Calculation of the width of the decay $\psi' \to \psi\eta$

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The width of the transition $\psi' \to \psi\eta$ in the charmonium system is calculated in a potential-model approach. A two-stage process is assumed in which the intermediate states are charmed mesons which in turn couple to $\psi\eta$ via a quark-exchange process. A width of 1.6 keV is obtained.

I. INTRODUCTION

A long-standing problem in the charmonium system is the unexpectedly large width of the decay $\psi' \to \psi\eta$. A branching ratio of 0.0218 ± 0.0049 was reported[1] for this decay in 1980. This result is surprisingly high, in comparison with the branching ratio of 0.49 ± 0.05 obtained[2] for the decay $\psi' \to \psi\pi\pi$. If it is assumed that the $\psi' \to \psi\eta$ and $\psi' \to \psi\pi\pi$ transitions proceed through gluonic couplings ($\psi' \to \psi gg$, $gg \to \eta$ or $\pi\pi$), then the $\psi\eta$ decay mode, which is $SU(3)_{\text{flavor}}$-inhibited and occurs in p wave with less available phase space, would be expected to have a width whose ratio to the width for the $\psi\pi\pi$ mode is considerably smaller than the value of 0.04 indicated by these experiments. Indeed, the SU(3)-singlet-octet mixing angle of the η ($\sin\theta \sim 0.2$) is sufficient to account for this observed ratio on its own, without even taking into account the angular momentum and phase-space considerations. One suspects, therefore, that there is some mechanism which enhances the $\psi' \to \psi\eta$ width. Goldberg[3] has suggested that the coupling of the η to two gluons is unusually strong. This, by enhancing the process $gg \to \eta$, could lead to a strengthening of the $\psi\eta$ mode relative to $\psi\pi\pi$. However, the calculation of the coupling $gg \to \eta$ is problematic.

An equivalent picture that can be used to explain the relative strength of $\psi' \to \psi\eta$ is $\eta\eta_c$ mixing. Strong pseudoscalar mixing between light-quark states and $c\bar{c}$ was proposed soon after the initial charmonium discoveries.[4] Harari[5] specifically suggested that this could account for the large $\psi' \to \psi\eta$ width by allowing this decay to proceed without Okubo-Zweig-Iizuka-rule suppression. However, a useful quantitative application of this idea requires a knowledge of the $c\bar{c}$ admixture in the η.

A third possible way to picture the enhancement of $\psi' \to \psi\eta$ is through the use of intermediate charmed-meson pairs.[6] In this paper, a calculation based on this picture within a specific potential model is presented. The decay $\psi' \to \psi\eta$ is viewed as a two-stage process, in which the ψ' is coupled by means of the pair production of light quarks to virtual intermediate charmed-meson states,

$$D\bar{D},\ \ \frac{1}{\sqrt{2}}(D^*\bar{D}+D\bar{D}^*),\ \ D^*\bar{D}^*\ ,$$

$$F\bar{F},\ \ \frac{1}{\sqrt{2}}(F^*\bar{F}+F\bar{F}^*),\ \ F^*\bar{F}^*\ ,$$

and these states are then coupled to $\psi\eta$ via the exchange of charmed and light quarks or charmed and light antiquarks. These exchanges are assumed to involve the total Hamiltonian for the relative motion and interaction between the two clusters (D and $\bar{D}$, for example) in the intermediate states, where the interaction between clusters is taken to be the sum of the two-body potentials between the quarks in one cluster and those in the other. Furthermore, these two-body potentials are assumed to have central, spin-spin, spin-orbit, and tensor terms. Thus, the exchange matrix elements involve a sum of many terms. It is found, however, that most of these terms either vanish or cancel in pairs. The only nonzero contributions come from: for $D\bar{D}$ and $F\bar{F}$, the spin-orbit potential between charmed and light quarks or antiquarks; for $(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}$ and $(F^*\bar{F}+F\bar{F}^*)/\sqrt{2}$, the spin-orbit potential between the charged quark and its antiquark; and, for $D^*\bar{D}^*$ and $F^*\bar{F}^*$, the spin-orbit and tensor potentials between charmed and light quarks or antiquarks. These contributions are found to be sufficient to give a $\psi\eta$ width, via this process, of 1.6 keV.

The plan of this paper is as follows. Section II is devoted to the pair-production process, which couples the primitive $c\bar{c}$ states of charmonium to the charmed-meson states, $D\bar{D}$, $F\bar{F}$, etc. This process yields charmonium states which include contributions from the charmed-meson sector and, therefore, results in nonvanishing amplitudes for charmed-meson states in charmonium states—$\langle D\bar{D} \mid \psi' \rangle \neq 0$, for example. The quark-exchange processes which couple the charmed-meson states to $\psi\eta$ are then discussed in Sec. III. Selected matrix elements for these exchange processes are considered in some detail to illustrate which terms vanish, which terms cancel, and which terms lead to nonzero contributions. In Sec. IV the amplitudes of Sec. II are combined with the matrix elements of Sec. III to yield a width for $\psi' \to \psi\eta$. The final section includes a discussion of this result and speculation about an extension of this mechanism to the $\psi' \to \psi\pi^0$ decay.

II. PAIR PRODUCTION

Pair production couples two-quark systems to four-quark systems, specifically a meson to a meson-antimeson pair. We use a model in which either quark in the original meson emits a gluon, which then produces a quark-antiquark pair. Recoupling between the original two quarks and the produced pair then yields the final meson-antimeson. The coupling operator for this process is obtained from a semirelativistic reduction of the Feynman amplitude for the relevant graphs. This is regarded as a good approximation for the production of light quarks (u, d, or s) off of heavy quarks (c or b), and therefore for the coupling of charmonium states ($c\bar{c}$) or b-quarkonium states ($b\bar{b}$) to charmed mesons ($c\bar{u}$, $c\bar{d}$, $c\bar{s}$) or bare b states ($b\bar{u}$, $b\bar{d}$, $b\bar{s}$), respectively.

The pair-production operator obtained is[7]

$$H_{\rm pair} = -i\frac{\sigma_l}{2E_l}\cdot\nabla V\ , \tag{1}$$

where σ_l is the Pauli operator for the produced light-quark pair, E_l is the total relativistic energy of either member of this pair (the energies of the produced quark and antiquark are assumed to be approximately equal), and V is the interquark potential. This is a modified version of the operator used by Eichten *et al.*[8] in similar calculations. The potential V is taken to be the central part of the potential of Stanley and Robson.[9]

In the charmonium system there are two $J^P=1^-$ states below the threshold for charmed-meson decay—the ψ and ψ'. These can couple virtually, by quark-pair production, to the charmed-meson states $D\bar{D}$, $(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}$, $D^*\bar{D}^*$, $F\bar{F}$, $(F^*\bar{F}+\bar{F}F^*)/\sqrt{2}$, and $F^*\bar{F}^*$. The relative motion between these mesons is conveniently expanded in radial oscillator functions. One thus obtains the matrix elements $\langle\psi\,|\,H_{\rm pair}\,|\,M_c\bar{M}_c,Nl\rangle$ and $\langle\psi'\,|\,H_{\rm pair}\,|\,M_c\bar{M}_c,Nl\rangle$, where $(M_c\bar{M}_c)$ symbolizes any of the charmed-meson states listed above and N,l are the quantum numbers of the radial oscillator. Because of spin-parity conservation, we have $l=1$ only, except for $D^*\bar{D}^*$ or $F^*\bar{F}^*$ coupled to total spin 2. Since ψ and ψ' states have $^3D_1$ components due to the tensor force of the Stanley-Robson potential, the pair-production operator, a tensor of rank one, can couple these components to $(D^*\bar{D}^*)_{S=2}$ or $(F^*\bar{F}^*)_{S=2}$ with $l=3$ relative motion also.

After calculation of the pair-production matrix elements, diagonalization in the total charmonium—charmed-meson space then yields slightly shifted values for the charmonium energies and leakage of the charmonium states into the charmed-meson space. For example, the state $|\,\psi'\rangle$ is now represented as

$$|\,\psi'\rangle=\sum_i \alpha_i\,|\,\psi_i^P\rangle+\sum_{\substack{(M_c\bar{M}_c)\\ N,l}}\gamma_{M_c\bar{M}_c,Nl}\,|\,M_c\bar{M}_c,Nl\rangle\ , \tag{2}$$

where the $|\,\psi_i^P\rangle$ are the primitive $\psi,\psi',\ldots,$ states, i.e., the states in the $c\bar{c}$ sector only. The amplitudes α_i and

$$\gamma_{M_c\bar{M}_c,Nl}\equiv\langle M_c\bar{M}_c,Nl\,|\,\psi'\rangle$$

are, of course, determined directly from the diagonalization.

III. QUARK EXCHANGE

The role of quark exchange in hadronic interactions has been studied by Robson[10] and others,[11] mostly with regard to nucleon-nucleon interactions. Here, we wish to consider a nondiagonal quark-exchange process, i.e., one in which the final and initial states are different. In particular, if a c quark and an n quark ($n=u$ or d) are exchanged between $D(c\bar{n})$ and $\bar{D}(\bar{c}n)$, one obtains a coupling of $D\bar{D}$ to the nonstrange part of the state $|\,\psi\eta\rangle$ ($\eta\approx n\bar{n}/\sqrt{2}-s\bar{s}/\sqrt{2}$); similarly, c and s exchange between $F(c\bar{s})$ and $\bar{F}(\bar{c}s)$ gives a coupling of $F\bar{F}$ to the strange part of $|\,\psi\eta\rangle$. Identical effects are obtained with the exchanges $\bar{c}\leftrightarrow\bar{n}$ and $\bar{c}\leftrightarrow\bar{s}$, respectively. A convenient way of determining the exchange operators appropriate for these processes is to use an $SU(4)_{\rm flavor}$ representation in which the four quarks u, d, s, and c are identical. Then, in evaluating the interaction between clusters of these quarks, we must use properly antisymmetrized states. For a state $M(c\bar{l})\bar{M}(\bar{c}l)$, with $l=n$ or s, such an antisymmetrized state is

$$\Psi=(1-P_{cl})(1-P_{\bar{c}\bar{l}})\phi(c\bar{l})\phi(\bar{c}l)R\ , \tag{3}$$

where ϕ is the internal wave function of M (or $\bar{M}$), R is the wave function of relative motion between M and $\bar{M}$ and $P_{cl},P_{\bar{c}\bar{l}}$ are the exchange operators in the space, spin, and color variables. Matrix elements of the Hamiltonian between states of this form, $\langle\Psi\,|\,H\,|\,\Psi\rangle$, include the exchange terms

$$-\langle\Phi\,|\,HP_{cl}\,|\,\Phi\rangle-\langle\Phi\,|\,HP_{\bar{c}\bar{l}}\,|\,\Phi\rangle\ , \tag{4}$$

where Φ is the cluster-symmetrized state

$$\Phi=(1+P_{cl}P_{\bar{c}\bar{l}})\Psi\ . \tag{5}$$

Thus, our exchange operator is

$$H_{\rm ex}=-HP_{\bar{c}\bar{l}}-HP_{cl} \tag{6}$$

taken between cluster-symmetrized states.

For the interaction between clusters, H, we take

$$H=T+\sum_{\substack{i=c,\bar{l}\\ j=\bar{c},l}}V_{ij}\ , \tag{7}$$

where T is the kinetic energy and V_{ij} is the two-quark potential. Thus the exchange operator contains the kinetic-energy terms $-TP_{cl}$ and $-TP_{\bar{c}\bar{l}}$, as well as the potential-energy terms $-(V_{cl}+V_{\bar{c}\bar{l}}+V_{c\bar{c}}+V_{l\bar{l}})P_{cl}$ and $-(V_{cl}+V_{\bar{c}\bar{l}}+V_{c\bar{c}}+V_{l\bar{l}})P_{\bar{c}\bar{l}}$.

For V_{ij}, we take the Stanley-Robson potential, with a suitably modified color factor. That is, instead of the $-\frac{4}{3}$ which these authors use for the quark-antiquark interaction in a color singlet, we take

$$I_{ij}^{\rm color}=\langle(c\bar{l})1(l\bar{c})1\,|\,\mathbf{F}_i\cdot\mathbf{F}_j(\tfrac{1}{3}+2\mathbf{F}_c\cdot\mathbf{F}_l)\,|\,(c\bar{l})1(l\bar{c})1\rangle\ , \tag{8}$$

where the $\mathbf{F}$'s are the $SU(3)_{\rm color}$ generators and $(c\bar{l})1$, for example, indicates that c and $\bar{l}$ are in a color-singlet state. $(\frac{1}{3}+2\mathbf{F}_c\cdot\mathbf{F}_l)$ is the color exchange operator and $\mathbf{F}_i\cdot\mathbf{F}_j$ is the assumed color dependence of V_{ij}. From Eq. (8) and a similar expression for $P_{\bar{c}\bar{l}}$, we find color factors of $\frac{4}{9}$ for $V_{cl}P_{cl},V_{\bar{c}\bar{l}}P_{cl},V_{cl}P_{\bar{c}\bar{l}},V_{\bar{c}\bar{l}}P_{\bar{c}\bar{l}}$ and $-\frac{4}{9}$ for $V_{c\bar{c}}P_{cl},V_{l\bar{l}}P_{cl}$, $V_{c\bar{c}}P_{\bar{c}\bar{l}},V_{l\bar{l}}P_{\bar{c}\bar{l}}$.

The potential V_{ij} contains central (V^C), spin-spin (V^{SS}), spin-orbit (V^{LS}), and tensor (V^T) terms. We find that not all of these terms contribute to each exchange integral $\langle M\bar{M},Nl\,|\,H_{\rm ex}\,|\,\psi\eta\rangle$. For example, consider the central or spin-spin potentials. These give exchange terms which conserve total quark spin and so can couple $\psi\eta$ only to $(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}$ or $(F^*\bar{F}+F\bar{F}^*)/\sqrt{2}$ (or $D^*\bar{D}^*$, $F^*\bar{F}^*$, each with total spin 1, but there is no pair production from ψ' to these states because of a vanishing spin factor and so we can ignore them in our $\psi'\to\psi\eta$ mechanism). But because of the p-wave nature of the relative motion between ψ and η, we can show that even these terms either vanish or cancel in pairs. Consider, for example, $V^C_{c\bar{c}}P_{cl}$ (in what follows, we shall consider only P_{cl}; identical results can be obtained for $P_{\bar{c}\bar{l}}$). The spin matrix element for $D^*\bar{D}$ or $F^*\bar{F}$ is

$$\langle \chi_{1M}(c\bar{l})\chi_{00}(l\bar{c})\,|\,P^{\sigma}_{cl}\,|\,\chi_{1M}(c\bar{l})\chi_{00}(l\bar{c})\rangle=\tfrac{1}{2}\ , \tag{9}$$

where χ_{SM} is the singlet or triplet spin function and $P^{\sigma}_{cl}=\frac{1}{2}(1+\boldsymbol{\sigma}_c\cdot\boldsymbol{\sigma}_l)$. Similarly, for $D\bar{D}^*$ or $F\bar{F}^*$,

$$\langle \chi_{1M}(c\bar{l})\chi_{00}(l\bar{c})\,|\,P^{\sigma}_{cl}\,|\,\chi_{00}(c\bar{l})\chi_{1M}(l\bar{c})\rangle=\tfrac{1}{2}\ . \tag{10}$$

But for the spatial integrals we have, for $D^*\bar{D}$,

$$I_{D^*D}(\mathbf{k})=\int d^3x_{c\bar{c}}d^3y_{l\bar{l}}d^3R\ \psi^*(\mathbf{x}_{c\bar{c}})\phi^*_{\eta}(\mathbf{y}_{l\bar{l}})e^{-i\mathbf{k}\cdot\mathbf{R}}V^C(\mathbf{x}_{c\bar{c}})\phi_{D^*}\left[-\mathbf{R}+\frac{\mathbf{x}_{c\bar{c}}}{2}+\frac{\mathbf{y}_{l\bar{l}}}{2}\right]$$
$$\times\phi_D\left[\mathbf{R}+\frac{\mathbf{x}_{c\bar{c}}}{2}+\frac{\mathbf{y}_{l\bar{l}}}{2}\right]R_{Nlm_l}(-\lambda\mathbf{x}_{c\bar{c}}+(1-\lambda)\mathbf{y}_{l\bar{l}})\ , \tag{11}$$

where ψ, ϕ_η, ϕ_{D^*}, and ϕ_D are the internal wave functions of the ψ, η, D^*, and D; $R_{Nlm_l}=R_{Nl}Y_{lm_l}$, a radial oscillator function times a spherical harmonic; and we have taken a plane wave for the relative motion between ψ and η. In Eq. (11), we have also used

$$\mathbf{R}=\frac{m_l\mathbf{y}_l+m_{\bar{l}}\mathbf{y}_{\bar{l}}}{m_l+m_{\bar{l}}}-\frac{m_c\mathbf{x}_c+m_{\bar{c}}\mathbf{x}_{\bar{c}}}{m_c+m_{\bar{c}}}\ ,$$

and the fact that, under the exchange $c\leftrightarrow l$,

$$\mathbf{x}_{c\bar{c}}\to\mathbf{x}_{l\bar{c}}=-\mathbf{R}+\mathbf{x}_{c\bar{c}}/2+\mathbf{y}_{l\bar{l}}/2,\quad \mathbf{y}_{l\bar{l}}\to\mathbf{x}_{c\bar{l}}=\mathbf{R}+\mathbf{x}_{c\bar{c}}/2+\mathbf{y}_{l\bar{l}}/2\ ,$$

$$\mathbf{R}\to\mathbf{R}'=\frac{m_c\mathbf{x}_c+m_{\bar{l}}\mathbf{y}_{\bar{l}}}{m_c+m_{\bar{l}}}-\frac{m_l\mathbf{y}_l+m_{\bar{c}}\mathbf{x}_{\bar{c}}}{m_l+m_{\bar{c}}}=-\lambda\mathbf{x}_{c\bar{c}}+(1-\lambda)\mathbf{y}_{l\bar{l}}\ ,$$

with $\lambda=m_c/(m_c+m_l)$.

Similarly, for $D\bar{D}^*$, we get

$$I_{DD^*}(\mathbf{k})=\int d^3x_{c\bar{c}}d^3y_{l\bar{l}}d^3R\ \psi^*(x_{c\bar{c}})\phi^*_{\eta}(y_{l\bar{l}})e^{-i\mathbf{k}\cdot\mathbf{R}}V^C(\mathbf{x}_{c\bar{c}})\phi_D\left[-\mathbf{R}+\frac{\mathbf{x}_{c\bar{c}}}{2}+\frac{\mathbf{y}_{l\bar{l}}}{2}\right]$$
$$\times\phi_{D^*}\left[\mathbf{R}+\frac{\mathbf{x}_{c\bar{c}}}{2}+\frac{\mathbf{y}_{l\bar{l}}}{2}\right]R_{Nlm_l}(-\lambda\mathbf{x}_{c\bar{c}}+(1-\lambda)\mathbf{y}_{l\bar{l}})\ . \tag{12}$$

Now take $\mathbf{R}\to-\mathbf{R}$ in Eq. (12) and compare with Eq. (11). We see that

$$I_{DD^*}(\mathbf{k})=I_{D^*D}(-\mathbf{k})\ . \tag{13}$$

But the final motion of ψ and η is p wave. Therefore, $I_{D^*D}(-\mathbf{k})=-I_{D^*D}(\mathbf{k})$ and

$$I_{DD^*}(\mathbf{k})=-I_{D^*D}(\mathbf{k})\ . \tag{14}$$

Since the spin factors for $D^*\bar{D}$ and $D\bar{D}^*$ are the same [see Eqs. (9) and (10)], we conclude that

$$\langle\psi\eta\,|\,V^C_{c\bar{c}}P_{cl}\,|\,(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}\rangle=\tfrac{1}{2}[I_{D^*D}(\mathbf{k})+I_{DD^*}(\mathbf{k})]/\sqrt{2}=0 \tag{15}$$

and similarly for $(F^*\bar{F}+F\bar{F}^*)/\sqrt{2}$.

In this same way, we can also show that $V^C_{l\bar{l}}P_{cl}$ vanishes and that $V^C_{cl}P_{cl}$ and $V^C_{\bar{c}\bar{l}}P_{cl}$ cancel. Identical results also follow for V^{SS}, and for the kinetic-energy terms. So we are left with the spin-orbit and tensor potentials. These give exchange operators which do not conserve total quark spin and thus can give contributions for all spin channels in the charmed-meson sector. However, for $D\bar{D}$ and $D^*\bar{D}^*$ ($F\bar{F}$ and $F^*\bar{F}^*$), the contributions due to $V_{c\bar{c}}P_{cl}$ and $V_{l\bar{l}}P_{cl}$, for $V=V^{LS}$ or V^T, vanish exactly as above, leaving only $V_{cl}P_{cl}$ and $V_{\bar{c}\bar{l}}P_{cl}$. By comparing the spin factors for these terms, e.g., for the spin-orbit terms,

$$\mathbf{S}_{cl}=\langle\chi_{1M}(c\bar{l})\chi_{00}(l\bar{c})\,|\,(\boldsymbol{\sigma}_c+\boldsymbol{\sigma}_l)P^{\sigma}_{cl}\,|\,[\chi_{s_1m_1}(c\bar{l})\chi_{s_2m_2}(l\bar{c})]_{SM'}\rangle \tag{16}$$

and

$$\mathbf{S}_{\bar{c}\bar{l}}=\langle\chi_{1M}(c\bar{l})\chi_{00}(l\bar{c})\,|\,(\sigma_{\bar{c}}+\sigma_{\bar{l}})P^{\sigma}_{cl}\,|\,[\chi_{s_1m_1}(c\bar{l})\chi_{s_2m_2}(l\bar{c})]_{SM'}\rangle \tag{17}$$

for charmed mesons of spins s_1 and s_2 coupled to S, we easily find that $\mathbf{S}_{\bar{c}\bar{l}}=-\mathbf{S}_{cl}$ for $s_1=0$, $s_2=0$ ($D\bar{D}$ or $F\bar{F}$) and for $s_1=1$, $s_2=1$ ($D^*\bar{D}^*$ or $F^*\bar{F}^*$), but $\mathbf{S}_{\bar{c}\bar{l}}=\mathbf{S}_{cl}=0$ for $s_1=1$, $s_2=0$ and $s_1=0$, $s_2=1$ $[(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}$ or $(F^*\bar{F}+F\bar{F}^*)/\sqrt{2}]$. The relative minus sign between the spin factors $\mathbf{S}_{cl}$ and $\mathbf{S}_{\bar{c}\bar{l}}$ for $s_1=s_2=0$ and $s_1=s_2=1$ cancels the relative minus sign between the spatial integrals and results in the spin-orbit exchange terms $V^{LS}_{cl}P_{cl}$ and $V^{LS}_{\bar{c}\bar{l}}P_{cl}$ adding constructively, for $D\bar{D}$ and $D^*\bar{D}^*$ (or $F\bar{F}$ and $F^*\bar{F}^*$). These terms are zero for $(D^*\bar{D}+D\bar{D}^*)/\sqrt{2}$ or $(F^*\bar{F}+F\bar{F}^*)/\sqrt{2}$. But we find that the term $V^{LS}_{c\bar{c}}P_{cl}$ has a relative minus sign in the spin factors between $s_1=1$, $s_2=0$ and $s_1=0, s_2=1$ and so contributes in this case. On the other hand, the matrix element of $V^{LS}_{l\bar{l}}P_{cl}$ vanishes—it is zero, in general, because the η has spin 0. For the tensor potential, we get a nonzero result only when $s_1=s_2=1$ and then only through the terms $V^{T}_{cl}P_{cl}$ and $V^{T}_{\bar{c}\bar{l}}P_{cl}$ which, like the spin-orbit terms, pick up a relative minus sign in the spin factor to cancel the relative minus sign in the spatial integral.

Since the $P_{\bar{c}\bar{l}}$ give the same results, we have, taking account of color factors, the exchange matrix elements

$$\langle\psi\eta\,|\,H_{\rm ex}\,|\,D\bar{D},Nl\rangle=\tfrac{16}{9}\langle\psi\eta\,|\,V^{LS}_{cl}P_{cl}\,|\,D\bar{D},Nl\rangle\ , \tag{18}$$

$$\langle\psi\eta\,|\,H_{\rm ex}\,|\,(D^*\bar{D}+D\bar{D}^*)/\sqrt{2},Nl\rangle=-\tfrac{8}{9}\langle\psi\eta\,|\,V^{LS}_{c\bar{c}}P_{cl}\,|\,(D^*\bar{D}+D\bar{D}^*)/\sqrt{2},Nl\rangle\ , \tag{19}$$

$$\langle\psi\eta\,|\,H_{\rm ex}\,|\,D^*\bar{D}^*,Nl\rangle=\tfrac{16}{9}\langle\psi\eta\,|\,(V^{LS}_{cl}+V^{T}_{cl})P_{cl}\,|\,D^*\bar{D}^*,Nl\rangle\ , \tag{20}$$

and similar results for the F's. In these expressions, $l=1$ only, except in the tensor of Eq. (20) where $l=1$ or 3.

As an example, consider $\langle\psi\eta\,|\,V^{LS}_{cl}P_{cl}\,|\,D\bar{D},N=1l=1\rangle$. The spin factor Eq. (16) is found to be the unit vector $\hat{\mathbf{e}}_M$ [where $\hat{\mathbf{e}}_0=\hat{\mathbf{z}}$, $\hat{\mathbf{e}}_{\pm1}=\mp(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})/\sqrt{2}$] and this must be dotted into the spatial integral

$$\mathbf{I}_{m_l}(\mathbf{k})=\int d^3x\,d^3y\,d^3R\,\psi(\mathbf{x})\phi_\eta(\mathbf{y})e^{-i\mathbf{k}\cdot\mathbf{R}}V_{LS}(\mathbf{x}_{cl})\mathbf{L}_{x_{cl}}\phi_D\left[-\mathbf{R}+\frac{\mathbf{x}}{2}+\frac{\mathbf{y}}{2}\right]\phi_D\left[\mathbf{R}+\frac{\mathbf{x}}{2}+\frac{\mathbf{y}}{2}\right]R_{11m_l}(-\lambda\mathbf{x}+(1-\lambda)\mathbf{y})\ , \tag{21}$$

where we have taken

$$V^{LS}_{cl}=V_{LS}(\mathbf{x}_{cl})(\sigma_c+\sigma_l)\cdot\mathbf{L}_{x_{cl}}\ .$$

But $\mathbf{x}_{cl}=\mathbf{x}_l-\mathbf{x}_c=\mathbf{R}+\mathbf{x}/2-\mathbf{y}/2$, where $\mathbf{x}\equiv\mathbf{x}_{c\bar{c}}$, $\mathbf{y}\equiv\mathbf{y}_{l\bar{l}}$, and $\mathbf{R}$ is defined above. So let $\rho=\mathbf{x}_{cl}=\mathbf{R}+\mathbf{x}/2-\mathbf{y}/2$ and change variables $\mathbf{x},\mathbf{y},\mathbf{R}\to\mathbf{x},\mathbf{y},\rho$ to get

$$\begin{aligned}\mathbf{I}_{m_l}(\mathbf{k})=\int & d^3x\,d^3y\,d^3\rho\,\psi(\mathbf{x})\phi_\eta(\mathbf{y})e^{-i\mathbf{k}\cdot\bar{R}}\\ &\times V_{LS}(\rho)\mathbf{L}_\rho\phi_D(\mathbf{x}-\rho)\phi_D(\mathbf{y}+\rho)\\ &\times R_{11m_l}(-\lambda\mathbf{x}+(1-\lambda)\mathbf{y})\ .\end{aligned} \tag{22}$$

Now take $-\lambda\mathbf{x}+(1-\lambda)\mathbf{y}\sim-\mathbf{x}$ (for the Stanley-Robson model $\lambda=0.88$ for the $m_l=m_n$, 0.79 for $m_l=m_s$), and use Gaussians for all meson wave functions. We chose Gaussian parameters by minimizing the expectation values of the Stanley-Robson potential in each case. Then Eq. (22) can be done analytically up to a final radial integral involving the potential. We find that

$$\begin{aligned}\hat{\mathbf{e}}_M\cdot\mathbf{I}_{m_l}(\mathbf{k})=&\,Cke^{-\alpha k^2}Y_{1,M-m_l}(\hat{\mathbf{k}})\\ &\times\int_0^\infty d\rho\,\rho^4e^{-a\rho^2}\\ &\times[j_0(bk\rho)+j_2(bk\rho)]V_{LS}(\rho)\ ,\end{aligned} \tag{23}$$

where C, α, a, and b are constants depending only on the Gaussian parameters of the wave functions and the oscillator parameter chosen for the $D\bar{D}$ relative motion. Similar expressions can be obtained for Eqs. (19) and (20).

IV. CALCULATION OF THE WIDTH

Our wave functions have nonrelativistic normalizations. So the phase-space factor for $\psi'\to\psi\eta$ is

$$\begin{aligned}P&=\frac{k^2d\Omega_k}{(2\pi)^3}\frac{dk}{dE}\\ &=\frac{E_\psi E_\eta}{M_{\psi'}}k\frac{d\Omega_k}{(2\pi)^3}\end{aligned} \tag{24}$$

since

$$E=(k^2+M_\psi{}^2)^{1/2}+(k^2+M_\eta{}^2)^{1/2}=M_{\psi'}\ ,$$

with $M_{\psi'}$, M_ψ, and M_η the masses of ψ', ψ, and η. Using Fermi's "golden rule No. 2," the width for the decay is

$$\Gamma=\frac{2\pi}{3}\sum_{m,m'}\int\frac{E_\psi E_\eta}{M_{\psi'}}k\frac{d\Omega_k}{(2\pi)^3}\,|\,\langle\psi_m\eta\,|\,H\,|\,\psi'_{m'}\rangle\,|^2\ , \tag{25}$$

where we have summed over final spins m and averaged over initial spins m'. For $\langle\psi_m\eta\,|\,H\,|\,\psi'_{m'}\rangle$ we take

$$\langle\psi_m\eta\,|\,H\,|\,\psi'_{m'}\rangle=\frac{1}{\sqrt{2}}\sum_{\substack{s_1s_2\\Nl}}(\langle\psi_m\eta_n\,|\,H_{\rm ex}\,|\,D_{s_1}\bar{D}_{s_2},Nl\rangle\langle D_{s_1}\bar{D}_{s_2},Nl\,|\,\psi'_{m'}\rangle-\langle\psi_m\eta_s\,|\,H_{\rm ex}\,|\,F_{s_1}\bar{F}_{s_2},Nl\rangle\langle F_{s_1}\bar{F}_{s_2},Nl\,|\,\psi'_{m'}\rangle)\ , \tag{26}$$

where $D_{s_1}\bar{D}_{s_2}$ and $F_{s_1}\bar{F}_{s_2}$ designate the various spin states $D\bar{D}$, $F^*\bar{F}^*$, etc., and η_n, η_s are the nonstrange and strange parts of the η. In this equation we have assumed that the set $\{ | D_{s_1}\bar{D}_{s_2},Nl\rangle, | F_{s_1}\bar{F}_{s_2},Nl\rangle \}$ is complete. We expect this to be a good approximation; more energetic charmed mesons, such as the p states, are expected to have little effect at the energy level of the ψ'. In Eq. (26) the unspecified Hamiltonian H reduces to H_{ex} when taken between $\psi\eta$ and the charmed-meson states. The amplitudes $\langle D_{s_1}\bar{D}_{s_2},Nl | \psi'\rangle$ and $\langle F_{s_1}\bar{F}_{s_2},Nl | \psi'\rangle$ are determined from pair production—they are, in fact, the coefficients $\gamma_{M_c\bar{M}_c,Nl}$ of Eq. (2). Taking these and evaluating the exchange integrals of Eqs. (18), (19), and (20), we obtain

$$\langle \psi_m \eta | H | \psi'_{m'}\rangle = - i\frac{4\pi}{3}\frac{16}{9}(0.0020) \times Y_{1,m-m'}(\hat{\mathbf{k}})\ \mathrm{GeV}^{-1/2} , \qquad (27)$$

where we have taken $N = 1, \ldots, 4$ for $l = 1$; $N = 1$ only for the $l = 3$ tensor term in the summation of Eq. (26). Using Eq. (25) with $M_{\psi'} = 3.684$, $M_\psi = 3.095$, $M_\eta = 0.549$, $k = 0.196$ GeV, this gives

$$\Gamma_{\psi' \to \psi\eta} = 1.6 \text{ keV} . \qquad (28)$$

V. CONCLUSION AND COMMENTS

Based on a total ψ' width of 215.2 ± 39.9 keV (Ref. 12), the branching ratio reported in Ref. 1 gives a $\psi' \to \psi\eta$ width of 4.7 ± 1.4 keV. In our calculation of this width, we have made a number of approximations—the use of Gaussian wave functions in exchange integrals, the simplification $-\lambda\mathbf{x} + (1-\lambda)\mathbf{y} \to -\mathbf{x}$ in these integrals, and the assumption of completeness in Eq. (26). Nevertheless, the result obtained in this calculation, 1.6 keV, falls approximately within the range of two standard deviations of the measured value. This is quite promising and indicates that a mechanism of this type can, in fact, contribute significantly to the width of the $\psi' \to \psi\eta$ decay.

We feel that the reason this mechanism works is that it is able to circumvent the $SU(3)_{flavor}$ inhibition. $SU(3)_{flavor}$ is broken at three stages in this process: first, in the energies and wave functions of the D and F mesons of the $q\bar{q}$ model; second, in the pair-production mechanism, which is mass-dependent; and third, in the exchange integrals.

It is possible that this same process can be used to explain the isospin-violating $\psi' \to \psi\pi^0$ decay. This decay has been reported[13] to have a branching ratio of $0.15 \pm 0.6\%$. This seems to be too large to be a pure electromagnetic decay and it has been suggested by Bhandari and Wolfenstein[6] that this might be caused by coupling through the electromagnetically split D^+-D^0 states. In this regard, a mechanism of the type presented in this paper with a three-tiered breaking of the SU(2) symmetry could be useful. A calculation would require, however, that we put the electromagnetic splittings into the Stanley-Robson model and into the pair-production process.

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