Nonleptonic kaon decay

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Current-algebra methods are used to analyze $K \to 2\pi$ decays. A study is made of the effects of terms which are usually made to vanish by the soft-pion procedure. For this purpose the pions are treated symmetrically and massless pions, $k^2=q^2=0$, are considered as an alternative to soft ones. Detailed numerical estimates are made for the various contributions arising from "left-left" and "left-right" operators.

I. INTRODUCTION

Progress in the understanding of nonleptonic strangeness-changing weak decays of hadrons has been achieved during the last decade.[1] This progress has been made possible by the advent of quantum chromodynamics (QCD) as a possible theory of strong interactions. Gaillard and Lee, and Altarelli and Maiani,[2] using renormalization-group arguments, were able to show that short-distance effects cause an enhancement of the Wilson coefficient function[3] of the $\Delta I=\frac{1}{2}$ operator and suppression of the coefficient of the $\Delta I=\frac{3}{2}$ operator, relative to the free-field value. This, however, was not enough by itself to account for the observed $\Delta I=\frac{1}{2}$ rule. In principle, evaluation of the matrix elements of operators could result in further enhancement for the $\Delta I=\frac{1}{2}$ amplitude. However, detailed calculations have not borne this out yet. Shifman, Vainshtein, and Zakharov[4] (SVZ) pointed out the existence of a new class of operators, the so-called "left-right" or "penguin" operators, in addition to the usual "left-left" operators considered in Ref. 2. These new operators are pure $\Delta I=\frac{1}{2}$ and thus could be of great help in the attempts to explain octet enhancement. Recently, Donoghue, Golowich, Ponce, and Holstein[5] have carried out a detailed analysis of $\Delta S=1$ nonleptonic decays employing the MIT bag model[6] to evaluate the hadronic matrix elements of the various operators.

We should also point out there have been other efforts to explain the $\Delta I=\frac{1}{2}$ rule using different approaches. Oneda and collaborators[7] have carried out detailed analysis based on the ansatz of the so-called level realization of asymptotic flavor SU(N) symmetry.[8] McKellar and Scadron[9] propose that for kaon decay the significant piece of the $\Delta I=\frac{1}{2}$ weak Hamiltonian is the s-d quark tadpole generated by W exchange that was considered earlier by Ross and the present author.[10] They argue that the Nambu-Goldstone nature of the kaon prevents this tadpole from being transformed away by higher QCD or QFD (quantum-flavour-dynamic) interactions.

In most of the works cited above the methods of current algebra, PCAC (partial conservation of axial-vector current), and soft pions play a significant role.[11] For processes involving more than one pion it has been suggested, rather than reduce one pion at a time and symmetrize, that it is more appropriate to treat all pions in the process symmetrically and on equal footing.[12] In this work we apply this symmetric treatment to nonleptonic kaon decay into two pions. Also instead of taking the soft-pion limits of $k^\alpha,q^\alpha \to 0$, we instead employ the massless-pion approximation and take $k^2=q^2=0$, keeping $k^\alpha,q^\alpha \neq 0$. These matters are taken up on Sec. II where PCAC and short-distance expansions are used to set up a representation for the kaon decay amplitude. It is our main aim in this work to study the effects of terms that would normally give a vanishing contribution in the usual treatment using soft pions. We are further motivated to do this by the rather appreciable momentum dependence of the $K \to 2\pi$ decay amplitude.[5,11] A major problem here is to determine the structure of an amplitude $M_{\alpha\beta}(k,q)$ (see Sec. II for notation) that involves the time-ordered product of two axial-vector currents and two hadronic weak currents. This problem is tackled in Sec. III and our approach consists of dividing contributions into those coming from "short" and "long" distances. The short-distance component is calculable and we propose a certain approximation for the long-distance component. In Sec. IV we put together the results of Secs. II and III, analyse further the kaon decay amplitude and give numerical estimates. Section V is devoted to some concluding remarks. In Appendix A we evaluate certain integrals connected with the short-distance contributions discussed in Sec. III while Appendix B is devoted to a discussion of some QFD renormalization effects connected with the axial-vector current operator. As a theoretical framework in this paper we take QCD to describe strong interactions while weak and electromagnetic interactions are described by the SU(2)⊗U(1) gauge theory of Weinberg and Salam extended to hadrons via the Glashow-Iliopoulos-Maiani mechanism and refer to it below as the standard theory. We confine ourselves to only four quark flavors in order to simplify the analysis.

II. PCAC, SHORT-DISTANCE EXPANSION, AND CURRENT ALGEBRA

We write the nonleptonic weak decay amplitude in the form

$$M=-\frac{g_w^2}{8}\int d^4z\, D_F(z;m_W^2)\langle F,\pi^i,\pi^j\,|\,\{\,T[J_{\mu N}^\dagger(z)J_S^\mu(0)]+\text{H.c.}\}\,|\,I\rangle\ , \tag{2.1}$$

where g_w is the electroweak coupling constant of the standard theory. The hadronic currents are given by

$$\begin{aligned}J_{\mu N}&=(\bar u\cos\theta_C-\bar c\sin\theta_C)\gamma_\mu(1-\gamma_5)d\ ,\\ J_{\mu S}&=(\bar u\sin\theta_C+\bar c\cos\theta_C)\gamma_\mu(1-\gamma_5)s\ .\end{aligned} \tag{2.2}$$

Contracting out the pions in Eq. (2.1) we get

$$M=\frac{g_w^2}{8}\int d^4z\, d^4x\, d^4y\, e^{i(k\cdot x+q\cdot y)}\times D_F(z;m_W^2)(\Box_x+m_\pi^2)(\Box_y+m_\pi^2)\times\langle F\,|\,T[\phi_\pi^i(x)\phi_\pi^j(y)J_{\mu N}^\dagger(z)J_S^\mu(0)]\,|\,I\rangle+\text{H.c.} \tag{2.3}$$

In Eq. (2.3), H.c. stands for a term as the first one with the product of the weak currents replaced by $J_{\mu N}(z)J_S^{\mu\dagger}(0)$. For ease of writing we shall not write this term in the equations that follow and it should be understood that it is there. Next we invoke PCAC in the form

$$\partial^\mu A_\mu^i=m_\pi^2F_\pi\phi_\pi^i\ . \tag{2.4}$$

Then for massless pions $k^2=q^2=0$ we obtain

$$M=\frac{g_w^2}{8F_\pi^2}\int d^4z\, d^4x\, d^4y\, e^{i(k\cdot x+q\cdot y)}D_F(z;m_W^2)\times\langle F\,|\,T[\partial^\alpha A_\alpha^i(x)\partial^\beta A_\beta^j(y)J_{\mu N}^\dagger(z)J_S^\mu(0)]\,|\,I\rangle\ . \tag{2.5}$$

Now let us consider the following quantity

$$\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}T[A_\alpha^i(x)A_\beta^j(y)J_{\mu N}^\dagger(z)J_S^\mu(0)]\ . \tag{2.6}$$

For the product of the two hadronic currents one follows Wilson[3] and expands in terms of a set of local operators O_k

$$T[J_{\mu N}^\dagger(z)J_S^\mu(0)]=\tfrac{1}{2}\sum_k C_k(z)O_k(0)\ , \tag{2.7}$$

where we have separated off a factor of $\frac{1}{2}$ for later convenience. In writing Eq. (2.7) we have absorbed all the Cabibbo-angle factors in the coefficient functions $C_k(z)$. The operator basis on the right-hand side (RHS) of Eq. (2.7) has been identified in Refs. (2) and (4). Using Eq. (2.7) in Eq. (2.6) we write

$$\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}T[A_\alpha^i(x)A_\beta^j(y)J_{\mu N}^\dagger(z)J_S^\mu(0)]=\tfrac{1}{2}\sum_k C_k(z)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}T[A_\alpha^i(x)A_\beta^j(y)O_k(0)]\ . \tag{2.8}$$

We now apply the standard techniques of current algebra to the T product on the RHS of Eq. (2.8). To simplify the ensuing analysis let us restrict ourselves to the neutral-pion case, $i=j=3$. The required identity then reads[12]

$$\begin{aligned}\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}T[A_\alpha^3(x)A_\beta^3(y)O_k(0)]={}&T[\partial^\alpha A_\alpha^3(x)\partial^\beta A_\beta^3(y)O_k(0)]+\delta(x^0-y^0)T\{[A_0^3(y),\partial^\alpha A_\alpha^3(x)]O_k(0)\}\\&+\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_0^3(y),[A_0^3(x),O_k(0)]]\\&+\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_0^3(x),[A_0^3(y),O_k(0)]]+\delta(y^0)T\{[A_0^3(y),O_k(0)]\partial^\alpha A_\alpha^3(x)\}\\&+\delta(x^0)T\{[A_0^3(x),O_k(0)]\partial^\beta A_\beta^3(y)\}\ .\end{aligned} \tag{2.9}$$

We substitute Eq. (2.9) into the RHS of Eq. (2.8), multiply both sides of the resultant equation by

$$\frac{g_w^2}{8F_\pi^2}e^{i(k\cdot x+q\cdot y)}D_F(z;m_W^2) \tag{2.10}$$

and integrate over x, y, and z. Taking matrix elements between a kaon initial state: $|\,I\rangle=|\,K\rangle$ and the vacuum as a final state we arrive at the equation

$$\frac{g_W{}^2}{8F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}D_F(z;m_W{}^2)\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial y^\beta}\langle 0\,|\,T[A^{3\alpha}(x)A^{3\beta}(y)J^\dagger_{\mu N}(z)J^\mu_S(0)]\,|\,K\rangle$$

$$=\sum_j\frac{g_W{}^2}{16F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}C_j(z)D_F(z;m_W{}^2)\langle 0\,|\,T[\partial_\alpha A^{3\alpha}(x)\partial_\beta A^{3\beta}(y)O_j(0)]\,|\,K\rangle$$

$$+\sum_j\frac{g_W{}^2}{16F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}C_j(z)D_F(z;m_W{}^2)\delta(x^0-y^0)\langle 0\,|\,T\{[A^3_0(y),\partial^\alpha A^3_\alpha(x)]O_j(0)\}\,|\,K\rangle$$

$$+\sum_j\frac{g_W{}^2}{32F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}C_j(z)D_F(z;m_W{}^2)\delta(x^0)\delta(y^0)$$

$$\times\langle 0\,|\,\{[A^3_0(y),[A^3_0(x),O_j(0)]]+[A^3_0(x),[A^3_0(y),O_j(0)]]\}\,|\,K\rangle$$

$$+\sum_j\frac{g_W{}^2}{16F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}C_j(z)D_F(z;m_W{}^2)$$

$$\times\langle 0\,|\,[\delta(y^0)T\{[A^3_0(y),O_j(0)]\partial^\alpha A^3_\alpha(x)\}+\delta(x^0)T\{[A^3_0(x),O_j(0)]\partial^\beta A^3_\beta(y)\}]\,|\,K\rangle\;. \tag{2.11}$$

The first term on the RHS of Eq. (2.11) is easily recognized to be the K transition complitude. Next performing the z integration in all but the term on the left-hand side (LHS) of Eq. (2.11) and making use of the standard relation

$$\frac{g_W{}^2}{8m_W{}^2}=\frac{G_F}{\sqrt{2}}\;, \tag{2.12}$$

we arrive at

$$M=-\frac{g_W{}^2}{8F_\pi{}^2}k_\alpha q_\beta\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}D_F(z;m_W{}^2)\langle 0\,|\,T[A^{3\alpha}(x)A^{3\beta}(y)J^\dagger_{\mu N}(z)J^\mu_S(0)]\,|\,K\rangle$$

$$+\frac{G_F}{2\sqrt{2}F_\pi{}^2}\sum_j C_j\int d^4x\,d^4y\,e^{i(k\cdot x+q\cdot y)}\delta(x^0-y^0)\langle 0\,|\,T\{[A^3_0(y),\partial^\alpha A^3_\alpha(x)]O_j(0)\}\,|\,K\rangle$$

$$+\frac{G_F}{4\sqrt{2}F_\pi{}^2}\sum_j C_j\int d^4x\,d^4y\,e^{i(k\cdot x+q\cdot y)}\delta(x^0)\delta(y^0)\langle 0\,|\,\{[A^3_0(y),[A^3_0(x),O_j(0)]]+[A^3_0(x),[A^3_0(y),O_j(0)]]\}\,|\,K\rangle$$

$$-\frac{iG_F}{2\sqrt{2}F_\pi}\sum_j C_j\left\{\int d^4y\,e^{iq\cdot y}\langle\pi^0(k)\,|\,\delta(y^0)[A^3_0(y),O_j(0)]\,|\,K\rangle+\int d^4x e^{ik\cdot x}\langle\pi^0(q)\,|\,\delta(x^0)[A^3_0(x),O_j(0)]\,|\,K\rangle\right\}, \tag{2.13}$$

where M is the $K\to 2\pi$ decay amplitude

$$M=-\frac{G_F}{2\sqrt{2}F_\pi{}^2}\sum_j C_j\int d^4x\,d^4y e^{i(k\cdot x+q\cdot y)}\langle 0\,|\,T[\partial_\alpha A^{3\alpha}(x)\partial_\beta A^{3\beta}(y)O_j(0)]\,|\,K\rangle\;. \tag{2.14}$$

The quantities C_j have been determined in Refs. (2) and (4) and our normalization corresponds to that of Ref. (5). We have used the contraction procedure in reverse to put one pion back into the final state in each of the last two terms of Eq. (2.13), corresponding to the last two terms of Eq. (2.11). On the RHS of Eq. (2.14) the sum extends over four-quark dimension-six operators. When one or two pions momenta are sent to zero the first term on the RHS of Eq. (2.13) gives a vanishing contribution and the resultant expression for M forms the starting point of the usual current-algebra analysis.[11] As announced in the introduction it is not our intention here to take k^α or $q^\beta\to 0$ and we shall retain the first term on RHS of Eq. (2.13) in our formula. We shall write this term in the form

$$k_\alpha q_\beta M^{\alpha\beta}(k,q)\;, \tag{2.15}$$

where $M^{\alpha\beta}$ is easily read from Eq. (2.13):

$$M^{\alpha\beta}(k,q)=-\frac{g_W{}^2}{8F_\pi{}^2}\int d^4x\,d^4y\,d^4z\,e^{i(k\cdot x+q\cdot y)}D_F(z;m_W{}^2)\langle 0\,|\,T[A^{3\alpha}(x)A^{3\beta}(y)J^\dagger_{\mu N}(z)J^\mu_S(0)]\,|\,K\rangle\;. \tag{2.16}$$

We shall take a closer look at the amplitude $M^{\alpha\beta}$ in the next section.

III. STRUCTURE OF THE AMPLITUDE $M^{\alpha\beta}$

Calculating the amplitude $M^{\alpha\beta}$ of Eq. (2.16) is a difficult problem in strong interaction theory. Our approach here will be to try and give an approximate treatment based upon separating $M^{\alpha\beta}$ into pieces, coming from "short distances" involving the variable x and y and from "long distances." We will try and make the meaning of this statement more precise in the sequel. The amplitude $M^{\alpha\beta}$ describes the "emission" of axial-vector currents $A^{3\alpha}$ and $A^{3\beta}$ carrying momenta $k^2=q^2=0$. The significant domain in the variable z is constrained by virtue of the presence of the propagator $D_F(z;m_W{}^2)$ to remain bounded by $\sim m_W{}^{-1}$. The short distances associated with x and y are taken to be those satisfying: $x,y \le z \sim m_W{}^{-1}$. In the absence of strong interactions this situation is described by the graphs of Fig. 1. These graphs, together with their radiative corrections when the QCD strong interactions are switched on, would then represent the short-distance piece of $M^{\alpha\beta}$ in our description. Let us then consider the T product for the four currents that appear in Eq. (2.16). At short distances and in the absence of QCD interactions we readily find that

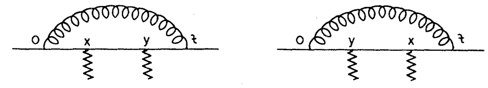

FIG. 1. Diagrams contributing to $M_S^{\alpha\beta}$. In this, and other diagrams, the wiggly line describes the W bosons while the zigzag line stands for the axial-vector current.

$$
\begin{aligned}
T[A^{3\alpha}(x)A^{3\beta}(y)J^{\dagger}_{\mu N}(z)J^{\mu}_{S}(0)] = -\frac{i}{4}\cos\theta_C\sin\theta_C[\,&\bar{d}(0)\Gamma_\mu S_F(z-x)\gamma^\alpha\gamma_5 S_F(x-y)\gamma^\beta\gamma_5 S_F(y)\Gamma^\mu s(0)\\
&+d(0)\Gamma_\mu S_F(z-y)\gamma^\beta\gamma_5 S_F(y-x)\gamma^\alpha\gamma_5 S_F(x)\Gamma^\mu s(0)\\
&+z_\nu\partial^\nu\bar{d}(0)\Gamma_\mu S_F(z-x)\gamma^\alpha\gamma_5 S_F(x-y)\gamma^\beta\gamma_5 S_F(y)\Gamma^\mu s(0)\\
&+z_\nu\partial^\nu\bar{d}(0)\Gamma_\mu S_F(z-y)\gamma^\beta\gamma_5 S_F(y-x)\gamma^\alpha\gamma_5 S_F(x)\Gamma^\mu s(0)+\cdots\,]\,. \qquad (3.1)
\end{aligned}
$$

In Eq. (3.1) $\Gamma_\mu=\gamma_\mu(1-\gamma 5)$ and the operators on the RHS are normal-ordered. Next using the short-distance form of the fermion propagators and making repeated use of the following identity for Dirac matrices

$$\gamma^\alpha\gamma^\beta\gamma^\sigma = g^{\alpha\beta}\gamma^\sigma - g^{\alpha\sigma}\gamma^\beta + g^{\beta\sigma}\gamma^\alpha - i\epsilon^{\alpha\beta\sigma\tau}\gamma_5\gamma_\tau\,, \qquad (3.1')$$

We note that the first two terms on the RHS of Eq. (3.1) reduce to a sum of terms each consisting of a coefficient function multiplying a vector or axial-vector current operator. Since these operators have zero anomalous dimensions we see that the form of the expansion of Eq. (3.1) continues to hold when we switch on the QCD strong interactions and no logarithmic corrections arise. We shall continue to use the compact expression given in Eq. (3.1) in the calculations below. Using Eq. (3.1) we can now write for the contribution to $M^{\alpha\beta}$ coming from these two terms

$$M_S^{(1)\alpha\beta}(k,q) = \frac{ig_w{}^2}{32F_\pi{}^2}\cos\theta_C\sin\theta_C\langle 0\,|\,\bar{d}(0)[I^{\alpha\beta}(k,q)+I^{\beta\alpha}(q,k)]s(0)\,|\,K\rangle\,, \qquad (3.2)$$

where the subscript S on $M_S^{(1)\alpha\beta}$ signifies the contribution from the region of short distances and

$$I^{\alpha\beta}(k,q) = \frac{1}{(2\pi)^4}\int d^4p\,\frac{\Gamma_\mu \not{p}\gamma^\alpha\gamma_5(\not{k}+\not{p})\gamma^\beta\gamma_5(\not{p}+\not{q}+\not{k})\Gamma^\mu}{(p^2-m_W{}^2+i\epsilon)(p^2+i\epsilon)[(p+k)^2+i\epsilon][(p+q+k)^2+i\epsilon]}\,. \qquad (3.3)$$

$I^{\beta\alpha}(q,k)$ is obtained from Eq. (3.1) by the simple exchange $\alpha\leftrightarrow\beta$, $k\leftrightarrow q$. A lengthy but straightforward calculation gives (see Appendix A for details)

$$
\begin{aligned}
I^{\alpha\beta}(k,q) = \frac{i}{4\pi^2 m_W{}^2}\Bigg[&\tfrac{1}{3}\left[\ln\frac{m_W{}^2}{\lambda^2}-\frac{11}{6}\right]k^\alpha\gamma^\beta + \left[-\tfrac{2}{3}\ln\frac{m_W{}^2}{\lambda^2}+\frac{8}{9}\right]k^\beta\gamma^\alpha - \frac{i}{3}\epsilon^{\alpha\beta\tau\lambda}k_\tau\gamma_\lambda + \left[\tfrac{2}{3}\ln\frac{m_W{}^2}{\lambda^2}-\frac{19}{18}\right]q^\alpha\gamma^\beta\\
&+\left[-\tfrac{1}{3}\ln\frac{m_W{}^2}{\lambda^2}+\frac{4}{9}\right]q^\beta\gamma^\alpha - \frac{i}{6}\epsilon^{\alpha\beta\tau\lambda}q_\tau\gamma_\lambda + \tfrac{1}{3}\left[\ln\frac{m_W{}^2}{\lambda^2}-\frac{11}{6}\right]g^{\alpha\beta}\not{k}\\
&+\left[-\tfrac{1}{3}\ln\frac{m_W{}^2}{\lambda^2}+\frac{4}{9}\right]g^{\alpha\beta}\not{q}\,\Bigg](1-\gamma_5)\,, \qquad (3.4)
\end{aligned}
$$

where λ is some mass such that $\lambda^2 \ll m_W{}^2$ (see Appendix A). The remarkable feature is that upon forming the sum $I^{\alpha\beta}(k,q)+I^{\beta\alpha}(q,k)$ all the logarithmic terms cancel. In fact contraction with $k_\alpha q_\beta$ yields a very simple

$$k_\alpha q_\beta[I^{\alpha\beta}(k,q)+I^{\beta\alpha}(q,k)]=\frac{-i}{12\pi^2 m_W{}^2}q\cdot k(\not{k}+\not{q})(1-\gamma_5) . \tag{3.5}$$

We can now finally write

$$k_\alpha q_\beta M_S^{(1)\alpha\beta}=\frac{G_F}{\sqrt{2}48\pi^2 F_\pi{}^2}\cos\theta_C\sin\theta_C(q\cdot k)(k+q)_\mu\langle 0\,|\,\bar{d}(0)\gamma^\mu(1-\gamma_5)s(0)\,|\,K\rangle . \tag{3.6}$$

We now specify the kaon state to be that of K^0 and note that in this case the contribution will come from the Hermitian-conjugate term of Eq. (2.3) that we have been suppressing all along. We then write

$$\langle 0\,|\,\bar{\lambda}\gamma^\mu\gamma_5 n\,|\,K^0(k+q)\rangle=if_{K^0}(k+q)^\mu , \tag{3.7}$$

and thereby obtain

$$k_\alpha q_\beta M_S^{(1)\alpha\beta}=\frac{-iG_F\cos\theta_C\sin\theta_C}{\sqrt{2}12\pi^2 f_\pi{}^2}(q\cdot k)^2 f_{K^0} , \tag{3.8}$$

where we have introduced the charged-pion decay constant $f_\pi=\sqrt{2}F_\pi$.

We have noted above that the logarithmic terms in Eq. (3.4) cancel in the sum $I^{\alpha\beta}(k,q)+I^{\beta\alpha}(q,k)$. It might be thought that this is peculiar to the case of two neutral pions because for two charged pions, $\pi^+\pi^-$, only one of the two diagrams of Fig. (1) contributes. Thus the symmetric combination indicated in Eq. (3.2) does not occur and we end up with one term with logarithms as given in Eq. (3.4). In fact, we find upon contraction with $k_\alpha q_\beta$, in the case of $\pi^+\pi^-$ the following expression:

$$k_\alpha q_\beta I^{\alpha\beta}(k,q)=\frac{i(q\cdot k)}{4\pi^2 m_W{}^2}\left[\tfrac{1}{3}\ln\left[\frac{m_W{}^2}{\lambda^2}\right](\not{q}-\not{k})+\tfrac{1}{18}(5\not{k}-11\not{q})\right](1-\gamma_5) . \tag{3.9}$$

Upon taking matrix elements, however, the logarithmic term being proportional to $q_\mu-k_\mu$ will give zero contribution since $q^2=k^2=0$. If we denote the charged pion amplitude by $M_{+-,S}^{(1)\alpha\beta}$ we then find

$$k_\alpha q_\beta M_{+-,S}^{(1)\alpha\beta}=-\frac{iG_F\cos\theta_C\sin\theta_C}{\sqrt{2}12\pi^2 f_\pi{}^2}(q\cdot k)^2 f_{K^0} , \tag{3.10}$$

which is identical to the neutral-pion case result of Eq. (3.8).

Next we turn to the third and fourth terms in Eq. (3.1). By similar manipulations we find that they make the following contribution to $M^{\alpha\beta}$:

$$M_F^{(2)\alpha\beta}=-\frac{iG_F\cos\theta_C\sin\theta_C}{\sqrt{2}48\pi^2 F_\pi{}^2}\langle 0\,|\,\partial^\nu\bar{d}(0)(g^{\alpha\beta}\gamma_\nu+g^\beta_\nu\gamma^\alpha+g^\alpha_\nu\gamma^\beta)(1-\gamma_5)s(0)\,|\,K\rangle , \tag{3.11}$$

where the subscript F signifies the absence of QCD interactions. We can cast Eq. (3.11) in an alternative form that involves the derivative of the d or s field operators

$$M_F^{(2)\alpha\beta}=\frac{iG_F\cos\theta_C\sin\theta_C}{\sqrt{2}48\pi^2 F_\pi{}^2}\langle 0\,|\,\bar{d}(0)\overleftrightarrow{\partial}^\nu(g^{\alpha\beta}\gamma_\nu+g^\beta_\nu\gamma^\alpha+g^\alpha_\nu\gamma^\beta)(1-\gamma_5)s(0)\,|\,K\rangle , \tag{3.12}$$

where we define

$$\bar{d}\overleftrightarrow{\partial}^\nu s=\tfrac{1}{2}(\bar{d}\partial^\nu s-\partial^\nu\bar{d}\,s) . \tag{3.13}$$

When the QCD interactions are turned on then upon using the identity of Eq. (3.1a) and the short-distance form of the fermion propagators we see that the third and fourth terms in the expansion of Eq. (3.1) involve the operator

$$O^{\nu\beta}=\bar{d}D^\nu\gamma^\beta(1-\gamma_5)s , \tag{3.14}$$

where now the color-covariant derivative D^ν appears here because of gauge invariance. In discussing the renormalization of tensor operators, as is well known, it is convenient to work with traceless operators that possess definite symmetry properties. Thus, our operator basis consists of the following two operators

$$O_2^{\alpha\beta}=\bar{d}(\overleftrightarrow{D}^\alpha\gamma^\beta+\overleftrightarrow{D}^\beta\gamma^\alpha-\tfrac{1}{2}g^{\alpha\beta}\overleftrightarrow{\not{D}})(1-\gamma_5)s , \tag{3.15}$$

$$O_0=\bar{d}\,\overleftrightarrow{\not{D}}(1-\gamma_5)s , \tag{3.16}$$

where

$$\overleftrightarrow{D}_\mu=\overleftrightarrow{\partial}_\mu-igT^aB^a_\mu \ , \tag{3.17}$$

with B^a_μ being the gluon field. The operator $O_2^{\alpha\beta}$ is the well-known nonsinglet twist-2 operator that enters in the description of deep-inelastic electroproduction.[13] The coefficient functions of these operators are modified over the free-field case by the appearance of functions $E_j(x,y,z;g,\mu)$, $j=0,2$, which obey renormalization-group equations whose solution reads

$$E_j(e^{-t}x,e^{-t}y,e^{-t}z;g,\mu)=E_j(x,y,z;\bar{g}(t),\mu)\exp\left[-\int_0^t dt'\gamma_j[\bar{g}(t')]\right] . \tag{3.18}$$

$\bar{g}(t)$ is the running coupling constant and γ_j is the anomalous dimension of the operator. The parameter t is taken to be

$$t=\ln\frac{m_W}{\mu} . \tag{3.19}$$

By writing the quantities γ_j in the form

$$\gamma_j(g)=\frac{g^2}{4\pi^2}\gamma_j^{(2)}+\cdots , \tag{3.20}$$

then for large t Eq. (3.18) is written in the form

$$E_j(e^{-t}x,e^{-t}y,e^{-t}z;g,\mu) \Longrightarrow \left[1-\frac{\alpha\beta_1}{\pi}t\right]^{\gamma_j^{(2)}/\beta_1} E_j(x,y,z;\bar{g}(t),\mu) . \tag{3.21}$$

The quantity β_1 is given by

$$\beta_1=-\tfrac{11}{6}C_2(G)+\tfrac{2}{3}T(R)N_f , \tag{3.22}$$

with N_f being the number of flavors and the group-theoretical quantities $C_2(G)$, etc., have their usual meaning. On the RHS of Eq. (3.21), and with t given by Eq. (3.19) we can set $\bar{g}$ equal to zero and replace the $E_j(\bar{g})$ by their free-field values. The quantity $\gamma_2^{(2)}$ is known from the results of Ref. (13) while for the operator O_0 we easily compute the anomalous dimension to one-loop order and find

$$\gamma_0(g)=\frac{g^2C_2(R)}{4\pi^2} . \tag{3.23}$$

Now in the case of four flavors that we are considering we finally have for the contribution to $M^{\alpha\beta}$ coming from the third and fourth terms of Eq. (3.1), and in the presence of strong interactions, the following expression:

$$M_S^{(2)\alpha\beta}=\frac{iG_F\cos\theta_C\sin\theta_C}{48\sqrt{2}\pi^2F_\pi{}^2}\left[\left[1+\frac{25g^2}{24\pi^2}\ln\frac{m_W}{\mu}\right]^{-32/75}\langle 0|O_2^{\alpha\beta}|K\rangle+\tfrac{3}{2}\left[1+\frac{25g^2}{24\pi^2}\ln\frac{m_W}{\mu}\right]^{-8/25}g^{\alpha\beta}\langle 0|O_0|K\rangle\right] . \tag{3.24}$$

Let us now specify the kaon state to be that of K^0. We use the equations of motion to cast O_0 in the form

$$O_0=-\frac{i}{2}(m_s+m_d)\bar{d}s-\frac{i}{2}(m_s-m_d)\bar{d}\gamma_5 s . \tag{3.25}$$

Only the second term on the RHS of Eq. (3.25) contributes when calculating the second matrix element in Eq. (3.24). The matrix element of $\bar{d}\gamma_5 s$ is next related to that of the divergence of the axial-vector current. For the operator $O_2^{\alpha\beta}$ we write

$$\langle 0|O_2^{\alpha\beta}|K^0(p)\rangle=(p^\alpha p^\beta-\tfrac{1}{4}g^{\alpha\beta}p^2)A_2 . \tag{3.26}$$

We then arrive at

$$k_\alpha q_\beta M_S^{(2)\alpha\beta}=\frac{-iG_F\cos\theta_C\sin\theta_C m_K{}^4}{\sqrt{2}384\pi^2F_\pi{}^2}\left[-\left[1+\frac{25g^2}{24\pi^2}\ln\frac{m_W}{\mu}\right]^{-32/75}A_2+3\left[1+\frac{25g^2}{24\pi^2}\ln\frac{m_W}{\mu}\right]^{-8/25}\frac{(m_s-m_d)}{m_s+m_d}f_{K^0}\right] . \tag{3.27}$$

Next we turn to a discussion of other possible contributions to $M^{\alpha\beta}$. The great virtue of the contribution $M_S^{\alpha\beta}$ is that it is calculable but there is no reason on general grounds to expect that it would dominate $M^{\alpha\beta}$. A contribution which is partially calculable arises when only one of the distances x or y is short in the sense that we are specifying here, with the other being outside the short-distance domain. In the absence of strong interactions this situation is described by the graph of Fig. 2. The T product of three currents that arises here is easily shown at short distances to be given by

$$T[A_3^{\alpha}(x)J_{\mu N}(z)J_S^{\mu\dagger}(s)]=-\frac{\cos\theta_C\sin\theta_C}{2\pi^2}\frac{(z-x)_\rho x_\tau}{[(z-x)^2-i\epsilon]^2(x^2-i\epsilon)^2}$$
$$\times[\bar d(0)\Gamma^{\tau\alpha\rho}(1-\gamma_5)s(0)+z_\nu\partial^\nu\bar d(0)\Gamma^{\tau\alpha\rho}(1-\gamma_5)s(0)+\tfrac{1}{2}z_\nu z_\sigma\partial^\nu\partial^\sigma\bar d(0)\Gamma^{\tau\alpha\rho}(1-\gamma_5)s(0)+\cdots]$$
$$-\frac{m_u{}^2\cos\theta_C\sin\theta_C}{8\pi^2}\frac{1}{[(z-x)^2-i\epsilon](x^2-i\epsilon)}\bar d(0)\gamma^\alpha(1-\gamma_5)s(0)+\cdots\ , \tag{3.28}$$

where

$$\Gamma^{\tau\alpha\rho}=\gamma^\tau\gamma^\alpha\gamma^\rho\ . \tag{3.29}$$

We shall denote the terms on the RHS of Eq. (3.28) successively by T_{iF}^{α}, $i=1,2,\ldots$. In the presence of strong interactions we designate them by T_i^{α}. We compute the contribution of each one of them in turn. Now the first term T_{1F}^{α} gives rise to a logarithmically divergent contribution to the amplitude $M^{\alpha\beta}$ which is of $O(g_W{}^2)$. We show in Appendix B that this contribution can be removed by QFD renormalization of the axial-vector current operator. A finite $O(g_W{}^2)$ contribution also arises from T_{1F}^{α}. This, however, does not represent a genuine weak-interaction effect and is also removed by renormalization of the axial-vector current. Moreover, the finite $O(G_F)$ contribution that arises from this term is proportional to k^α and hence vanishes upon contraction with k_α. When QCD interactions are switched on, no logarithmic modification of T_{1F}^{α} occurs and $T_1^{\alpha}=T_{1F}^{\alpha}$. This is so since by use of Eq. (3.1a) T_{1F}^{α} can be expressed as a sum of terms each involving the SU(3) current operator

$$\bar q\gamma^\alpha(1-\gamma_5)\frac{\lambda_{6+i7}}{2}q=\bar d\gamma^\alpha(1-\gamma_5)s\ , \tag{3.30}$$

and this has zero anomalous dimension. Hence, our conclusions above are unchanged and no contribution to $k_\alpha q_\beta M^{\alpha\beta}$ results from T_1^{α}.

Next we turn to the term T_{2F}^{α}. In terms of the derivative operator of Eq. (3.13) we find that it contributes the following

$$k_\alpha\int d^4z\,d^4x\,e^{ik\cdot x}D_F(z;m_W{}^2)T_{2F}^{\alpha}$$
$$=-\frac{\cos\theta_C\sin\theta_C}{24\pi^2m_W{}^2}k_\alpha k_\nu\bar d(0)\overleftrightarrow{\partial}^\nu\gamma^\alpha(1-\gamma_5)s(0)\ . \tag{3.31}$$

When QCD interactions are switched on $T_{2F}^{\alpha}\to T_2^{\alpha}$. The coefficient function occuring in T_2^{α} is modified over the free-field value that appears in T_{2F}^{α} and the operators that enter now are $O_2^{\alpha\beta}$ and O_0 of Eqs. (3.15) and (3.16). Using by now familiar arguments we find for the contribution of T_2^{α}

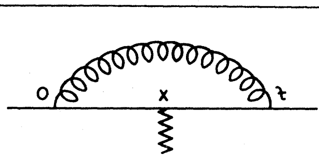

FIG. 2. Diagram associated with Eq. (3.28) of the text.

$$k_\alpha\int d^4z\,d^4x\,e^{ik\cdot x}D_F(z;m_W{}^2)T_2^{\alpha}$$
$$=-\frac{\cos\theta_C\sin\theta_C}{48\pi^2m_W{}^2}\left[1+\frac{25g^2}{24\pi^2}\ln\frac{m_W}{\mu}\right]^{-32/75}k_\nu k_\alpha O_2^{\nu\alpha}\ . \tag{3.32}$$

Note that the masslessness condition $k^2=0$ eliminates the contribution coming from the operator O_0.

Next we turn to the term T_{3F}^{α} and find for its contribution

$$k_\alpha\int d^4z\,d^4x\,e^{ik\cdot x}D_F(z;m_W{}^2)T_{3F}^{\alpha}$$
$$=-\frac{i\cos\theta_C\sin\theta_C}{96\pi^2m_W{}^2}k_\alpha[2\bar d(0)\overleftrightarrow{\not\partial}\overleftrightarrow{\partial}^\alpha(1-\gamma_5)s(0)$$
$$+\bar d(0)\overleftrightarrow{\partial}^2\gamma^\alpha(1-\gamma_5)s(0)]\ . \tag{3.33}$$

In the presence of QCD interactions the following operator enters the operator-product expansion:

$$O^{\sigma\nu\tau}=\bar dD^\sigma D^\nu\gamma^\tau(1-\gamma_5)s\ . \tag{3.34}$$

As before we work with operators of definite symmetry properties. It turns out that the contribution from the symmetric and traceless twist-two operator $O_3^{\sigma\nu\tau}$ as well as that coming from antisymmetric operators vanishes and the nonzero contribution from T_3^{α} involves the following two vector operators:

$$Q_1^{\alpha}=\bar d\,\overleftrightarrow{\not D}\overleftrightarrow{D}^\alpha(1-\gamma_5)s\ , \tag{3.35}$$
$$Q_2^{\alpha}=\bar d\,\overleftrightarrow{D}_\mu\overleftrightarrow{D}^\mu\gamma^\alpha(1-\gamma_5)s\ . \tag{3.36}$$

These operators mix under renormalization. No mixing with pure gluon operators occurs because of the nonsinglet nature of Q_1^{α} and Q_2^{α}. We calculate the anomalous-dimension matrix to one-loop order and find

$$\gamma^{(2)}=\frac{g^2C_2(R)}{6\pi^2}\begin{bmatrix}\frac{5}{4} & -\frac{13}{8}\\ 1 & -1\end{bmatrix}. \tag{3.37}$$

We then write the contribution of T_3^{α} as

$$k_\alpha \int d^4z\, d^4x\, e^{ik\cdot x} D_F(z;m_W{}^2) T_3^\alpha = -\frac{i\cos\theta_C \sin\theta_C}{48\pi^2 m_W{}^2} \sum_{i,j=1}^{2} k_\alpha \widetilde{E}_i \left[\left[1 + \frac{25 g^2}{24\pi^2} \ln\frac{m_W}{\mu} \right]^{-6\gamma^{(2)}/25} \right]_{ij} Q_j^\alpha(0) , \tag{3.38}$$

where

$$\widetilde{E}_1 = 1, \;\; \widetilde{E}_2 = \tfrac{1}{2} . \tag{3.39}$$

Finally, we observe that the last term on the RHS of Eq. (3.28) gives rise to an $O(G_F)$ contribution. However, this is suppressed due to the smallness of the u-quark mass. In fact, with the estimate $m_u \simeq 5$ MeV (Ref. 14) we have $G_F m_u{}^2 \simeq 2.9\times 10^{-10}$. This suppression continues to hold when QCD interactions are turned on and we shall ignore this contribution.

We shall denote the contribution to $M^{\alpha\beta}$ that involves Eq. (3.28), and the similar term obtained from it by the replacements $x \to y$ and $\alpha \to \beta$ by $M_{SL}^{\alpha\beta}$, where the subscript L describes the situation that one of the distances x or y is large. Recalling Eq. (2.16) for $M^{\alpha\beta}$ and using our results in Eqs. (3.32) and (3.38) we can now write

$$\begin{aligned} k_\alpha q_\beta M_{SL}^{\alpha\beta} = \frac{G_F \cos\theta_C \sin\theta_C}{\sqrt{2}\, 48\pi^2 F_\pi{}^2} k_\alpha q_\beta \Bigg\{ & \left[1 + \frac{25 g^2}{24\pi^2} \ln\frac{m_W}{\mu} \right]^{-32/75} k_\nu \int d^4y\, e^{iq\cdot y} \langle 0 \,|\, T[A^{3\beta}(y) O_2^{\nu\alpha}(0)] \,|\, K \rangle \\ & + i \sum_{i,j=1}^{2} \widetilde{E}_i \left[\left[1 + \frac{25 g^2}{24\pi^2} \ln\frac{m_W}{\mu} \right]^{-6\gamma^{(2)}/25} \right]_{ij} \int d^4y\, e^{iq\cdot y} \langle 0 \,|\, T[A^{3\beta}(y) Q_j^\alpha(0)] \,|\, K \rangle \\ & + (q \leftrightarrow k, \alpha \leftrightarrow \beta) \Bigg\} . \end{aligned} \tag{3.40}$$

Next we consider contributions associated with both variables x,y being large compared with $z \sim m_W{}^{-1}$. We denote the corresponding piece of $M^{\alpha\beta}$ by $M_L^{\alpha\beta}$. In gengeneral, $M_L^{\alpha\beta}$ would be given by the class of diagrams depicted in Fig. 3 with multiparticle intermediate states. Here we shall approximate this contribution by retaining only single-particle intermediate states. Dominance of pole diagrams is frequently invoked in phenomenological descriptions of weak decays.[12] Of course, it is particularly effective when the intermediate particle states are close in mass to the decaying state. Let us also recall here that it is our declared aim in this work to study the effects of terms that are usually thrown away by the soft-pion procedure. These effects are not simply confined to the retention of the $k_\alpha q_\beta M^{\alpha\beta}$ term as we shall see in the next section. While we are able to give a fairly general analysis for the contributions $M_S^{\alpha\beta}$ and $M_{SL}^{\alpha\beta}$, we have necessarily to settle for an approximate treatment for $M_L^{\alpha\beta}$. We shall then represent $M_L^{\alpha\beta}$ by the diagrams of Figs. 4 and 5. In these diagrams S_n and P_n denote intermediate scalar or pseudoscalar meson states. In Fig. 4 the kaon undergoes a weak transition to a scalar meson state which is, in turn, coupled to a pseudoscalar state through the action of one of the axial-vector currents. The pseudoscalar meson then couples to the vacuum state through the second axial-vector current. Since, however, the contribution from each of the two diagrams of Fig. 4 is proportional to either k^α or q^β, then they would give zero upon contraction with $k_\alpha q_\beta$.

Next we turn to the graphs of Fig. 5 where the internal meson state can be scalar or pseudoscalar. We note immediately, however, that for the pseudoscalar case the amplitude coupling the meson state to two axial-vector current is proportional to the covariant

$$\epsilon^{\alpha\beta\gamma\delta} q_\gamma k_\delta . \tag{3.41}$$

Hence, upon contraction with $k_\alpha q_\beta$ we again find a vanishing contribution. We are thus left with the case of scalar-meson intermediate states and hence we write $M_L^{\alpha\beta}$ as

$$M_L^{\alpha\beta} = \frac{1}{F_\pi{}^2} \sum_n T_n^{\alpha\beta}(k,q) \frac{1}{m_K{}^2 - m_n{}^2} \langle n \,|\, \mathscr{H}_w(0) \,|\, K^0 \rangle , \tag{3.42}$$

where m_n denotes the intermediate meson mass and the sum extends over all scalar meson states of positive G parity. $T_n^{\alpha\beta}(k,q)$ is the amplitude describing the coupling of the state n to be the two axial-vector currents:

$$T_n^{\alpha\beta}(k,q) = i \int d^4z\, e^{i(k-q)\cdot z/2} \times \langle 0 \,|\, T[A^{3\alpha}(z/2) A^{3\beta}(-z/2)] \,|\, n \rangle . \tag{3.43}$$

The effective nonleptonic weak Hamiltonian is given as usual by

$$\mathscr{H}_w(0) = \frac{G_F \cos\theta_C \sin\theta_C}{2\sqrt{2}} \sum_j C_j O_j + \text{h.c.} , \tag{3.44}$$

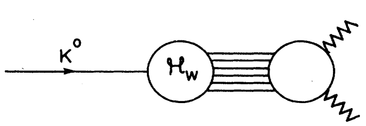

FIG. 3. General class of diagrams contributing to $M_L^{\alpha\beta}$.

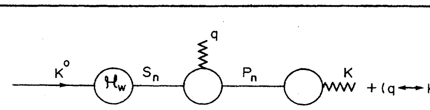

FIG. 4. Two-pole diagram contributing to $M_L^{\alpha\beta}$.

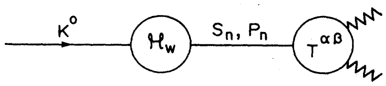

FIG. 5. Pole diagram contributing to $M_L^{\alpha\beta}$.

where we now make the $\cos\theta_C \sin\theta_C$ factor explicit.

By means of the standard methods of PCAC and current algebra we can relate the amplitude $T_n^{\alpha\beta}(k,q)$ to the amplitude $T_n(k,q)$ that describes the strong decay $n \to 2\pi^0$. To show this we start from the identity

$$\frac{\partial}{\partial y^\beta}\frac{\partial}{\partial x^\alpha}T[A_\alpha^3(x)A_\beta^3(y)] = T[\partial^\alpha A_\alpha^3(x)\partial^\beta A_\beta^3(y)]+[A_0^3(y),\partial^\alpha A_\alpha^3(x)]\delta(x^0-y^0) . \quad (3.45)$$

we then easily derive that

$$k_\alpha q_\beta T_n^{\alpha\beta} = -F_\pi{}^2 T_n(k,q) - i\langle 0 | [Q_5^3, \partial^\alpha A_\alpha^3(0)] | n\rangle , \quad (3.46)$$

where Q_5^3 is the axial-vector charge:

$$Q_5^3 = \int d^3x\, A_0^3(x) . \quad (3.47)$$

The equal-time commutator (ETC) occuring in Eq. (3.46) will be discussed further in the next section. The amplitude $T_n(k,q)$ is given by

$$T_n(k,q) = \frac{i}{F_\pi{}^2}\int d^4z\, e^{i(k-q)\cdot z/2} \times\langle 0 | T[\partial^\alpha A_\alpha^3(z/2)\partial^\beta A_\beta^3(-z/2)] | n\rangle . \quad (3.48)$$

As we mentioned earlier $T_n(k,q)$ describes the strong decay process $n \to 2\pi^0$ where the emitted pions have zero masses. We note from Eq. (3.42) that the decaying meson state is also off-shell since its momentum p satisfies $p^2 = m_K{}^2$.

This then concludes our analysis of the quantity $M^{\alpha\beta}$.

IV. THE $K \to 2\pi$ AMPLITUDE

Assuming the absence of q-number Schwinger terms in the ETC's occuring in Eq. (2.13) we now write the $K^0 \to 2\pi^0$ amplitude in the form

$$M = k_\alpha q_\beta M^{\alpha\beta} + \frac{1}{F_\pi{}^2}\int d^4x\, e^{i(k+q)\cdot x}\langle 0 | T\{[Q_5^3, \partial^\alpha A_\alpha^3(x)]\mathscr{H}_w(0)\} | K^0\rangle$$
$$+\frac{1}{F_\pi{}^2}\langle 0 | [Q_5^3,[Q_5^3,\mathscr{H}_w(0)]] | K^0\rangle - \frac{i}{F_\pi}\{\langle \pi^0(k) | [Q_5^3,\mathscr{H}_w(0)] | K^0\rangle + \langle \pi^0(q) | [Q_5^3,\mathscr{H}_w(0)] | K^0\rangle\} . \quad (4.1)$$

We shall now analyze each of the terms appearing on the RHS of Eq. (4.1) which we denote successively by $M_1, \ldots, M_4$. From the results of Sec. III we have

$$M_1 = k_\alpha q_\beta M^{\alpha\beta} = k_\alpha q_\beta (M_S^{(1)\alpha\beta} + M_S^{(2)\alpha\beta} + M_{SL}^{\alpha\beta})$$
$$-\sum_n \left[T_n(k,q) + \frac{i}{F_\pi{}^2}\langle 0 | [Q_5^3, \partial^\alpha A_\alpha^3(0)] | n\rangle \right]\frac{1}{m_K{}^2 - m_n{}^2}\langle n | \mathscr{H}_w(0) | K^0\rangle , \quad (4.2)$$

where we have used SU(3) symmetry to replace f_{K^0} by f_K, the charged-kaon decay constant. The terms M_1 and M_2 involve the ETC between the axial-vector charge and the divergence of the axial-vector current. In general, one writes

$$[A_0^i(y), \partial^\mu A_\mu^j(x)]\delta(x^0-y^0) = -i\sigma_{ij}(x)\delta^4(x-y) , \quad (4.3)$$

where i,j denote isospin indices. The quantity $\sigma_{ij}(x)$ in general will, in the limit of isospin symmetry, have $I=0$ and $I=2$ components. Here we shall ignore the $I=2$ component and write

$$[A_0^3(y), \partial^\mu A_\mu^3(x)]\delta(x^0-y^0) = -im_\pi{}^2 F_\pi \sigma(x)\delta^4(x-y) , \quad (4.4)$$

where $\sigma(x)$ is an isoscalar scalar field and we have taken note of the PCAC relation, Eq. (2.6) so as to extract the factor $m_\pi{}^2 F_\pi$. Now the divergence of the axial-vector current, in the limit of SU(2) symmetry, is given by

$$\partial^\mu A_\mu^3 = i\hat{m}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) , \quad (4.5)$$

where $\hat{m}$ is the common mass for the u and d quarks. Hence one can evaluate the ETC and find[15]

$$[A_0^3(y), \partial^\mu A_\mu^3(x)]\delta(x^0-y^0) = -i\hat{m}[\bar{u}(x)u(x) + \bar{d}(x)d(x)]\delta^4(x-y) . \quad (4.6)$$

We note that the term on the RHS of Eq. (4.6) is purely isoscalar. By comparing Eqs. (4.4) and (4.6) we see that we have the equivalence relation[16]

$$m_\pi{}^2 F_\pi \sigma(x) \equiv \hat{m}[\bar{u}(x)u(x) + \bar{d}(x)d(x)] . \quad (4.7)$$

We can now write the quantity M_2 as

$$M_2 = -\frac{im_\pi{}^2}{F_\pi{}^2}\int d^4x\, e^{i(k+q)\cdot x}\langle 0 | T[\sigma(x)\mathscr{H}_w(0)] | K^0\rangle . \quad (4.8)$$

If the field $\sigma(x)$ is an interpolating field for a physical particle of mass m_σ then using the reduction formalism one can write for the integral occuring on the RHS of Eq. (4.8)

$$\int d^4x\, e^{i(k+q)\cdot x}\langle 0\,|\,T[\delta(x)\mathscr{H}_w(0)]\,|\,K^0\rangle = \frac{-i\langle\sigma(k+q)\,|\,\mathscr{H}_w(0)\,|\,K^0\rangle}{m_\sigma{}^2-(k+q)^2}\ , \quad (4.9)$$

where the σ particle has momentum $k+q$. However, in general, unlike the case of the divergence of the axial-vector current and the pion field, the field $\sigma(x)$ is not expected to be dominated by a single physical particle and we have to replace the RHS of Eq. (4.9) by

$$-i\sum_n \frac{Z_n{}^{1/2}\langle n(k+q)\,|\,\mathscr{H}_w(0)\,|\,k^0\rangle}{m_n{}^2-(k+q)^2}\ , \quad (4.10)$$

where the quantity $Z_n{}^{1/2}$ is defined by

$$\langle 0\,|\,\sigma\,|\,n\rangle = Z_n{}^{1/2}\ , \quad (4.11)$$

and the sum extends over all possible scalar isoscalar physical particle states n. Combining Eqs. (4.8) and (4.10) and noting that $(k+q)^2=m_K{}^2$ we can now finally write

$$M_2=-\frac{m_\pi{}^2}{F_\pi}\sum_n Z_n{}^{1/2}\frac{\langle n(k+q)\,|\,\mathscr{H}_w(0)\,|\,K^0\rangle}{m_n{}^2-m_K{}^2}\ . \quad (4.12)$$

We next turn to the quantity M_3 which reads

$$M_3=\frac{1}{F_\pi{}^2}\langle 0\,|\,[Q_5^3,[Q_5^3,\mathscr{H}_w(0)]]\,|\,K^0\rangle\ . \quad (4.13)$$

To evaluate the double commutator on the RHS let us recall that $\mathscr{H}_w(0)$ is given by Eq. (3.19) where the operators O_k are given by[4]

$$\begin{aligned} O_1&=\mathscr{H}_A-\mathscr{H}_B\ ,\\ O_2&=\mathscr{H}_A+\mathscr{H}_B+2\mathscr{H}_C+2\mathscr{H}_D\ ,\\ O_3&=\mathscr{H}_A+\mathscr{H}_B+2\mathscr{H}_C-3\mathscr{H}_D\ ,\\ O_4&=\mathscr{H}_A+\mathscr{H}_B-\mathscr{H}_C\ , \end{aligned} \quad (4.14)$$

where

$$\begin{aligned} \mathscr{H}_A&=\bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)d\ ,\\ \mathscr{H}_B&=\bar{s}\gamma_\mu(1-\gamma_5)d\bar{u}\gamma^\mu(1-\gamma_5)u\ ,\\ \mathscr{H}_C&=\bar{s}\gamma_\mu(1-\gamma_5)d\bar{d}\gamma^\mu(1-\gamma_5)d\ ,\\ \mathscr{H}_D&=\bar{s}\gamma_\mu(1-\gamma_5)d\,\bar{s}\gamma^\mu(1-\gamma_5)s\ . \end{aligned} \quad (4.15)$$

In addition, we have the so-called "penguin" operators originally introduced by SVZ:

$$\begin{aligned} O_5&=\bar{s}\gamma_\mu(1-\gamma_5)t^A d\bar{Q}\gamma^\mu(1+\gamma_5)t^A Q\ ,\\ O_6&=\bar{s}\gamma_\mu(1-\gamma_5)d\bar{Q}\gamma^\mu(1+\gamma_5)Q\ . \end{aligned} \quad (4.16)$$

(The operators O_j are all understood to be normal ordered.) O_1, O_2, O_5, and O_6 transform like SU(3) octets and carry isopin $I=\frac{1}{2}$, while O_3 and O_4 are 27-plets under SU(3) and carry $I=\frac{1}{2}$ and $\frac{3}{2}$, respectively. In Eq. (4.16) the field operator Q is summed over all three light-quark flavors u,d,s. The matrices t^A generate the color gauge group SU(3) and satisfy $\mathrm{tr}\,t^A t^B=2\delta^{AB}$. Now as Tanuma *et al.*[7] have pointed out, we have for the double commutators

$$[Q_5^3,[Q_5^3,O_k(0)]]=[Q^3,[Q^3,Q_k(0)]]\ ,\qquad k=1,\ldots,6 \quad (4.17)$$

where Q^3 denotes the third isospin generator. Hence, evaluation of the matrix element of the double commutator can be effected through the known action of the isospin operator Q^3 and we have

$$M_3=\frac{1}{4F_\pi{}^2}\langle 0\,|\,\mathscr{H}_w(0)\,|\,K^0\rangle\ . \quad (4.18)$$

Next we turn to the quantity M_4. The ETC's between the axial-vector charge and the operators O_j read as[5,17]

$$[Q_5^3,O_j(0)]=-[Q^3,O_j(0)],\ \ j=1,\ldots,4 \quad (4.19)$$

$$[Q_5^3,O_6(0)]=-\tfrac{1}{2}O_6(0)+O_6^{(c)}(0)\ , \quad (4.20)$$

where $O_6^{(c)}$ is given by

$$O_6^{(c)}=\tfrac{2}{3}\langle 0\,|\,\bar{d}d\,|\,0\rangle{:}\bar{s}(1+\gamma_5)d{:}\ . \quad (4.21)$$

Similarly,

$$\begin{aligned} [Q_5^3,O_5(0)]&=-\tfrac{1}{2}O_5(0)+O_5^{(c)}(0)\ ,\\ O_5^{(c)}&=\tfrac{16}{3}O_6^{(c)}\ . \end{aligned} \quad (4.22)$$

In the following we shall denote the contribution to M_4 arising from the operator O_k by $M_4(O_k)$ and write

$$M_4=\sum_{k=1}^{6}M_4(O_k)\ . \quad (4.23)$$

Using Eqs. (4.19)—(4.22) we easily deduce that

$$M_4(O_k)=\frac{iG_F\cos\theta_C\sin\theta_C}{4\sqrt{2}F_\pi}[C_k\langle\pi^0(k)\,|\,O_k\,|\,K^0(p)\rangle+(k\to q)],\ \ k=1,\ldots,4 \quad (4.24)$$

$$M_4(O_5)=\frac{iG_F\cos\theta_C\sin\theta_C C_5}{4\sqrt{2}F_\pi}[\langle\pi^0(k)\,|\,O_5\,|\,K^0(p)\rangle-\tfrac{64}{9}\langle 0\,|\,\bar{d}d\,|\,0\rangle\langle\pi^0(k)\,|\,\bar{s}d\,|\,K^0(p)\rangle+(k\to q)]\ , \quad (4.25)$$

$$M_4(O_6)=\frac{iG_F\cos\theta_C\sin\theta_C C_6}{4\sqrt{2}F_\pi}[\langle\pi^0(k)\,|\,O_6\,|\,K^0(p)\rangle-\tfrac{4}{3}\langle 0\,|\,\bar{n}n\,|\,0\rangle\langle\pi^0(k)\,|\,\bar{s}d\,|\,K^0(p)\rangle+(k\to q)]\ , \quad (4.26)$$

where $p=k+q$ is the momentum of the kaon.

Next we consider the evaluation of the matrix elements of the operators that appear in the expressions given for the quantities M_j. We compute the matrix elements using the vacuum-insertion method as usually done in the literature.[18,4,19] Now the coefficients C_k have the following numerical values:[4] $C_1=2.5$, $C_2=0.08$, $C_3=0.08$, $C_4=0.4$, $C_5=(-0.06)\rightarrow(-0.14)$, and $C_6=(-0.01)\rightarrow(-0.05)$. In view of this, and bearing in mind the nature of the accompanying operators O_j, we shall give below explicit results only for the operators O_1, O_4, and O_5.

For the operators O_k we consider, as is usually done, the matrix element between K^+ and π^+ states. The matrix element between K^0 and π^0 is then obtained by standard SU(2) considerations. We start by considering the quantity M_4. We then easily find for O_1:

$$\langle \pi^+(k) \,|\, O_1 \,|\, K^+(p)\rangle = \tfrac{2}{3} f_\pi f_K k\cdot p \; . \tag{4.27}$$

We immediately observe an important difference from the case of SVZ. In their corresponding result $k\cdot p$ is replaced by k^2. This is so because they consider matrix elements between states of equal momenta which arise as a result of applying the soft-pion method. When comparing the matrix element of O_1 and O_5, SVZ take $k^2=m_K{}^2$. There is some ambiguity here because it is not clear what happens to four-momenta when one of them is taken to zero.[5] In our case $k\cdot p$ is unambiguously given as $\frac{1}{2}m_K{}^2$ because the kaon is on its mass-shell. Recalling that there are two terms on the RHS of Eq. (4.24) then they add up to give an $m_K{}^2$ factor. For the K^0-to-π^0 matrix element we thus find

$$\begin{aligned}\langle \pi^0(k) \,|\, O_1 \,|\, K^0(p)\rangle &= -\frac{1}{\sqrt{2}}\langle \pi^+(k) \,|\, O_1 \,|\, k^+(p)\rangle \\ &= -\frac{1}{3\sqrt{2}} f_\pi f_K m_K{}^2 \; .\end{aligned} \tag{4.28}$$

For the operator O_4 we find

$$\begin{aligned}\langle \pi^-(k) \,|\, 0_4 \,|\, K^0(p)\rangle &= \sqrt{2}\langle \pi^+(k) \,|\, O_4 \,|\, K^+(p)\rangle \\ &= \frac{2\sqrt{2}}{3} f_\pi f_k m_K{}^2 \; .\end{aligned} \tag{4.29}$$

Next we turn to O_5. One applies the vacuum-insertion method to the rearranged form obtained after a Fierz transformation, namely,

$$\begin{aligned}O_5 = &-4\bar{\lambda}(1+\gamma_5)Q\bar{Q}(1-\gamma_5)n \\ &+\tfrac{4}{3}\bar{s}_r(1+\gamma_5)Q_s\bar{Q}_s(1-\gamma_5)d_r \; ,\end{aligned} \tag{4.30}$$

where r,s are color indices. One then finds that

$$\begin{aligned}\langle \pi^0(k) \,|\, O_5 \,|\, K^0(p)\rangle &= -\frac{1}{\sqrt{2}}\langle \pi^+(k) \,|\, O_5 \,|\, K^+(p)\rangle \\ &= \frac{32 f_\pi f_k m_\pi{}^2 m_K{}^2}{9\sqrt{2}(m_u+m_d)(m_s+m_u)} \; .\end{aligned} \tag{4.31}$$

Equation (4.31) was first derived by SVZ. In establishing it one uses the equations of motion to relate the pseudoscalar densities $\bar{q}_a\gamma_5 q_b$ (a,b are flavor indices), to the divergence of the corresponding axial-vector current $\bar{q}_a\gamma_\mu\gamma_5 q_b$. This then gives rise to the quark-mass factors appearing in Eq. (4.31). Using Eqs. (4.28)—(4.31) we can now write

$$M_4(O_1) = -i\frac{G_F\cos\theta_C\sin\theta_C}{6\sqrt{2}} C_1 f_K m_K{}^2 \; . \tag{4.32}$$

where use has been made of the reaction $\sqrt{2}F_\pi = f_\pi$. Next we have

$$M_4(O_4) = -i\frac{\sqrt{2}G_F\cos\theta_C\sin\theta_C}{3} C_4 f_k m_K{}^2 \; . \tag{4.33}$$

For the case of $M_4(O_5)$ we need also the matrix element of the scalar operator $\bar{s}d$ between K^0 and π^0. Again using the quark equations of motion $\bar{s}d$ can be related to the divergence of the vector current $\bar{s}\gamma_\mu d$. We find

$$\langle \pi^0(k) \,|\, \bar{s}d \,|\, K^0(p)\rangle = -\frac{m_K{}^2 f_+(0)}{\sqrt{2}(m_s-m_d)} \; , \tag{4.34}$$

where $f_+(0)$ is the K_{l3} form factor. Hence, we arrive at

$$\begin{aligned}M_4(O_5) = &\frac{iG_F\cos\theta_C\sin\theta_C C_5}{2\sqrt{2}F_\pi} \\ &\times\left[\frac{32 f_\pi f_K m_\pi{}^2 m_K{}^2}{9\sqrt{2}(m_u+m_d)(m_s+m_u)}\right. \\ &\left.+\frac{64}{9\sqrt{2}}\langle 0 \,|\, \bar{d}d \,|\, 0\rangle \frac{m_K{}^2 f_+(0)}{m_s-m_d}\right] \; .\end{aligned} \tag{4.35}$$

The first term on the RHS is twice that found by SVZ for the quantity $M_4(O_5)$. The reason for this is that we have two matrix elements in Eq. (4.25) in contrast to the one matrix element situation of the soft-pion method. Our result for $M_4(O_5)$ also differs by the presence of the second term on the RHS of Eq. (4.35), the origin of which is the additional term $O_5^{(c)}$ in the commutator of Eq. (4.22).

Next we turn to the quantity M_3. We note from Eq. (4.18) that only the $\Delta I=\frac{1}{2}$ component of $\mathscr{H}_w$ would make a contribution here. It is interesting to observe that, in the vacuum intermediate state approximation, all the left-left operators give zero contribution. The nonvanishing contribution comes from the left-right or Penguin operators only. We find that

$$\begin{aligned}M_3 = &\frac{4iG_F}{9\sqrt{2}(m_\lambda+m_n)F_\pi{}^2}\cos\theta_C\sin\theta_C m_K{}^2 f_K \\ &\times C_5[\langle 0 \,|\, \bar{d}d \,|\, 0\rangle - \langle 0 \,|\, \bar{s}s \,|\, 0\rangle] \; .\end{aligned} \tag{4.36}$$

To determine the vacuum expectation values (VEV's), of the quark densities one usually resorts to the methods employed in the treatment of chiral symmetry breaking by Glashow and Weinberg, and by Gell-Mann, Oakes, and Renner.[20] These involve the use of PCAC for the pseudoscalar-meson octet and the current-density commutation relations so as to derive

$$\begin{aligned}m_\pi{}^2 f_\pi{}^2 &= -(m_d+m_u)\langle 0 \,|\, \bar{u}u+\bar{d}d \,|\, 0\rangle \; , \\ m_K{}^2 f_K{}^2 &= -(m_s+m_u)\langle 0 \,|\, \bar{u}u+\bar{s}s \,|\, 0\rangle \; .\end{aligned} \tag{4.37}$$

Then the VEV on the RHS of Eq. (4.36) is determined to be

$$\langle 0 \,|\, (\bar{d}d-\bar{s}s) \,|\, 0\rangle = \frac{m_K{}^2 f_K{}^2}{m_s+m_u} - \frac{m_\pi{}^2 f_\pi{}^2}{m_d+m_u} \; . \tag{4.38}$$

Next we turn to the quantities M_2 and M_1. We observe that when the ETC of Eq. (4.4) is used in Eq. (4.2), that M_2 cancels against the third term on the RHS of the equation for M_1. The combined contribution then reads

$$M_1+M_2=k_\alpha q_\beta(M_S^{(1)\alpha\beta}+M_S^{(2)\alpha\beta}+M_{SL}^{\alpha\beta}) - \sum_n T_n(k,q)\frac{1}{m_K{}^2-m_n{}^2}\langle n\,|\,\mathscr{H}_w(0)\,|\,K^0\rangle \;. \tag{4.39}$$

Using the vacuum-insertion method we obtain

$$\langle n\,|\,O_1\,|\,K^0\rangle=\frac{2i}{3}f_{K^0}f_n m_K{}^2$$

$$\langle n\,|\,O_5\,|\,K^0\rangle=\frac{32im_K{}^2f_{K^0}}{9(m_s+m_d)}(\langle n\,|\,\bar{d}d\,|\,0\rangle-\langle n\,|\,\bar{s}s\,|\,0\rangle)\;, \tag{4.40}$$

where f_n is defined by

$$\langle 0\,|\,\bar{u}\gamma^\mu u\,|\,n\rangle=f_n p^\mu \;. \tag{4.41}$$

Now we try to carry out numerical estimates for the various contributions M_j that make up the $K^0\to 2\pi^0$ decay amplitude M. For this purpose we shall take $f_\pi=130$ MeV, $f_K=1.15f_\pi$, and $\cos\theta_C=0.97$. We also need values for the "current" quark masses. These are known only to within 30% accuracy.[21] We shall use the following values

$$m_u=5\text{ MeV},\quad m_d=9\text{ MeV},\quad m_s=175\text{ MeV}\;. \tag{4.42}$$

We begin by considering M_4 and find

$$M_4(O_1)=-i2.92\times10^{-8}\text{ GeV}\;. \tag{4.43}$$

In what follows we shall refer to $M_4(O_1)$ as the basic term. We shall express our results for each of the remaining contributions in the form of a ratio to the basic term. We write $M_4(O_5)$ as

$$M_4(O_5)=M_4^{(p)}(O_5)+M_4^{(c)}(O_5) \tag{4.44}$$

corresponding to the two terms that appear on the RHS of Eq. (4.35). We then find that

$$\frac{M_4^{(p)}(O_5)}{M_4(O_1)}=-\frac{83C_5}{C_1}\;. \tag{4.45}$$

We remark that this is less than twice the value found by SVZ [see the remark following Eq. (4.35)], simply because we use a larger value for m_λ. Next we have

$$\frac{M_4^{(c)}(O_5)}{M_4(O_1)}=\frac{55.3C_5}{C_1}\;. \tag{4.46}$$

For M_3 we obtain

$$\frac{M_3}{M_4(O_1)}=-\frac{11.4C_5}{C_1}\;. \tag{4.47}$$

Next we turn to M_1+M_2. For the first term on the RHS of Eq. (4.39) we find that

$$\frac{k_\alpha q_\beta M_S^{(1)\alpha\beta}}{m_4(O_1)}=7.4\times10^{-2}\;, \tag{4.48}$$

viz., the contribution from the first short-distance piece of $M^{\alpha\beta}$ is small. In Eq. (3.27) there enters the quantity A_2 defined by Eq. (3.26), the magnitude of which is unknown *a priori*. There is no reason to expect an abnormally large value for it and for $A_2\sim O(f_\pi)$ we see that the contribution of $k_\alpha q_\beta M_S^{(2)\alpha\beta}$ is suppressed to the same extent as that of $k_\alpha q_\beta M_S^{(1)\alpha\beta}$.

Next we turn to the quantity $k_\alpha q_\beta M_{SL}^{\alpha\beta}$. We focus on the first term on the RHS of Eq. (3.40) and to estimate its contribution we use the following relation:

$$q^\beta\int d^4y\,e^{iq\cdot y}\langle 0\,|\,T[A_\beta^3(y)O^{\mu\nu}(0)]\,|\,K^0(p)\rangle = F_\pi\langle\pi^0(q)\,|\,O^{\mu\nu}(0)\,|\,K^0(p)\rangle + i\langle 0[Q_5^3(q),O^{\mu\nu}(0)]\,|\,K^0(p)\rangle\;, \tag{4.49}$$

where

$$Q_5{}^3(q)\int d^3y\,e^{-i\mathbf{q}\cdot\mathbf{y}}A_0^3(0,\mathbf{y})\;. \tag{4.50}$$

We parametrize the matrix element of $O^{\mu\nu}$ as follows

$$\langle\pi^0(q)\,|\,O^{\mu\nu}(0)\,|\,K^0(p)\rangle = \left[p^\mu p^\nu-\frac{m_K{}^2}{4}g^{\mu\nu}\right]F_1+q^\mu q^\nu F_2 + (p^\mu q^\nu+p^\nu q^\mu-\tfrac{1}{2}p\cdot q g^{\mu\nu})F_3\;, \tag{4.51}$$

and obtain

$$k_\mu k_\nu\langle\pi^0(q)\,|\,O^{\mu\nu}(0)\,|\,K^0(p)\rangle=m_K{}^4F\;, \tag{4.52}$$

with

$$F=\tfrac{1}{4}(F_1+F_2+2F_3)\;. \tag{4.53}$$

Using Eq. (4.49) in the first term of the RHS of Eq. (3.40) and denoting the contribution that involves only the matrix element $\langle\pi^0\,|\,O^{\mu\nu}\,|\,K^0\rangle$ by ϕ we find that

$$\phi\simeq1.16\times10^{-9}F\text{ GeV}\;. \tag{4.54}$$

The quantity F is again unknown *a priori* but we expect $F\sim 0(f_+)$, where f_+ is the K_{l3} form factor. With this estimate we find

$$\frac{\phi}{M_4(O_1)}\simeq2.8\times10^{-2}\;. \tag{4.55}$$

We thus conclude that ϕ is suppressed. It is clear that this suppression continues to hold for a range of larger values of F. Similar arguments would lead us to conclude that the contributions from the remaining terms in Eq. (3.40) are also suppressed and hence the quantity $k_\alpha q_\beta M_{SL}^{\alpha\beta}$ is small. Denoting the remaining term on the RHS of Eq. (4.39) by $M_{1L}+M_{2L}$, we note that a glance at the meson data[22] reveals that the lowest lying scalar mesons of even G parity are the isoscalar $S(975)$ and $\epsilon(1300)$. The contribution to $M_{1L}+M_{2L}$ arising from them will be purely $\Delta I=\frac{1}{2}$ in nature. As an illustration we compute this contribution. To do this we need the decay amplitude $T_n(k,q)$. We determine T_n from the known widths of these resonances:[22]

$$\Gamma_S=33\pm6 \text{ MeV}, \quad \Gamma_\epsilon=200\text{—}600 \text{ MeV} . \tag{4.56}$$

Note that for ϵ the width is poorly determined. For S the partial decay made into the $\pi\pi$ channel is $78\pm3\%$ while for ϵ it is $\sim 90\%$. The data describes the combined $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes and upon use of the isospin invariance of the strong interactions we compute that

$$\begin{aligned} |\, T_S \,| &\simeq 130 \text{ MeV} , \\ |\, T_\epsilon \,| &\simeq 300\text{—}519 \text{ MeV} . \end{aligned} \tag{4.57}$$

In computing $|\,T_S\,|$ we have taken $\Gamma_S \simeq 33$ MeV for simplicity and the range of values for $|\,T_\epsilon\,|$ in Eq. (4.57) corresponds to the range in the experimental determination of the widths in Eq. (4.56). We shall use these values to give us an idea about the size of the term $M_{1L}+M_{2L}$. Note that in Eq. (4.39), $T_n(k,q)$ is extrapolated from the physical mass $m_n{}^2$ of the resonance of $m_k{}^2$. We do not attempt to study the effects of mass extrapolation here and simply use the results of Eq. (4.57). In order to exhibit the ratio of $M_{1L}+M_{2L}$ to $M_4(O_1)$ it is convenient to introduce the following quantities:

$$R(O_j;n)=\frac{M_{1L}(O_j;n)+M_{2L}(O_j;n)}{M_4(O_1)} , \tag{4.58}$$

in which the numerator represents the contribution of the resonance n to $M_{1L}+M_{2L}$. We can only determine the magnitudes here since the phase of T_n is unknown. We then find that

$$\begin{aligned} |\, R(O_1;S) \,| &= 3.7\times 10^{-4}\,|\,f_S\,| , \\ |\, R(O_1;\epsilon) \,| &= (4.2\text{—}7.2)\times 10^{-4}\,|\,f_\epsilon\,| , \end{aligned} \tag{4.59}$$

$$\begin{aligned} |\, R(O_5;S) \,| &= \left|\frac{C_5}{C_1}\right| (10.44\times 10^{-6})\,|\,\langle S\,|\,(\bar{d}d-\bar{s}s)\,|\,0\rangle\,| , \\ |\, R(O_5;\epsilon) \,| & \\ &= \left|\frac{C_5}{C_1}\right| (11.7\text{—}20.3)10^{-6}\,|\,\langle \epsilon\,|\,(\bar{d}d-\bar{s}s)\,|\,0\rangle\,| . \end{aligned} \tag{4.60}$$

Thus, the ratios appear to be parametrized by the "decay" constants f_n and the matrix elements $\langle n\,|\,(\bar{d}d-\bar{s}s)\,|\,0\rangle$. If we take f_π to be a representative value for f_n we see from Eq. (4.59) that the contribution to $M_{1L}+M_{2L}$ coming from the operator O_1 is suppressed relative to $M_4(O_1)$. In order to be able to proceed further with Eq. (4.60) we have to be able to determine the value of the matrix element on the RHS. To do this we follow the same line of reasoning that led to Eq. (4.38) and consider the identity

$$\begin{aligned} &\frac{\partial}{\partial y^\beta}\frac{\partial}{\partial x^\alpha} T[A^8_\alpha(x)A^8_\beta(y)] \\ &\quad = T[\partial^\alpha A^8_\alpha(x)\partial^\beta A^8_\beta(y)] + [A^8_0(y),\partial^\alpha A^8_\alpha(x)]\delta(x^0-y^0) , \end{aligned} \tag{4.61}$$

where 8 denotes the value of the SU(3) index. With PCAC extended to describe the meson octet we write

$$\partial^\alpha A^8_\alpha = m_\eta{}^2 F_\eta \phi_\eta , \tag{4.62}$$

where ϕ_η stands for the η-meson field and we are ignoring the question of η-η' mixing. Then we readily derive the following equation:

$$\begin{aligned} k_\alpha q_\beta T^{\alpha\beta}_{8,n}(k,q) &= -F_\eta{}^2 T_{8,n}(k,q) \\ &\quad - i\langle 0\,|\,[Q^8_5,\partial^\alpha A^8_\alpha(0)]\,|\,n\,\rangle , \end{aligned} \tag{4.63}$$

with

$$\begin{aligned} T^{\alpha\beta}_{8,n}(k,q) &= i\int d^4z\, e^{i(k-q)\cdot z/2} \\ &\quad \times \left\langle 0 \left| T\left[A^{8\alpha}\left[\frac{z}{2}\right] A^{8\beta}\left[-\frac{z}{2}\right]\right]\right| n \right\rangle , \end{aligned} \tag{4.64}$$

$$\begin{aligned} T_{8,n}(k,q) &= \frac{i}{F_\eta{}^2}\int d^4z\, e^{i(k-q)\cdot z/2} \\ &\quad \times \left\langle 0 \left| T\left[\partial^\alpha A^8_\alpha\left[\frac{z}{2}\right] \partial^\beta A^8_\beta\left[-\frac{z}{2}\right]\right]\right| n \right\rangle . \end{aligned}$$

$T_{8,n}(k,q)$ is the amplitude for the off-shell strong decay process $n\rightarrow 2\eta$. Next the divergence of the axial-vector current A^8_α reads

$$\partial^\alpha A^8_\alpha = \frac{i}{\sqrt{3}}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s) . \tag{4.66}$$

We use Eq. (4.66) to compute the ETC appearing in Eq. (4.63). In the SU(3) limit and with $\hat{m}$ standing for the common u,d,s-quark mass we have

$$\begin{aligned} [Q^8_5,\partial^\alpha A^8_\alpha(0)] &= -i\frac{\hat{m}}{3}[\bar{u}(0)u(0)+\bar{d}(0)d(0) \\ &\quad +4\bar{s}(0)s(0)] . \end{aligned} \tag{4.67}$$

Using Eq. (4.67) into Eq. (4.63) we then have

$$\begin{aligned} k_\alpha q_\beta T^{\alpha\beta}_{8,n}(k,q) &= -F_\eta{}^2 T_{8,n}(k,q) \\ &\quad -\frac{\hat{m}}{3}\langle 0\,|\,\bar{u}u+\bar{d}d+4\bar{s}s\,|\,n\,\rangle . \end{aligned} \tag{4.68}$$

Next using Eq. (4.5) into Eq. (3.46) we also obtain

$$\begin{aligned} k_\alpha q_\beta T^{\alpha\beta}_n(k,q) &= -F_\pi{}^2 T_n(k,q) \\ &\quad -\hat{m}\langle 0\,|\,\bar{u}u+\bar{d}d\,|\,n\,\rangle . \end{aligned} \tag{4.69}$$

From Eqs. (4.68) and (4.69) we then deduce that

$$\begin{aligned} \langle 0\,|\,(\bar{d}d-\bar{s}s)\,|\,n\,\rangle &= \frac{3}{4\hat{m}}\{ F_\eta{}^2 T_{8n}(k,q) - F_\pi{}^2 T_n(k,q) \\ &\quad + k_\alpha q_\beta [\, T^{\alpha\beta}_{8,n}(k,q) \\ &\quad - T^{\alpha\beta}_n(k,q)]\} . \end{aligned} \tag{4.70}$$

Again, as in the case of Eq. (4.37), one resorts to the soft-meson limit in order to get an estimate for the quantity on the LHS of Eq. (4.70). This then gives

$$\langle 0|(\bar{d}d-\bar{s}s)|n\rangle \simeq \frac{3}{4\hat{m}}[F_\eta{}^2 T_{8n}(0,q)-F_\pi{}^2 T_n(0,q)] \ . \qquad (4.71)$$

The $\eta\eta$ decay mode has not been reported for the $S(975)$ and has possibly been seen for the $\epsilon(1300)$. So for these two states at least we can set $T_{8n}(0,q)\simeq 0$ and thereby arrive at

$$\langle 0|(\bar{d}d-\bar{s}s)|n\rangle \simeq -\frac{3F_\pi{}^2}{4\hat{m}} T_n(0,q) \ . \qquad (4.72)$$

Taking $\hat{m}=\frac{1}{3}(m_u+m_d+m_s)$ we then obtain

$$\langle 0|(\bar{u}u-\bar{s}s)|n\rangle \simeq -100 T_n(0,q) \ . \qquad (4.73)$$

The numerical estimates of Eq. (4.57) then finally give

$$|\langle 0|(\bar{d}d-\bar{s}s)|S\rangle| \simeq 130\times 10^2 \text{ MeV}^2 \ ,$$
$$|\langle 0|(\bar{d}d-\bar{s}s)|\epsilon\rangle| \simeq (300\text{—}519)\times 10^2 \text{ MeV}^2 \ . \qquad (4.74)$$

Using Eq. (4.74) in Eq. (4.60) we obtain

$$|R(O_5;S)| \simeq \left|\frac{C_5}{C_1}\right| (13.6)\times 10^{-2} \ ,$$
$$|R(O_5;\epsilon)| \simeq \left|\frac{C_5}{C_1}\right| (35\text{—}105)\times 10^{-2} \ . \qquad (4.75)$$

We observe that the mass extrapolations involved in the quantities T_n will most likely bring the values below the experimental values of Eq. (4.57). Hence, the numerical estimates of Eqs. (4.59), (4.60), and (4.75) should represent some sort of upper bounds with the actual values being less than the quoted ones. Let us also observe with Eq. (4.72) one can determine the ratio $R(O_5;n)$ and not just the absolute values if the sign of $T_n(k,q)$ is not changed by the extrapolation to $k^\alpha=0$. In fact, one would have

$$R(O_5;n)=\frac{8C_5F_\pi{}^2T_n(k,q)T_n(0,q)}{C_1\hat{m}(m_s+m_d)(m_n{}^2-m_K{}^2)}<0 \qquad (4.76)$$

since $C_5/C_1<0$.

Collecting together the numerically significant contributions which come from $M_4(O_1)$, $M_4(O_5)$, and M_3 we find from Eqs. (4.43)—(4.47) that the $K^0\to 2\pi^0$ amplitude reads

$$M=-i\left[1+\frac{C_5}{C_1}(-83+55.3-11.4)\right] \times(2.92\times 10^{-8}) \text{ GeV} \ . \qquad (4.77)$$

V. CONCLUSIONS

In this work, we have studied the consequences for the $K\to 2\pi$ amplitude of the procedure in which one avoids taking soft-pion limits. It is important for the consistency of the soft-pion procedure that when quantities like $k_\alpha q_\beta M^{\alpha\beta}$ are made to vanish to sending k_α or q_β to zero, that the quantity itself be rather small for physical momenta since otherwise large extrapolations will be involved. In the approximation of the treating the final pions as massless, $k^2=q^2=0$, we have computed the short-distance contribution $k_\alpha q_\beta M_S^{\alpha\beta}$, and found that it is indeed small in comparison to the basic term generated by the Lee-Gaillard operator O_1. We have argued that the quantity $k_\alpha q_\beta M_{SL}^{\alpha\beta}$ is also small. For the long-distance component $k_\alpha q_\beta M_L^{\alpha\beta}$, we could only given an approximate treatment in which we retain single-particle intermediate states. We have found that the contribution to $k_\alpha q_\beta M_L^{\alpha\beta}+M_2$, where M_2 is the so-called σ term, coming from the operators O_j, $j=1,\ldots,4$, is indeed small. Recalling that

$$\frac{C_5}{C_1}=-(2.4\text{—}5.6)\times 10^{-2} \ , \qquad (5.1)$$

we see from Eq. (4.70) that the contribution from the operator O_5 is suppressed due to the smallness of the ration in Eq. (5.1). We gave explicit results for the resonant states $S(975)$ and $\epsilon(1300)$ but presumably similar results hold for higher-mass states.

Treating the two pions symmetrically and keeping $k^\alpha, q^\beta \neq 0$, has the advantage that the contribution from the operator O_5 is doubled. It has turned out that application of the vacuum-insertion approximation is more favorable to the symmetric case. From Eq. (4.77) we see that the $K^0\to 2\pi^0$ amplitude has the value

$$M=-i[1+(0.9\text{—}2.19)]\times(2.92\times 10^{-8}) \text{ GeV} \ . \qquad (5.2)$$

The first term in the square bracket corresponds to the Lee-Gaillard operator O_1 while the second one represents the Penguin contribution. Thus, for $K_S^0\to 2\pi^0$ we have the amplitude

$$M(K_S^0\to 2\pi^0)=-i(0.80\text{—}1.32)\times 10^{-7} \text{ GeV} \ . \qquad (5.3)$$

When compared with the experimental value of the $\Delta I=\frac{1}{2}$ amplitude $a_{1/2}$ (Ref. 9)

$$a_{1/2}=(3.84\pm 0.01)\times 10^{-7} \text{ GeV} \ , \qquad (5.4)$$

we see that 21%—34% of the experimental result is accounted for. As is well known the theoretical estimate is quite sensitive to the value of C_5. For $C_5=-0.2$, which is not an unreasonable value,[23] the theoretical prediction is increased to 44%. Agreement with experiment would require $C_5=-0.53$.

Recently Dupont and Pham[23] criticized the SVZ use of the vacuum-insertion method to compute the K-π matrix element of the penguin operator on the grounds that it does not lead to a term quadratic in momenta as required by chiral-symmetry constraints. Let us recall that in our notation the operator O_5 is normal ordered. For the non-normal-ordered form $\overline{O}_5$ Donghue[24] has, however, shown that the inclusion of a previously omitted diagram restores the consistency of the K-π matrix element of $\overline{O}_5$ with chiral-symmetry constraints and hence the criticism of Pham and Dupont of the vacuum-insertion method is not tenable. In a recent paper Gavela *et al.*[25] have reached similar conclusions. Moreover, these authors argue that the vacuum-insertion method becomes exact in the chiral limit and is thus quite reliable as far as the computation of the K-π matrix element of O_5 is concerned. In fact, in the chiral limit and for zero-momentum pion and kaon one finds using the reduction procedure, and the vacuum approximation in the case of O_5, that

$$\langle \pi^0 | O_5 | k^0 \rangle = \frac{64}{9F_\pi^2} \langle 0 | \bar\psi\psi | 0 \rangle^2 ,$$
$$\langle \pi^0 | \bar s d | K^0 \rangle = \frac{1}{F_\pi^2} \langle 0 | \bar\psi\psi | 0 \rangle , \tag{5.5}$$

where ψ stands for any of the quark flavors. Now since

$$\bar O_5 = O_5 - \tfrac{32}{9} [\langle 0 | \bar d d | 0 \rangle :\bar s (1+\gamma_5) d: + \langle 0 | \bar s s | 0 \rangle :\bar s (1-\gamma_5) d:] , \tag{5.6}$$

we readily see that

$$\langle \pi^0 | \bar O_5 | K^0 \rangle = 0 . \tag{5.7}$$

In Refs. (24) and (25) attempts were made to go beyond the chiral limit in including terms in $P_\pi \cdot P_K$. The resultant amplitude given by Donghue,[25] for example, for $K_S^0 \to \pi^+\pi^-$ can account at most for 9% or 23% of the experimental result, depending on the sign of the term quadratic in momenta. In our approach we retained symmetry-breaking effects in computing the constant pieces of the various matrix elements and were able to get better agreement. Inclusion of terms quadratic in momenta could conceivably improve our estimates further.

In conclusion, we find it reasonable to believe that the symmetric treatment of pions and working with $k^2 = q^2 = 0$ instead of k^α or $q^\beta \to 0$, as well as attempting to improve upon the vacuum-approximation technique in the future, and by systematically including the effects of terms quadratic in momenta, could very well lead to a better understanding of the $\Delta I = \frac{1}{2}$ rule.

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APPENDIX A

In this Appendix we consider the evaluation of the integral given by Eq. (3.3) of the text:

$$I^{\alpha\beta}(k,q) = \frac{1}{(2\pi)^4} \int d^4p \frac{S^{\alpha\beta}}{(p^2 - m_W{}^2 + i\epsilon)(p^2 + i\epsilon)[(p+k)^2 + i\epsilon][(p+k+q)^2 + i\epsilon]} , \tag{A1}$$

where

$$S^{\alpha\beta} = \Gamma_\mu \not p \gamma^\alpha \gamma_5 (\not k + \not p) \gamma^\beta \gamma_5 (\not p + \not k + \not q) \Gamma^\mu . \tag{A2}$$

We easily find that

$$S^{\alpha\beta} = 8(p^\beta p_\sigma p_\tau - p^\beta p_\sigma k_\tau - k^\beta p_\sigma p_\tau)\Gamma^{\tau\alpha\sigma} - 4(p^2 - 2p\cdot k) p_\sigma \Gamma^{\beta\alpha\sigma} - 8 p_\alpha p_\sigma \Gamma^{\tau\beta\sigma} + 4p^2 q_\tau \Gamma^{\tau\beta\alpha} , \tag{A3}$$

where

$$\Gamma^{\tau\alpha\sigma} = \gamma^\tau \gamma^\alpha \gamma^\sigma (1-\gamma_5) . \tag{A4}$$

If we denote by D the denominator appearing in the integral of Eq. (A1) then we have, using Feynman's formula,

$$\frac{1}{D} = \Gamma(4) \int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \frac{1}{(M^2 - 2p\cdot A - p^2)^4} , \tag{A5}$$

where

$$M^2 = m_W{}^2 (1-x_1) + 2(x_2 - x_1) k\cdot q , \tag{A6}$$

$$A = (x_3 - x_1) k + (x_2 - x_1) q . \tag{A7}$$

Using Eqs. (A3) and (A5) in Eq. (A1) we generate a number of integrals. The most involved of the momentum integrations has the form

$$I_{\mu\nu\sigma} = \int d^4p \frac{p_\mu p_\nu p_\sigma}{(M^2 - 2A\cdot p - p^2)^4} . \tag{A8}$$

The n-dimensional integration techniques then give

$$\int d^np \frac{p_\mu p_\nu p_\sigma}{(M^2 - 2A\cdot p - p^2)^\alpha} = \frac{i\pi^{n/2}}{(M^2 + A^2)^{\alpha - n/2}} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} (-A_\mu A_\nu A_\sigma) + \frac{i\pi^{n/2}}{(M^2 + A^2)^{\alpha - 1 - n/2}} \frac{\Gamma(\alpha - 1 - n/2)}{\Gamma(\alpha)} \tfrac{1}{2} (g_{\mu\nu} A_\sigma + g_{\mu\sigma} A_\nu + g_{\nu\sigma} A_\mu) . \tag{A9}$$

Using Eq. (A9), and other results of similar nature, we can do all the momentum integrations and this is then followed by performing the x_i integrations. Let us illustrate by considering the contribution to $I^{\alpha\beta}$ arising from the first term on the RHS of Eq. (A3) for $S^{\alpha\beta}$. Denoting this contribution by $I^{\alpha\beta}_{(1)}$ we have

$$I^{\alpha\beta}_{(1)}=\frac{8\Gamma(4)}{(2\pi)^4}\int_0^1 dx_1\int_0^{x_1}dx_2\int_0^{x_2}dx_3\int d^4p\frac{p^\beta p_\sigma p_\tau \Gamma^{\tau\alpha\sigma}}{(M^2-2A\cdot p-p^2)^4} . \tag{A10}$$

Doing the momentum integration using Eq. (A9) and retaining the term that involves $(M^2+A^2)^{-1}$ only since it is the one that gives rise to an $O(G_F)$ contribution [the term in $(M^2+A^2)^{-2}$ gives a contribution suppressed relative to previous one by a factor of $m_W{}^2$], we obtain

$$I^{\alpha\beta}_{(1)}=\frac{i}{4\pi^2}\int_0^1 dx_1\int_0^{x_1}dx_2\int_0^{x_2}dx_3\frac{(g^\beta_\tau A_\sigma+g^\beta_\sigma A_\tau+g_{\sigma\tau}A^\beta)\Gamma^{\tau\alpha\sigma}}{M^2+A^2} . \tag{A11}$$

Next we observe that

$$M^2+A^2=m_W{}^2(1-x_1-\delta) , \tag{A12}$$

where

$$\delta=\frac{2k\cdot q(x_1-x_2)}{m_W{}^2}[1-(x_1-x_3)] . \tag{A13}$$

Thus in Eq. (A12) the term δ in the bracket on the RHS is suppressed by a factor of $m_W{}^2$ relative to the term $1-x_1$ and we are going to drop it in the following. The integrations over x_i are greatly simplified in this way. However, to regularize the infrared divergences resulting because of this (and also because of the fact we have set the u-quark mass to zero) we shall introduce a mass scale λ such that $\lambda^2 \ll m_W{}^2$ and write

$$M^2+A^2=m_W{}^2(1-x_1-\lambda^2) . \tag{A14}$$

Performing the x_i integrations one then finally obtains for $I^{\alpha\beta}_{(1)}$:

$$I^{\alpha\beta}_{(1)}=-\frac{i}{12\pi^2 m_W{}^2}\left[\ln\frac{m_W{}^2}{\lambda^2}-\frac{11}{6}\right][(k_\tau+\tfrac{1}{2}q_\tau)\Gamma^{\tau\alpha\beta}+(k_\tau+\tfrac{1}{2}q_\tau)\Gamma^{\beta\alpha\tau}+(k^\beta+\tfrac{1}{2}q^\beta)g_{\sigma\tau}\Gamma^{\tau\alpha\sigma}] . \tag{A15}$$

As we have noted in the text the dependence on λ disappears from the result for the physical quantity $k_\alpha q_\beta M^{\alpha\beta}_S$. Next we employ Eq. (3.2) of the text to reduce the product of three γ matrices. The remaining contributions to $I^{\alpha\beta}$ coming from the rest of the terms in $S^{\alpha\beta}$ are similarly handled. In this way we arrive finally at Eq. (3.4) of the text.

APPENDIX B

In this Appendix we consider the T product of one axial-vector current with the two weak hadronic currents:

$$T[A^{3\alpha}(x)J_{\mu N}(z)J^{\mu\dagger}_S(0)] . \tag{B1}$$

If at first we ignore QCD interactions we readily find the following expression for the T product of Eq. (B1) at short distances $[x \lesssim z, z\lesssim O(m_W{}^{-1})]$

$$T[A^{3\alpha}(x)J_{\mu N}(z)J^{\mu\dagger}_S(0)]=-\frac{\cos\theta_C\sin\theta_C}{2\pi^4}\frac{(z-x)_\rho x_\tau}{[(z-x)^2-i\epsilon]^2(x^2-i\epsilon)^2}$$
$$\times(g^{\tau\alpha}g^{\rho\lambda}-g^{\tau\rho}g^{\alpha\lambda}+g^{\alpha\rho}g^{\tau\lambda}-i\epsilon^{\tau\alpha\rho\sigma}g^\lambda_\sigma)\bar d\gamma_\lambda(1-\gamma_5)s+\cdots . \tag{B2}$$

It is then easily established that the term on the RHS of Eq. (B2) would give rise to a logarithmically divergent contribution to the amplitude $M^{\alpha\beta}$ which is of order $g_w{}^2$. This contribution does not represent a genuine weak interaction effect and can be removed by QFD renormalization of the axial-vector current operator as we shall now explain.

We find it convenient to work in momentum space and with operators of definite chirality. We define the following set of electrically neutral operators:

$$\begin{aligned}
O^\alpha_1&=\tfrac{1}{2}[\bar u\gamma^\alpha(1-\gamma_5)u-\bar d\gamma^\alpha(1-\gamma_5)d] ,\\
O^\alpha_2&=\bar c\gamma^\alpha(1-\gamma_5)c ,\\
O^\alpha_3&=\bar c\gamma^\alpha(1-\gamma_5)c ,\\
O^\alpha_4&=d\gamma^\alpha(1-\gamma_5)s ,\\
O^\alpha_5&=O^{\alpha\dagger}_2 ,\\
O^\alpha_6&=O^{\alpha\dagger}_4 .
\end{aligned} \tag{B3}$$

When we consider renormalization of Green's functions with an insertion of one of these operators O_i, we realize that mixing with other members of the set occurs. Let $\Gamma_{O_i}^{(q_a q_b)}$ denote the proper renormalized Green's function with external quark states q_a and q_b (a,b are flavor indices), and one insertion of the operator O_i. We then have

$$\Gamma_{O_i}^{(q_a q_b)} = Z_{q_a}^{1/2} Z_{q_b}^{1/2} \sum_j Z_{ij} \Gamma_{0,O_j}^{(q_a q_b)} , \tag{B4}$$

where Z_{q_a} and Z_{q_b} are the wave-function-renormalization constants of the quark fields and Z_{ij} is the renormalization matrix for the set of operators O_i. On the RHS, $\Gamma_{0,O_j}^{(q_a q_b)}$ denotes the unrenormalized Green's function. Here we are primarily concerned with insertion of the operator O_1 between external n and λ states. In Fig. 6 we show, as an illustration, the $O(g_w^2)$ graphs corresponding to O_1^α and O_2^α. Similar diagrams describe the operators O_3^α and O_5^α. The contribution of the operator O_4^α begins at zeroth order while O_6^α enters at fourth order in the electroweak coupling constant q_w. We write

$$\Gamma_{0,O_j}^{(ds)} = (g_w^2 b_j)\gamma^\alpha(1-\gamma_5) + \cdots , \quad j=1,2,3,5 \tag{B5}$$

$$\Gamma_{0,O_4}^{(ds)} = \gamma^\alpha(1-\gamma_5) + O(\alpha) . \tag{B6}$$

In Eq. (B5) the b_j are logarithmically divergent functions of the ultraviolet cutoff Λ. For the quantities Z_{ij} we write

$$Z_{ij} = \delta_{ij} + g_w^2 b_{ij} \ \cdots , \tag{B7}$$

where b_{ij} are arbitrary functions at this stage. For the quark wave-function constants we write

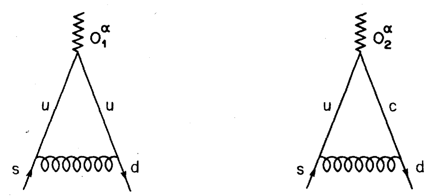

FIG. 6. One-loop graphs contributing to the Green's functions $\Gamma_{O_i}^{(ds)}$ for $i=1,2$.

$$Z_d = 1 + d_d g_w^2 + \cdots ,$$
$$Z_s = 1 + d_s g_w^2 + \cdots . \tag{B8}$$

Next we substitue Eqs. (B5)–(B8) into Eq. (B4), with $i=1$, and obtain

$$\Gamma_{O_1}^{(ds)} = g_w^2(b_1 + b_{14})\gamma^\alpha(1-\gamma_5) + O(g_w^4) . \tag{B9}$$

Hence, by choosing

$$b_{14} = -b_1$$

we cancel the $O(g_w^2)$ contribution completely. When the QCD interactions are turned on, the term on the RHS of Eq. (B2) is unchanged since only vector and axial-vector current operators appear on both sides of the equation and these have zero anomalous dimensions as far as QCD renormalization goes. Thus, our conclusions remain the same.

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