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Quantum chromodynamics with colored Higgs mechanism

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We derive, in the context of a previously proposed chiral soliton bag model with colored Higgs mechanism, the field equations for the eight gluons and the remaining four Higgs particles. It is pointed out that the boundary condition used by Jändel for supporting his claim about the lack of a suppression mechanism for open-color states does not follow from these highly coupled nonlinear field equations, unless nonlinearities appearing in these equations are neglected. It is also pointed out that the possible presence of a large surface energy in the proposed model does not necessarily give rise to the so-called "fine-tuning" problem. Finally, we also outline briefly a few additional important aspects which may be useful for further understanding the proposed model.

In the preceding paper,[1] Jändel has used a certain gluon boundary condition [his Eq. (10)] to argue that bag states with open color may be present for the recently proposed chiral bag model with colored Higgs mechanism.[2] Owing to introduction of a large vacuum expectation value outside the bag, he has also argued [on the basis of his Eq. (11)] that an extraordinary large surface energy cannot be avoided in the proposed model. Since the proposed model[2] necessarily gives rise to different field equations for different gluons, he has also proposed a modification scheme which maintains global color SU(3) symmetry.

In this paper, we describe, in a specific version of the previously proposed model,[2] the field equations for the eight gluons and the remaining four Higgs particles. It is pointed out that these equations do not necessarily lead to the gluon boundary condition used by Jändel[1] [his Eq. (10)]. Thus, there is no ground to suspect the possible presence of the generic problem of the lack of natural suppression of open-color bag states for the proposed model.[2] It is also pointed out that, since the physical Higgs particles can be chosen to vanish identically for ordinary bag solutions, the possible presence of a large surface energy does not necessarily give rise to the so-called "fine-tuning" problem. Accordingly, there is no urgent need to call for his proposed global-color-SU(3)-invariant scheme, which is considerably more complicated than the proposed model.[2]

To set the record straight, we wish to point out that Gross and Wilczek,[3] and Cheng, Eichten, and Li[4] have investigated the problem of Higgs phenomena in asymptotically free gauge theories, except that the Higgs mechanism in question does not exhibit spatial dependence. However, it is clear that some violation of global color SU(3) symmetry can in principle be present. In addition, the model of Pati and Salam[5] allows severe violation of color symmetry but, yet, no serious disagreement with high-energy phenomena is anticipated.[6] The idea of invoking Higgs mechanism to confine a U(1) gauge boson to within the bag was discussed by Creutz and Soh.[7] Interestingly enough, the present author has come to realize the usefulness of invoking the Higgs mechanism to confine non-Abelian gauge bosons to within the bag in the context of the Friedberg-Lee model,[8] without knowing the related important developments mentioned above.[3–7]

At this juncture, we wish to reiterate for a general reader the importance of investigating any possible *renormalizable* soliton bag models, including the model of Friedberg and Lee,[8] the previously proposed model,[2] and Jändel's modified scheme.[1] The contemporary machinery developed for quantum field theories can only make systematic predictions out of a renormalizable model, whether it is an *effective* theory or not. Further, any theory, including the Glashow-Weinberg-Salam electroweak theory, is an *effective* theory at some level. Therefore, we actually do not have the luxury to argue, as Jändel did in his Comment,[1] that "as the soliton field is used only as an effective description for the low-energy features of the full theory there is no reason to require a renormalizable model," unless we are either able to improve our modern field-theoretic technique to allow nonrenormalizable models or willing to give up the quest of finding a quantitative theory.

To investigate important questions such as those raised by Jändel in the preceding paper,[1] it is essential to set down the field equations appropriate for the proposed model.[2] Considering first the gluon sector, we find[9]

$$\partial_\nu G^a_{\mu\nu} + g f_{abc} G^b_\nu G^c_{\mu\nu} = i\frac{g}{2}\bar{\psi}\gamma_\mu\lambda_a\psi + \delta L_H/\delta G^a_\mu \quad . \tag{1}$$

Here the Higgs Lagrangian L_H has been chosen as[2]

$$L_H = L^0_H - V_\Phi \quad , \tag{2a}$$

$$L^0_H = -[(D_\mu\Phi_+)^\dagger(D_\mu\Phi_+) + (D_\mu\Phi_-)^\dagger(D_\mu\Phi_-)] \quad , \tag{2b}$$

$$V_\Phi = \frac{\nu^2}{2}[2(\chi/\chi_\infty)^2 - 1][(\Phi^\dagger_+\Phi_+) + (\Phi^\dagger_-\Phi_-)] + \frac{\eta^2}{4}[(\Phi^\dagger_+\Phi_+)^2 + (\Phi^\dagger_-\Phi_-)^2 + 2(\Phi^\dagger_+\Phi_-)(\Phi^\dagger_-\Phi_+)] \quad , \tag{2c}$$

with

$$D_\mu \equiv \partial_\mu - ig\frac{\lambda_a}{2}G^a_\mu \quad ,$$
$$v^2 \equiv -\nu^2/\eta^2 >> m_p{}^2 \quad , \tag{2d}$$
$$\eta^2 > 0 \quad .$$

As indicated earlier,[2] it is possible to choose a gauge, i.e., the unitarity gauge, such that the two Higgs triplets are given by

$$\Phi_+ = \begin{pmatrix} \sin\theta(\eta_1 + i\eta_2) + \cos\theta\left(\eta_3 + \frac{1}{\sqrt{3}}\tilde{\eta}_8\right) \\ \cos\theta(\eta_1 - i\eta_2) - \sin\theta\left(\eta_3 - \frac{1}{\sqrt{3}}\tilde{\eta}_8\right) \\ 0 \end{pmatrix} , \tag{3a}$$

$$\Phi_- = \begin{pmatrix} \cos\theta(\eta_1 + i\eta_2) - \sin\theta\left(\eta_3 + \frac{1}{\sqrt{3}}\tilde{\eta}_8\right) \\ -\sin\theta(\eta_1 - i\eta_2) - \cos\theta\left(\eta_3 - \frac{1}{\sqrt{3}}\tilde{\eta}_8\right) \\ 0 \end{pmatrix} , \tag{3b}$$

with

$$\tilde{\eta}_8 = \eta_8 + \omega(\mathbf{r}) \ . \tag{3c}$$

Here $\{\eta_1, \eta_2, \eta_3, \eta_8\}$ are four real Higgs particles and $\omega(\mathbf{r})$ is the *spatial-dependent* vacuum expectation value characterizing the unique feature of the proposed Higgs mecahnism. Substituting Eqs. (3a) and (3b) into Eqs. (2b) and (2c), we obtain

$$\begin{aligned} L_H^0 = & -2[(\partial_\mu\eta_1)^2 + (\partial_\mu\eta_2)^2 + (\partial_\mu\eta_3)^2 + \tfrac{1}{3}(\partial_\mu\tilde{\eta}_8)^2] \\ & -2g\{G_\mu^1[(\partial_\mu\eta_2)\eta_3 - \eta_2(\partial_\mu\eta_3)] + G_\mu^2[(\partial_\mu\eta_1)\eta_3 - \eta_1(\partial_\mu\eta_3)] + G_\mu^3[(\partial_\mu\eta_1)\eta_2 - \eta_1(\partial_\mu\eta_2)]\} \\ & -g^2(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \tfrac{1}{3}\tilde{\eta}_8{}^2)[\tfrac{1}{2}(G_\mu^1G_\mu^1 + G_\mu^2G_\mu^2 + G_\mu^3G_\mu^3) + \tfrac{1}{4}(G_\mu^4G_\mu^4 + G_\mu^5G_\mu^5 + G_\mu^6G_\mu^6 + G_\mu^7G_\mu^7) + \tfrac{1}{6}G_\mu^8G_\mu^8] \\ & -g^2\eta_1\frac{1}{\sqrt{3}}\tilde{\eta}_8\left(\frac{2}{\sqrt{3}}G_\mu^1G_\mu^8 + G_\mu^4G_\mu^6 + G_\mu^5G_\mu^7\right) + g^2\eta_2\frac{1}{\sqrt{3}}\tilde{\eta}_8\left(\frac{2}{\sqrt{3}}G_\mu^2G_\mu^8 - G_\mu^4G_\mu^7 + G_\mu^5G_\mu^6\right) \\ & -g^2\eta_3\frac{1}{\sqrt{3}}\tilde{\eta}_8\left(\frac{2}{\sqrt{3}}G_\mu^3G_\mu^8 + \tfrac{1}{2}(G_\mu^4G_\mu^4 + G_\mu^5G_\mu^5) - \tfrac{1}{2}(G_\mu^6G_\mu^6 + G_\mu^7G_\mu^7)\right) \ . \end{aligned} \tag{4a}$$

$$V_\Phi = \frac{\nu^2}{2}[2(\chi/\chi_\infty)^2 - 1]2(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^3 + \tfrac{1}{3}\tilde{\eta}_8{}^2) + \frac{\eta^2}{4}[2(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \tfrac{1}{3}\tilde{\eta}_8{}^2)^2 + \tfrac{8}{3}(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2)\tilde{\eta}_8{}^2] \ . \tag{4b}$$

Equations (1), (4a), and (4b) allow us to set down the field equation explicitly for each gluon field. For instance, we have

$$\partial_\nu G_{\mu\nu}^1 + gf_{1bc}G_\nu^bG_{\mu\nu}^c = i\frac{g}{2}\bar{\psi}\gamma_\mu\lambda_1\psi - 2g[(\partial_\mu\eta_2)\eta_3 - \eta_2(\partial_\mu\eta_3)] - g^2(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \tfrac{1}{3}\tilde{\eta}_8{}^2)G_\mu^1 - \tfrac{2}{3}g^2\eta_1\tilde{\eta}_8G_\mu^8 \ , \tag{5a}$$

$$\partial_\nu G_{\mu\nu}^4 + gf_{4bc}G_\nu^bG_{\mu\nu}^c = i\frac{g}{2}\bar{\psi}\gamma_\mu\lambda_4\psi - \frac{g^2}{2}(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \tfrac{1}{3}\tilde{\eta}_8{}^2)G_\mu^4 - g^2\frac{1}{\sqrt{3}}\tilde{\eta}_8(\eta_3G_\mu^4 + \eta_1G_\mu^6 + \eta_2G_\mu^7) \ , \tag{5b}$$

$$\partial_\nu G_{\mu\nu}^8 + gf_{8bc}G_\nu^bG_{\mu\nu}^c = i\frac{g}{2}\bar{\psi}\gamma_\mu\lambda_8\psi - \frac{g^2}{3}(\eta_1{}^2 + \eta_2{}^2 + \eta_3{}^2 + \tfrac{1}{3}\tilde{\eta}_8{}^2)G_\mu^8 - \tfrac{2}{3}g^2\tilde{\eta}_8(\eta_3G_\mu^3 - \eta_2G_\mu^2 + \eta_1G_\mu^1) \ . \tag{5c}$$

Here it is essential to note that the field equations for the gluon fields contain a number of nonlinear terms. These nonlinearities make it extremely difficult, if not impossible, to solve *ordinary* QCD (i.e., QCD without colored Higgs mechanism) even in the classical approximation. The presence of nonlinearities in a non-Abelian gauge theory, contrary to a U(1) gauge theory such as QED, has also made it plausible to conjecture that these equations can be solved in conjunction with the boundary condition

$$\hat{\mathbf{n}}\cdot\mathbf{E}^a = 0, \quad \hat{\mathbf{n}}\times\mathbf{B}^a = 0, \quad a = 1, \ldots, 8 \ , \tag{6}$$

where $\hat{\mathbf{n}}$ is the unit vector normal to the confining surface. In other words, nonlinearities make it impossible to derive the boundary condition from the field equations, unlike what one can do in an Abelian U(1) gauge theory.[7] On the other hand, since we can already find nontrivial solutions satisfying the boundary condition (6) (as in glueball studies), it is clear that the problem of the boundary condition versus the field equations is far more complicated than Jändel has argued in his paper.[1]

In our opinion, there are at least three possible scenarios as we modify QCD by incorporating the colored Higgs mechanism.

(A) The nonlinearities involving only gluons, as expressed by the term $gf_{abc}G_\nu^bG_{\mu\nu}^c$ in Eq. (1), dictate formation of a bag. In other words, gluons are confined to within a radius R_G, which is somewhat smaller than the confining radius R_Q for quarks. Here we assume $\chi(\mathbf{r}) = \chi_\infty$ for $r \geq R_Q$.

(B) The nonlinearies mentioned in scenario A are not strong enough to dictate formation of a bag but, as augmented by additional nonlinearities induced by the colored Higgs mechanism, gluons get confined to within a radius R_G which is roughly the same as R_Q.

(C) All the nonlinearities mentioned above in A and B have nothing to do with gluon confinement. In this case, gluons are confined because of large masses outside the bag. Jändel's argument regarding the boundary condition may be justified if this is indeed the case.

We note that, in both cases B and C, conventional QCD fails to provide adequate confinement for gluons but introduction of colored Higgs mechanism helps in resolving the problem. It is clear that, in this paper, Jändel has chosen to ignore the potential important role played by nonlinearities and, consequently, scenario C has been his only focus. As indicated above, however, the boundary condition does not follow from the field equation except in special cases. As a matter of fact, even in the simple Friedberg-Lee model,[8] the boundary condition for quarks does not follow strictly from the field equation for quarks, since one can always make the transition region as soft as he wishes. Preliminary investigation suggests that the same may be true for gluons.

We turn our attention to the Higgs sector, in which Jändel has argued in his paper that a large surface energy is inevitable for a bag state. The field equations for Higgs particles can readily be obtained from Eqs. (4a) and (4b). For instance, we find

$$\partial_\mu\partial_\mu\eta_1+\frac{g}{2}\partial_\mu(G_\mu^2\eta_3+G_\mu^3\eta_2)+\frac{g}{2}(G_\mu^2\partial_\mu\eta_3+G_\mu^3\partial_\mu\eta_2)$$
$$-\tfrac{1}{2}g^2\eta_1[\tfrac{1}{2}(G_\mu^1G_\mu^1+G_\mu^2G_\mu^2+G_\mu^3G_\mu^3)+\tfrac{1}{4}(G_\mu^4G_\mu^4+G_\mu^5G_\mu^5+G_\mu^6G_\mu^6+G_\mu^7G_\mu^7)+\tfrac{1}{6}G_\mu^8G_\mu^8]$$
$$-\tfrac{1}{4}g^2\frac{1}{\sqrt{3}}\tilde{\eta}_8\left(\frac{2}{\sqrt{3}}G_\mu^1G_\mu^8+G_\mu^4G_\mu^6+G_\mu^5G_\mu^7\right)-\frac{\nu^2}{2}\eta_1[2(\chi/\chi_\infty)^2-1]-\frac{\eta^2}{2}\eta_1(\eta_1{}^2+\eta_2{}^2+\eta_3{}^2+\tilde{\eta}_8{}^2)=0\ , \quad (7a)$$

$$\partial_\mu\partial_\mu\tilde{\eta}_8-\tfrac{1}{2}g^2\tilde{\eta}_8[\tfrac{1}{2}(G_\mu^1G_\mu^1+G_\mu^2G_\mu^2+G_\mu^3G_\mu^3)+\tfrac{1}{4}(G_\mu^4G_\mu^4+G_\mu^5G_\mu^5+G_\mu^6G_\mu^6+G_\mu^7G_\mu^7)+\tfrac{1}{6}G_\mu^8G_\mu^8]$$
$$-\tfrac{1}{4}g^2\Bigg(2(G_\mu^3\eta_3+G_\mu^1\eta_1-G_\mu^2\eta_2)G_\mu^8-\frac{\sqrt{3}}{2}\eta_3(G_\mu^4G_\mu^4+G_\mu^5G_\mu^5-G_\mu^6G_\mu^6-G_\mu^7G_\mu^7)$$
$$+\sqrt{3}\eta_1(G_\mu^4G_\mu^6+G_\mu^5G_\mu^7)+\sqrt{3}\eta_2(G_\mu^4G_\mu^7-G_\mu^5G_\mu^6)\Bigg)-\frac{\nu^2}{2}\tilde{\eta}_8[2(\chi/\chi_\infty)^2-1]-\tfrac{3}{2}\eta^2\tilde{\eta}_8(\eta_1{}^2+\eta_2{}^2+\eta_3{}^2+\tfrac{1}{9}\tilde{\eta}_8{}^2)=0\ . \quad (7b)$$

It is clearly an exremely difficult task to solve these highly coupled field equations. For the purpose of this paper, we may consider, among others, a specific solution as given by

$$\eta_1=\eta_2=\eta_3=\eta_8=0\ , \quad (8a)$$

$$G_\mu^i=0 \text{ for } i\geq 4\ . \quad (8b)$$

We then find, in the absence of quarks,

$$\partial_\nu G_{\mu\nu}^1+g(G_\nu^2G_{\mu\nu}^3-G_\nu^3G_{\mu\nu}^2)+\tfrac{1}{3}g^2\omega^2G_\mu^1=0\ , \quad (8c)$$

$$\partial_\nu G_{\mu\nu}^2+g(G_\nu^3G_{\mu\nu}^1-G_\nu^1G_{\mu\nu}^3)+\tfrac{1}{3}g^2\omega^2G_\mu^2=0\ , \quad (8d)$$

$$\partial_\nu G_{\mu\nu}^3+g(G_\nu^1G_{\mu\nu}^2-G_\nu^2G_{\mu\nu}^1)+\tfrac{1}{3}g^2\omega^2G_\mu^3=0\ , \quad (8e)$$

$$\partial_\mu\partial_\mu\omega-\tfrac{1}{4}g^2\omega(G_\mu^1G_\mu^1+G_\mu^2G_\mu^2+G_\mu^3G_\mu^3)-\frac{\nu^2}{2}[2(\chi/\chi_\infty)^2-1]\omega-\tfrac{1}{6}\eta^2\omega^3=0\ . \quad (8f)$$

Equations (8c)–(8e) are very much the same field equations suitable for an SU(2) gauge theory. We note that Eq. (8f) is satisfied inside the bag [where $\omega(\mathbf{r})=0$] as well as outside the bag [where $\chi(\mathbf{r})=\chi_\infty$]. In the transition region where $\omega(\mathbf{r})$ rises from zero to some large value, we may choose the model parameters or even the field $\chi(\mathbf{r})$ just to ensure the validity of the constraint (8f).

The important aspect here is that all Higgs particles are absent for the bag being considered [Eq. (8a)]. Accordingly, there is no large surface energy associated with the physical Higgs particles. Nevertheless, there is a term in L_H^0 which is proportional to $(\partial_\mu\omega)^2$. It appears that this term has led Jändel to speculate the existence of the so-called fine-tuning problem. However, this term does not involve any physical field at all, so it can contribute only to the displacement of the overall mass scale. At the worst, it is the difference of such contributions to different hadrons which is of physical significance. If there would be some fine-tuning to be done, it should have been carried out for such differences. Thus, the problem does not seem to be as serious as Jändel has speculated. In passing, it is important to note that, in the MIT bag model[10] or the flux-tube model,[11] the "absolute" position of the hadron spectrum has also been fine tuned to the "physical" location.

To conclude this paper, we wish to append some remarks which are also relevant for the proposed model.[2]

(1) The running coupling constant $\alpha_S(Q^2)$ for the proposed model[2] is given by[3–6]

$$\alpha_S(Q^2)=\alpha_S(Q_0{}^2)\left(1+\alpha_S(Q_0{}^2)b_0\ln\frac{Q^2}{Q_0{}^2}\right)^{-1}\ , \quad (9)$$

with

$$b_0=11-\tfrac{2}{3}N_f-\tfrac{1}{6}N_\Phi\ . \quad (10)$$

Here N_f is the number of flavors and N_Φ is the number of complex Higgs triplets ($N_\Phi=2$ for the proposed model[2]). It is clear from Eqs. (2) and (10) that the grand unification scale, if such unification does occur, is modified only slightly.

(2) The various symmetry-breaking mechanisms, including quark confinement, chiral-symmetry breaking, and colored Higgs mechanism, are expected to disappear as Q^2 becomes large enough. This aspect can be investigated in some detail via the method developed by Gross and Wilczek[3] and by Cheng, Eichten, and Li.[4] These authors have considered the problem of Higgs phenomena in asymptotically free gauge theories. Here it is useful to note that the temperature for the deconfinement phase transition or restoration of chiral symmetry depends on the model parameters and thus is adjustable in the proposed model (as opposed to ordinary QCD). As Q^2 becomes large enough, all spontaneous-symmetry-breaking phenomena proposed in the model get washed out so that it should be possible to implement the model into a grand unification scheme.

(3) Although two color-triplet Higgs fields have been proposed, it is unlikely that such Higgs fields can be combined with quarks to form an overall color singlet but fractionally charged object. (This is due to the fact that eight components of these Higgs fields get sucked away by the Higgs mechanism.) Questions of this kind are currently under investigation.

Notes added in proof. It is also useful to note that the term proportional to $(\partial\omega_\mu)^2$ as appearing in L_H^0 [Eq. (4a)] is *not* the only big number in the proposed model. In fact, there are additional "compensating" big numbers associated with V_Φ and $L_{\sigma\pi}$, where potentials simulating spontaneous symmetry breakings of desired characteristics have been proposed. This aspect is essential for resolving any "fine-tuning" problem as pointed out by Jändel.

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