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Multipole moments of (composite) W's and the magnetic moment of the muon

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The contribution of W to the magnetic moment of the muon $(g-2)_{\text{muon}}$ is calculated for non-gauge-(composite-) model values of the magnetic dipole (κ) and electric quadrupole (λ) moments of the W; the comparison of this contribution to the experimental value of $(g-2)_{\text{muon}}$ provides a constraint on κ and λ.

The discovery[1] of the $W^{\pm}$ and Z^0 has been successfully achieved at CERN, with masses, spins, and cross sections in good agreement with the predictions of the standard model of Glashow-Salam-Weinberg.[2] The experimental study of the self-couplings of the electroweak bosons remains now to be investigated (mainly with the production of pairs of these bosons), and is very important, for it will finally enable us to know whether the weak bosons are gauge particles with self-interactions constrained by the expected $SU(2)\times U(1)$ gauge symmetry or nongauge (e.g., composite) particles whose self-interactions at present energies would be described by an effective (nonrenormalizable) theory. The process $p\bar{p}\to W^-\gamma X$ (Ref. 3) will certainly offer a good possibility to study the γWW vertex in the near future, leading to a nonmarginal number of events per run at the $p\bar{p}$ Fermilab collider.

The most general local γWW vertex, compatible with C, P, and T invariance and electromagnetic-current conservation is known[4] to depend on two independent free parameters κ and λ, related to the magnetic dipole moment μ_W,

$$\mu_W = e(1+\kappa+\lambda)/2M_W \ ,$$

and electric quadrupole moment Q_W,

$$Q_W = -e(\kappa-\lambda)/M_W^2 \ ,$$

of the W. The $SU(2)\times U(1)$ gauge symmetry of the standard model constrains severely the κ and λ parameters, giving them definite values at the tree level ($\kappa=1$ and $\lambda=0$) with radiative corrections of order of α. However, if the W is not a gauge particle, it may well be that its magnetic dipole or (and) electric quadrupole moment(s) is (are) quite different[5] from the standard-model prediction. (For instance, this would be in the case of composite W's, interacting strongly among themselves.)

In this short paper, I calculate the additional contribution that anomalous (i.e., nongauge) values of κ and λ give to the magnetic moment of the muon.[6] Imposing the condition that this additional contribution does not exceed the difference between the experimental[7] and the most recent theoretical standard-model value[8] of the magnetic moment of the muon, I get a relation between κ and λ which may be used to constrain these parameters.

The only relevant diagram for the calculation is depicted on Fig. 1. The corresponding vertex amplitude reads

$$\Lambda_\mu^{(M+Q)} = -\frac{e^3}{4\sin^2\theta_W}\int\frac{d^4k}{(2\pi)^4}\gamma_\rho \not{k}\gamma_\sigma(1-\gamma_5)[g^{\alpha\sigma}-M_W^{-2}(p-k)^\alpha(p-k)^\sigma][g^{\rho\beta}-M_W^{-2}(p'-k)^\rho(p'-k)^\beta]$$
$$\times(\Delta M_{\mu\alpha\beta}+\Delta Q_{\mu\alpha\beta})\frac{1}{[(p-k)^2-M_W^2][(p'-k)^2-M_W^2]k^2} \ , \tag{1}$$

where θ_W is the Weinberg angle ($\sin^2\theta_W\simeq 0.22$), $e^2=4\pi\alpha$, M_W is the W mass, the various four-momenta are labeled on Fig. 1, and $\Delta M_{\mu\alpha\beta}$ ($\Delta Q_{\mu\alpha\beta}$) includes all the additional terms appearing in the γWW vertex for nongauge values of κ (λ). From the most general form of the γWW vertex, compatible with the C, P, and T invariance and electromagnetic-current conservation, $\Delta M_{\mu\alpha\beta}$ and $\Delta Q_{\mu\alpha\beta}$ can be cast into the form

$$\Delta M_{\mu\alpha\beta} = (\kappa-1)(g_{\mu\alpha}q_\beta - g_{\mu\beta}q_\alpha) \ , \tag{2a}$$

$$\Delta Q_{\mu\alpha\beta} = (\lambda M_W^{-2})\{k_\mu(k_\beta q_\alpha - k_\alpha q_\beta) + g_{\mu\alpha}(k^2 q_\beta - k\cdot q\, k_\beta) + g_{\mu\beta}(k\cdot q\, k_\alpha - k^2 q_\alpha)$$
$$+ k_\mu(p'_\alpha q_\beta - q_\alpha p_\beta) - p'_\mu q_\alpha k_\beta + p_\mu k_\alpha q_\beta + g_{\mu\alpha}[k\cdot q\, p_\beta - (p\cdot k + p'\cdot k)q_\beta]$$
$$+ g_{\mu\beta}[(p\cdot k + p'\cdot k)q_\alpha - p'_\alpha k\cdot q] + g_{\alpha\beta}k\cdot q\, q_\mu + p'_\mu q_\alpha p_\beta - p_\mu p'_\alpha q_\beta + m^2(g_{\mu\alpha}q_\beta - g_{\mu\beta}q_\alpha)\} \tag{2b}$$

where $q=p'-p$ is the four-momentum of the photon and m is the muon mass.

The different steps of the calculation are standard.[6] From the expression of the electromagnetic current of the muon,

$$J_\mu^{(\text{muon})} = -ie\bar{u}(p')\left[F_1(q^2)\gamma_\mu + \frac{i}{2m}F_2(q^2)\sigma_{\mu\nu}q^\nu + \text{parity-violating terms}\right]u(p) \ , \tag{3}$$

where $F_1(q^2)$ and $F_2(q^2)$ are, respectively, the charge and anomalous magnetic form factors, the magnetic moment of the muon is given by

$$\mu^{(\text{muon})} = (e/2m)F_2(q^2=0) = (e/2m)a^{(\text{muon})} \ . \tag{4}$$

To get the additional weak contribution to $a^{(\text{muon})}$, one has to sandwich (1) between the wave functions of the external muon lines and use the on-shell condition $[\bar{u}(p')\not{p}' = \bar{u}(p')m$ and $\not{p}u(p) = mu(p)]$ to reduce the number of γ matrices, after the contraction of the Lorentz indices. Applying this procedure to (1), one finds that the most divergent contribution to $F_2(q^2=0)$ in (1) is logarithmic in nature.[9] Then, using a cutoff Λ to regularize the integrand and a Feynman parametrization to combine the factors in the denominator of (1), and shifting the origin of momentum integration, the additional weak contribution to the magnetic moment of the muon can be written as

$$a^{(\text{muon})}(\kappa-1;\lambda;\mu^2) = \left(\frac{\alpha}{\pi}\right)\frac{1}{8\sin^2\theta_W}\left(\frac{m}{M_W}\right)^2 A(\kappa-1;\lambda;\mu^2)\ , \tag{5a}$$

with

$$A(\kappa-1;\lambda;\mu^2) \simeq (\kappa-1)\left[\frac{1-3\mu^2}{(\mu^2-1)^3}\ln\mu^2 + \frac{2\mu^2}{(\mu^2-1)^2} + O\left(\frac{m^2}{M_W{}^2}\ln\mu^2\right)\right] + \lambda\left[\frac{2\mu^2}{(1-\mu^2)^3}\ln\mu^2 + \frac{\mu^2+1}{(\mu^2-1)^2} + O\left(\frac{m^2}{M_W{}^2}\ln\mu^2\right)\right]\ , \tag{5b}$$

where $\mu^2 = M_W{}^2/\Lambda^2$.

An accurate measurement of the muon magnetic moment has been performed by the CERN Muon Storage Ring Collaboration:[7]

$$a_{\text{expt}}^{(\text{muon})} \simeq (11\,659\,240 \pm 81)\times 10^{-10}\ .$$

On the other hand, the most recent theoretical standard-model calculation[8] gives

$$a_{\text{theor}}^{(\text{muon})} \simeq (11\,659\,200 \pm 20)\times 10^{-10}\ .$$

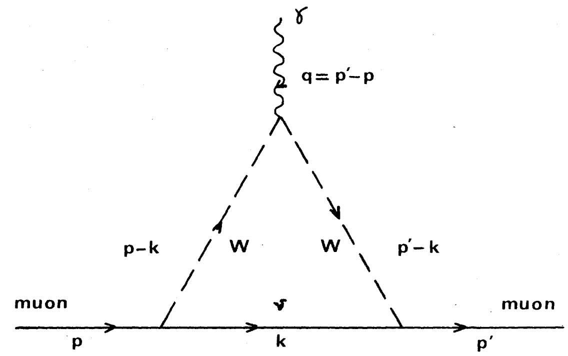

FIG. 1. γWW vertex contribution to the magnetic moment of the muon.

The requirement that $a^{(\text{muon})}(\kappa-1;\lambda;\mu^2)$ not exceed the difference between the experimental and the theoretical value of $a^{(\text{muon})}$ provides a constraint on κ and λ. I get the relation

$$(-2.7) < A(\kappa-1;\lambda;\mu^2) < 6.4\ . \tag{6}$$

Figure 2 gives an illustration of the relation (6): the allowed upper and lower limits on κ are varied with respect to Λ, for different values of λ. For low values of the (compositeness) scale Λ (e.g., $\Lambda \sim 1$ TeV), relatively large devia-

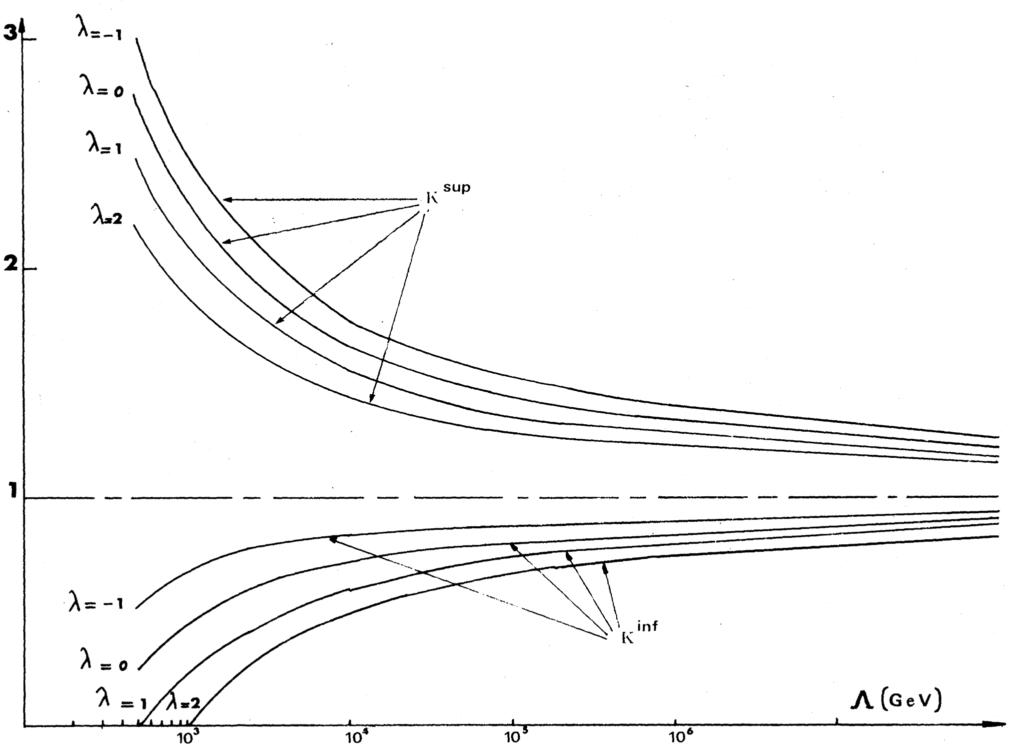

FIG. 2. Allowed upper (lower) limit κ^{sup} (κ^{inf}) vs Λ, for different values of λ.

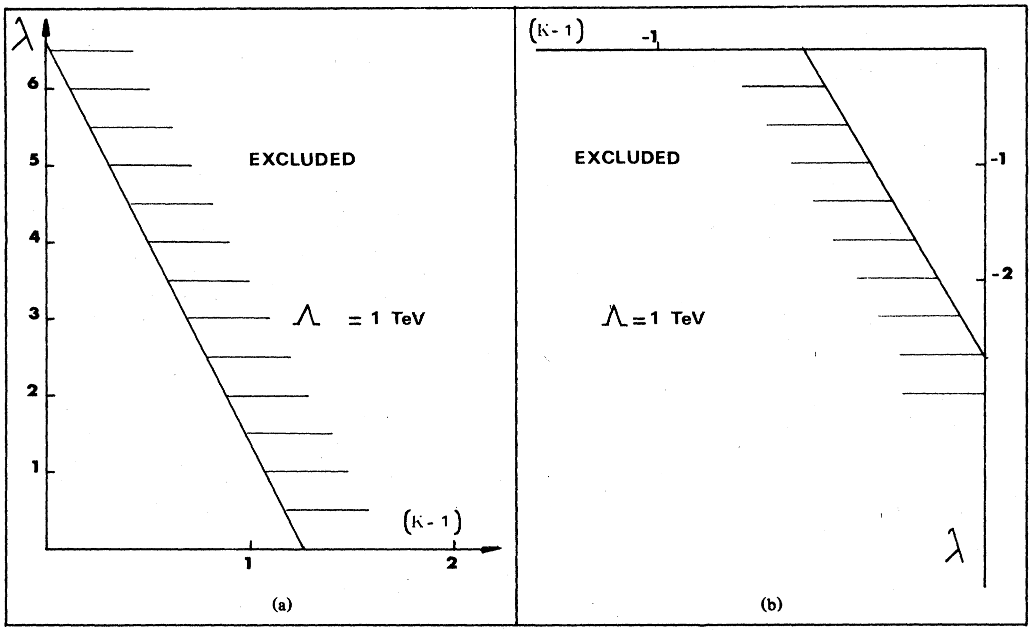

FIG. 3. (a) Allowed domain for κ and λ, assuming $\Lambda=1$ TeV and $(\kappa-1)$ and λ positive. (b) Same as (a) with $(\kappa-1)$ and λ negative.

tions of κ from its standard-model value are consistent with the measured muon magnetic moment and would induce noticeable effects in processes such as $p\bar{p}\rightarrow\gamma WX$ (Ref. 3) or $p\bar{p}\rightarrow W^+W^-X$ (Ref. 10). On the other hand, if Λ is much larger than 1 TeV, the situation becomes more similar to the standard-model one, as one may expect; for $\Lambda\gtrsim10^6$ GeV, relation (6) implies $\delta\kappa|\kappa\lesssim$20–40%.

Besides a possible measurement of κ and λ, future experiments at the multi-TeV colliders should reveal whether some manifestations of compositeness occur in the $\sim$1-TeV region. Assuming for definiteness that $\Lambda=1$ TeV, it is interesting to see in what range κ and λ may lie.

Figure 3(a) [3(b)] shows the portion of the $\{\kappa,\lambda\}$ plane, allowed by relation (6) if both $(\kappa-1)$ and λ are positive (negative). If they have opposite signs, (6) is no longer sufficient and one has to look for an additional constraint which, combined with (6), may limit both κ and λ. The study of the radiative decay modes of the W ($W\rightarrow\nu l\gamma$) should be helpful to reach this goal, as soon as a comparison of the experimental and standard-model values of the radiative width of the W becomes possible.

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