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Quark confinement in quantum chromodynamics

Suraj N. Gupta and Stanley F. Radford
Department of Physics, Wayne State University, Detroit, Michigan 48202
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A quark-confinement mechanism inspired by renormalization-group-improved perturbative quantum chromodynamics is proposed. We obtain a quark-antiquark confining potential in the lowest approximation, show that it yields reasonable results for the $b\bar{b}$ and $c\bar{c}$ spin-averaged energy levels, and comment on the significance of the higher-order effects.

I. INTRODUCTION

Quark confinement in quantum chromodynamics has been extensively investigated, and various techniques have been suggested to deal with this phenomenon.[1] It is also hoped that this problem may eventually be resolved with the help of the lattice gauge theory, which is still in a developmental stage.[2] We shall here propose another approach inspired by the renormalization-group-improved perturbative treatment. We shall obtain a quark-antiquark confining potential in the lowest approximation, show that it yields reasonable results for the $b\bar{b}$ and $c\bar{c}$ spin-averaged energy levels, and comment on the significance of the higher-order effects. An interesting feature of our treatment is that we do not calculate and combine the potentials at short and long distances, but obtain the resulting potential directly at all distances.

II. QUARK-ANTIQUARK CONFINING POTENTIAL

Let us consider the well-known dependence of the quark-gluon coupling constant g on the renormalization-scale parameter μ, which is given in the one-loop approximation by

$$\alpha_s' = \frac{\alpha_s}{1+(\alpha_s/12\pi)(33-2n_f)\ln(\mu'^2/\mu^2)} , \tag{2.1}$$

where $\alpha_s = g^2/4\pi$. The above relation, which can also be expressed as

$$\alpha_s' = \frac{12\pi}{(33-2n_f)\ln(\mu'^2/\Lambda^2)} \tag{2.2}$$

with

$$\Lambda^2 = \mu^2 \exp\left[-\frac{12\pi}{(33-2n_f)\alpha_s}\right] , \tag{2.3}$$

sets a restriction on the allowed values of the renormalization-scale parameter. For, let μ be some allowed value of this parameter, and let us see whether μ' can take values lower than μ. Since the Lagrangian density for physical fields must be Hermitian, it is necessary that the quark-gluon coupling constant g' be real, and therefore α_s' must remain non-negative. But, according to (2.2), this is possible only if

$$\mu' \geq \Lambda , \tag{2.4}$$

and it is to be noted that α_s' becomes infinite for the lowest value of μ' allowed by (2.4).

Now, the static spin-averaged one-gluon exchange quark-antiquark potential is

$$V_2(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(-16\pi\alpha_s)}{3}\left[\frac{1}{\mathbf{k}^2} - \frac{1}{4m^2}\right] , \tag{2.5}$$

which consists of a Coulomb term and a $\delta(\mathbf{r})$ term. Renormalization-group improvement is usually carried out by the replacement of α_s by

$$\alpha_{s,\mathrm{eff}} = \frac{12\pi}{33-2n_f}\frac{1}{\ln(\mathbf{k}^2/\Lambda^2)} , \tag{2.6}$$

where Λ^2 is given by (2.3), and the purpose of this improvement is to incorporate certain higher-order contributions into a lower-order perturbative result. We, however, observe that the higher-order contributions should be selected in such a way that $\alpha_{s,\mathrm{eff}}$ remains positive to ensure that g_{eff} is real, which implies that (2.6) is valid only for $|\mathbf{k}| \geq \Lambda$. Moreover, since $\alpha_{s,\mathrm{eff}}$ becomes infinite for $|\mathbf{k}| = \Lambda$, renormalization-group improvement of perturbative results becomes invalid in the vicinity of $|\mathbf{k}| = \Lambda$. We, therefore, conclude that the replacement of α_s by $\alpha_{s,\mathrm{eff}}$ in (2.5) can lead to physically sensible results only for

$$|\mathbf{k}| \geq (1+\epsilon)\Lambda , \tag{2.7}$$

where a small parameter ϵ has been introduced to exclude the vicinity of $|\mathbf{k}| = \Lambda$. Thus, our renormalization-group-improved quark-antiquark potential takes the form

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3}\int_{|\mathbf{k}| \geq (1+\epsilon)\Lambda} d\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(-64\pi^2)}{33-2n_f} \times \frac{1}{\ln(\mathbf{k}^2/\Lambda^2)}\left[\frac{1}{\mathbf{k}^2} - \frac{1}{4m^2}\right] , \tag{2.8}$$

which gives upon angular integrations, with $|\mathbf{k}|/\Lambda = q$,

$$V(r) = V_0(r) + V'(r) , \tag{2.9}$$

where

$$V_0(r)=-\frac{16}{33-2n_f}\int_{1+\epsilon}^{\infty}dq\frac{\sin(q\Lambda r)}{rq\ln q}\ , \qquad (2.10)$$

$$V'(r)=\frac{4}{33-2n_f}\frac{\Lambda^2}{m^2}\int_{1+\epsilon}^{\infty}dq\frac{q\sin(q\Lambda r)}{r\ln q}\ , \qquad (2.11)$$

and, as usual for a confining potential, we are allowed to add a constant C.

III. $b\bar{b}$ AND $c\bar{c}$ SPECTRA

We have investigated the $b\bar{b}$ and $c\bar{c}$ spectra with the use of the Hamiltonian

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}' \qquad (3.1)$$

with

$$\mathscr{H}_0=2m+\mathbf{p}^2/m+V_0(r)+C\ , \qquad (3.2)$$

$$\mathscr{H}'=V'(r)\ . \qquad (3.3)$$

Our wave functions were of the form

$$\psi_{nl}^{m}(\mathbf{r})=\sum_{k=0}^{K}a_{L,nl}\left[\frac{r}{R}\right]^{L}e^{-r/R}Y_l^m(\Omega_r)\ ,\quad L=k+l\ , \qquad (3.4)$$

with the Lth component

$$\psi_L(\mathbf{r})=\left[\frac{r}{R}\right]^{L}e^{-r/R}Y_l^m(\Omega_r)\ . \qquad (3.5)$$

The coefficients $a_{L,nl}$ were determined by the variational technique of minimizing the expectation value of $\mathscr{H}_0$, while the optimum value of the parameter R was determined by satisfying the virial theorem. Finally, the contribution of $\mathscr{H}'$ to the energy levels was included by first-order perturbation theory.

It should be noted that the above procedure requires evaluation of the matrix elements

$$\langle L\mid V\mid L'\rangle=\int_0^{\infty}r^2dr\,V(r)\left[\frac{r}{R}\right]^{L+L'}e^{-2r/R}\ , \qquad (3.6)$$

which is facilitated by substituting (2.10) and (2.11) into (3.6), and performing integration over r. Thus,

$$\langle L\mid V_0\mid L'\rangle=-\frac{16R^2}{33-2n_f}\frac{\Gamma(2+L+L')}{2^{2+L+L'}}\int_{1+\epsilon}^{\infty}dq\frac{\sin[(2+L+L')\tan^{-1}(\Lambda Rq/2)]}{q\ln q[1+(\Lambda Rq/2)^2]^{(2+L+L')/2}}\ , \qquad (3.7)$$

$$\langle L\mid V'\mid L'\rangle=\frac{R^2\Lambda^2}{(33-2n_f)m^2}\frac{\Gamma(2+L+L')}{2^{2+L+L'}}\int_{1+\epsilon}^{\infty}dq\frac{q\sin[(2+L+L')\tan^{-1}(\Lambda Rq/2)]}{\ln q[1+(\Lambda Rq/2)^2]^{(2+L+L')/2}}\ , \qquad (3.8)$$

and upon changing the variable of integration from q to $x=\tan^{-1}(\Lambda Rq/2)$, computation of these integrals is not too difficult.

Our values for the spin-averaged energy levels of $b\bar{b}$ and $c\bar{c}$ below the bottom and charm thresholds, together with the values of the parameters, are given in Table I. In order to coordinate the results for $b\bar{b}$ and $c\bar{c}$, we chose the same value of ϵ for both systems. We also ensured that $\Lambda_{b\bar{b}}$ and $\Lambda_{c\bar{c}}$ satisfy the constraint resulting from the relations

$$\alpha_s(b\bar{b})=\frac{6\pi}{(33-2n_f)\ln(\mu/\Lambda_{b\bar{b}})}\ ,$$

$$\alpha_s(c\bar{c})=\frac{\alpha_s(b\bar{b})}{1+[\alpha_s(b\bar{b})/6\pi](33-n_f-n_f')\ln(\mu'/\mu)}\ , \qquad (3.9)$$

$$\Lambda_{c\bar{c}}=\mu'\exp\left[-\frac{6\pi}{(33-2n_f')\alpha_s(c\bar{c})}\right]\ ,$$

with $\mu=m_b$, $n_f=4$, $\mu'=m_c$, and $n_f'=3$.

The theoretical energy levels in Table I are in reasonable agreement with the experimental results.[3] Since the masses of the ground states in this table are inputs, it

TABLE I. $b\bar{b}$ and $c\bar{c}$ spin-averaged energy levels with $m_b=4.75$ GeV, $\Lambda_{b\bar{b}}=0.370$ GeV, $m_c=1.55$ GeV, $\Lambda_{c\bar{c}}=0.445$ GeV, and $\epsilon=0.03$. Masses of the ground states are inputs.

$b\bar{b}$ state	Mass (GeV)	$c\bar{c}$ state	Mass (GeV)
$1S$	9.45	$1S$	3.07
$2S$	10.02	$2S$	3.67
$3S$	10.35	$1P$	3.50
$1P$	9.89		
$2P$	10.26		

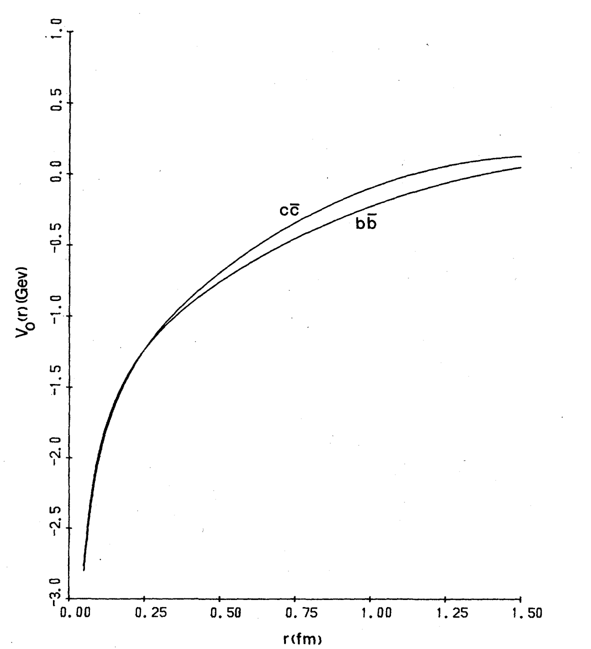

FIG. 1. $V_0(r)$ potential corresponding to the $b\bar{b}$ and $c\bar{c}$ parameters given in Table I.

should be noted that the spin-averaged value of the $b\bar{b}$ ground-state was determined by using a theoretical value[4] for the experimentally unknown hyperfine splitting.

IV. CONCLUSION

We have proposed a quark-confinement mechanism and derived a quark-antiquark confining potential, which is given by (2.9) and consists of two terms. The dominant term $V_0(r)$, which arises from the Coulomb term in (2.5), is shown in Fig. 1. The other term $V'(r)$, which arises from the $\delta(\mathbf{r})$ term in (2.5), is a rapidly oscillating function of r with large amplitudes at short distances, while it practically vanishes at long distances. Our potential yields results in reasonable agreement with experiments.

The approach, which we have applied in the lowest approximation, becomes increasingly complex with the inclusion of higher-order effects, and an accurate calculation of the quark-antiquark confining potential remains a difficult problem. However, according to the treatment described by us in Sec. II, quarks cannot exchange low-momentum gluons even when higher-order terms are included in the effective coupling constant (2.6) through the renormalization-group equations.[5] We, therefore, expect that the basic features of our quark-confinement mechanism will be preserved upon inclusion of the higher-order contributions.

It is interesting that our results for the spin-averaged energy levels of $b\bar{b}$ and $c\bar{c}$ are in close agreement with those of Buchmüller and Tye,[6] who have proposed a potential model which incorporates linear confinement and asymptotic freedom. Our values of the quark masses and the parameter Λ are also not too different from those of these authors.

The success of our treatment shows that our confinement mechanism is able to account for the gross features of a quark-antiquark system in the lowest approximation. This seems to imply that the dominant effects of higher-order contributions can be absorbed into the lowest-order result through a redefinition of the basic parameters.

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