Some dynamical properties of the pion in $\pi^+ \to e^+ \nu\gamma$ and $\pi^+ \to e^+ e^+ e^- \nu$ decays

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We investigate some of the dynamical properties of the pion by explicitly calculating the γ and ξ parameters in $\pi^+ \to e^+ \nu\gamma$ and $\pi^+ \to e^+ e^+ e^- \nu$ decays within the relativistic quark model, nucleon-loop approximation, and the σ model. One of the purposes in this study is to clarify a sign ambiguity raised recently in the prediction of γ within the quark model. The second purpose is to give some precise predictions for ξ, which has not yet been discussed extensively. The last purpose in this analysis involves a possible connection between ξ and chiral-symmetry breaking. Within the σ model, this parameter is found to depend logarithmically upon m_π, the explicit-breaking parameter of the symmetry.

I. INTRODUCTION

The pion triplet is known to be the lightest hadronic multiplet, with an overall mass of the order 140 MeV. It is much lighter than other hadrons, especially the baryons. This has led to the general belief that pions are the Nambu-Goldstone bosons associated with spontaneous chiral-symmetry breaking of the underlying strong-interaction theory. In combination with current algebras and PCAC (partial conservation of axial-vector current), it produced many useful results in the 1960s through the realization in the soft-pion limit, though with an exception in dealing with the $\pi^0 \to 2\gamma$ decay. Naive PCAC predicts that the pion cannot decay in the soft-pion limit. The efforts in trying to explain this puzzle resulted in the discovery of the now well-known Adler-Bell-Jackiw anomalies[1] in the context of perturbation theory. This is an important step toward a better understanding of the field theories, especially the gauge field theories which have become the basic language in modern-day elementary-particle physics. The pions have evidently played a very special role in this particular development. Despite the fact that the consideration of chiral symmetry has been very fruitful concerning pion physics, the nature of how it is actually broken in the real world still remains as a mystery in particle physics, though some hint of spontaneous breaking has been reported by lattice gauge calculations.[2]

One of the possibilities in looking for some useful insight in this respect is in the $\pi^+ \to e^+ \nu\gamma$ and $\pi^+ \to e^+ e^+ e^- \nu$ decays.[3] The structure-dependent γ parameter in both decays contains important information about u- and d-quark masses, regardless of the nature of symmetry breaking. Nevertheless, the parameter ξ, which manifests itself only in the four-body decay mode, does contain an additional piece of information; it tells whether the chiral symmetry is spontaneously or explicitly broken. Therefore, the magnitude of ξ should give us an important understanding concerning this particular mystery.

In the past, the determination of the γ parameter has drawn much attention both in the theoretical and the experimental aspects. Two possible values have been extracted from the experiment since the 1960s without further improvement up to now. The first experiment, performed in 1963 by Depommier *et al.*,[4] gave $\gamma = 0.26$ or -1.98, after using the updated value of the π^0 lifetime. About a decade later, a second experiment by Stetz *et al.*[5] gave $\gamma = 0.44$ or -2.36. At this moment, there are two experiments going on at SIN and TRIUMF. We expect that their results shall become available in the near future. Theorists have also given a wide range of predictions by using a variety of models. The situation of the predicted results has been quite confusing, especially, when compared with the two existing values measured. The soft-pion-limit prediction from current algebra,[6] after applying the Weinberg sum rule,[7] gave a value for γ that sensitively depends upon $\langle r_\pi{}^2 \rangle$ (Ref. 8), the pion's mean square radius. A small change in the experimental determination of $\langle r_\pi{}^2 \rangle$ would result in a large change in the value of $|\gamma|$. It varies from 0.09 to 1.88, depending upon which experimental value for $\langle r_\pi{}^2 \rangle$ is used.[3,9] In the hard-pion approach,[10] one introduces an additional parameter δ associated with the anomalous magnetic moment of the A_1 meson. Though $\langle r_\pi{}^2 \rangle$ can be related to δ in this procedure, the value of γ thus predicted now depends on δ and is still not a very stringent result. The static quark model and the σ model both predict[11,12] $\gamma = 0$. In the relativistic quark model, the calculations performed by Moreno and Pestieau,[13] as well as Montemayor and Moreno[14] gave $\gamma = -1$, while that by Paver and Scadron[15] gave $\gamma = 1$. These values are referring to the convention we shall use later in defining γ. Apparently, there is a contradiction between their signs of the two results. Besides, both values are not close to any of the observed ones. After all, this model does not respect the chiral symmetry which manifests itself in the low-energy processes involving the pion. On the other hand, the nucleon-loop approximation suggests $\gamma = \frac{1}{3}$, which is quite close to the positive one of the values measured. The general belief about this agreement is that it comes out merely as an accident, similar to the situation of $\pi^0 \to 2\gamma$ decay.

The ξ parameter associated with the mass of the virtual photon, manifests itself only in the decay process

$\pi^+\to e^+e^+e^-\nu$. This parameter has not been discussed extensively except that it can be related to $\langle r_\pi^2\rangle$. Its value was explicitly given only in the vector-dominance model,[3] which predicts $|\xi|=2.4$. Because this model does not respect chiral symmetry, this number also lacks good justification. A more careful study of this parameter from other models is apparently necessary.

Therefore, we carried out our calculations within the quark model, nucleon-loop approximation, and the σ model[16] in order to clear up some of the confusion associated with the γ parameter and to look for some more stringent predictions for the ξ parameter. In Sec. II, we shall first present the results from the quark model and the nucleon-loop approximation. In Sec. III, the detail obtained from the σ model will be discussed.

II. THE QUARK MODEL AND NUCLEON-LOOP APPROXIMATION

Now we come to the calculation of the γ and ξ parameters within the quark model where the u quark and the d quark may have different masses. We start with the Lagrangian

$$\mathscr{L}=\mathscr{L}_S+\mathscr{L}_{\rm EW} \tag{1}$$

with

$$\mathscr{L}_S=\bar{\psi}_c(i\partial\!\!\!/-\hat{m})\psi_c+\tfrac{1}{2}(\partial_\mu\boldsymbol{\phi}\cdot\partial^\mu\boldsymbol{\phi}-m_\pi{}^2\boldsymbol{\phi}\cdot\boldsymbol{\phi})+ig_S\bar{\psi}_c\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5\psi_c\ , \tag{2}$$

and $\mathscr{L}_{\rm EW}$ is the familiar Lagrangian of the standard Glashow-Salam-Weinberg model, which we shall not repeat here. The above Lagrangian is to be understood to define an effective action for QCD at low energies. The triplet pion field $\boldsymbol{\phi}$ is not to be regarded as fundamental. ψ_c is the doublet of the quark fields with color index c, and $\hat{m}$ the quark mass matrix. g_S and $\boldsymbol{\tau}$ are the effective strong coupling constant and the Pauli matrices, respectively.

In this model, the neutral component of the vector current is always conserved whether $m_d=m_u$ or not. While for the charged components, they are conserved only when $m_d=m_u$. The axial-vector current is clearly not conserved. Though CVC (conserved-vector-current hypothesis) is explicitly violated in the theory by the quark mass difference, it by no means implies that this would necessarily result in a large observable effect. This is because that the quark masses we have here are not definite to be the current quark mass or the constituent quark mass.[17] This is not clear at this stage. They may possibly be neither one. It seems to us that the notion of quark masses is somehow dependent upon what type of process we are dealing with. The current and constituent masses which are defined in other contexts may not be proper for the present situation because the nonperturbative behaviors involved for each case are not clear to be unique. In other words, we cannot expect to deduce a same and single effective mass parameter for the quark from among diverse QCD effects involved in different physical processes. It is quite apparent that the concept of a free quark propagator is not adequate to describe the complete dynamics within the pions. It should be modified in a certain way not only at small momentum transfers but also at high energies. The high-energy behavior has been studied since the last decade by various people without conclusive result.[18] Only recently were Chang and Chang able to settle this issue through a careful renormalization-group analysis.[19] The nonperturbative low-energy behavior still remains as an open question.

In the present model, the quark propagators are treated as free ones, while leaving the poles unexplained. In order to prevent the amplitude from developing an absorptive part, we demand that $m_u+m_d>m_\pi$. Whether this requirement is really consistent with the confinement is certainly a problem that merits further study, but we merely assume it to be the case for our purpose here.

We shall now turn our attention to the specific process we are interested in, namely, the general radiative pion decays in which the photon is allowed to be virtual.

The most general form of the amplitude for the process (we let the photon convert into e^+e^- here) can be written as

$$\mathscr{M}={\rm IB}+{\rm SD}\ , \tag{3}$$

where

$${\rm IB}=\frac{m_l G f_\pi e^2}{\sqrt{2}k^2}\bar{u}(q)\left[\frac{2t\cdot j+k\!\!\!/ j\!\!\!/}{2t\cdot k+k^2}-\frac{(2p-k)\cdot j}{2p\cdot k-k^2}\right]\times(1+\gamma_5)v(t)\ , \tag{4}$$

$${\rm SD}=\frac{Ge}{\sqrt{2}k^2}\{ai\epsilon_{\mu\nu\lambda\rho}L^\mu p^\nu j^\lambda k^\rho+b[(p\cdot k)(L\cdot j)-(k\cdot L)(p\cdot j)]+ck^2(L\cdot j)\}\ , \tag{5}$$

which are called the inner-bremsstrahlung term and the structure-dependent term, respectively. They satisfy gauge invariance separately. a and b are the vector and axial-vector form factors, c is the third form factor associated with the mass of the virtual photon. f_π is the pion decay constant. The four-vector

$$L^\mu=\bar{u}(q)\gamma^\mu(1-\gamma_5)v(t)$$

is the usual $V-A$ charged weak current, and

$$j^\lambda=\bar{u}(l)\gamma^\lambda v(s)$$

is the electromagnetic current. It is understood that the corresponding antisymmetrized parts due to e^+e^+ exchange should be taken into account. This amplitude can be obtained in this model by the set of Feynman diagrams as shown in Fig. 1.

In this model, f_π cannot be extracted. It is divergent and cannot be renormalized. We concentrate only on the predictions of the vector and axial-vector form factors a and b as well as the third form factor c, which are all finite and well defined. Especially, the parameters $\gamma\equiv b/a$ and $\xi\equiv c/a$ can be given unambiguously without having to know what the exact value of the effective quark-pion coupling constant g_S is. We shall give directly in the following the explicit expressions for a, b, and c calculated from the Feynman diagrams in Fig. 1.

The vector form factor a is given by

$$a=\frac{\sqrt{2}n_c e_u g_S}{4\pi^2}\int_0^1 dx\int_0^{1-x}dy\frac{(m_d-m_u)x+m_u}{\mathscr{D}}$$

$$+(e_u\to e_d, m_u\leftrightarrow m_d)\ , \tag{6}$$

where

$$\mathscr{D}=p_1{}^2y(1-y)+p_2{}^2x(1-x)+2p_1\cdot p_2xy$$
$$-(m_d{}^2-m_u{}^2)x-m_u{}^2 \tag{7}$$

and, the second term in (6) is obtained from the first term by replacing e_u with e_d and exchanging m_u and m_d. When $m_u=m_d$, as could be shown, the Vaks-Ioffe relation that connects a to the π^0 decay rate is true. Therefore, CVC is good as has been expected. When $m_u\neq m_d$, this relation is violated and CVC breaks down. But, as mentioned earlier and will be discussed later, this does not mean that it should give rise to a large observable effect.

Similarly, we obtain the explicit expressions for form factors b and c. They are, separately,

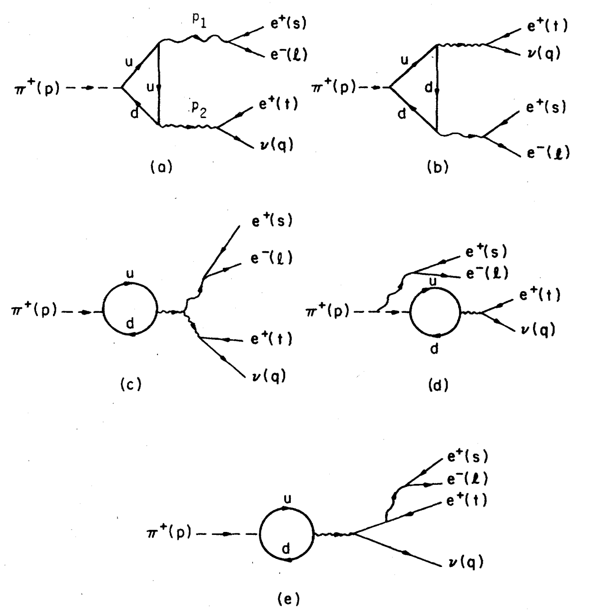

FIG. 1. The Feynman diagrams that contribute to the $\pi^+\to e^+e^+e^-\nu$ decay within the quark model. It is understood that their antisymmetrized parts due to e^+e^+ exchange should be taken into account.

$$b=\frac{\sqrt{2}n_c e_u g_S}{4\pi^2}\int_0^1 dx\int_0^{1-x}dy\frac{2(m_d-m_u)xy-(m_d+m_u)x+m_u}{\mathscr{D}}-(e_u\to e_d, m_u\leftrightarrow m_d)\ , \tag{8}$$

$$c=\frac{\sqrt{2}n_c e_u g_S}{4\pi^2}\int_0^1 dx\int_0^{1-x}dy\frac{-2(m_d-m_u)(y^2+xy)+2m_dy+(m_d+m_u)x-m_u}{\mathscr{D}}-(e_u\to e_d, m_u\leftrightarrow m_d)\ . \tag{9}$$

In the on-shell limit for the photon, we have

$$a=-\frac{\sqrt{2}n_c e_u g_S}{4\pi^2}[m_u\mathscr{I}_0+(m_d-m_u)\mathscr{I}_1]+(e_u\to e_d, m_u\leftrightarrow m_d)\ , \tag{10}$$

$$b=-\frac{\sqrt{2}n_c e_u g_S}{4\pi^2}\left[m_u(\mathscr{I}_0-2\mathscr{I}_1)-(m_d-m_u)\left[\mathscr{I}_1+\frac{1}{p\cdot k}[(p-k)^2(\mathscr{I}_1-\mathscr{I}_2)-(m_d{}^2-m_u{}^2)\mathscr{I}_1-m_u{}^2\mathscr{I}_0+\tfrac{1}{2}]\right]\right]$$

$$+(e_u\to e_d, m_u\leftrightarrow m_d)\ , \tag{11}$$

where

$$\mathscr{I}_i=\frac{1}{2p\cdot k}\int_0^1 dx\, x^{i-1}\ln\frac{(p-k)^2x(1-x)-(m_d{}^2-m_u{}^2)x-m_u{}^2}{p^2x(1-x)-(m_d{}^2-m_u{}^2)x-m_u{}^2}\ ,\quad k\equiv p_1\ , \tag{12}$$

which agree with the result given in Ref. 14, except that we have a relative sign difference between a and b from theirs after we have taken care of the convention difference. This results in a sign difference in the prediction of the γ parameter.

From Eqs. (6) and (8), we obtain $\gamma\equiv b/a$ as a function of m_u and m_d. As could be seen from the expressions, both form factors a and b are essentially proportional to m_u and m_d, therefore, when we take the ratio b/a for γ, it is basically sensitive to m_d/m_u only. The statement is more obvious when we look at the soft-pion limit. The value of γ as a function of $R\equiv m_d/m_u$ is shown in Fig. 2. When $m_d=m_u$, we have $\gamma=1$. As mentioned above, our result has an opposite sign from that given in Ref. 14, but it agrees with the number given in Ref. 15. According to our result, γ decreases from 1 to 0.6 if m_d/m_u increases

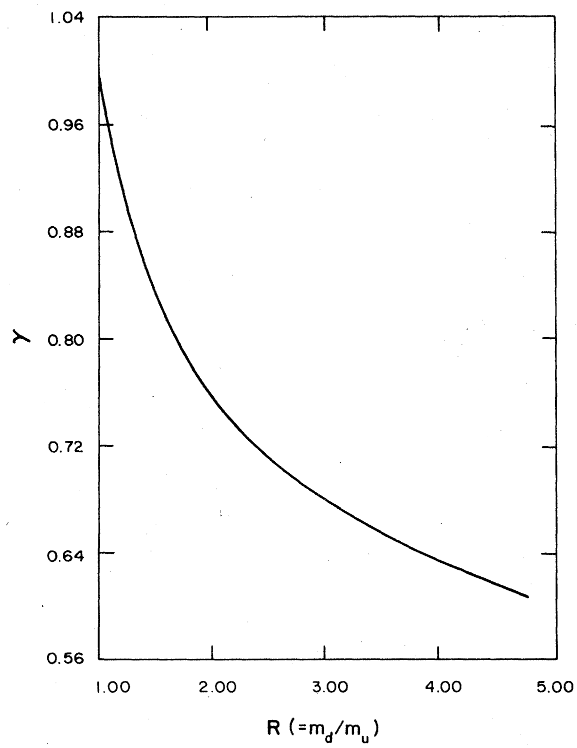

FIG. 2. The γ parameter predicted within the quark model as a function of R, the ratio between the d- and u-quark masses.

from 1 to the values near 5. Though when m_d/m_u gets large enough the parameter γ predicted here approaches the positive one of the observed values; the meaning of this agreement, however, is ambiguous. There seems no good justification either theoretically or experimentally concerning why m_d/m_u should be greater than the order of 5. Actually, we have another reason to believe that this prediction has not taken into account all the important contributions of QCD because the gluonic corrections are apparently very important due to the strong coupling involved in this low-energy process. To take care of this matter we have to know how to deal with the nonperturbative phenomena. This is of course a difficult issue that has stood in the way since the beginning of QCD.

It may be wondered if we should reestimate the experimental numbers as was done in Ref. 14, because these results were obtained by explicitly using the Vaks-Ioffe relation, which breaks down in this model when $m_d \neq m_u$. However, because the meaning of the masses m_u and m_d we use here is not yet clear (also, no indication of any severe violation of CVC has been observed), we feel that the numbers given by these experiments should be close to reality. Though we are not definite if the masses used here have properly summarized the nonperturbative effects, we believe that the difference between m_d and m_u should be small compared with m_d and m_u themselves, especially when they are regarded as the constituent masses. This can be considered by the conjecture that, starting from the current quark masses $m_u \approx 5$ MeV and $m_d \approx 8$ MeV, the QCD effects modify their values in a unique way in the sense that m_u and m_d are increased in exactly the same way because the color interaction should, in principle, make no distinction with respect to the different components of the quark isodoublet. The careful renormalization-group analysis by Chang and Chang appears to agree with this conjecture.[19] Hence we believe that whether the masses here are the constituent mass (~ 350 MeV) or not, the value $(m_d - m_u)/m_u$ should be small as long as $m_u + m_d > m_\pi$. Moreover, Weinberg[20] has established a theorem, which states that no isospin violation is expected in any purely pionic low-energy matrix element regardless what the u-d-quark mass difference is. We regard, therefore, that CVC is approximately good in the pion decays studied here.[21]

Similarly, we plot the value of $\xi \equiv c/a$ as a function of $R \equiv m_d/m_u$. As can be seen from Fig. 3, ξ increases as m_d/m_u increases, in contrast to the situation of γ. When $m_d = m_u$, $\xi = 1$, which is very close to γ. In order to compare with the results from the nucleon-loop approximation, we shall concentrate on the situation where $m_d = m_u \equiv m_q$, i.e., CVC is good, and take the soft-pion limit of the complete expressions for form factors a, b, and c. In this case, they become

$$a = -\frac{\sqrt{2} n_c g_S}{8\pi^2 m_q}(e_u + e_d) , \tag{13}$$

$$b = -\frac{\sqrt{2} n_c g_S}{24\pi^2 m_q}(e_u - e_d) , \tag{14}$$

$$c = -\frac{\sqrt{2} n_c g_S}{24\pi^2 m_q}(e_u - e_d) . \tag{15}$$

After applying the Goldberger-Treiman relation, they turn out as

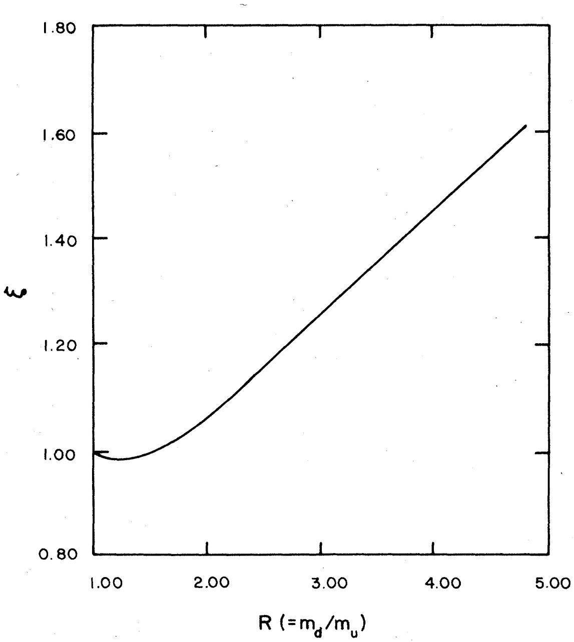

FIG. 3. The ξ parameter predicted within the quark model as a function of R, the ratio between the d- and u-quark masses.

$$a=-\frac{\sqrt{2}}{8\pi^2}\frac{e_p}{f_\pi}\,, \tag{16}$$

$$b=-\frac{\sqrt{2}}{8\pi^2}\frac{e_p}{f_\pi}\,, \tag{17}$$

$$c=-\frac{\sqrt{2}}{8\pi^2}\frac{e_p}{f_\pi}\,, \tag{18}$$

where we have used the quantum numbers $e_u=\frac{2}{3}e_p$, $e_d=-\frac{1}{3}e_p$, and $n_c=3$. Clearly, from the above results, we also obtain directly $\gamma=\xi=1$, as given earlier from Figs. 2 and 3.

The results from the nucleon-loop approximation, on the other hand, read

$$a=-\frac{\sqrt{2}}{8\pi^2}\frac{e_p}{f_\pi}\,, \tag{19}$$

$$b=-\frac{\sqrt{2}}{24\pi^2}\frac{e_p}{f_\pi}\,, \tag{20}$$

$$c=-\frac{\sqrt{2}}{24\pi^2}\frac{e_p}{f_\pi}\,. \tag{21}$$

As could be compared from Eqs. (16) and (19), for the form factor a, the results are in agreement with each other from both the quark model and the nucleon-loop approach as is the case in $\pi^0\to2\gamma$ decay due to the same anomaly. However, regarding form factors b and c, the quark model and the nucleon approximation give different answers. This is because, unlike the case of form factor a and π^0 decay, there does not exist any Adler-Bardeen type of theorem that guarantees the absence of the higher-order corrections besides the fact that there is not any anomaly to start with in this case to provide unique predictions for these two form factors.

In the nucleon-loop approximation, we see that $\gamma\equiv b/a=\frac{1}{3}=0.33$, which was already given in Ref. 22. Similarly, we find $\xi\equiv c/a=\frac{1}{3}=0.33$ also. As mentioned in the Introduction, the prediction of γ in this particular approximation agrees quite well with the positive one of the observed values. The parameter ξ has not yet been measured. Here, the value of ξ predicted is also the same as γ. If the future experiments do not obtain this particular number, it would imply some inconsistency within the nucleon-loop approach. Because in both the quark model and the nucleon approximation we have used here, one important feature relating to the pion physics, namely, the Nambu-Goldstone nature, is not respected, we therefore cannot regard the above predictions to be well justified even some are close to the experimental values. In the next section, we shall calculate these parameters within the framework of the σ model which does respect the chiral symmetry.

III. THE σ MODEL

Though the numerical simulations in the study of the lattice gauge theory has indicated it to be the case, no analytical proof has ever been given to the conjecture that the chiral symmetry possessed by the original QCD Lagrangian is spontaneously broken. For several reasons it has been a general belief that this conjecture is true. First of all, the existence of the light hadronic triplet, the pions, has indicated that they are very likely the Goldstone bosons associated with this symmetry breaking. After all, the vector current is well conserved, and no light scalar with the mass scale of the pion has ever been observed, in contrast to the nonconserved axial-vector current and an associated light pseudoscalar, the pion triplet. The success of the low-energy theorems applied to a variety of physical processes since the 1960s has beautifully confirmed this belief. On the other hand, the standard electroweak theory of Glashow-Salam-Weinberg, which has been proven to be quite successful also up to this moment, requires that the fermions should be massless in the original Lagrangian. This gives further support for believing that the original QCD Lagrangian is chiral-invariant. Though it is not well understood concerning the detailed mechanism of the symmetry breaking, the σ model,[16] nonetheless, is believed to summarize most of the important features associated with this particular perspective concerning QCD at low energies. This model satisfies current algebras and PCAC, and also predicts many important low-energy results which are in good agreement with the experiments.

Hence, here we shall present the results for the γ and ξ parameters predicted within this particular model. The parameter γ has been obtained in Ref. 12; we concentrate at the prediction and the meaning of the parameter ξ. The σ-π-loop contribution to the structure-dependent part of this decay process is given by the set of Feynman diagrams as shown in Fig. 4. The contribution from the nu-

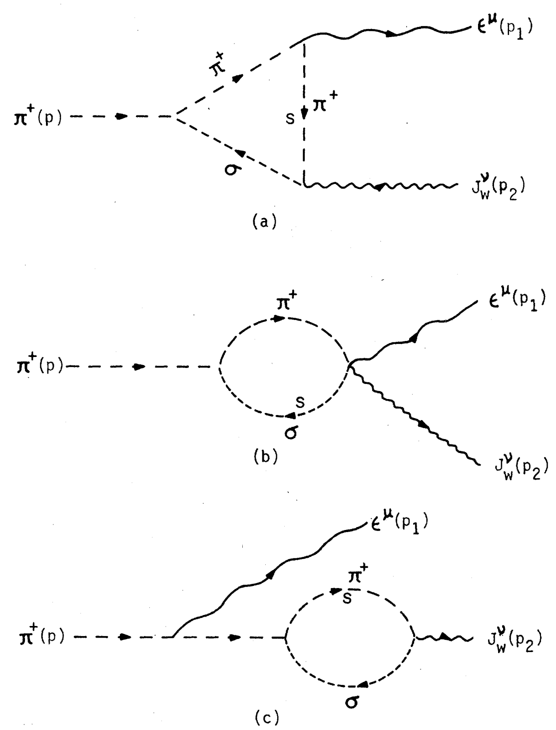

FIG. 4. The additional σ-π-loop contributions in the σ model to the general radiative pion decay $\pi^+\to e^+\nu\gamma$ where the photon may be off-shell.

cleon loop is already given in the last section. The σ-π loops in Fig. 4 do not give any contribution to the vector form factor a as pointed out in Ref. 14. However, they contribute to form factors b and c significantly. They also give the one-loop corrections to the Ward identity

$$p_1^\mu \mathcal{T}_{\mu\nu} = -ief_\pi p_\nu \ . \tag{22}$$

The detailed proof of this identity will be given in the Appendix. Now we write down directly the expressions for form factors b and c in the σ model, including both the contributions from the nucleon loops and the σ-π loops. They are, separately,

$$b = -\frac{\sqrt{2}e_p}{24\pi^2}\left[\frac{g_{\pi NN}}{m_N} + \frac{g_{\sigma\pi\pi}}{m_\sigma{}^2}\right] \simeq 0 \ , \tag{23}$$

$$\begin{aligned} c &= -\frac{\sqrt{2}e_p}{24\pi^2}\left[\frac{g_{\pi NN}}{m_N} - \frac{g_{\sigma\pi\pi}}{m_\sigma{}^2} + \frac{3}{2}\frac{g_{\sigma\pi\pi}}{m_\sigma{}^2}\left[\frac{17}{18} - \frac{2}{3}\ln\frac{m_\sigma}{m_\pi}\right]\right] \\ &= -\frac{\sqrt{2}e_p}{24\pi^2 f_\pi}\left[\frac{7}{12} + \ln\frac{m_\sigma}{m_\pi}\right] \ , \end{aligned} \tag{24}$$

where, as could be seen, b is independent of m_π, while c contains m_σ/m_π. Note that the above result for c is obtained under the assumption that m_σ is far larger than m_π, e.g., $m_\sigma = 5m_\pi$. The parameters γ and ξ now turn out as

$$\gamma = 0 \ , \tag{25}$$

$$\xi = \frac{1}{3}\left[\frac{7}{12} + \ln\frac{m_\sigma}{m_\pi}\right] . \tag{26}$$

Apparently, ξ does contain important information concerning the pattern of chiral-symmetry breaking through its dependence upon the explicit-breaking parameter m_σ/m_π. If we choose m_σ to be 700 MeV, the parameter $\xi = \frac{2}{3}$.

IV. SUMMARY AND DISCUSSION

In the previous sections we have calculated the γ and ξ parameters in the general radiative pion decay $\pi^+ \to e^+ \nu\gamma$ from the relativistic quark model, the nucleon-loop approximation, and the σ model. In the soft-pion limit, we obtain $\gamma = \xi = 1$ for the quark model, in contrast to the result $\gamma = \xi = \frac{1}{3}$ from the nucleon-loop approximation. The prediction from the σ model gives $\gamma = 0$, $\xi = \frac{2}{3}$ for $m_\sigma = 700$ MeV, which is quite different from the previous two cases not only numerically but also qualitatively in the sense that $\gamma \neq \xi$ in general. In the case of the quark model, the large deviation from the observed values for γ is quite conceivable because we have neglected the gluonic corrections, which are apparently very important in low-energy processes such as this, as we have mentioned. In addition, the high-energy behavior should also be modified for the quark propagator, for which we have used a free propagator. Furthermore, we are reminded that the effects due to spin-1 mesons such as ρ and A_1 are not yet properly included in this model. Goldman and Wilson[23] has taken this into account by changing the denominator of relevant meson propagators through a phenomenological fit within the static quark model. In the relativistic quark model here as specified by the Lagrangians in Eqs. (1) and (2) in Sec. II, it is not clear to us at this moment how a proper modification along the line given by Goldman and Wilson could be done in a consistent way. This is currently under investigation.

If ξ that will be measured in the future is quite different from the value of γ, the σ-model prediction is probably favored. As can be seen from Eq. (26), the parameter ξ obtained here involves a mass singularity in m_π associated with the infrared property of a spontaneously broken symmetry as already discussed by Li and Pagels.[24] In conclusion, we feel that a careful measurement of this parameter is very crucial to our understanding concerning the dynamics inside the pion.

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APPENDIX

To treat carefully the additional σ-π-loop contributions to the γ and ξ parameters within the σ model, we shall prove the Ward identity (22) for the set of diagrams shown in Fig. 4. Dimensional regularization is employed in this calculation.

We first write down the three-point function (though only two Lorentz indices appear) in the following way:

$$\mathcal{T}_{\mu\nu} = \mathcal{T}^{a}_{\mu\nu} + \mathcal{T}^{b}_{\mu\nu} + \mathcal{T}^{c}_{\mu\nu} \tag{A1}$$

where $\mathcal{T}^{a}_{\mu\nu}$, $\mathcal{T}^{b}_{\mu\nu}$, and $\mathcal{T}^{c}_{\mu\nu}$ are the contributions from diagrams (a), (b), and (c), respectively, in Fig. 4. Explicitly,

$$\mathcal{T}^{a}_{\mu\nu} = \frac{e g_{\sigma\pi\pi}}{(2\pi)^n}\int d^n s \frac{(2s_\mu + p_{1\mu})(2s_\nu - p_{2\nu})}{[(s-p_2)^2 - m_\sigma{}^2](s^2 - m_\pi{}^2)[(s+p_1)^2 - m_\pi{}^2]} \tag{A2}$$

which is logarithmically divergent by itself,

$$\mathscr{T}^{b}_{\mu\nu}=\frac{eg_{\sigma\pi\pi}}{(2\pi)^n}\int d^ns\frac{g_{\mu\nu}}{(s^2-m_\sigma{}^2)[(s+p)^2-m_\pi{}^2]} \tag{A3}$$

which is also logarithmically divergent, and

$$\mathscr{T}^{c}_{\mu\nu}=\frac{eg_{\sigma\pi\pi}}{(2\pi)^n}\frac{p_\mu+p_{2\mu}}{p_2{}^2-m_\pi{}^2}\times\int d^ns\frac{2s_\nu-p_{2\nu}}{[(s-p_2)^2-m_\sigma{}^2](s^2-m_\pi{}^2)} \tag{A4}$$

which appears divergent naively; nonetheless, it turns out to be finite after the integration is performed, as will be pointed out later. Integrating over the loop momentum s, we can rewrite (A1) in the form

$$\mathscr{T}_{\mu\nu}=C_{11}p_{1\mu}p_{1\nu}+C_{12}p_{1\mu}p_{2\nu}+C_{21}p_{2\mu}p_{1\nu}+C_{22}p_{2\mu}p_{2\nu}+C_gg_{\mu\nu}\ , \tag{A5}$$

where

$$C_{11}=\frac{i\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}eg_{\sigma\pi\pi}\int_0^1dx\int_0^{1-x}dy\frac{4y^2-2y}{\mathscr{D}_\sigma{}^{3-n/2}}\ , \tag{A6}$$

$$C_{12}=\frac{i\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}eg_{\sigma\pi\pi}\left[\int_0^1dx\int_0^{1-x}dy\frac{2x+2y-4xy-1}{\mathscr{D}_\sigma{}^{3-n/2}}+\frac{1}{2-n/2}\frac{1}{p_2{}^2-m_\pi{}^2}\int_0^1dx\frac{2x-1}{[p_2{}^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2]^{2-n/2}}\right]\ , \tag{A7}$$

$$C_{21}=\frac{i\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}eg_{\sigma\pi\pi}\int_0^1dx\int_0^{1-x}dy\frac{-4xy}{\mathscr{D}_\sigma{}^{3-n/2}}\ , \tag{A8}$$

$$C_{22}=\frac{i\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}eg_{\sigma\pi\pi}\left[\int_0^1dx\int_0^{1-x}dy\frac{4x^2-2x}{\mathscr{D}_\sigma{}^{3-n/2}}+\frac{1}{2-n/2}\frac{1}{p_2{}^2-m_\pi{}^2}\int_0^1dx\frac{2(2x-1)}{[p_2{}^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2]^{2-n/2}}\right]\ , \tag{A9}$$

$$C_g=\frac{i\pi^{n/2}\Gamma(2-n/2)}{(2\pi)^n}eg_{\sigma\pi\pi}\left[\int_0^1dx\int_0^{1-x}dy\frac{2}{\mathscr{D}_\sigma{}^{2-n/2}}-\int_0^1dx\frac{1}{[p^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2]^{2-n/2}}\right]\ , \tag{A10}$$

with

$$\mathscr{D}_\sigma\equiv p_1{}^2y(1-y)+p_2{}^2x(1-x)+2p_1\cdot p_2xy-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2\ . \tag{A11}$$

In C_{12} and C_{22}, there appear poles proportional to $1/(2-n/2)$; however, they vanish identically because the integration over x gives zero. This is exactly the reason why diagram (c) is in fact finite. In C_g the two poles coming from diagram (a) and (b), respectively, cancel against each other. Therefore, the whole result is finite.

Next, we proceed to prove that the Ward identity (22) is satisfied. To see this, we multiply p_1^μ by (A5) and have

$$p_1^\mu\mathscr{T}_{\mu\nu}=-ie(Ap_{1\nu}+Bp_{2\nu})\ , \tag{A12}$$

where

$$A=-\frac{\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}g_{\sigma\pi\pi}(a_1+a_2+a_3+a_4) \tag{A13}$$

with a_1, a_2, a_3, and a_4, after the integration over y, given by

$$a_1=\int_0^1 dx\left[-4(1-x)+\frac{2\beta}{\lambda}\ln\left[\frac{\alpha+\beta(1-x)+\lambda(1-x)^2}{\alpha}\right]-\frac{2(\beta^2-2\alpha\lambda)}{\lambda\sqrt{q}}\ln\frac{[2\lambda(1-x)+\beta-\sqrt{q}\,](\beta+\sqrt{q}\,)}{[2\lambda(1-x)+\beta+\sqrt{q}\,](\beta-\sqrt{q}\,)}\right],$$

$$a_2=\int_0^1 dx\left[-\frac{\beta}{\lambda}\ln\left[\frac{\alpha+\beta(1-x)+\lambda(1-x)^2}{\alpha}\right]+\frac{\beta^2}{\lambda\sqrt{q}}\ln\frac{[2\lambda(1-x)+\beta-\sqrt{q}\,](\beta+\sqrt{q}\,)}{[2\lambda(1-x)+\beta+\sqrt{q}\,](\beta-\sqrt{q}\,)}\right],$$

$$a_3=\int_0^1 dx\left[4(1-x)-\frac{\beta}{\lambda}\ln\left[\frac{\alpha+\beta(1-x)+\lambda(1-x)^2}{\alpha}\right]-2(1-x)\ln[\alpha+\beta(1-x)+\lambda(1-x)^2]\right.$$
$$\left.+\frac{\sqrt{q}}{\lambda}\ln\frac{[2\lambda(1-x)+\beta-\sqrt{q}\,](\beta+\sqrt{q}\,)}{[2\lambda(1-x)+\beta+\sqrt{q}\,](\beta-\sqrt{q}\,)}\right],$$

$$a_4=\int_0^1 dx\,\ln[\alpha+\beta(1-x)+\lambda(1-x)^2]\,. \tag{A14}$$

In the above expressions, we have introduced for convenience the set of constants

$$\alpha\equiv p_2{}^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2,\quad \beta\equiv p_1{}^2+2p_1\cdot p_2x,\quad \lambda\equiv -p_1{}^2,\quad q\equiv\beta^2-4\alpha\lambda\,.$$

As can be seen, proper cancellations do occur among a_i's to give a result for A in a simple form:

$$A=\frac{\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}g_{\sigma\pi\pi}\int_0^1 dx(1-2x)\ln\left[\frac{p^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2}{m_\sigma{}^2}\right]. \tag{A15}$$

B can be similarly obtained as

$$B=\frac{\pi^{n/2}\Gamma(3-n/2)}{(2\pi)^n}g_{\sigma\pi\pi}\int_0^1 dx(1-2x)\ln\left[\frac{p^2x(1-x)-(m_\sigma{}^2-m_\pi{}^2)x-m_\pi{}^2}{m_\sigma{}^2}\right]$$
$$=A\,. \tag{A16}$$

Hence (A12) can be written as

$$p_1^\mu\mathscr{T}_{\mu\nu}=-ieA(p_{1\nu}+p_{2\nu})=-ief_\pi^{(1)}p_\nu\,, \tag{A17}$$

which satisfies the Ward identity (22) with

$$f_\pi^{(1)}\equiv A \tag{A18}$$

as the one-loop correction to the pion decay constant f_π.

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