Abelian model for mesons

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We present an Abelian model for mesons based on the interaction of a charged Nambu string with a U(1) gauge field. The concept of effective quarks is introduced and the model leads to the semirelativistic equation $[(p^2+m_1{}^2)^{1/2}+(p^2+m_2{}^2)^{1/2}+ar-b/r]\psi(\mathbf{r})=E\psi(\mathbf{r})$ to describe mesons.

Nonrelativistic quarkonium models[1] meet a great deal of success in explaining the salient features of the heavy-meson spectroscopy, and their semirelativistic generalization $[m+p^2/2m\rightarrow(p^2+m^2)^{1/2}]$ seems to be quite promising for light mesons.[2,3] However, in these approaches the choice of the analytical expression of the potential, and in particular that of the spin interactions, is very unsatisfactory as a result of our lack of a theory accounting simultaneously for confinement and asymptotic freedom. It is therefore natural in order to avoid this problem to try to build simple models satisfying our previous requirement of unification. This is what we do in this paper, where we present, in the restricted case of a meson model, the basic physical ideas and the key points of a model which will be generalized to describe mesons and baryons in the following paper.[4]

As our physical ingredients will be borrowed from the well-known flux-tube and string models of mesons,[5–12] which were quite fashionable a decade ago, and as some of the results of the present paper are already more or less known, let us first discuss in what respects our point of view is original. Before this discussion and as a proof of the relevance and usefulness of this new point of view we want to mention that all the results to be obtained in the following paper by a subsequent use of the technique we develop in Secs. I to III of this paper are new. Moreover, a further use of this technique has recently led to another new result which can be abstracted as follows: "We obtained effective quarks with spin from the supersymmetric string."[12]

So what is our point of view (we will consider here the simplest situation and thus ignore the spin degree of freedom)? First of all, we believe that the behavior of a hadron should be described, at the classical level, by the relativistic string introduced by Nambu,[13] exactly as an electron is described at that level by a relativistic point. Such a belief is not at all revolutionary, as it was also adopted by Bars and Hanson[9] (see also the work of Kikkawa *et al.*[11]), but our point of view will shortly depart from theirs. Thus the string is the fundamental object of a Lagrangian formalism for the theory of hadrons and is not to appear after either a convenient effective formulation of a London-type theory of the superconducting vacuum in which the quarks, thought of as the fundamental entities, are embedded as in Ref. 7, or a suitable regularization of some singular terms in the action of a Dirac monopole theory used as a possible theory for quarks binding as in Refs. 6 and 8 or, at last, a regularization of a nonlocal four-fermion interaction as in Ref. 10.

Next we think that just as the electron is charged relatively to the $U(1)_{em}$ gauge field, the string also is charged relatively to some gauge field. In the restricted case of the meson model considered in this paper a simple U(1) gauge field is suitable [this U(1) is not the $U(1)_{em}$ mentioned above]; however, in the following paper, where we will want room to accommodate simultaneously mesons and baryons, the U(1) field will be replaced by an SU(3) field.

The interaction term we chose (see Sec. I) to completely specify what "charged" means for a string is not new either as it was already obtained by Balachandran *et al.* in their effective Lagrangian[8] and was also considered by Barut and Bornzin in their electromagnetic string model,[6] where even the complete action S was written but not studied and, *a fortiori*, not interpreted as we will do below. Nambu[7] also used it when he tried to first quantize his model, in a manner which is, however, not completely consistent with the point of view he adopted in the first part of his paper (see Ref. 6). However, in spite of so many appearances of our interaction term, the interesting physics we will now explain has not yet been discussed nor have the results of Sec. III been exhibited.

The interesting physics we found concerns the quarks: where are they in this model? First of all what characterizes the quarks? In our opinion, the fundamental property which characterizes what are usually called quarks is that they bear the internal quantum numbers such as spin, color, electromagnetic charge, flavor, etc., ... (In the case of the present oversimplified model, the only quantum number to appear will be the magnetic charge which will stand for color.) On this basis what we will show is that there are indeed quarks in the model and that they are localized at the ends of the string, but we will also show that they are "effective" in the sense that (1) they are not present at the Lagrangian level where only a charged string appears and that (2) it is simpler and clearer, in an advanced formulation of the dynamics (at the level of the equations of motion, for instance), to think in terms of charged ends (the quarks) interacting with each other than to think in terms of a charged string. The key sign of the existence of effective quarks will be recognized in Eqs. (4), (5), and (6) and a clear realization of this concept will appear in Sec. III, see Eqs. (13)–(16).

Here we strongly depart from the point of view adopted by Bars and Hanson.[9] We can even say that our point of view is opposite to theirs. Indeed, what they did is to keep an uncharged string and to attach quantum numbers at the ends of the string by a suitable reduction of conventional quark fields to a world line. In so doing they were able to understand the flavor symmetry breaking induced by the quark masses, a problem which is not solved in our model, but they do not deduce any analog to the asymptotically free part of the QCD potential, a problem which was one of our motivations and which is solved below at both the classical and the first-quantized level [see the term $-M_0{}^2/r$ in Eqs. (7) and (14)]. Generalizing their technique they were also able to introduce a spin for their quarks; however, their procedure does not allow any contact with the spinning string as seems to be required by experiment.[2] On the contrary, our method naturally leads to quarks with spin provided we use the spinning string instead of the Nambu one.[12] To be complete we should mention that other authors succeeded in finding an analog to the asymptotically free QCD potential, in particular, Balachandran *et al.*[8] deduced a classical Yukawa potential between their quarks and Sugawara[10] obtained as we do a classical Coulomb potential.

So much for the discussion of our physical ideas. Let us now turn to the model itself.

As a further motivation for our work let us sum up the specific results to be obtained in the next sections.

(1) As was already mentioned above, our physical system will be a Nambu string coupled to a U(1) gauge field with no quarks introduced at the Lagrangian level. Nevertheless, they will be shown to emerge as effective objects in an advanced formulation of the dynamics.

(2) After first quantization the two views of our system, namely, that of a string coupling to a U(1) gauge field and that of a pair of magnetically charged quarks interacting with each other, will be seen to be unitarily equivalent.

(3) The mesons will be shown to be described in a suitable approximation by a semirelativistic Schrödinger equation for the effective quarks provided with the potential of Eichten *et al.* corrected, at large distances only, by an additional K/r term.

The paper is organized as follows: In Sec. I we introduce the action that defines the model. In Sec. II the classical results are exhibited, a first evidence of the effective quarks is seen, and the potential of Eichten *et al.* is recovered. Section III is devoted to first quantization; it ends with a semirelativistic Schrödinger equation for mesons. Finally we give our conclusions. Throughout the mathematics is not fully developed but it can be found in Ref. 3.

I. ACTION OF THE MODEL

The fundamental fields of our Lagrangian will be a U(1) gauge field $A_\mu(x)$ and a vector field $y^\mu(\tau^0,\tau^1)$ where τ^0,τ^1 are two parameters which define a point on the sheet spanned in space-time by a string of finite length (an example of parametrization is $\tau^0=t,\tau^1=s$ where t is the time and s the normal coordinate along the string) and where $y^\mu(\tau^0,\tau^1)$ are the Minkowski coordinates of that point. τ^1 will be chosen in [0,1]. It will be useful to introduce $u_\alpha{}^\mu=\partial y^\mu/\partial\tau^\alpha$, $\alpha=0,1$.

The action for the system is defined as the sum of three contributions: (a) a kinetic term for the string, (b) a kinetic term for the U(1) field, and (c) an interaction term.

(a) A kinetic term for the string

The first idea coming to mind is to use the kinetic term introduced by Nambu,[13]

$$S_0=\eta\int d\tau^0 d\tau^1\sqrt{-|g|}\ , \tag{1}$$

where $|g|=\mathrm{Det}(g_{\alpha\beta})$ and

$$g_{\alpha\beta}=u_\alpha{}^\mu u_{\beta\mu},\ \ \alpha,\beta\in\{0,1\}\ .$$

η is a finite parameter of dimension (length)$^{-2}$. However, it was argued by Salisbury and Sundermeyer[14] that this term alone does not lead consistently (that is, for arbitrary variations of the string coordinates $y^\mu(\tau^0,\tau^1),\tau^1\in[0,1]$) to both the equation of motion of the string and the boundary conditions; moreover, it was also noticed in Ref. 14 that such a problem does not appear if some masses (m) are attached to the ends of the string. In this case the action S_0 is changed to S_S,

$$S_S=\eta\int d\tau^0 d\tau^1\sqrt{-|g|}+m\int_{l_i}ds_i+m\int_{l_f}ds_f\ . \tag{2}$$

Besides that it was shown by Bardeen *et al.* in Ref. 15 that the physical system described by the Nambu action (1) is truly different from the one described by the limit $m\to 0$ of the action (2) which, on the other hand, leads to an encouraging equation for the light-meson spectrum.[15]

Altogether, these arguments thus give us the feeling that the action (1) is not complete and suggest, in addition, a way of describing the relevant physical system. It is, however, most unsatisfactory that the limit $m\to 0$ in the action (2) be singular[15] which make us suspect that (2) is not the correct way of modifying (1). In our opinion the correct way might be to work in the tetrad formalism where the action of a massive particle just differs from the action of a massless one by a cosmological term[16] so that the action of a massless particle is the regular limit $m\to 0$ of the action of a massive one. [To be specific, this last action becomes[16] $\int(\dot y^2/e+me)d\tau$, e being the einbein.]

Therefore a good candidate for the complete action for a string with massless ends might be, in the orthogonal gauge (in this gauge the zweibein $e^a_\mu=\eta^a_\mu$, where η^a_μ is the flat two-dimensional Minkowski metric and $e=1$), something like

$$\frac{\eta}{2}\int d\tau^0 d\tau^1\eta^{\alpha\beta}\partial_\alpha y^\mu\partial_\beta y_\mu+\tfrac{1}{2}\int d\tau^0\dot y_i{}^2+\tfrac{1}{2}\int d\tau^0\dot y_f{}^2\ .$$

Unfortunately, the form of the correct generalization of (1) in this formalism, which should be invariant under general coordinate transformations in the parameter space, is not yet clear to us, this problem still being under investigation in the case of the supersymmetric extension of this model.[12] This is why we will use here for definiteness the ansatz of massive ends to reach the correct equation (obtained in a regular limit $m\to 0$) to describe mesons [see Eq. (12)]. We, however, emphasize that this does not mean we are introducing quarks at the ends of the string and only reveals a poor understanding of the problem stressed above. We also stress that we need the general gauge-invariant action and not only the action in a particular gauge in order to deduce the Dirac constraints of the model which are the interesting equations in the tetrad formalism.[16] So we proceed from S_S.

Incidentally, the missing kinetic contributions of the ends of the string may be responsible for the existence of tachyons in the free string spectrum.

(b) A kinetic term for U(1) field

We take

$$S_F=\frac{1}{16\pi}\int F_{\mu\nu}(x)F^{\mu\nu}(x)d^4x$$

with

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

everywhere.

(c) An interaction term

We take

$$S_I=\frac{M_0}{2}\int \widetilde{G}_{\mu\nu}(x)F^{\mu\nu}(x)d^4x\ ,$$

where $\widetilde{G}_{\mu\nu}$ is the dual of $G_{\mu\nu}$, $\widetilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$, and where $G_{\mu\nu}(x)$ is the tensor field introduced by Dirac in his theory of magnetic monopoles:[17]

$$G_{\mu\nu}(x)=\int_{\text{sheet}}\frac{\partial(y^\mu,y^\nu)}{\partial(\tau^0,\tau^1)}\delta^4(x-y)d\tau^0d\tau^1 \tag{3}$$

$[G_{\mu\nu}(x)=0$ for x not being in the interior of the sheet noted $\dot{S}]$. This term requires some explanation as it seems to violate parity and time reversal if $A_\mu(x)$ is to be a vector field, but this is not so as M_0 should be understood in this preliminary model as a pseudoscalar quantity. But what could that mean?

As will be shown in the following paper[4] $G_{\mu\nu}(x)$ should in fact be written

$$G_{\mu\nu}(x)=\int d\tau^0d\tau^1\frac{\partial(y^\mu,y^\nu)}{\partial(\tau^0,\tau^1)}\delta^4(x-y)\rho(\tau^0,\tau^1)\ ,$$

where $\rho(\tau^0,\tau^1)$ is an additional pseudoscalar field which is, in the present case, in the particular configuration $\rho(\tau^0,\tau^1)=$const and which therefore is absorbed in M_0 so that the pseudoscalar nature of ρ reflects itself in M_0. A more physical way to understand the pseudoscalar nature of M_0 will be given shortly.

Please note that S_I *could not* be written as an interaction term involving only the ends of the string because in the language of forms S_I is just $\int_{\text{sheet}}\widetilde{F}$ so that if we had $\int_{\text{sheet}}\widetilde{F}=\int_{\partial(\text{sheet})}B$ for all sheets, where B is a one-form then we would have $\widetilde{F}=\mathbf{d}B$ and next $\mathbf{d}\widetilde{F}=0$ everywhere which associated with $\mathbf{d}F=0$ gives $F=0$ everywhere.

II. CLASSICAL RESULTS

In this section we compute, in the nonrelativistic approximation, the classical potential between a pair of effective quarks through a solution of the equations of motion. These equations are obtained by varying the action with respect to $A_\mu(x)$, $y^\mu(\tau^0,\tau^1)$ with $\tau^1\in\,]0,1[$, $y_f^\mu=y^\mu(\tau^0,1)$, and $y_i^\mu=y^\mu(\tau^0,0)$.

The variation of $A^\mu(x)$ gives

$$\partial_\mu F^{\mu\nu}=-4\pi M_0\partial_\mu\widetilde{G}^{\mu\nu}\ .$$

To show the role of the U(1) symmetry as the key to effective magnetic charges (there are no magnetic charges in the model as we have $\partial_\mu\widetilde{F}^{\mu\nu}=0$) we introduce $H^{\mu\nu}$ such that

$$F^{\mu\nu}=-4\pi M_0\widetilde{G}^{\mu\nu}+H^{\mu\nu}\ , \tag{4}$$

where

$$\partial_\mu H^{\mu\nu}=0\ . \tag{5}$$

Now

$$\partial_\mu\widetilde{F}^{\mu\nu}=0\Longrightarrow\partial_\mu\widetilde{H}^{\mu\nu}=-4\pi M_0\partial_\mu G^{\mu\nu}$$

but

$$\partial_\mu G^{\mu\nu}=\int_{l_i}\frac{dy_f^\nu}{ds_f}\delta^4(x-y_f)d_y-\int_y\frac{d_y v_i}{ds_i}\delta^4(x-y_i)ds_i=d_f^\nu-d_i^\nu\ .$$

Thus,

$$\partial_\mu\widetilde{H}^{\mu\nu}=-4\pi M_0(d_f^\nu-d_i^\nu)\ ,\qquad \partial_\mu H^{\mu\nu}=0\ . \tag{6}$$

Consequently, it is possible to interpret M_0 as the effective magnetic charge of the end i and $-M_0$ as that of the end f. Moreover Eqs. (3) and (4) reveal the physical interpretation of $M_0G^{\mu\nu}$, together with the physical content of what "charged" means for a string: it is the tensor field which describes the fields along an infinitely thin idealized (without edge effects) solenoid. Thus in the case of a straight charged string at rest $M_0G^{0i}(x)$ is just the opposite of its "inner magnetic field" so that, as $G^{0i}(x)$ is a three-vector and as a magnetic field should be an axial vector, M_0 should be a pseudoscalar quantity.

From the equations of motion of the U(1) field, our charged string just *appears* as a flux tube without edge effects associated with two magnetic charges localized at its ends. These magnetically charged ends are what we interpret as the effective colored quarks of the model.

The variation of $y^\mu(\tau^0,\tau^1)$ and the induced one of y_i^μ,y_f^μ lead to

$$M_0u_0^\mu u_1^\nu\left[\frac{\partial\widetilde{F}_{\mu\nu}}{\partial y^\rho}+\frac{\partial\widetilde{F}_{\rho\mu}}{\partial y^\nu}+\frac{\partial\widetilde{F}_{\nu\rho}}{\partial y^\mu}\right]-\eta\frac{\partial}{\partial\tau^\alpha}(g^{\alpha\beta}\sqrt{-|g|}\,u_{\beta\rho}=0 \quad \text{for } (\tau^0,\tau^1)\in\dot{S} \tag{7}$$

and

$$-m\frac{d^2y_i^\mu}{ds_i{}^2}-\frac{dy_i^\nu}{ds_i}M_0F_\nu{}^\mu +\eta\frac{1}{\sqrt{-|g|_i}}\frac{dy_i^\nu}{ds_i}\frac{\partial(y_\nu,y^\mu)}{\partial(s_i,\tau_1)}\bigg|_i=0\,,$$

$$-m\frac{d^2y_f^\mu}{ds_f{}^2}+\frac{dy_f^\nu}{ds_f}M_0F_\nu{}^\mu -\eta\frac{1}{\sqrt{-|g|_f}}\frac{dy_f^\nu}{ds_f}\frac{\partial(y_\nu,y^\mu)}{\partial(s_f,\tau_1)}\bigg|_f=0\,.$$

As in usual electrodynamics these equations contain some infinite self-interactions which have been already taken into account in a renormalized form through the parameters m and η and which have to be removed to get sensible results. This removal is done by analogy with the electrodynamics so that for the string, the term

$$u_0^\mu u_1^\nu\left[\frac{\partial\tilde F_{\mu\nu}}{\partial y^\rho}+\frac{\partial\tilde F_{\rho\mu}}{\partial y^\nu}+\frac{\partial\tilde F_{\nu\rho}}{\partial y^\mu}\right]=u_0^\mu u_1^\nu(\mathbf{d}\tilde F)_{\mu\nu\rho}$$

has to be interpreted entirely as string self-interaction, being the analog of the term $u_0^\mu(\mathbf{d}A)_{\mu\nu}$ for a charge particle ($\mathbf{d}$ is the exterior derivative). This was not remarked in Ref. 8 where Eq. (7) was also obtained.

For the string ends $\tilde F^{\mu\nu}$ has to be replaced by $\tilde H^{\mu\nu}_{f(i)}$ for the end $i(f)$ where $\tilde H^{\mu\nu}_f(i)$ is the field created by the end $f(i)$. Here also our treatment differs from the one of Balachandran *et al.*[8] as they kept the term $4\pi M_0G^{\mu\nu}|_{i\text{ or }f}$.

Therefore, the equations of motion simplify to

$$F_{\mu\nu}=-4\pi M_0\tilde G_{\mu\nu}+H_{\mu\nu}\,, \tag{4}$$

$$\partial_\mu H^{\mu\nu}=0\,, \tag{5}$$

$$\partial_\mu\tilde H^{\mu\nu}=-4\pi M_0(d_f^\nu-d_i^\nu)\,, \tag{6}$$

$$\frac{\partial}{\partial\tau^\alpha}(g^{\alpha\beta}\sqrt{-|g|}\,u_\beta^\mu)=0 \text{ on } \dot S\,, \tag{7'}$$

which reduces to the usual noncoupled Nambu string equation, and

$$\begin{aligned}&-m\frac{d^2y_i^\mu}{ds_i{}^2}-\frac{dy_i^\nu}{ds_i}M_0H_\nu^\mu(f) +\eta\frac{1}{\sqrt{-|g|_i}}\frac{dy_i^\nu}{ds_i}\frac{\partial(y_\nu,y^\mu)}{\partial(s_i,\tau^1)}\bigg|_i=0\,,\\ &-m\frac{d^2y_f^\mu}{ds_f{}^2}+\frac{dy_f^\nu}{ds_f}M_0H_\nu^\mu(i) -\eta\frac{1}{\sqrt{-|g|_f}}\frac{dy_f^\nu}{ds_f}\frac{\partial(y_\nu,y^\mu)}{\partial(s_f,\tau^1)}\bigg|_f=0\,.\end{aligned} \tag{8}$$

We made no attempt to find the general solution of this system and we will just discuss a particular solution which is, for our effective quark pair, the analog of the Bohr mode of the hydrogen atom. This solution is the rigid motion of the free string defined in Ref. 18:

$$\tau^0=y^0\,,$$
$$y^1=A(\tau^1-\tfrac12)\cos\omega\tau^0\,,$$
$$y^2=A(\tau^1-\tfrac12)\sin\omega\tau^0\,,$$
$$y^3=0\,.$$

In the nonrelativistic limit, the above motion is still a solution of the coupled system [Eqs. (4)—(7′)] provided

$$\left[\frac{\omega A}{2}\right]^2=\frac{1+\dfrac{M_0{}^2}{A\eta}}{\dfrac{2m}{A\eta}-\dfrac{M_0{}^2}{2A^2\eta}}$$

which replace the usual $(\omega A/2)^2=1$ constraint in the free case ($M_0=0, m=0$). In that limit the interaction potential between the charged ends of the string can be written

$$V(r)=\eta\,|\mathbf{y}_f-\mathbf{y}_i|-\frac{M_0{}^2}{|\mathbf{y}_f-\mathbf{y}_i|} \tag{9}$$

since

$$\eta\frac{1}{\sqrt{-|g|_i}}\frac{dy_i^\nu}{d\tau^0}\frac{\partial(y_\nu,y_\mu)}{\partial(\tau^0,\tau^1)}\bigg|_i$$

is just

$$\frac{\partial}{\partial y_i^\mu}\{\eta[-(y_f-y_i)^2]^{1/2}\}\,.$$

It is a confining potential since η is positive as implied by the positivity of the string energy.

So we recover, at the classical level, the nonrelativistic potential used by Eichten *et al.*[19] As we will see in the next section this potential will appear again in the static approximation of the first-quantized model.

III. FIRST QUANTIZATION

In what follows we will not quantize the U(1) gauge field. In order to quantize our model let us first compute the relevant conjugate momenta.

We have as a momentum conjugate to the variable $y^\mu(\tau^0,\tau^1),\tau'\in\,]0,1[$,

$$\mathscr{P}_\mu=\eta\frac{\partial(\sqrt{-|g|})}{\partial u_0^\mu}+u_1^\nu M_0\tilde F_{\mu\nu}\,,$$

and as a momentum conjugate to the variation y_i^μ and y_f^μ,

$$P_{i,\mu}=m\frac{dy_{i,\mu}}{ds_i}$$

and

$$P_{f,\mu}=m\frac{dy_{f,\mu}}{ds_f}\,.$$

As is well known the invariance of a Lagrangian under local reparametrization leads to primary constraints which can be written

$$(\mathscr{P}_\mu-M_0\tilde F_{\mu\nu}u_1^\nu)^2=-\eta^2u_1^\nu u_{1\nu}\,, \tag{10}$$

$$u_{1\mu}\mathscr{P}^{\mu}=0 , \qquad (11)$$

generalizing the usual free string constraints and

$$P_i^{\mu}P_{\mu}^{i}=m^2=P_f^{\mu}P_{\mu}^{f} .$$

Instead of using the Dirac quantization formalism for constrained systems,[20] we will invert them in the particular gauge

$$y^0(\tau^0,\tau^1)=\tau^0=y_i^0(\tau^0)=y_f^0(\tau^0)$$

which leads to

$$\mathscr{P}^0=[(\mathscr{P}^j-M_0\widetilde{F}^{j\nu}u_{1\nu})^2+\eta^2u_1^2]^{1/2}+u_{1\nu}M_0\widetilde{F}^{0\nu} ,$$

$$P_i^0=(\mathbf{P}_i^2+m^2)^{1/2} ,$$

$$P_j^0=(\mathbf{P}_f^2+m^2)^{1/2} ,$$

and next to a nontrivial Hamiltonian

$$H=P_i{}^0+P_f{}^0+\int_{\tau^1=0}^{\tau^1=1}\mathscr{P}^0(\tau^1)d\tau^1 .$$

Remark: There is an ambiguity in the extraction of $\mathscr{P}^0(\tau^1)$ due to (11) [an arbitrary multiple of which could have been added to (10) before inversion]. Nevertheless, the meaning of (11) [it is associated to the remaining invariance by reparametrization of the string in that gauge $\tau^1\rightarrow\tau^1(\widetilde{\tau}^1)$] and usual classical formulas make the above choice perfectly natural.

The quantization is then obtained by the following identifications

$$P_i^{\mu}\rightarrow -i\frac{\partial}{\partial y_i^{\mu}} ,$$

$$P_f^{\mu}\rightarrow -i\frac{\partial}{\partial y_f^{\mu}}, \quad \mu=1,\ 2,\ \text{or}\ 3$$

$$\mathscr{P}\rightarrow -i\frac{\delta}{\delta y^{\mu}(\tau^1)} ,$$

and leads to a Schrödinger-type eigenvalue problem, namely,

$$H\,|\,\psi\rangle=E\,|\,\psi\rangle \Longleftrightarrow \Big[(P_i{}^2+m^2)^{1/2}+(P_f{}^2+m^2)^{1/2}+\int_{\tau^1=0}^{\tau^1=1}d\tau^1[(\mathscr{P}^j-M_0\widetilde{F}^{j\nu}u_{1\nu})^2+\eta^2u_1{}^2]^{1/2} - \int_{\tau^1=0}^{\tau^1=1}d\tau^1M_0\widetilde{F}^{0j}u_{ij}+E\Big]\psi(\mathbf{y}_i,\mathbf{y}_f,\Gamma)=0 , \qquad (12)$$

where $\psi(\mathbf{y}_i,\mathbf{y}_f,\Gamma)$ is a generalized wave function, it is a function of $\mathbf{y}_i,\mathbf{y}_f$ and a functional of $\mathbf{y}(\tau^1),\tau^1\in\,]0,1[$, i.e., of the string curve Γ.

Let us come back here on the ansatz (2) for S_S. Firstly, we note that the limit $m\rightarrow 0$ is regular in (12) so that the role and the motivation of the ansatz (2) for S_S is to provide (12) with the kinetic terms $(P_i{}^2)^{1/2}+(P_f{}^2)^{1/2}$ which would have been missing if we had used (1) for S_S (in the tetrad formalism these two terms would appear in the first class constraints of the model). In this limit, Eq. (12) will be our starting point for a description of mesons. So we proceed from (13)

$$\Big[(P_i{}^2)^{1/2}+(P_f{}^2)^{1/2}+\int_{\tau^1=0}^{\tau^1=1}d\tau^1[(\mathscr{P}^j-M_0\widetilde{F}^{j\nu}u_{1\nu})^2+\eta^2u_1{}^2]^{1/2}-\int_{\tau^1=0}^{\tau^1=1}d\tau^1M_0\widetilde{F}^{0j}u_{1j}+E\Big]\psi(\mathbf{y}_i,\mathbf{y}_f,\Gamma)=0 . \qquad (13)$$

The integral in (12) and (13) is defined through a discretization of the curve, using

$$u_{1\nu}(\tau_k{}^1)=\frac{y_{k+1}-y_k}{\epsilon}$$

and

$$\frac{\delta}{\delta y^{\nu}(\tau_k{}^1)}=\frac{1}{\epsilon}\frac{\partial}{\partial y_k^{\nu}},\quad k\in\{0,\ldots,N-1\}$$

where $y_i=y_0$, $y_f=y_N$, and $\epsilon=1/N$.

Equation (13) is named "the string-field eigenvalue equation" to recall that its form emphasizes the string coupling to the U(1) field and by opposition to the unitarily equivalent "quark-field eigenvalue equation," which we will now introduce. The physical interpretation of this last equation is much simpler than that of Eq. (13) thus stressing the nature of the effective quarks as the relevant objects of an advanced formulation of the dynamics.

We obtain it using the transformation

$$\psi=\exp\Big[i\int_{\tau^1=0}^{\tau^1=1}M_0C^{\mu}u_{1\mu}d\tau^1\Big]\psi'=U\psi' ,$$

where we wrote

$$\widetilde{H}^{\mu\nu}=\partial^{\mu}C^{\nu}-\partial^{\nu}C^{\mu}$$

[C^{μ} exists because of (4)] which gives

$$\Big[[(\mathbf{P}_i-M_0\mathbf{C}_i)^2]^{1/2}+[(\mathbf{P}_f-M_0\mathbf{C}_f)^2]^{1/2}+\int_{\tau^1=0}^{\tau^1=1}d\tau^1(\mathscr{P}^2+\eta^2u_1{}^2)^{1/2}+M_0(C_f{}^0-C_i{}^0)-E\Big]\psi'(\mathbf{y}_i,\mathbf{y}_f,\Gamma)=0 . \qquad (14)$$

Everywhere, self-interactions subtracted quantities are used.

The physical content of our system is now clear, we have two effective massless magnetically charged particles (the quarks) interacting with each other via the U(1) field an via a string whose dynamics translates into a complicated potential term

$$\Big[\int_{\tau^1=0}^{\tau^1=1}d\tau^1(\mathscr{P}^2+\eta^2u_1{}^2)^{1/2}\Big]\psi' .$$

Therefore, (14) is suitable to describe mesons in the chiral limit. To get more insight into the interaction between our two quarks we will concentrate on its static limit.

In that limit we reduce the string potential through the Born-Oppenheimer method therefore considering the curve Γ to be frozen into the curve $\Gamma_{\min}$ which minimizes the integral in the absence of quantum fluctuations, namely, which minimizes

$$\int_{\tau^1=0}^{\tau^1=1} (\eta^2 u_1{}^2)^{1/2} d\tau^1 \; .$$

We thus have

$$\Gamma_{\min} = [\mathbf{x}(\tau^1) = \mathbf{y}_i + \tau^1(\mathbf{y}_f - \mathbf{y}_i)]$$

leading to the classical potential

$$V_S = \eta r_{fi} = \eta \, | \mathbf{y}_f - \mathbf{y}_i |$$

so that the overall quark potential in that approximation is simply

$$V = \eta r_{fi} - \frac{M_0{}^2}{r_{fi}}$$

and the quark-field eigenvalue equation becomes

$$\left[(\mathbf{P}_i{}^2)^{1/2} + (\mathbf{P}_f{}^2)^{1/2} + \eta r_{fi} - \frac{M_0{}^2}{r_{fi}} \right] \psi'_{\min} = E \psi'_{\min} \; . \qquad (15)$$

Now from Eq. (15), and although the flavor symmetry breaking is not understood yet in our model, it is obvious how to incorporate it by hand.

An *ad hoc* manner is clearly to modify the kinetic term $(\mathbf{P}_i{}^2)^{1/2} + (\mathbf{P}_f{}^2)^{1/2}$ to give masses to the effective quarks. Thus, our final equation to describe mesons with an *ad hoc* account of flavor symmetry breaking will be

$$\left[(\mathbf{P}_i{}^2 + m_i{}^2)^{1/2} + (\mathbf{P}_f{}^2 + m_f{}^2)^{1/2} + \eta r_{fi} - \frac{M_0{}^2}{r_{fi}} \right] \psi'_{\min} = E \psi'_{\min} \; . \qquad (16)$$

[We could obviously have used $m_i \int ds_i + m_f \int ds_f$ without any limit to obtain the result (16); however, to emphasize the role of the ansatz (2) which should not be thought as introducing quarks at the Lagrangian level we refrain to do so.] So as announced in the Introduction our model allows us to derive in a well-prescribed approximation a semirelativistic generalization of the nonrelativistic equation used by Eichten *et al.* to describe heavy quarkonia.[19] Such a generalization is quite suitable for a unified description of light and heavy mesons and is even required experimentally as suggested in Ref. 2.

Remark: We can refine the potential $V(r)$ a bit by taking into account the quantum fluctuations of the string which should write $k/r(\hbar c)$ (by dimensional analysis). Known results of string theory[21] then suggest that this term is only present at large enough distances and is attractive ($k<0$).

IV. CONCLUSIONS

What may have been learned from such an oversimplified model is in our opinion threefold: first of all, it seems that to consider a charged Nambu string is quite relevant to meson physics; secondly, we have seen that the quarks may not be present in the Lagrangian and nevertheless appear as relevant effective objects; and, thirdly, we have seen that it is possible to recover a satisfactory[2] phenomenological equation in a single model. Obviously, what should be done now is to try to generalize the above model to a realistic one and a first step in that direction is to describe simultaneously mesons and baryons as classical colorless objects made of effective colored quarks. That is the problem we address ourselves to in the following paper and we will see that its solution gives us clues on how to pursue toward a realistic model. However, the crucial problem of how to build a quantum color singlet and not only a classical colorless state may be far more difficult and is not attacked yet. The problem of the quark spin has recently been resolved in this framework through a supersymmetric extension of the present model.[12]

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[1]Let us mention the earliest work of E. Eichten *et al.* [Phys. Rev. Lett. **34**, 369 (1975)] for nonrelativistic models and that of J. Kang and H. Schnitzer [Phys. Rev. D **12**, 841 (1975)] for semirelativistic models. A more complete list of references is given in C. Quigg, in *Gauge Theories,* 1981 Les Houches Lectures, edited by M. K. Gaillard and R. Stora (North-Holland, Amsterdam, 1982).

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