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$q\bar{q}$ pair creation: A field-theory approach

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Quark-antiquark ($q\bar{q}$) pair creation is reexamined in quantum chromodynamics. The pair-creation-probability expression is derived in a Lorentz-covariant formalism and its Abelian specialization is shown to give Schwinger's original quantum-electrodynamics results.

Fermion-antifermion pair creation in quantum electrodynamics (QED) is a well understood phenomenon. A plethora of work since the appearance of Schwinger's original paper[1] has found applications in contemporary questions of physics. Additionally, the recent decade enjoyed the discoveries of quark-antiquark ($q\bar{q}$) bound states from e^+e^- annihilation. Analogous to e^+e^- annihilation, $q\bar{q}$ annihilation from hadronic collisions can result in a different $q\bar{q}$ pair-creation process that requires a study from the quantum-chromodynamics (QCD) point of view. Although Schwinger's original work addressed several important questions including the first derivation of the axial anomaly, his pair-creation expression is not sufficiently general to be extended to QCD promptly. This is mainly due to the non-Abelian nature of QCD and the resulting color forces between quark-antiquark pairs. The interquark potential is found[2] to be Coulomb-type at short distances, but it becomes linear at large distances, thus confining quarks permanently. Furthermore, in QED the background fields are under laboratory control; either $E_{\rm ext}$ or $H_{\rm ext}$ fields can be eliminated from the physical picture, whereas color background fields E_a and H_a are expected necessarily to exist in the interior of hadrons. Several authors have succeeded in deriving a rate of $q\bar{q}$ pair creation by the use of the chromodynamic flux-tube picture: an initial $q\bar{q}$ pair with large relative momentum forms a flux tube and through quantum tunneling, a new $q\bar{q}$ pair is created by fission of the tube.[3] However, the results of these investigations lack the generality that a proper field theory would provide. In this Brief Report, we show that a field-theoretical method of approach produces a general QCD result.

In earlier studies of QCD-vacuum behavior it was noted that the QCD vacuum is unstable[4] in the presence of background color fields. Such instability, similar to QED, implies $q\bar{q}$ pair creation. This pair-creation mechanism can be outlined briefly. The QCD background field is described by the non-Abelian field strengths

$$F^a_{\mu\nu}=\partial_\mu B^a_\nu-\partial_\nu B^a_\mu+gf^{abc}B^b_\mu B^c_\nu \quad , \tag{1}$$

where B^a_μ is the background gauge field and f^{abc} are the structure constants of the gauge group. Fermion fields are included in the theory in a color-group representation with representation matrices T_a, obeying

$$[T_a,T_b]=if_{abc}T_c \quad .$$

The assumption of approximately constant background field is generalized to

$$D^{ab}_\lambda F^b_{\mu\nu}=0 \tag{2}$$

involving the covariant derivative, $D_\lambda=\partial_\lambda-igt_aB^a_\lambda$, where the representation matrix t_a may be either T_a or $(f_a)_{bc}=-if_{abc}$; it obeys $[D_\mu,D_\nu]=igF_{\mu\nu}$, where $F_{\mu\nu}=t_aF^a_{\mu\nu}$. A corollary of the constant-background-field condition is

$$[F_{\kappa\lambda},F_{\mu\nu}]=0 \quad . \tag{3}$$

The $f\bar{f}$ pair creation is obtained by investigating the vacuum-persistence amplitude

$$\langle 0_+|0_-\rangle=e^{iW} \quad , \tag{4}$$

where pair creation is indicated by an imaginary part of the action expression $W=\int d^4x\,L_{\rm eff}$ resulting from a single particle loop propagating in a constant-background-field configuration.

The quantum-correction contribution to the effective action is expressed by

$$\delta W=-i\,{\rm Tr}G\,\delta G^{-1}=i\,{\rm Tr}\delta\ln G \quad , \tag{5}$$

where G is the fermion propagator obtained from

$$L_{\rm eff}=\bar{\psi}(\gamma^\mu D_\mu+m)\psi \quad . \tag{6}$$

The trace Tr includes coordinate-space as well as internal-symmetry and Dirac indices. Using the proper-time techniques introduced by Schwinger, the vacuum-fluctuation contribution due to a single fermion loop may be written as

$$\Delta W=-\frac{2}{(4\pi)^2}{\rm Tr}\int_{0+i\epsilon}^{\infty+i\epsilon}\frac{ds}{s^3}e^{-m^2s}\frac{\epsilon s}{\sin(\epsilon s)}\frac{hs}{\sinh(hs)}\times\cos(\epsilon s)\cosh(hs) \quad , \tag{7}$$

where the color matrices ϵ and h identify the four eigenvalues ($\pm\epsilon$, $\pm ih$) of $gF_{\mu\nu}$ regarded as a Lorentz-index matrix:

$$(\epsilon,h)=\frac{g}{\sqrt{2}}\{[(E^2-H^2)^2+4(\mathbf{E}\cdot\mathbf{H})^2]^{1/2}\pm(E^2-H^2)\}^{1/2} \quad , \tag{8}$$

where as usual $E_i=F^{0i}$, $H_i=\frac{1}{2}\epsilon_{ijk}F^{jk}$. [Equation (3) implies that all these matrices commute.] Only the imaginary part

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of ΔW is of interest for pair creation:

$$\text{Im}\Delta W = \frac{2}{(4\pi)^2}\text{Tr}_Q \sum_{n=1}^{\infty} \frac{\epsilon h}{n}\coth\left[\frac{h}{\epsilon}n\pi\right] e^{(m^2/\epsilon)n\pi}\int d^4x \ , \tag{9}$$

where Tr_Q indicates a trace over color matrices (ϵ and h).

The case of Abelian symmetry can be recovered by letting $T_a = 1$; effectively, by treating ϵ and h as numbers and omitting Tr_Q, we easily recover the well known pair-creation probability in an electric field ($h = 0$, $\epsilon = e|\mathbf{E}|$),

$$\text{Im}\Delta W = \frac{(eE)^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-(m^2/eE)n\pi}\int d^4x \ , \tag{10}$$

which was derived by Schwinger.

We may remark on the factor [in Eq. (8)]

$$\epsilon h = g^2|\mathbf{E}_a\cdot\mathbf{H}_a| = \frac{g^2}{4}|F^a_{\mu\nu}\tilde{F}^{a\mu\nu}| \ .$$

$F\tilde{F}$ terms a dynamical action are a source of *CP* violation; here, however, their background-field nature makes them at most an effect of a *CP*-asymmetric environment, while the absolute value removes their noninvariance.

The confinement question needs to be reexamined in the light of the covariant formalism followed here. Previous authors have attributed confinement solely to electric flux lines. However, in the present Lorentz-covariant form, the magnetic-field effects are nontrivial.

An interesting situation arises when electric- and magnetic-field amplitudes become equal ($\epsilon = h$):

$$P = \frac{1}{8\pi^2}\text{Tr}_Q\epsilon^2\sum_{n=1}^{\infty}\frac{1}{n}\coth(n\pi)e^{-(m^2/\epsilon)n\pi} \ , \tag{11}$$

where

$$\epsilon^2 = \epsilon h = g^2|\mathbf{E}_a\cdot\mathbf{H}_a| = \tfrac{1}{4}g^2|F^a_{\mu\nu}\tilde{F}^{a\mu\nu}| \ .$$

In conclusion, a field-theoretical approach in the derivation of $q\bar{q}$ pair creation provides a much more complete picture than that of flux-tube studies.

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[1]J. Schwinger, Phys. Rev. **82**, 664 (1951). The background-field techniques first appeared in this paper.

[2]The linear plus Coulomb potential was first introduced in the charmonium-spectroscopy studies by B. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. **34**, 168 (1975); **34**, 706 (1975), and independently by other groups. For references, see T. Applequist, R. M. Barnett, and K. Lane, Annu. Rev. Nucl. Part. Sci. **28**, 387 (1978).

[3]E. Brezin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970), derived Schwinger's results by using the WKB method in a vacuum-tunneling model. A flux-tube model was first studied by J. Kogut and L. Susskind [Phys. Rev. D **11**, 395 (1975)]. A. Casher, H. Neuberger, and S. Nussinov [Phys. Rev. D **20**, 179 (1979)] investigated pair production in a chromoelectric-flux-tube model. N. K. Glendenning and T. Matsui [Phys. Rev. D **28**, 2890 (1983)] and A. Bialas and W. Czyz [Phys. Rev. D **31**, 198 (1985)] generalized the pair-creation mechanism to QCD by taking into consideration the mutual interaction of the pair and an arbitrary field-strength flux tube between two arbitrary color charges.

[4]Vacuum instability in QCD has been investigated in a Lorentz-covariant form by A. Yildiz and P. H. Cox, Phys. Rev. D **21**, 1095 (1980); M. Claudson, A. Yildiz, and P. H. Cox, *ibid.* **22**, 2022 (1980).