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From Regge trajectories to a static sine-Gordon-type equation

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We present a model of mesons and baryons as colorless bound states of *effective* colored quarks. Their dynamics is in both cases that of a single string, and as a result light mesons and baryons are shown to lie on parallel Regge trajectories. A baryon appears as a soliton on the string (with a favored quark-diquark structure), the underlying equation of which reduces, in a suitable approximation, to a static sine-Gordon-type equation. A baryonic number differentiating between baryons and mesons is defined.

In this article we will generalize the preliminary Abelian model introduced in Ref. 1 to an SU(3) non-Abelian one allowing a unified description of mesons and baryons. Thanks to the key structure of our preliminary model, that is, to the appearance of effective quarks as "condensed" magnetic charges, we will again be able to match a single-string dynamics with the usual quark description of mesons and baryons. That is, a meson will be made of an effective-quark—effective-antiquark pair and a baryon will be made of three effective quarks which in turn will dynamically cluster to a quark-diquark pair. As a result, the light mesons and baryons will be shown to lie on linear and parallel Regge trajectories, and the origin of the translation of the baryonic trajectories relatively to the mesonic ones will be identified. Moreover, the coupling between the effective quarks and the SU(3) Yang-Mills field will, as in Ref. 1, account for the necessary short-distance interaction.

This is not the first attempt to describe baryons in the framework of string models. Several models having been constructed up to now;[2-6] however, none of these models is directly related to our work although we may share some results. In particular, it has been known for a long time that linear baryons lead to Regge trajectories with the same slope as mesons.

This paper is organized as follows:

In the first section we show how to build a meson and a baryon as colorless composites of effective colored quarks and antiquarks. At this stage, the discrimination between baryons and mesons will be achieved by using a nondynamical pseudoscalar and SU(3)-valued field ρ defined on the world sheet of the string. In so doing, we will suggest a one-soliton nature of baryons as opposed to a zero-soliton nature of mesons and define a baryonic number (for a first appearance of this idea, see Ref. 7). Unfortunately, the light-baryon Regge trajectories will appear, in that scheme, to lie slightly below the light-meson ones in contradiction with experiment and thus obviously asking for a cure. We will end this first section in showing that a very natural cure to this problem is to promote the nondynamical field ρ to a dynamical one.

The second section will be devoted to the introduction and the study of the dynamics of this field and to its consequences. To begin with, we will show the field ρ possesses a single degree of freedom θ and will introduce a canonical kinetic term for this last field as the only additional piece to the action of Sec. I. Next we will look at the equation of motion for θ and show it reduces in a suitable approximation to a static sine-Gordon-type equation so that the mesons will be recovered as the trivial solution of this equation and the baryons will be described by the one-soliton solution of the same equation. These last will be seen to exhibit a quark-diquark structure. Then we will explain how the θ contribution to energy acts to translate the baryon trajectories in the right direction. Unfortunately, the equation for θ being far too complicated to be solved analytically, some of our last proofs will remain qualitative. Nevertheless, these heuristic reasonings are, in our opinion, convincing. We obviously intend to publish a complete numerical treatment of the equation for θ as soon as available. The paper will end with our conclusions. Throughout we use the same notation as in Ref. 1.

I. BARYON AND MESON STRUCTURE

To begin with, the fundamental fields of our Lagrangian will be an SU(3) Yang-Mills field $A_\mu^a(x)$, $a \in \{1,\ldots,8\}$ and a vector field $y^\mu(\tau^0,\tau^1)$. From this last field, it will be convenient to define the SU(3)-valued tensor field

$$G^{\mu\nu}(x)=\int_{\text{sheet}}\frac{\partial(y^\mu,y^\nu)}{\partial(\tau^0,\tau^1)}\delta^4(x-y)\rho(\tau^0,\tau^1)d\tau^0 d\tau^1 ,$$

where $\rho(\tau^0,\tau^1)$ is a mapping from the parameter space $]-\infty,+\infty[x[0,1]$ to the Lie algebra su(3) of SU(3) which will be made precise as we go along. Depending on what $\rho(\tau^0,\tau^1)$ we choose we will describe either a meson or a baryon.

An analogous tensor was introduced by Eguchi.[8] Under a local SU(3) transformation $g(x)$ the fields introduced transform as follows.

We define

$$A_\mu(x)=A_\mu^a(x)T^a ,$$

where T^a are the eight Hermitian generators of SU(3): we use $T^a = \lambda^a/2$ then

$$^gA_\mu(x) = g(x)A_\mu(x)g^{-1}(x) + \frac{i}{e}g(x)\partial_\mu g^{-1}(x) ,$$

e is the SU(3) coupling constant, and

$$^g\rho(\tau^0,\tau^1) = g[y^\mu(\tau^0,\tau^1)]\rho(\tau^0,\tau^1)g^{-1}[y^\mu(\tau^0,\tau^1)] \tag{1}$$

such that

$$^gG_{\mu\nu}(x) = g(x)G_{\mu\nu}(x)g^{-1}(x) .$$

Note the implicit dependence of a general ρ in $y^\mu(\tau^0,\tau^1)$. The action for these fields then can be written as a trivial generalization of the action in Ref. 1:

$$S = \eta \int \sqrt{-g}\, d\tau^0 d\tau^1 + m \int_{l_i} ds_i + m \int_{l_f} ds_f + \frac{1}{8\pi} \int \mathrm{Tr}[F_{\mu\nu}(x)F^{\mu\nu}(x)]d^4x + M_0 \int \mathrm{Tr}(\widetilde{G}_{\mu\nu}F^{\mu\nu})d^4x .$$

We recall that the two terms $m\int_{l_i} ds_i + m\int_{l_f} ds_f$ are just an ansatz to get, through the limit $m \to 0$, all the kinetic contributions for the string as is explained in Ref. 1. By the way, note that in spite of the fact we will describe baryons together with mesons we have introduced only two masses which further stresses their ansatz character. This action is clearly invariant under gauge transformations whatever are M_0 and e and also under these local reparametrizations which map $(\tau^0,0) \to (\tilde{\tau}^0,0)$ and $(\tau^0,1) \to (\tilde{\tau}^0,1)$. In order for it to be also invariant under the complete Lorentz group, we have to require that $\rho(\tau^0,\tau^1)$ changes sign under parity and time reversal so that $G_{\mu\nu}(x)$ is a pseudoscalar tensor field and M_0 a usual scalar coupling constant. Note that $\rho(\tau^0,\tau^1)$ is not considered as a dynamical variable at that stage.

In what follows we will further impose the gauge-invariant condition

$$\mathrm{Tr}[\rho^2(\tau^0,\tau^1)] = N \quad \forall(\tau^0,\tau^1) \in]-\infty,+\infty[x[0,1] , \tag{2}$$

where N is an arbitrary constant. An early motivation for it was to realize an analogy with the previous Abelian model. Indeed, it is well known (see Ref. 9 for an early proof) that in the Abelian case $\int G_{\mu\nu}G^{\mu\nu}d^4x$ is formally proportional to $\int \sqrt{-g}\; d\tau^0 d\tau^1$ which appears in the action of the Abelian model; moreover, it is easy to prove that, similarly,

$$\int \mathrm{Tr}[G_{\mu\nu}(x)G\mu\nu(x)]d^4x$$

is formally proportional to

$$\int \sqrt{-g}\; \mathrm{Tr}(\rho^2)d\tau^0 d\tau^1$$

so that, as it is also $\int \sqrt{-g}\; d\tau^0 d\tau^1$ which appears in the action of the non-Abelian model, we get a complete analogy if we impose $\mathrm{Tr}\rho^2 = \mathrm{const}$.

A more pragmatic reason for this condition is that it leaves only one degree of freedom in $\rho(\tau^0,\tau^1)$ (see below), which will be enough for our purposes. Then we follow the same steps as in Ref. 1 in order to choose the distributions allowing the usual interpretation of mesons and baryons as multiquark bound states. That is, we first compute the $A^a_\mu(x)$ equation of motion which can be written

$$D_\mu F^{\mu\nu} = -4\pi M_0 D_\mu \widetilde{G}^{\mu\nu} , \tag{3}$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ie[A^\mu, A^\nu] ,$$

$$D_\mu(F^{\mu\nu} \text{ or } \widetilde{G}^{\mu\nu}) = (\partial_\mu - ie[A_\mu, \cdot])(F^{\mu\nu} \text{ or } \widetilde{G}^{\mu\nu}) .$$

To get more insight in this equation we can, using (1), specialize to a gauge where $\rho(\tau^0,\tau^1)$ is a diagonal matrix thus developing on the Cartan subalgebra generated by T^3 and T^8. Note that $\rho(\tau^0,\tau^1)$ is not completely fixed by this condition as $^g\rho(\tau^0,\tau^1)$ is as good as $\rho(\tau^0,\tau^1)$ provided $g(x)$ belongs to the Weyl group of SU(3). Then assuming the Abelian ansatz for $A_\mu(x)$ in this gauge, that is,

$$a_\mu(x) = a^3_\mu(x)T^3 + a^8_\mu(x)T^8 ,$$

Eq. (3) further simplifies to

$$\partial_\mu F^{\mu\nu} = -4\pi M_0 \partial_\mu \widetilde{G}^{\mu\nu} ,$$

where

$$F^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu .$$

Such an Abelian ansatz is *a priori* a very restrictive one; however, a work of Patkos[10] suggests that it may well be sufficient to catch most of the correct physics of our model. What Patkos proved is that for a general class of Higgs models which allow non-Abelian flux-tube configurations it is always possible to find a gauge where these solutions take an Abelian form thus showing that there exist situations not very far from ours where an Abelian ansatz exhausts the entire physics. This fact gives us the hope that our Abelian ansatz may indeed be relevant especially if the physics to be deduced from it is consistent with experiment as it will appear to be. Thus, we proceed from the Abelian form of the Eqs. (3).

We are now in a good position to choose $\rho(\tau^0,\tau^1)$ distributions. Indeed, we remember from Ref. 1 that the form of $\partial_\mu G^{\mu\nu}$ was the key toward effective Abelian quarks so that we will look for ρ's implying, respectively,

$$(\partial_\mu G^{\mu\nu})_M \propto d^\nu_{q_i} + d^\nu_{\bar{q}_i}, \quad i = V \text{ or } R \text{ or } B$$

and

$$(\partial_\mu G^{\mu\nu})_B \propto d^\nu_{q_R} + d^\nu_{q_V} + d^\nu_{q_B} ,$$

where d^ν stands for the q_i dimensionless current density, namely,

$$d^\nu_i = \rho_i \int_{\substack{\text{world}\\ \text{line of } q_i}} d\tau^0 \frac{dy^\nu}{d\tau^0} \delta^4(x-y)$$

$\{i\} = \{V,R,B\}$ indicates the color of the effective quarks and

$$\rho_V=\tfrac{1}{2}T^3+\frac{1}{2\sqrt{3}}T^8\ ,$$

$$\rho_R=-\tfrac{1}{2}T^3+\frac{1}{2\sqrt{3}}T^8,\quad \rho_B=-\frac{1}{\sqrt{3}}T^8,\quad \rho_{\bar{i}}=-\rho_i\ .$$

The unique suitable solutions to these equations, up to a sign and up to the residual gauge transformations still possible for a diagonal ρ, are, for mesons (to appear as an effective $q\bar{q}$ pair),

$$\rho_M=\sqrt{2N}\,T^8 \tag{4}$$

which is visualized on Fig. 1, and, for baryons (to appear as three effective qs),

$$\begin{aligned}\rho_B(\tau^0,\tau^1)&=\sqrt{2N}\,T^8\ \text{ for } (\tau^0,\tau^1)\in\,]-\infty,+\infty[\times]0,\tau_d^1[\ ,\\ \rho_B(\tau^0,\tau^1)&=-P_1(\sqrt{2N}\,T^8)\\ &\qquad\text{for } (\tau^0,\tau^1)\in\,]-\infty,+\infty[\times]\tau_d^1,1[\ .\end{aligned} \tag{5}$$

See Fig. 2 for a well-chosen parametrization of the sheet. τ_d^1 will be specified later and the operator P_1 operates the permutation $123\to231$ on the eigenvalues of $(\sqrt{2N}\,T^8)$. The proof of this statement is straightforward from the explicit general expression of $\partial_\mu G^{\mu\nu}(x)$, i.e.,

$$\begin{aligned}\partial_\mu G^{\mu\nu}(x)=&\int d\tau^0\delta^4(x-y)\frac{\partial y^\nu}{\partial\tau^0}(\tau^0,1)\rho(\tau^0,1)\\ &-\int d\tau^0\delta^4(x-y)\frac{\partial y^\nu}{\partial\tau^0}(\tau^0,0)\rho(\tau^0,0)\\ &+\int d\tau^0 d\tau^1\delta^4(x-y)\\ &\quad\times\left[\frac{\partial y^\nu}{\partial\tau^1}\frac{\partial}{\partial\tau^0}\rho-\frac{\partial y^\nu}{\partial\tau^0}\frac{\partial}{\partial\tau^1}\rho\right]\ .\end{aligned} \tag{6}$$

It is easy to verify that residual gauge transformations transform the $B-\bar{B}$, $R-\bar{R}$ and $V-\bar{V}$ mesons among themselves and the $B-V-R$ baryon to any other color arrangement. The charge-conjugate states are obtained through $\rho_M\to-\rho_M$ and $\rho_B\to-\rho_B$.

The above solutions for the ρ matrix strongly suggest an analogy between baryon and meson on one side and the configurations of two-dimensional field theory with and without soliton on the other side. Such a solitonic interpretation of baryons, which was first introduced by Skyrme,[7] further mentioned by Goldstone and Jackiw,[11] and which has recently received a regain of interest in the context of bag models,[12] will naturally strengthen in the next section where ρ will be given a dynamics the necessity of which we will now demonstrate. As the ρ distributions (4) and (5) cannot be the definitive ones, we will be content in looking at the classical equations of motion to derive the static effective potential between the ends of the string which, in turn, will imply the light-baryon Regge trajectories to be linear and parallel to the mesonic ones but also to be slightly below them in contradiction with experiment, so that something must be modified in the model.

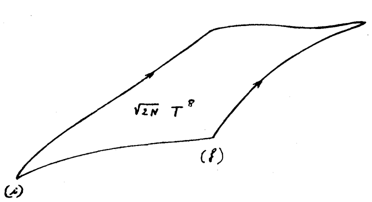

FIG. 1. Visualization of the ρ distribution on the world sheet swept out in space-time by the string in the case of a meson.

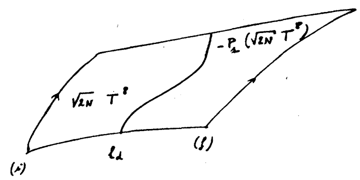

FIG. 2. Same as Fig. 1 in the case of a baryon.

It is easy to verify, proceeding as in Ref. 1, that the rigid-string motion[13] is a solution in both the meson and the baryon cases, provided we choose the intermediate quark to stand in the middle of the rotating rigid string (that fixes τ_d^1 once the parametrization of the string is chosen). For that motion the static effective potentials between the ends can be written, respectively, as

$$\begin{aligned}V_M&=\eta r-\frac{(M_0\sqrt{2N}\,)^2}{r}\ ,\\ V_B&=\eta r-\frac{5}{2}\frac{(M_0\sqrt{2N}\,)^2}{r}\ .\end{aligned} \tag{7}$$

Then, plugging them into the semirelativistic Schrödinger equation for the ends immediately leads[14] to the above-mentioned result as $M_0\sqrt{2N}$ should be nonzero.

A way out of this contradiction is simply to allow the motion of the intermediate quark along the string. Indeed, we know the zero-point energy of a relativistic particle in a one-dimensional box amounts to π/L (L: box length) so that, taking also into account the zero-point energy coming from the fluctuations of the string (i.e., $\pi/2r$ as suggested by both experiment[14] and the Neveu-Schwarz model[15]) we are led to

$$\begin{aligned}V_M&=\eta r-\frac{\pi}{2r}-\frac{(M_0\sqrt{2N}\,)^2}{r}\ ,\\ V_B&=\eta r-\frac{\pi}{2r}-\frac{(M_0\sqrt{2N}\,)^2}{r}+\frac{[\pi-\frac{3}{2}(M_0\sqrt{2N}\,)^2]}{r}\ .\end{aligned}$$

Thus the leading baryon trajectory appears to be translated above the mesonic one by

$$\begin{aligned}&(\text{meson intercept})-(\text{baryon intercept})\\ &\qquad=\tfrac{1}{2}[\pi-\tfrac{3}{2}(M_0\sqrt{2N}\,)^2]=\tfrac{1}{2}(3.14-0.24)\simeq1.5\end{aligned}$$

if we use the value of $M_0\sqrt{2N}$ suggested in Ref. 14 in the case of $m_u=0.1$ GeV. Such a value is quite of the right order of magnitude as (ρ intercept)$-$(Δ intercept)$\simeq1$.

The aim of the remaining part of this paper is to implement a dynamics for the intermediate quark.

Before leaving this section let us define a baryon number differentiating between mesons and baryons. We pose

$$B=\frac{2\sqrt{3}}{(2N)^{3/2}}\int_C\left[\frac{\partial}{\partial\tau^0}(\mathrm{Tr}\rho^3)d\tau^0+\frac{\partial}{\partial\tau^1}(\mathrm{Tr}\rho^3)d\tau^1\right], \quad (8)$$

where C is a curve going from one edge of the sheet spanned in space-time by the string to the other. B is easily seen to be gauge invariant, reparametrization invariant, conservative, and to be 0 for a meson and 1 for a baryon.

II. THE MODEL

To allow the motion of the intermediate quark along the string the most natural idea is to promote the nondynamical field $\rho(\tau^0,\tau^1,y^\mu)$ to a dynamical one, in such a way its equation of motion admits meson-like and baryon-like solutions. Moreover, we must pay a close attention to the ρ contribution to the static potential in order to not spoil the linearity of the mesonic and baryonic Regge trajectories. This last requirement will in turn give us clues toward the right Lagrangian for ρ as we shall see below.

So let us address ourselves the problem of implementing a dynamics for the field ρ. First of all we notice that it possesses only a single degree of freedom. Indeed, once we choose the diagonal gauge the ρ field can be written as

$$\begin{aligned}\rho(\tau^0,\tau^1)&\\&=2\sqrt{N/6}\,\mathrm{diag}(\cos(\theta+\pi/3),\cos(\theta-\pi/3),\cos(\theta+\pi))\\&=\sqrt{2N}(\cos\theta T^8-\sin\theta T^3)\end{aligned} \quad (9)$$

as a result of the constraints $\mathrm{Tr}(\rho)=0$ and $\mathrm{Tr}(\rho^2)=N$, where $\theta(\tau^0,\tau^1)$ is a mapping from the parameter space to R.

Under the remaining gauge transformations (those which permutate the eigenvalues) $\theta(\tau^0,\tau^1)\to{}^g\theta(\tau^0,\tau^1)$ such that in all cases

$${}^g\theta(\tau^0,\tau^1)=\theta(\tau^0,\tau^1)[2\pi/3]$$

or

$${}^g\theta(\tau^0,\tau^1)=-\theta(\tau^0,\tau^1)[2\pi/3]\ .$$

Apart from these permutating transformations, $\theta(\tau^0,\tau^1)$ is considered as gauge invariant as should be the eigenvalues of a matrix transforming as in (1). Moreover, $\theta(\tau^0,\tau^1)$ is a scalar under the proper Lorentz group and becomes

$${}^{P,T}\theta(\tau^0,\tau^1)=\theta(\tau^0,\tau^1)+\pi$$

under P or T. We also note that

$$\mathrm{Tr}\rho^3=-\frac{N^{3/2}}{\sqrt{6}}\cos3\theta(\tau^0,\tau^1)$$

so that we should have

$$\sin3\theta(\tau^0,0)\partial_0\theta(\tau^0,0)=0=\sin3\theta(\tau^0,1)\partial_0\theta(\tau^0,1) \quad (10)$$

in order to insure the baryon number (8) conservation.

Then the dynamics of the ρ field is implemented through a *simple kinetic* term in the action (no potential for θ is needed, neither is it suitable for our present purpose as we shall see shortly)

$$S_\theta=-\int\frac{\sqrt{-g}}{2}\partial_\alpha\theta\partial^\alpha\theta d\tau^0d\tau^1$$

which is manifestly Lorentz, gauge and reparametrization invariant. In addition, we will require the following set of boundary conditions for the model to describe physical cases:

$$[\theta(\tau^0,0)=0 \text{ and } \theta(\tau^0,1)=0][\pi/3]$$

or (11)

$$[\theta(\tau^0,0)=0 \text{ and } \theta(\tau^0,1)=\pi/3][\pi/3]\ .$$

From (10) we see that these conditions insure the baryon-number conservation. In our opinion, these boundary conditions appear to be imposed by hands only because the model in its present state describes classical colorless objects and not quantum color singlets and as such should disappear in a quantum version. (Moreover our model of effective quarks with spin[16] suggests they may be intimately related to the boundary conditions which characterize the Ramond and the Neveu-Schwarz sectors of the supersymmetric string model.) Thus the complete action describing our model can be written as

$$\begin{aligned}S_T=\eta\int d\tau^0d\tau^1\sqrt{-|g|}+\frac{1}{8\pi}\int\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})d^4x\\-\int\sqrt{-|g|}\,\frac{\partial\alpha\theta\,\partial^d\theta}{2}d\tau^0d\tau^1\\+M_0\int\mathrm{Tr}[\widetilde{G}_{\mu\nu}(x)F^{\mu\nu}(x)]d^4x\ ,\end{aligned}$$

where the last term may also be written as

$$M_0\int d\tau^0d\tau^1\mathrm{Tr}[\rho(\tau^0,\tau^1)\widetilde{F}^{\mu\nu}(y(\tau^0,\tau^1))]\frac{\partial(y_\mu,y_\nu)}{\partial(\tau^0,\tau^1)}\ .$$

We are now going to show that this action describes both mesons and baryons respectively, as, the zero-soliton and the one-soliton solution of the equation of motion of the $\theta(\tau^0\tau^1)$ field and simultaneously preserves the linearity of both the light-meson and light-baryon Regge trajectories. We will also exhibit the origin of the relative position of these trajectories. So let us first report the coupled system of equations of motion for the fields A^μ, y^μ, and θ (we report here the self-interactions subtracted equations in the diagonal gauge, using the Abelian ansatz for A^μ) [$\partial_\mu G^{\mu\nu}$ is given in (6) and $\rho(\tau^0,\tau^1)$ is defined in (9)]

$$A^\mu:\begin{cases}\partial_\mu H^{\mu\nu}=0\Longleftrightarrow\widetilde{H}^{\mu\nu}=\partial^\mu C^\nu-\partial^\nu C^\mu,\ \ \partial_\mu C^\mu=0\\ \partial_\mu\widetilde{H}^{\mu\nu}=-4\pi M_0\partial_\mu G^{\mu\nu},\end{cases} \quad (12)$$

$$\begin{aligned}y^\mu:\ &\frac{\partial}{\partial\tau^\delta}(\sqrt{-g}\,u^\nu_\gamma T^{\gamma\delta})\\&-2M_0\mathrm{Tr}\left[\left[u_{\mu,0}\frac{\partial}{\partial\tau^1}\rho-u_{\mu,1}\frac{\partial}{\partial\tau^0}\rho\right]\widetilde{H}^{\mu\nu}\right]=0\ ,\end{aligned} \quad (13)$$

$$-\epsilon m \frac{\partial}{\partial \tau^0}\left[\frac{dy^\nu}{ds}\right]+2M_0 u_{\mu,0}\mathrm{Tr}(\rho\tilde{H}^{\mu\nu}) -\sqrt{-g}\,T^{\gamma 1}u^{\nu}\gamma\,|_{\,\mathrm{Edges}}=0\ , \tag{14}$$

where $T^{\alpha\beta}$ is minus the stress tensor taken with $A_\mu=0$, i.e.,

$$T^{\alpha\beta}=\left[\tfrac{1}{2}\partial_\gamma\theta\,\partial^\gamma\theta-\frac{\mu M_0{}^2}{8\pi}N\right]g^{\alpha\beta}-g^{\alpha\gamma}g^{\beta\delta}\partial_\gamma\theta\partial_\delta\theta\ , \tag{15}$$

and where $\epsilon m=-m$ for $\tau^1=0$ and $\epsilon m=+m$ for $\tau^1=1$.

θ: the variation $\delta\theta$ should verify $\delta\theta(\tau^0,0)=\delta\theta(\tau^0,1)=0$ as we require the specific boundary conditions (11) so that we get

$$\partial_\alpha(\sqrt{-g}\,g^{\alpha\beta}\partial_\beta\theta)+M_0\frac{\partial(y^\mu,y^\nu)}{\partial(\tau^0,\tau^1)}\mathrm{Tr}\left[\frac{\partial\rho}{\partial\theta}\tilde{H}_{\mu\nu}\right]=0\ . \tag{16}$$

Obviously, these equations are far too complicated to be solved exactly so that we will use the following strategy based on the experience acquired with the preliminary model of Ref. 1. We will concentrate on the nonrelativistic approximation of its solution and consider it leads to a relevant approximation to a static potential to be used in a semirelativistic equation for our system as it was the case in Ref. 1.

In such an approximation the coupled set of equations [(12)–(16)] admits a particular solution where the string is rigidly rotating, i.e., where

$$y^\mu=\begin{cases}\tau^0,\\ r(\tau^1-\overline{\tau})\cos\omega\tau^0,\\ r(\tau^1-\overline{\tau})\sin\omega\tau^0,\\ 0,\end{cases}$$

and where $\theta(\tau^0,\tau^1)$ is a static field satisfying (17):

$$\frac{d^2\theta}{d\tau^{12}}+(\sqrt{2N}M_0)^2\left[\frac{1}{(1-\tau^1)^2}\sin[\theta(\tau^1)-\theta(1)] +\frac{1}{(\tau^1)^2}\sin[\theta(\tau^1)-\theta(0)]-R\left[\int_0^1 d\sigma\frac{\tau^1-\sigma}{|\tau^1-\sigma|}\frac{d\theta}{d\sigma}\cos[\theta(\tau^1)-\theta(\sigma)]\right]\right]=0\ . \tag{17}$$

$\{\theta(1),\theta(0)\}$ should satisfy (11) in order to describe physical cases.

The parameters ω and τ will be fixed through the boundary conditions (14) in term of r, $\sqrt{2N}M_0$, η, and eventually m once a solution for $\theta(\tau^1)$ is chosen and the R operator in (17) is used to regulate the integral

$$\int_0^1 d\sigma\frac{\tau^1-\sigma}{|\tau^1-\sigma|^3}\frac{d\theta}{d\sigma}\cos[\theta(\tau^1)-\theta(\sigma)]$$

which is divergent in the general case of a nontrivial $\theta(\tau^1)$. Physically this divergence occurs because of the chromomagnetic flux characterized by θ is distributed in a three-dimensional space along an infinitely thin one-dimensional string thus leading to a singular field $\tilde{H}_{0i}(\tau^1)$ for any point inside the distribution itself. One way to define the R operation may be, for instance, to introduce a transverse extension for the string characterized by some $R_\perp$ so that

$$R\left[\int_0^1 d\sigma\frac{\tau^1-\sigma}{|\tau^1-\sigma|^3}\frac{d\theta}{d\sigma}\cos[\theta(\tau^1)-\theta(\sigma)]\right]=\frac{2}{R_\perp{}^2}\int_0^{R_\perp}r_\perp dr_\perp\int d\sigma\frac{\tau^1-\sigma}{\{[r_\perp{}^2+(\tau^1-\sigma)^2]^{1/2}\}^3}\frac{d\theta}{d\sigma}\cos[\theta(\tau^1)-\theta(\sigma)]$$
$$=\int d\sigma\frac{d\theta}{d\sigma}\frac{\cos[\theta(\tau^1)-\theta(\sigma)]2\,\mathrm{sign}(\tau^1-\sigma)}{\{|\tau^1-\sigma|+[R_\perp{}^2+(\tau^1-\sigma)^2]^{1/2}\}[R_\perp{}^2+(\tau^1-\sigma)^2]^{1/2}}\ .$$

To obtain any explicit solution of Eq. (17) will unavoidably require an involved numerical treatment because of the occurrence of the last integral term, numerical treatment which is not yet done; however, a lot of relevant information may be obtained without such a complete solution. Indeed, we notice first of all that $\theta(\tau^1)=\theta(1)=\theta(0)=0[\pi/3]$ is a trivial solution of (17) so that the mesons are obviously described by the action S_T. Secondly, we notice that if $\theta(\tau^1)$ is a nontrivial solution of (17) then $\phi(\tau^1)=\theta(1)+\theta(0)-\theta(1-\tau^1)$ is another nontrivial one satisfying the same boundary conditions. These two solutions are compared on Fig. 3. The appearance of this symmetry is quite natural from the results of the first section as once the ρ field becomes dynamical the

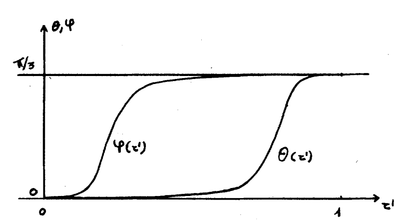

FIG. 3. A typical solution of Eq. (17): $\theta(\tau^1)$ is compared with the solution $\phi(\tau^1)=\theta(1)+\theta(0)-\theta(1-\tau^1)$ which follows by symmetry.

equilibrium of the intermediate quark in the middle of the string becomes unstable so that the intermediate quark will fall toward either one end or the other with equal chance leading to the above two mostly quark-diquark configurations of the θ field. Such a quark-diquark structure of baryons has been put forward for a long time, in various fields of baryon dynamics, where it appears to have some advantages (see Ref. 17 for a short list of references), but to may knowledge, it was used so far on phenomenological grounds only. (The only place where a flavored quark-diquark structure of baryons is deduced is in Ref. 4 where such configurations realize the two normal modes of the longitudinal motion of a string with three quarks attached to it.) Here, on the contrary, such a substructure will emerge from a well-defined model once a nontrivial nonsymmetric solution of Eq. (17) will be shown to exist.

We now turn to this problem and intend, in looking at two extreme approximations of Eq. (17) to convince the reader that such solutions are indeed very likely to exist. So let us consider as a first extreme approximation, Eq. (17) without the integral term. It is written as

$$\frac{d^2\theta}{d\tau^{12}}+(\sqrt{2N}M_0)^2\left\{\frac{1}{(1-\tau^1)^2}\sin[\theta(\tau^1)-\theta(1)]+\frac{1}{(\tau^1)^2}\sin[\theta(\tau^1)-\theta(0)]\right\}=0 \quad (18)$$

and specify to $\theta(0)=0$ and $\theta(1)=\pi/3$. This equation is just a static sine-Gordon-type equation, i.e.,

$$\frac{d^2\theta}{d\tau^{12}}+f(\tau^1)\sin[\theta-\phi(\tau^1)]=0 \ .$$

Thanks to the singularities for $\tau^1=0$ and $\tau^1=1$, the usual uniqueness theorems, for the nonlinear two-point boundary-value problem do not apply. Moreover, these singularities impose that any solution of Eq. (18) verifies $\theta(\tau^1=1)=\theta(1)$ and $\theta(\tau^0=0)=\theta(0)$ if $\theta(\tau^1=1)$ and $\theta(\tau^1=0)$ are to be finite.

Next look at the most general behavior of $\theta(\tau^1)$ for $\tau^1\to 0^+$ (the behavior for $\tau^1\to 1^-$ will follow by symmetry). There are three possibilities:

if $(\sqrt{2N}M_0)^2=\alpha<\frac{1}{4}$,

$$\theta(\tau^1)=a(\tau^1)^{(1-\beta)/2}+b(\tau^1)^{(1+\beta)/2}+\frac{\alpha(\sqrt{3}/2)}{2+\alpha}(\tau^1)^2 \ ,$$

where $\beta=(1-4\alpha)^{1/2}$;
if $\alpha=\frac{1}{4}$,

$$\theta(\tau^1)=a\sqrt{\tau^1}+b\sqrt{\tau^1}\ln\tau^4+\frac{\alpha(\sqrt{3}/2)}{2+\alpha}\tau^{12} \ ;$$

if $\alpha>\frac{1}{4}$,

$$\theta(\tau^1)=\sqrt{\tau^1}[a\cos(\tfrac{1}{2}\beta\ln\tau^1)+b\sin(\tfrac{1}{2}\beta\ln\tau^1)]+\frac{\alpha(\sqrt{3}/2)}{2+\alpha}-\tau^{12} \ ,$$

where $\beta=(\,|1-4\alpha|\,)^{1/2}$. On the other hand, for small enough α the developable solution can be written as (for small enough τ^1)

$$\theta(\tau^1)=-\alpha\frac{\sqrt{3}}{2}[\ln(1-\tau^1)+\tau^1]+\alpha^2\frac{\sqrt{3}}{2}\left[-\tfrac{7}{2}\tau^1+(\tfrac{3}{2}\tau^1-\tfrac{5}{2})\ln|1-\tau^1|-\tfrac{1}{4}\ln^2|1-\tau^1|+\sum_1^\infty\frac{\tau^{12}}{n^2}\right] .$$

All together, this suggests the following pattern of solution curves for Eq. (18) [see Fig. 4 where only curves satisfying $d\theta/d\tau^1(\tau^1=0)=0$ are drawn] which, in turn, suggests that for all finite values of α the baryonic configuration of the θ field are indeed quark-diquark like.

From the experimental data,[14] it follows that the curves we are interested in have

$$0.15\le\alpha=(\sqrt{2N}M_0)^2\le 0.55 \ .$$

Among these values, only those greater than $\frac{1}{4}$ lead to curves the gross shape of which (it is step distribution like) is satisfactory to describe baryons (forget about their embarassing oscillating behavior in the neighborhood of $\tau^1=1$). However, those smaller than $\frac{1}{4}$ should not be discarded yet, as they obviously do not give a small integral term as required by the consistency of our approximation so that the corresponding curves of Fig. 4 may be only very poor approximations of the correct ones.

The oscillatory behavior of the curves with $\alpha>\frac{1}{4}$ as well as the one of the curves with $\alpha<\frac{1}{4}$ are clearly very embarassing because the related singularity of $d\theta/d\sigma(\sigma)$ will lead to an infinite energy as we shall see shortly. Fortunately, we do not expect these behaviors to survive the introduction of the so far neglected integral term as such terms usually act to smooth singularities away.

To substantiate this argument, let us now look at the other extreme approximation of Eq. (17), that is at the one obtained by using the ansatz $\theta(\sigma)=\theta_B(\sigma)$ in the integral term [$\theta_B(\tau^1)$ is the particular configuration of the field $\theta(\tau^1)$ leading to the distribution $\rho_B(\tau^1)$ defined in (5)]. In this case Eq. (17) simplifies to

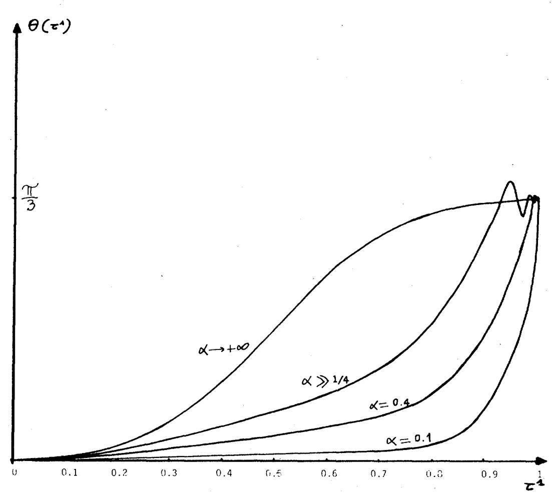

FIG. 4. Pattern of solution curves of Eq. (18) subjected to the boundary conditions $\theta(0)=0$ and $d\theta/d\tau^1(0)=0$.

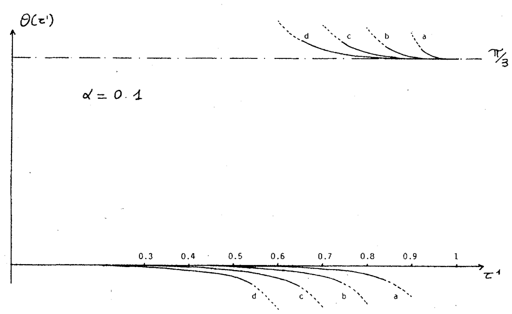

FIG. 5. Pattern of solution curves of Eq. (20) consistent with the ansatz $\theta(\tau^1)=\theta_B(\tau^1)$. We used $\alpha=0.1$ and the curve (a) corresponds to $\tau_d{}^1=0.9$, the curve (b) to $\tau_d{}^1=0.8$, the curve (c) to $\tau_d{}^1=0.7$ and the curve (d) to $\tau_d{}^1=0.6$.

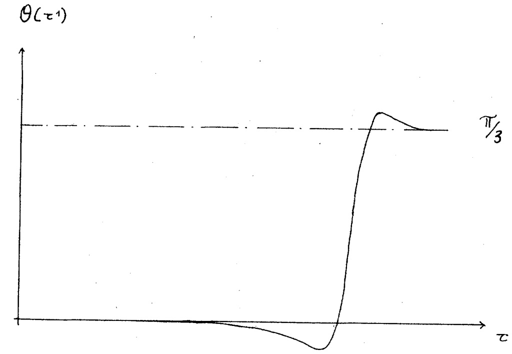

FIG. 6. Suggested shape of a solution of Eq. (17) suitable to describe a baryon.

$$\frac{d^2\theta}{d\tau^{12}}+\alpha\left[\frac{1}{(1-\tau^1)^2}\sin\left[\theta(\tau^1)-\frac{\pi}{3}\right]+(1/\tau^{12})\sin\theta(\tau^1)+\frac{2\,\mathrm{sign}(\tau^1-\tau_d{}^1)\sin[\theta(\tau^1)-2\pi/3]}{[R_\perp{}^2+(\tau^1-\tau_d{}^1)^2]^{1/2}\{\,|\tau^1-\tau_d{}^1|+[R_\perp{}^2+(\tau^1+\tau_d{}^1)^2]^{1/2}\}}\right]=0 \tag{19}$$

which is still a static sine-Gordon-type equation.

Proceeding as before we easily obtain, when α is small enough, the pattern of solution curves consistent with the ansatz $\theta(\sigma)=\theta_B(\sigma)$. This pattern, drawn on Fig. 5 for $\alpha=0.1$, clearly shows the disappearance of the singularity near $\tau^1=1$. Moreover, it strongly suggests the true solution for baryons looks like the curve of Fig. 6. Thus, we believe the above two extreme approximations [Eqs. (18) and (20)] match well together to give a convincing clue in favor the existence of nontrivial nonsymmetric solutions of finite energy of Eq. (17). Fourthly, we notice that Eq. (17) does not depend on any dimensionful parameter which will be crucial for the θ contribution to the energy to not spoil the linearity of baryon Regge trajectories.

Indeed, in the nonrelativistic approximation, the θ contribution to the energy can be written as

$$E_\theta=\frac{1}{r}\int_0^1 d\tau^1\left[\frac{d\theta}{d\tau^1}\right]^2 \tag{20}$$

so that a nontrivial solution $\theta(\tau^1)$ of (17) which do not depend on r will, as usual, provide a positive (of the sign required by experiment) translation of the baryon Regge trajectories. Such a translation would not have been easy to realize at all if we had introduced a potential term

$$S_v=\int\sqrt{-g}\,V(\theta)d\tau^0 d\tau^1$$

in addition to the kinetic term S_θ. Indeed at least one further dimensional parameter η' of dimension (length)$^{-2}$ would have appeared in S_v and consequently an r-dependent dimensionless parameter (r^2/η') would have entered in (17) because of the $dV/d\theta$ term so that any nontrivial solution of (17) would have been implicitly r dependent. Moreover E_θ would have written

$$E_\theta=\frac{1}{r}\int_0^1 d\tau^1\left[\frac{d\theta}{d\tau^1}\right]^2+r\int_0^1 d\tau^1 V(\theta)$$

and thus would have had a quite intricate r dependence. Therefore, altogether, an S_v term is not suitable at all for our present purpose of translating the baryonic Regge trajectories. However, such a term may be of some use in order to introduce natural quark masses in the model so that we must keep it in mind till we introduce the flavor.

What will be the final position of the baryonic trajectories we cannot tell without the exact solution $\theta(\tau^1)$ [which is also needed to compute the short-range chromointeraction analogous to (7)]; nevertheless, we think we are progressing in the right direction especially as we have not yet taken into account the quantum fluctuations contributions to the energy which are specific to baryons. These contributions include in particular a tunneling effect between the two symmetric quark-diquark configurations we missed at the end of Sec. I.

Remark: From (20) we see that the singular behavior of $d\theta/d\sigma$ near $\tau^1=1$ for solution curves of Eq. (18) with α around $\frac{1}{4}$ leads to a logarithmically divergent energy as stated above.

III. CONCLUSION

In spite of the fact that some of our last reasonings are for the moment only heuristic, we think we have convinced the reader that it is possible to merge in a single model some features of the hadronic spectroscopy which were up to now disconnected. To be specific we succeeded in merging together a description of common hadrons as quarks composites as suggested by the quark model of Gell-Mann and Zweig[18] (the quarks are effective here), a description of them as strings as first introduced for mesons by Nambu,[19] here with the additional information that baryons should be also straight strings and not y-shaped ones as considered by some authors[2,20] (an analogous result was obtained by Eguchi[3]), and the description of mesons relative to baryons, respectively, as, the zero and one soliton solution of a two-dimensional field theory as explicitly suggested in Refs. 7 and 11. Moreover, our model appears to be quite economical and in some sense straightforward once the physical idea of Ref. 1 is realized. In particular, no arbitrary potential for θ is needed to accommodate baryons as one soliton. However, we are still very far from anything realistic, spin, flavor, and quark masses remain to be introduced and, above all, we should go beyond the present classical field theoretical study of the action S_T. Nevertheless, we think the results obtained are encouraging.

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[1] D. Olivier, preceding paper, Phys. Rev. D **31**, 483 (1985).
[2] X. Artru, Nucl. Phys. **B85**, 442 (1975).
[3] T. Eguchi, Phys. Lett. **59B**, 457 (1975).
[4] W. Bardeen, I. Bars, A. J. Hanson, and R. D. Peccei, Phys. Rev. D **13**, 2364 (1976).
[5] K. Kikkawa, T. Kotani, M. Sato, and M. Kenmoku, Phys. Rev. D **19**, 1011 (1979).
[6] C. J. Burden and L. Tassie, Nucl. Phys. **B204**, 204 (1982).
[7] T. H. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. 31, 556 (1962).
[8] T. Eguchi, Phys. Lett. **59B**, 73 (1975).
[9] P. Olesen and H. C. Tze, Phys. Lett. **50B**, 482 (1974).
[10] A. Patkos, Nucl. Phys. **B129**, 339 (1977).
[11] J. Goldstone and R. Jackiw, Phys. Rev. D **11**, 1486 (1975).
[12] A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983); M. Rho, A. S. Goldhaber, and G. E. Brown, *ibid.* **51**, 747 (1983); J. Goldstone and R. L. Jaffe, *ibid.* **51**, 1518 (1983); E. Witten, Nucl. Phys. **B223**, 433 (1983).
[13] C. Rebbi, Phys. Rep. **12C**, 1 (1973).
[14] D. Olivier, Z. Phys. C **27**, 315 (1985).
[15] A. Neveu and J. H. Schwarz, Nucl. Phys. **B31**, 86 (1971).
[16] D. Olivier, Report No. LPTHE 84.29, 1984 (unpublished).
[17] A quark-diquark model for baryon was first introduced by D. B. Lichtenberg and L. Tassie, Phys. Rev. **155**, 1601 (1967). More recently it was used by T. Eguchi, Phys. Lett. **59B**, 457 (1975); M. Zralek *et al.*, Phys. Rev. D **19**, 820 (1979); L. F. Abbott *et al.*, Phys. Lett. **88B**, 157 (1979); D. B. Lichtenberg *et al.*, Phys. Rev. Lett. **48**, 1653 (1982); S. Fredriksson, Talk at the XIXth Rencontre de Moriond, La Plagne, Savoie, France (unpublished). We quoted a few references on miscellaneous fields of baryon dynamics where a more complete list of references may be found.
[18] M. Gell-Mann, Phys. Lett. **8**, 214 (1964); G. Zweig, in *Symmetries in Elementary Particle Physics,* proceedings of the 1964 International School of Physics "Ettore Majorana," edited by A. Zichichi (Academic, New York, 1965), p. 192.
[19] Y. Nambu, in *Symmetries and Quark Models,* proceedings of the International Conference, Detroit, Michigan, 1969, edited by R. Chand (Gordon and Breach, New York, 1970).
[20] S. Mandelstam, Phys. Lett. **53B**, 476 (1975); Z. F. Ezawa and H. C. Tze, Nucl. Phys. **B100**, 1 (1975).