) and (3.6) again, the second term on the left of Eq. (3.3) may be written as

$$\gamma^\mu eA_\mu\psi=(-\gamma_1\cdot e\,\mathbf{A}_1+\gamma_2\cdot e\,\mathbf{A}_2)\psi\ , \tag{3.7}$$

where $\mathbf{A}_1$ is the usual three-vector potential and $\mathbf{A}_2$ is a vector potential associated with the multitemporal axes. Using Eqs. (3.4) and (3.7) in Eq. (3.3) gives

$$(-\gamma_1\cdot\mathbf{p}+\gamma_2\cdot\mathbf{E}+\gamma_1\cdot e\,\mathbf{A}_1-\gamma_2\cdot eA_2-m)\psi=0 \tag{3.8}$$

or

$$[\gamma_1\cdot(\mathbf{p}-e\,\mathbf{A}_1)+\gamma_2\cdot e\,\mathbf{A}_2+m]\psi=\gamma_2\cdot\mathbf{E}\psi\ . \tag{3.9}$$

We now use Eq. (1.2) to express the right-hand side of Eq. (3.9) as

$$\gamma_2\cdot\mathbf{E}\psi=\gamma_2\cdot(\eta_4\hat{\mathbf{e}}_4+\eta_5\hat{\mathbf{e}}_5+\eta_6\hat{\mathbf{e}}_6)E\psi \tag{3.10}$$

or

$$\gamma_2\cdot\mathbf{E}\psi=\Theta E\psi\ , \tag{3.11}$$

where

$$\Theta=\eta_4\gamma^4+\eta_5\gamma^5+\eta_6\gamma^6 \tag{3.12}$$

or

$$\Theta=\begin{bmatrix} I\eta_4 & 0 & -iI\eta_5 & I\eta_6\\ 0 & I\eta_4 & -I\eta_6 & iI\eta_5\\ iI\eta_5 & -I\eta_6 & -I\eta_4 & 0\\ I\eta_6 & -iI\eta_5 & 0 & -I\eta_4 \end{bmatrix}\ . \tag{3.13}$$

Thus,

$$[\gamma_1\cdot(\mathbf{p}-e\,\mathbf{A}_1)+\gamma_2\cdot e\,\mathbf{A}_2+m]\psi=\Theta E\psi \tag{3.14}$$

but

$$\Theta=\Theta^\dagger=\Theta^{-1}\ , \tag{3.15}$$

so

$$[\Theta\gamma_1\cdot(\mathbf{p}-e\,\mathbf{A}_1)+\Theta\gamma_2\cdot e\,\mathbf{A}_2+\Theta m]\psi=E\psi\ , \tag{3.16}$$

from which we identify the Hamiltonian

$$H=\Theta\gamma_1\cdot(\mathbf{p}-e\,\mathbf{A}_1)+\Theta\gamma_2\cdot e\,\mathbf{A}_2+\Theta m\ . \tag{3.17}$$

However, there is a problem. The first and third terms of the Hamiltonian are Hermitian, but the middle term is neither Hermitian nor anti-Hermitian. It may be split to give

$$\Theta\gamma_2\cdot\mathbf{A}_2=\mathbf{\Lambda}_a\cdot\mathbf{A}_2+\mathbf{\Lambda}_h\cdot\mathbf{A}_2\equiv\mathbf{\Lambda}\cdot\mathbf{A}_2\ , \tag{3.18}$$

where $\mathbf{\Lambda}_a\cdot\mathbf{A}_2$ is an anti-Hermitian operator and $\mathbf{\Lambda}_h\cdot\mathbf{A}_2$ is a Hermitian operator. We note that $\mathbf{\Lambda}_h\cdot\mathbf{A}_2$ consists of the diagonal terms of $\Theta\gamma_2\cdot\mathbf{A}_2$ and $\mathbf{\Lambda}_a\cdot\mathbf{A}_2$ consists of the off-diagonal terms of this same operator.

Recalling that $\mathbf{A}_2$ is real, we now consider the physically realizable conditions necessary and sufficient to make H a Hermitian operator. We may set all components of $\mathbf{A}_2$ equal to zero. However, we note that this is not a realistic general assumption for matter obeying Eq. (3.2). We may set two components of $\mathbf{A}_2$ equal to zero. This alone is not enough to ensure that H is Hermitian. This follows from the retention of one term from $\mathbf{\Lambda}_a\cdot\mathbf{A}_2$. If additional terms of $\mathbf{A}_2$ are retained as nonzero, the problem simply becomes more complicated. Thus, a restriction on the field with which the particles interact is not sufficient. We now consider $\mathbf{\Lambda}_a$. If we set $\eta_i=\eta_j=0$ and $\eta_k=1$ for some combination of $i,j,k=4$, 5, or 6 and $i\neq j\neq k\neq i$, then all $\mathbf{\Lambda}$ components containing η_i and η_j vanish. However, this action alone is not sufficient to eliminate $\mathbf{\Lambda}_a$ and $\mathbf{\Lambda}_a\cdot\mathbf{A}_2$. Further, other possible values of the direction cosines again only complicate the situation. Thus, a simple restriction on the direction of the time vector of the interaction particles is not sufficient. The final case involves a combination. Consider $A_i\neq0, A_j=A_k=0$ and $\eta_i=1,\eta_j=\eta_k=0$. This combined restriction on both the applied field and on the time vector of the interacting particles is necessary and sufficient to ensure that $\mathbf{\Lambda}_a\cdot\mathbf{A}_2=0$.

Up to this point, we have restricted our derivation to the viewpoint of a K-type observer who is examining interactions involving only K-type matter. Although the covariant form of Eq. (3.2) makes it clear that we should be able to find a transformation that carries our results into the case of a J-type observer who is considering interactions involving only J-type matter, it is now a simple and instructive process to actually construct the Hamiltonian for this case. There are two choices as to the specific method used in this construction: we may search for the actual transformation, or we may take advantage of the work already performed and construct the J-type Hamiltonian directly. We choose the latter approach.

Recalling that a J-type observer has a viewpoint inverted with respect to that of a K-type observer and thus considers the axes x^s, $s=1,2,3$ to be timelike and the axes x^t,

$t=4,5,6$ to be spacelike, we note that the J-type observer writes the energy operator as

$$\begin{aligned}\mathbf{E}\psi&=i(\hat{e}^{1}\partial_{1}+\hat{e}^{2}\partial_{2}+\hat{e}^{3}\partial_{3})\psi\\&=(\hat{e}^{1}E_{1}+\hat{e}^{2}E_{2}+\hat{e}^{3}E_{3})\psi\\&=(\hat{e}^{1}e\eta_{1}+\hat{e}^{2}\eta_{2}+\hat{e}^{3}\eta_{3})E\psi\ ,\end{aligned} \tag{3.19}$$

where the η_s are again the direction cosines of $\mathbf{E}$ and $\mathbf{t}$. There is also a corresponding adjustment in the three-momentum vector operator. Further, J-type matter, which is matter a J-type observer views as subluminal, must obey

$$(p^{\mu}P_{\mu}+m^{2})\psi=0\ , \tag{3.20}$$

rather than Eq. (1.3). Equations (2.1) and (2.2) are not directly affected by the shift of viewpoints—beyond the meaning now assigned by the observer to the coordinates—to that of a J-type observer, but Eq. (3.20) will only be satisfied if

$$\{\gamma^{\mu},\gamma^{\nu}\}=-2g^{\mu\nu}I_{n}\ , \tag{3.21}$$

as compared to Eq. (2.3).

It is obvious that a representation of the γ^{μ} which satisfied Eq. (3.21) can be found. The most direct route to the desired representation is the simple permutation $\gamma^1\rightleftarrows\gamma^6$, $\gamma^2\rightleftarrows\gamma^5$, and $\gamma^3\rightleftarrows\gamma^4$. Of course, this is not the only acceptable permutation. It is equally obvious that the new γ^{μ} will serve as the basis for generating an alternative to our previous Dirac ring. In that we have demonstrated the uniqueness of the K-type representation of the ring, up to a similarity transformation, the fact that it is possible to construct the J-type representation demonstrates the existence of that transformation which was our other possible approach to arrive at a J-type H.

We now insert a potential A_{μ} as in Eq. (3.2) but note that an exchange of meaning of the components of A_{μ}, in direct analogy to the permutation of the γ^{μ}, is necessary to conform to the J-type nature of the problem we are considering. Repeating the procedures used to develop H for the K-type observer yields

$$H=\Theta\boldsymbol{\gamma}_{2}\cdot(\mathbf{p}-e\,\mathbf{A}_{2})+\Theta\boldsymbol{\gamma}_{1}\cdot e\,\mathbf{A}_{1}-\Theta m\ , \tag{3.22}$$

where $\boldsymbol{\gamma}_1$, $\boldsymbol{\gamma}_2$, $\mathbf{A}_1$, and $\mathbf{A}_2$ have also been redefined according to the permutation procedure given above and have components consistent with the J-type viewpoint. To eliminate the very real possibility of confusion, it is best to give the explicit expressions for the γ vectors. They are

$$\boldsymbol{\gamma}_{1}=\gamma^{1}\hat{e}_{1}+\gamma^{2}\hat{e}_{2}+\gamma^{3}\hat{e}_{3} \tag{3.23}$$

and

$$\boldsymbol{\gamma}_{2}=\gamma^{4}\hat{e}_{4}+\gamma^{5}\hat{e}_{5}+\gamma^{6}\hat{e}_{6}\ , \tag{3.24}$$

which are identical in form to Eqs. (3.5) and (3.6) but have components given according to the premutation rules stated above [e.g., γ^1 of Eq. (3.23) is the γ^6 of Eq. (3.6)]. Also, $\mathbf{A}_2$ is now the usual three-vector potential and $\mathbf{A}_1$ is now the vector potential associated with the J-type multitemporal axes. The only term remaining to be identified is the Θ matrix. It is

$$\Theta=\begin{bmatrix}I\eta_{3}&0&-iI\eta_{2}&I\eta_{1}\\0&I\eta_{3}&-I\eta_{1}&iI\eta_{2}\\iI\eta_{2}&-I\eta_{1}&-I\eta_{3}&0\\I\eta_{1}&-iI\eta_{2}&0&-I\eta_{3}\end{bmatrix}, \tag{3.25}$$

as would be expected.

Again, there is a term which is composed of a Hermitian and an anti-Hermitian part: $\Theta\boldsymbol{\gamma}_1\cdot e\,\mathbf{A}_1$. Reasoning identical to that used in finding the conditions necessary to make the K-type Hamiltonian (H_K) a Hermitian operator may be applied. The conditions necessary to make the J-type Hamiltonian (H_J) a Hermitian operator are $A_i\neq 0, A_j=A_k=0$ and $\eta_i=1,\eta_j=\eta_k=0$ for some combination of the index values 1,2,3.

There remains one point that, for clarity, should be made explicit. Throughout the construction of the J-type H the components of the various matrices and the elements of Eq. (3.25) have altered. From this, it is possible to reach the conclusion that the basis vectors $\hat{e}_{\mu}$ have also altered. However, in that we wish to retain the metric of Eq. (1.1), this is not the case and the $\hat{e}_{\mu}$ of the K-type coordinate system are identical to the $\hat{e}_{\mu}$ of the J-type coordinate system. The only change associated with the basis vectors is the view of their timelike or spacelike nature as taken by a particular observer type. Of course, this will be consistent with the observer's view of the meaning of the coordinates. Other approaches to the notation used could have been selected. Our choice was made to place emphasis on the significant relationships which exist between the two observer types and the importance in a (3+3) spacetime of ensuring that the meaning of physical quantities, as understood by the observer under consideration, is carefully preserved.

This completes the derivation of H_K and H_J for the cases of a K-type observer viewing K-type matter and a J-type observer viewing J-type matter. In that our concern in this paper is with the mechanism of temporal vector parallelism preservation for a collection of interacting subluminal matter within a (3+3) spacetime, in fact that these purely subluminal cases are sufficient for the task eliminates that need for the calculation of the superluminal cases of a K-type observer viewing J-type matter or a J-type observer viewing K-type matter. Thus, we end our derivations at this point and leave the inessential and considerably more complex question of superluminal motion for future consideration.

IV. DISCUSSION

In the preceding section we established the necessary and sufficient conditions for the (3+3) H_K and H_J to be Hermitian operators describing subluminal matter obeying the extended Dirac equation. Further, we established, by explicit construction, the physically equivalent nature of the K-type and J-type viewpoints. The covariance of Eq. (3.2) aside, this equivalence is not particularly unexpected. Examination of the definition of the K-type observer demonstrates that it is in direct analogy to the formulation of special relativity based on a (3 + 1) metric given by

$$g_{\mu\nu}=\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \quad (4.1)$$

A similar examination of the definition of the J-type observer demonstrates that it is in direct analogy to the formulation of special relativity based on a $(3+1)$ metric given by

$$g_{\mu\nu}=\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \quad (4.2)$$

It is well known that these are physically equivalent approaches. It should be noted that our work, restricted to the case of subluminal matter, does not demonstrate that the K-type and J-type viewpoints will have simultaneous physical existence within the same $(3+3)$ spacetime. We have only demonstrated a mathematical possibility. Resolution of this question requires consideration for the superluminal Hamiltonian, which might be formulated within the framework of a $(3+3)$ spacetime, but which is beyond the stated scope of our present work.

We now examine the consequences of the restrictions placed on H_K and H_J by the requirement of Hermiticity. We begin with some notational definitions. Consider a K-type observer. Let there exist a collection of interacting matter, within the $(3+3)$ spacetime, for which the initial state is such that the temporal vectors of the particles constituting this matter are paralled. Further, let this collection of matter have its rest frame as a K-type reference frame. Clearly, the initial state of this collection of matter defines a $(3+1)$ subspace of the $(3+3)$ spacetime. It is not necessary that the K-type observer start with his temporal coordinates configured in such a way that the collection of matter is unitemporal with respect to this coordinate system: he is free to orient these axes as he chooses. However, by Eq. (3.17) and its associated restrictions, the observer can only obtain a Hermitian H_K with which to describe the Dirac particles within this matter if he aligns one of his time axes with the time axis of the matter. Such an action would be natural and is accomplished by a rotation of the observer's temporal axes. Let the observer carry out such a coordinate transformation. We introduce a label for K-type $(3+1)$ subspaces: they are designated Ω_t where $t=4$, 5, or 6 and indicates the temporal axis selected by the observer for alignment with the temporal vector of the $(3+1)$ subspace. Let our observer select the axis $\hat{\mathrm{k}}$ for alignment. Then the subspace under observation will be $\Omega_{\hat{k}}$ and we further designate a $(3+1)$ observer composed of matter belonging to $\Omega_{\hat{k}}$ as a $K_{\hat{k}}$-type observer. This $(3+1)$ observer would use a metric of the form given in Eq. (4.1). Of course, the $K_{\hat{k}}$ observer has the usual option of using Eq. (4.2), but we eliminate this choice to permit a consistent notation within which to evaluate our results. Accepting the reinterpretation principle, we do not consider a negative-time vector particle as belonging to a separate subspace.

Notational considerations and initial conditions stated, we now proceed with our discussion. Consider interactions involving only $\Omega_{\hat{k}}$ matter. The K-type observer notes that the proper set of restrictions for the subspace $\Omega_{\hat{k}}$ Hamiltonian are $\eta_{\hat{k}}=1, \eta_{\hat{i}}=\eta_{\hat{j}}=0$ and $A_{\hat{k}}\neq 0$, $A_{\hat{i}}=A_{\hat{j}}=0$, where $\hat{i}\neq\hat{j}\neq\hat{k}\neq\hat{i}$. The observer further notes, by explicit examination for the Hamiltonian, that the interactions described by this Hamiltonain can never lead to the creation of a multitemporal state for the particles involved in the interaction. In that this Hamiltonian is only applicable to Dirac particles, it is natural to consider particles of other spins and the possibility of an interaction within the subspace resulting in multitemporal behavior of these particles. However, that such interactions are also eliminated may be quickly determined by considering the necessity for the conservation of six-momentum. The K-type and, thus, $K_{\hat{k}}$-type observer will never see multitemporal motion resulting from subluminal interactions that occur between matter contained within this subspace. Identical arguments may be applied to a J-type observer and will result in the same conclusion concerning the behavior of $\Omega_{\hat{s}}$ matter observed by a $J_{\hat{s}}$-type observer.

Before proceeding to consider the remaining possible interactions, we should determine if the physics described by at least one of the Hamiltonians of the $(3+3)$ K-type observer and at least one of the Hamiltonians of the $(3+3)$ J-type observer is identical to that of ordinary $(3+1)$ Dirac physics. We examine Ω_4 in the K-type subspace set and Ω_3 in the J-type subspace set. Inserting the proper restrictions on the Hamiltonians for these subspaces allows, by direct inspection, confirmation that no new physical behavior has been introduced.

In addition to set the states associated with the $\Omega_{\hat{k}}$ subspace, the $(3+3)$ K-type observer will also note two additional subluminal sets. These belong to the subspaces $\Omega_{\hat{i}}$ and $\Omega_{\hat{j}}$, where the indices are as previously defined. If no Dirac matter occupies the states of $\Omega_{\hat{i}}$ and $\Omega_{\hat{j}}$, these may be considered to be defined only to the degree that their temporal vectors must lie in a plane perpendicular to the $\Omega_{\hat{k}}$ temporal vector and that the $\Omega_{\hat{i}}$ and $\Omega_{\hat{j}}$ temporal vectors must be mutually perpendicular. This requirement follows directly from Eq. (3.17) and its associated restrictions combined with the fact that there is no particular reason for the K-type observer to retain the chosen orientation of these $\hat{\mathrm{i}}$ and $\hat{\mathrm{j}}$ axes.

It is natural for the K-type observer to consider what, if any, conditions might lead to the transfer of matter initially in one of the $\Omega_{\hat{k}}$ states to a state belonging to either $\Omega_{\hat{i}}$ or $\Omega_{\hat{j}}$. We have established that an interaction which is internal to $\Omega_{\hat{k}}$ is insufficient. But mathematically, there could be other $(3+1)$ subspaces within the $(3+3)$ spacetime. Further, matter could exist in either $\Omega_{\hat{i}}$ or $\Omega_{\hat{j}}$. Let us consider the interaction of a particle belonging to $\Omega_{\hat{k}}$ with a particle of the K-type but not belonging to $\Omega_{\hat{k}}$. Let the interaction be electromagnetic in nature and let the $\Omega_{\hat{k}}$ particle be a Dirac particle. The K-type observer will attempt to write a two-particle Hamiltonian to describe the interaction. But it is clear that no coordinate transformation of the K-type observer's temporal axes will permit the elimination of the anti-Hermitian part of this Hamiltonian. However, by Eq. (3.18), the Hamiltonian describing such an interaction may be split into a Hermitian and an anti-Hermitian part. That is,

$$(H_h + H_a)\psi = E\psi \ . \tag{4.3}$$

The eigenvalues of H_h will be real and the eigenvalues of H_a will be imaginary. Physically, this represents a damped state for the particle originally contained in $\Omega_{\hat{k}}$. This damped state will decay into an available stable state. This may be any state described by the three possible K-type Hermitian Hamiltonians associated with the K-type observer's original choice of temporal coordinate transformation as determined by the initial configuration of $\Omega_{\hat{k}}$. The process should be considered as a quantum transition of a bound particle. Prior to the interaction the $\Omega_{\hat{k}}$ particle, constrained to remain in $\Omega_{\hat{k}}$ if not disturbed by an external force, should be termed to be in a temporally bound state. During the transition, this particle was not temporally bound. After the transition was completed, the particle was again in a temporally bound state. As is normal for transitions between bound states, one does not expect the actual transition state to be observable. This, in part, eliminates the difficulty created by the fact that Hermiticity fails during the interaction, but a problem remains: the K-type observer, although a $(3+3)$ observer, is not able to write a Hermitian Hamiltonian for $\Omega_{\hat{k}}$ and, without transforming his temporal coordinates in such a way as to destroy the Hermiticity of his description of $\Omega_{\hat{k}}$, also obtain a Hermitian Hamiltonian with which to describe matter not contained within either $\Omega_{\hat{k}}$ or $\Omega_{\hat{i}}$ or $\Omega_{\hat{j}}$.

The Hermiticity problem described above may be resolved in several ways. We advance a tentative choice as being both that implied by a mathematical interpretation of our results and that consistent with the combined postulates of quantum mechanics and relativity. We allow $(3+3)$ K-type observers and $(3+3)$ J-type observers to retain the option of selecting any orientation of their temporal axes. With each orientation selected, we associate a threefold subluminal set of temporally bound $(3+1)$ states. Each $(3+1)$ subset of unitemporal states is termed a possible $(3+1)$ subspace of the $(3+3)$ spacetime. The $(3+1)$ elements of each threefold set are described by the three Hermitian Hamiltonians associated with the particular orientation selected. These states may be fully, partially, or not occupied. But a $(3+1)$ subspace is only defined when some states within that subspace are occupied. We did not make this distinction in the preceding discussion but now introduce it to provide separate terms for describing mathematical possibility and physical reality. We note that the union of one threefold subluminal set and a second threefold subluminal set is not necessarily the null set. With respect to a particular subspace $\Omega_{\hat{k}}$, the rotational freedom about the temporal axis of $\Omega_{\hat{k}}$ available to the $\hat{i}$ and $\hat{j}$ temporal axes gives an infinite number of threefold sets. Union between any two of these sets yields the $\Omega_{\hat{k}}$ states. We consider all threefold sets of subluminal states associated with such a union as states available to an $\Omega_{\hat{k}}$ particle. Further, if the union of the $\Omega_{\hat{k}}$ states and the threefold set of states associated with a given orientation of the $(3+3)$ observer's temporal axes is the null set, then we consider the states of this threefold set to be inaccessible to the $\Omega_{\hat{k}}$ particle. Thus, the Hermiticity conditions for H_K and H_J are interpreted as defining those states available, and those states not available, to a particular Dirac particle within the $(3+3)$ spacetime. We call this effect temporal polarization.

As a final point, if the K-type and J-type observers are physically able to exist simultaneously within the $(3+3)$ spacetime, then the orientation of the spatial axes of a particular K-type observer corresponds to the orientation of the temporal axes of a set of J-type observers. When the orientation of the K-type observer's temporal axes is also taken into consideration a subset of J-type observers, each differing only in relative velocity with respect to the K-type observer, is fully determined. It is under the assumption of simultaneous physical existence and the above specification of the meaning of subset and subspace, that we describe the effect of the introduction of quantum mechanics and electromagnetic fields into the relativity theory of a $(3+3)$ spacetime as the division of that $(3+3)$ spacetime into six $(3+1)$ Lorentzian subspaces.

V. CONCLUDING REMARKS

We have demonstrated that there are no contradictions between the observed unitemporal behavior of subluminal matter and a six-dimensional formulation of special relativity. Further, we have determined that the removal of the possibility of multitemporal behavior, which has been the primary problem with obtaining a physically realistic classical formulation, is a consequence of the quantum-mechanical behavior of Dirac particles in an applied electromagnetic field. In using the $(3+3)$ Dirac equation, we have found that there is some ambiguity in the interpretation of the Hermiticity requirements that apply to the six-dimensional Hamiltonian, but a form of temporal polarization accompanies any possible interpretation and results in the unitemporal behavior necessary for consistency with physical reality. This is sufficient to resolve the problem

Our next task is to examine the possibility of interaction between K-type matter and J-type matter. In that these are inherently in a state of superluminal motion with respect to each other, this amounts to examining the superluminal Hamiltonian. We will consider this problem in our next paper.

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