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Comment on bag models with spontaneously broken color symmetry

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A recently suggested field-theoretic bag model, where gluons are confined via a Higgs mechanism, is discussed. It is found that the proposed model creates gluon boundary conditions that break global $SU_c(3)$ invariance. A modified scheme that removes this anomaly is suggested. However, some severe generic problems remain. Examples are the lack of a suppression mechanism for states with open color and the large surface energy of the bag states.

In a recent paper,[1] Hwang proposed a new bag-model scheme where a description of important nonperturbative effects such as the confinement of quarks and gluons and chiral-symmetry breaking is attempted. The idea behind Hwang's model is that conventional quantum chromodynamics (QCD) should be extended by the addition of scalar fields in a way that preserves the renormalizability of the theory. At a hadronic energy scale the usual bag-model picture should be recovered, so that cavity QCD is the effective theory inside the hadronic bag, while pion dynamics dominates outside the bag. At high energies the new degrees of freedom are released, and it is hoped that models of this type may be incorporated in a grand unified theory.

It is interesting to compare this approach with that of the Friedberg-Lee soliton bag model,[2] where an auxiliary scalar soliton field is introduced as an effective source for quark confinement. The soliton field is thought to be related to some gluon condensate so that no extra degrees of freedom appear in the high-energy limit. As the soliton field is used only as an effective description for the low-energy features of the full theory there is no reason to require a renormalizable model.

The salient feature of Hwang's model is that gluons are confined because of a colored Higgs mechanism that generates large and unequal gluon masses outside the bag. This element is essential for the renormalization properties of the model but it also introduces some severe problems in the phenomenology at hadronic energies.

Such problems in Hwang's model are the absence of an exact global $SU_c(3)$ classification symmetry, the lack of a suppression mechanism for colored hadronic states, and the residual effect on the hadron masses of the large energy scale in the gluon-confinement mechanism. It is argued in this Comment that some of these anomalies can be removed if Hwang's model is properly modified, but the remaining difficulties cause some doubts concerning the usefulness of models of this type.

The model of Ref. 1 is defined by the Lagrangian

$$\mathscr{L}=\mathscr{L}_{\rm QCD}+\mathscr{L}_{\chi}+\mathscr{L}_{\rm int}+\mathscr{L}_{H} , \qquad (1)$$

where for simplicity the pionic degree of freedom has been excluded. The QCD Lagrangian is written in the usual notation

$$\mathscr{L}_{\rm QCD}=i\bar{\psi}\gamma^{\mu}(\partial_{\mu}-ig\tfrac{1}{2}\lambda_a G_{\mu}^{a})\psi-m\bar{\psi}\psi-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} . \qquad (2)$$

Quark confinement is enforced as in the Friedberg-Lee soliton bag model[2] by introducing a scalar soliton field χ according to

$$\mathscr{L}_{\chi}=\tfrac{1}{2}\partial_{\mu}\chi\,\partial^{\mu}\chi-U(\chi) ,$$
$$U(\chi)=p+\tfrac{1}{2}a\chi^2+\tfrac{1}{3!}b\chi^3+\tfrac{1}{4!}c\chi^4 . \qquad (3)$$

The color-singlet χ field is coupled to the quarks ψ in the simplest possible way:

$$\mathscr{L}_{\rm int}=-f\bar{\psi}\chi\psi . \qquad (4)$$

With a suitable choice of the soliton-field potential $U(\chi)$ one finds that χ takes a nonzero vacuum expectation value χ_{∞} and that the quarks are confined within a depression in the soliton field. The soliton bag model has been studied by many groups[3-5] and realistic solutions show a rather soft transition from a small and usually negative χ in the central part of the hadronic bag to a large χ that asymptotically approaches χ_{∞} as the quark fields vanish at a large distance from the bag. It is sufficient for the present qualitative discussion to use the approximation of Ref. 1 that $\chi=0$ inside a volume V in which the quarks are confined and that $\chi=\chi_{\infty}$ outside this region.

Gluons are thought to be massless within the bag. Outside the bag large gluon masses are generated via a Higgs mechanism that is defined by

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$$\mathscr{L}_H=(D_\mu\phi_+)^\dagger(D^\mu\phi_+)+(D_\mu\phi_-)^\dagger(D^\mu\phi_-)-V(\phi)\ ,$$
$$V(\phi)=\frac{\nu^2}{2}\left\{[2(\chi/\chi^\infty)^2-1](\phi_+^\dagger\phi_++\phi_-^\dagger\phi_-)+\frac{\eta^2}{4}[(\phi_+^\dagger\phi_+)^2+(\phi_-^\dagger\phi_-)^2+2(\phi_+^\dagger\phi_-)(\phi_-^\dagger\phi_+)]\right\}\ . \quad (5)$$

The complex scalar fields ϕ_+ and ϕ_- are color triplets. It was found that spontaneous symmetry breaking outside the bag generates nonzero vacuum expectation values for these fields. The resulting gluon masses are

$$M_{1,2,3}=gv,\ \ M_8=gv/\sqrt{3},\ \ M_{4,5,6,7}=gv/\sqrt{2},\ \ v=-(\nu/\eta)^2>>m_p{}^2\ . \quad (6)$$

It should be noted that even global $SU_c(3)$ invariance is broken outside the bag boundary. The scalars ϕ_+ and ϕ_- have, due to the χ dependence of the Higgs potential $V(\phi)$, vanishing vacuum expectation values inside the bag so that gluons are massless in this region and explicit $SU_c(3)$ gauge invariance appears to be restored.

At first sight this model seems to be able to produce a sensible phenomenology at a hadronic energy scale since quarks and gluons are confined within the bag where the Lagrangian is invariant with respect to $SU_c(3)$ transformations. At a closer look it appears, however, that although gluons are massless inside the bag, they are subject to boundary conditions that are not $SU_c(3)$ invariant. This fact is most easily understood by considering the classical equation of motion for the gluon field G_ν^a outside the bag,

$$(\partial_\mu\partial^\mu+M_a{}^2)G_\nu^a=0\ . \quad (7)$$

The region outside the bag is evidently acting as a color superconductor[6] and the penetration depth δ_a of the color-magnetic fields $\bar{B}_a$ is simply $\delta_a=M_a{}^{-1}$. As the gluon mass M_a, in Hwang's model, is different for different gluons we find that the boundary condition for the confined gluon depends on which particular gluon we chose. This means that global $SU_c(3)$ is not a good symmetry of the bound state and hence not a useful classification symmetry. It will be argued in the following that the gluon masses cannot be taken to be infinitely large in a realistic model but must be chosen as low as possible without violating experimental limits.

The model of Ref. 1 can, however, be improved by applying the general methods that can be found in the literature[7] of breaking a local $SU(N)$ symmetry in a renormalizable way while still preserving a global classification symmetry.

Let the quarks, gluons, and soliton scalar be in, respectively, the triplet, octet, and singlet representations of the local symmetry group $SU'(3)$. Spontaneous symmetry breaking is generated by three $SU'(3)$ triplets of Lorentz scalars ϕ_a^α, where the subscript a refers to the $SU'(3)$ symmetry and the superscript α is acted on by an auxiliary global symmetry $SU''(3)$. The ϕ_a^α scalars are in the conjugate triplet representation of $SU''(3)$ while all other fields are $SU''(3)$ singlets. Equation (5) is consequently redefined as

$$\mathscr{L}_H=(D_\mu\phi_a^\alpha)^\dagger(D^\mu\phi_a^\alpha)-\{\alpha[2(\chi/\chi_\infty)^2-1]\,\mathrm{Tr}(\phi^\dagger\phi)+\beta[\mathrm{Tr}(\phi^\dagger\phi)]^2+\gamma\,\mathrm{Tr}(\phi^\dagger\phi\phi^\dagger\phi)+\delta\det(\phi)\}\ . \quad (8)$$

The total Lagrangian (1) is now symmetric with respect to both local $SU'(3)$ and global $SU''(3)$ transformations.

Using "the global-symmetry theorem" by Mohapatra, Pati, and Salam[7] we find that the Higgs potential is minimized by choosing $\langle\phi_a^\alpha\rangle_{\text{vac}}=\kappa\delta_a^\alpha$. All gluons acquire the same mass, $M_G=g\kappa$, so that the anomaly of Hwang's model is removed. Both the $SU'(3)$ and the $SU''(3)$ symmetry are fully broken but a new "$SU_c(3)$" group is defined by the generators $F_i^c=F_i'+F_i''$. $SU_c(3)$ remains as an exact global symmetry even after the spontaneous symmetry breaking.

We find that this improved version of Hwang's model generates a bag model of confined quarks and gluons, where both the Lagrangian of the confined fields and their boundary conditions are invariant with respect to a global $SU_c(3)$ classification symmetry so that the consistency with hadronic physics is restored.

Both the model of Ref. 1 and the improved scheme suggested here still suffer, however, from several serious problems. There is no obvious mechanism that eliminates non-color-singlet bag states in this model. In the MIT bag model[8] the boundary condition for the gauge field is assumed to be

$$\hat{\mathbf{n}}\cdot\mathbf{E}^a=0,\ \ \hat{\mathbf{n}}\times\mathbf{B}^a=0,\ \ a=1,\ldots 8\ , \quad (9)$$

where $\hat{\mathbf{n}}$ is the normal to the bag surface. The total color charge of the allowed states must evidently vanish according to Gauss' law. This result is also obtained by the explicit gluon-confinement mechanism of the Friedberg-Lee model.

In the present model, gluons are confined because of the appearance of a large gluon mass M_G outside the bag. The gauge field must then vanish like $\exp(-M_Gd)$ with an increasing normal distance d from the bag surface. This leads, in the limit $M_G\to\infty$, directly to a set of boundary conditions for the color-electric and -magnetic fields that are independent of the details of the mass-generating mechanism. A full discussion of this issue has been presented by Creutz and Soh,[9] and it was found that the gluon boundary conditions in any model with a color Higgs mechanism can be written

$$\hat{\mathbf{n}}\times\mathbf{E}^a=0,\ \ \hat{\mathbf{n}}\cdot\mathbf{B}^a=0,\ \ a=1,\ldots 8\ . \quad (10)$$

These relations are derived simply by applying Gauss' law for the color-magnetic fields to a volume enclosing the bag surface and similarly by applying Stokes' law for the color-electric fields. The color-magnetic field is excluded from the region outside the bag. This is the well-known result of Nielsen and Olesen[6] that the Higgs mechanism is a relativistic generalization of the Landau-Ginzburg model for superconductivity. As the color-electric field is normal to the bag surface there is no reason why states with open color should be excluded form the physical spectrum.

It seems also that it might be difficult to avoid that hadronic color-singlet states take a mass of the order of the large "free" gluon mass M_G. This effect is due to the fact that the Higgs field ϕ is strictly zero inside the bag while some of its components have large values κ outside the bag. The contribution to the hadron mass from the surface energy of the ϕ field can be estimated in the mean-field approximation as

$$\int |\nabla\phi|^2 d^3x \sim \frac{\kappa^2}{\Delta R} R^2 = C_s R^2 \;, \tag{11}$$

where R is the bag radius and ΔR is the size of the transition region. As $\Delta R < R \sim m_p{}^{-1}$ we find that this contribution must be larger than M_G and hence much larger than any physical hadron mass. In order to improve this situation we must find a large negative contribution to the total bag energy that can compensate for the surface energy.

As the bag states have a finite size we would expect a finite contribution from the zero-point energy of the Higgs field. Let us treat this effect phenomenologically by adding a term $C_H'/R^2 + C_H/R$ to the energy of the bag state. Suppressing terms much smaller than the gluon mass we can write the hadron-mass formula:

$$M = C_H'/R^2 + C_H/R + C_S R^2 \;.$$

Stable bag states with a realistic mass can be obtained only if $M = dM/dR = 0$. A simple calculation shows that this is possible only if $C_s = \frac{27}{256} C_H{}^4 C_H'{}^{-3}$. The bag radius is fixed by the dynamics of the Higgs field and we obtain $R = (C_H'/3C_s)^{1/4}$. It is obvious that only very detailed calculations can settle the question whether such an exact cancellation between surface and zero-point energies is possible. It appears, however, that the gluon mass should be chosen to be as low as possible in order to improve the chances of success for such a scheme.

In conclusion, we find that although Hwang's model can be modified so that global $SU_c(3)$ symmetry is preserved several severe problems are still unresolved. In particular it appears that the large surface energy of the colored Higgs field would generate very large masses even for normal color-singlet hadrons. If zero-point-energy corrections are included this anomaly can at best be reduced to a fine-tuning problem vaguely reminiscent of that in some subquark models. A firm prediction of any such model would be that the bag radius is essentially the same for all hadrons. Some further modification of the model would probably also be needed in order to suppress the appearance of hadrons with open color at fairly low energies. Much further effort is evidently needed if models of this type shall be made useful for phenomenological applications.

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