Looking for weak-boson compositeness via form factors

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We examine the possibility that the composite structure of weak gauge bosons may first reveal itself through form-factor-like effects. We consider modifying the propagators and gauge-boson self-couplings with a simple parametrization of these composite-structure form factors, and examine a large number of processes where deviations from the standard model may be observed. Present data indicate the W, Z composite scale $\Lambda \gtrsim 0.1$ TeV but future data from CERN LEP II may push $\Lambda > 1$ TeV. We find the process most sensitive to these form factors resulting from compositeness to be the reaction $e^+e^- \to W^+W^-$ because of the unique cancellations which take place in gauge theories.

I. INTRODUCTION

In recent years it has become popular to consider the possibility that some or all of the particles of the standard electroweak model[1] (fermions as well as gauge bosons and Higgs scalars) are composite objects,[2–4] i.e., bound states of more fundamental constituents. On the experimental side there is currently no direct evidence that any of these particles are less than fundamental. In the case of fermions, leptons in particular, the limits[5] from SLAC (PEP) and DESY (PETRA) on tests of QED and the agreement between theory and experiment on the $g-2$ of the muon and electron[6] apparently indicate that the appropriate scale of compositeness for these particles exceeds[7] 1 TeV and probably exceeds $\simeq 10$ TeV. Although care should be taken in interpreting this $\simeq$10-TeV limit, it appears that we need energies in the range of the Superconducting Super Collider (SSC) ($\sqrt{s}=40$ TeV) to explore fermion composite structure—the CERN collider, the Fermilab Tevatron, CERN LEP, and the Stanford Linear Accelerator (SLC) apparently are inadequate.

On the other hand, there exists no real data constraining the composite scale of gauge bosons to be as large as that for fermions. Basically, this is due to the fact that we still know very little about their properties and there is very poor information on the interaction of *virtual* gauge bosons for Q^2 reasonably comparable to their M^2 values. Even at[5] PEP and PETRA, $s/M_Z{}^2 \simeq \frac{1}{6}-\frac{1}{5}$ for $\sqrt{s} \gtrsim 40$ GeV which is quite small. It does not seem impossible that the gauge-boson composite scale Λ could be in the 0.1–1-TeV range and is accessible to the CERN collider, Tevatron, LEP, or SLC.

The purpose of this paper is to examine the possibility that the gauge-boson composite scale is in the above range, the limits on this scale that can be found from existing data, and the modifications such compositeness would make in the predictions of the standard model. There are many approaches one can take in this analysis; one can, for example, consider the contribution of excited gauge bosons to low-energy phenomena[8] or the existence of new couplings between gauge bosons.[9] In our approach, we will consider the possibility that the gauge-boson composite structure leads to a modification in the gauge-boson propagator and to form factors in the trilinear and quartic gauge-boson couplings. These modifications, as we will see below, will involve the gauge-boson composite scale Λ. We will assume the fermion composite scale to be much greater than Λ; otherwise, there would appear additional form-factor effects from the fermion vertices, etc.

Making these modifications of the standard model (and leaving all other couplings intact) we will examine a set of processes in order to determine what limits can be placed on Λ. In Sec. II, we will discuss limits on W-boson propagator modifications from "low-energy" ($Q^2 \ll M_W{}^2$) interactions. Section III consists of an analysis of the modifications for the prediction for the processes $e^+e^- \to f\bar{f}$ (where $f=\mu$, u, or d) including the total cross section and the forward-backward symmetry. Section IV examines the reaction $e^+e^- \to W^+W^-$ which, in principle, involves both propagator and coupling form-factor modifications. This reaction is particularly sensitive to such modifications in gauge theories since as $s \to \infty$ this reaction violates unitarity unless the delicate cancellation present in gauge theories takes place. In Sec. V we examine the limits on Λ coming from the decay $Z \to Hl^+l^-$ and the muon anomalous magnetic moment. Section VI contains our conclusions; we will see that the present limits that can be placed on Λ are quite poor and that the suggested modifications can lead to significant alterations in the predictions of the standard model at high energies.

II. MODIFICATION OF GAUGE-BOSON INTERACTIONS

The simplest form of a modification of the W and Z gauge-boson propagators is of the multiplicative form

$$\frac{1}{p^2-M^2} \to \frac{1}{p^2-M^2} F(p^2,\Lambda)\ , \tag{2.1}$$

where $F(p^2,\Lambda)$ must satisfy $F(p^2,\Lambda)\to 1$ as $p^2/\Lambda^2 \to 0$. Obviously, since we do not want to *increase* the divergences in gauge theories (via power counting, say) we clearly do not want $F(p^2,\Lambda)$ to be an increasing function of p^2; thus we expect $F(p^2,\Lambda)\to 0$ as $p^2 \to \infty$. Also, we do not want to modify the pole structure of the above propagator since we wish to define particle mass as via the pole of the relevant propagator. This set of criteria, while not uniquely defining $F(p^2,\Lambda)$, is quite constraining; we will assume $F(p^2,\Lambda)$ to be of the form

$$F(p^2,\Lambda)=(1+\lambda p^2/\Lambda^2)^{-1},\quad \lambda=\pm 1 \tag{2.2}$$

such that $\lambda p^2>0$ and Λ is the gauge-boson composite scale which we will assume to be ≥ 0.1 TeV. This form is crude at best but will be sufficient to put some bound on the scale Λ. Now since $\Lambda > M_{W,Z}$ it is clear that getting constraints on the value of Λ will be difficult for small values of p^2 where, unfortunately, most of the data is available. The data from the usual high-precision experiments, such as μ, π, or K decay are quite insensitive to this possibility since we expect so large a value of Λ compared with the momentum transfer in these reactions. Obviously, this crude parametrization will not suffice for $|p^2|$ values $>\Lambda^2$. In this region, multiparticle production will occur as it does in the case of the nucleon.

The only place where we can currently constrain Λ in charged-current interactions (apart from W decay itself) is in high-energy ν scattering where Q^2 values can exceed 200 GeV^2 or so. Even here, this will lead only to a change in the cross section of only a few percent (at most) even if Λ is only 100 GeV. It is not even clear at the moment whether the experimental ν data is sensitive to the normal $Q^2/M_W{}^2$ propagator correction.[10] This is partly due to the uncertainty in the experimental cross-section normalization; it may be possible, however, that the higher-energy ν data from the Tevatron with its extended Q^2 range may show the usual propagator $Q^2/M_W{}^2$ correction and, perhaps, the substructure modification (2.2) as well.

As a first approximation, one would see corrections to the usual structure functions given by

$$F_i^\nu(x,Q^2)\to F_i^\nu(x,Q^2)\left[1-\frac{2Q^2}{M_W{}^2}(1+M_W{}^2/\Lambda^2)+\cdots\right] \tag{2.3}$$

so that the value of M_W extracted from the ν data M_W^ν will differ from the experimental value [from, say, UA1 (Ref. 11) and UA2 (Ref. 12)] by an amount

$$M_W^\nu \simeq M_W\left[1+\frac{M_W{}^2}{\Lambda^2}\right]^{-1/2} . \tag{2.4}$$

This difference would then allow us to constrain Λ, perhaps substantially. Note that $\Lambda\simeq 0.1$ TeV would yield $M_W^\nu/M_W\simeq 0.77$ which is quite a sizable effect so that high-precision measurements in this range of Q^2 ($\simeq$500–600 GeV^2) could push Λ above $\simeq$0.1 TeV but not much further.

A similar situation occurs when considering deep-inelastic ν neutral-current reactions; because of small overall uncertainties in the overall cross-section normalization and the very difficult job of disentangling propagator $Q^2/M_Z{}^2$ (and Q^2/Λ^2) effects from perturbative QCD and higher-twist effects it seems unlikely that one could obtain any reasonably strong constraints of Λ from this data. Indeed, one needs very accurate data at higher values of Q^2 if any nontrivial limits on Λ are to be obtained; at present, there are four Tevatron experiments planned to study this higher-Q^2 range. The resulting data may be useful in pushing the lower limit on the value of Λ above 100 GeV.

III. $e^+e^-\to f\bar{f}$

In the last few years, e^+e^- reactions have become an excellent testing ground for electroweak interactions.[13] Measurements of the total cross section and the forward-backward asymmetry $A_{\rm FB}$ can be used to constrain the standard model as well as alternative theories. Here we hope to show that present e^+e^- data for $\sqrt{s}\lesssim 45$ GeV cannot at present be used to push the value of Λ up above 100 GeV. Mainly, this is due to the large experimental uncertainties in $A_{\rm FB}$ and the very small deviation expected in the total cross section for this value of Λ. We now turn to a discussion of the modification of the differential cross section for $e^+e^-\to\bar{f}f$ produced by a finite scale parameter Λ.

As is well known, the cross section for $e^+e^-\to f\bar{f}$ can be written in the form

$$\frac{d\sigma}{d\cos\theta}=\frac{\pi\alpha^2}{2s}[A(1+\cos^2\theta)+2B\cos\theta] , \tag{3.1}$$

where, if we allow for the above modification of the Z propagator, we find

$$\begin{aligned} A&=Q_f{}^2-2Q_f v_e v_f (g/e)^2 s(s-M_Z{}^2)PF \\ &\quad +(v_e{}^2+a_e{}^2)(v_f{}^2+a_f{}^2)(g/e)^4 s^2 PF^2 , \\ B&=-2Q_f a_e a_f (g/e)^2 s(s-M_Z{}^2)PF \\ &\quad +4v_e v_f a_e a_f (g/e)^4 s^2 PF , \end{aligned} \tag{3.2}$$

where F is defined above and where

$$P=[(s-M_Z{}^2)^2+\Gamma_Z{}^2M_Z{}^2]^{-1} \tag{3.3}$$

with v_e,a_e (v_f,a_f) are the couplings of the electron (fermion) to the Z^0 boson defined via ($g\equiv e/\sin\theta_W$)

$$g\bar{f}\gamma_\mu(v_f-a_f\gamma_5)fZ^\mu . \tag{3.4}$$

Hence,

$$\begin{aligned} v_f&=\frac{1}{2\cos\theta_W}(T_{3f}-2x_W Q_f) , \\ a_f&=\frac{1}{2\cos\theta_W}T_{3f} . \end{aligned} \tag{3.5}$$

We take $x_W\equiv\sin^2\theta_W\simeq 0.220$, $M_Z=93$ GeV, and $\Gamma_Z=2.9$ GeV as input in our numerical calculation. Q_f and T_{3f} are the charge and third component of the left-handed weak isospin; the couplings for the electron are given with the replacement $e\to f$. In our analysis we will concentrate on the total cross section (i.e., the value of A) and the forward-backward asymmetry $A_{\rm FB}=3B/4A$ for the particular case $f=\mu$ since at present, this process has the most accurate experimental data.[13]

Figure 1 shows the modification of the predicted cross section for $e^+e^-\to\mu^+\mu^-$ as compared to the prediction

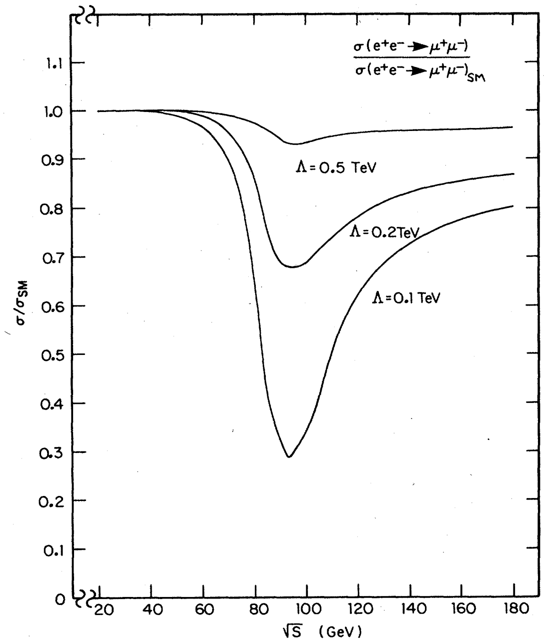

FIG. 1. Cross section for $e^+e^- \to \mu^+\mu^-$ relative to its standard-model value when composite corrections to the gauge-boson propagator are made.

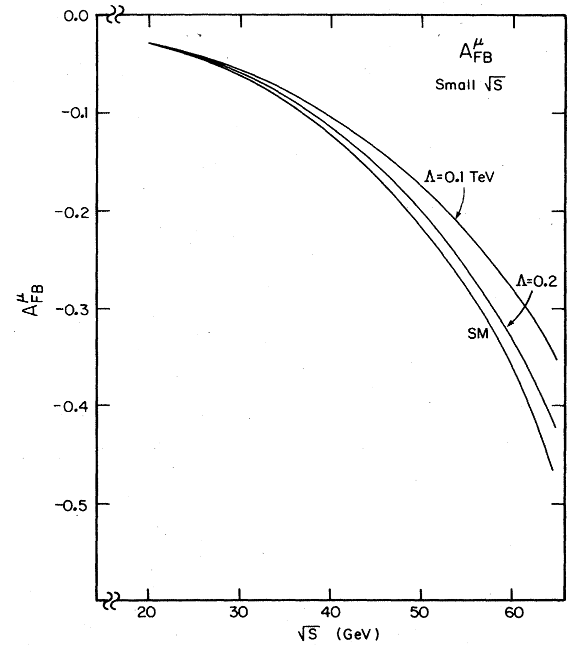

FIG. 2. The forward-backward asymmetry A_{FB} predicted for $e^+e^- \to \mu^+\mu^-$ in the SM and with $\Lambda = 0.1$ and 0.2 TeV for small values of $\sqrt{s}$.

of the standard model, i.e., σ/σ_{SM}. We see that for $\sqrt{s} \lesssim 45$ GeV or so the deviation is less than 1% even for $\Lambda = 100$ GeV; in fact, for $\sqrt{s} = 40$ GeV and $\Lambda = 100$ GeV the ratio differs from unity only by $\simeq 0.46\%$. The current data,[13] however, are not this accurate; Mark J, for example, finds $\sigma/\sigma_{SM} = 1.00 \pm 0.02$ in the $\sqrt{s} = 35$-GeV region. We see that the errors are at present too large to constrain Λ to be greater than 100 GeV—the lower limit considered here.

At higher energies, as we approach the Z pole σ/σ_{SM} decreases greatly from unity with the deviation reaching a maximum at $\sqrt{s} = M_Z$. This is due to the dominance of the Z^0 contribution near M_Z and the fact that the Z propagator is being modified by $(1 + M_Z{}^2/\Lambda^2)^{-2}$ for $\sqrt{s} = M_Z$; this substantially reduces this contribution hence leading to smaller values of σ/σ_{SM}. For $\sqrt{s} > M_Z$ the deviation from unity decreases although it remains sizable. An accurate measurement of σ near the Z^0 pole may be able to push $\Lambda > 1$ TeV (if no effect is observed) since for $\Lambda = 1$ TeV we expect σ/σ_{SM} to be $\simeq 0.983$ at $\sqrt{s} = M_Z$.

The explanation for the insensitivity of σ/σ_{SM} for reasonably small $\sqrt{s}$ values is clear; the pure QED term dominates and the γZ interference term is suppressed because $v_e v_\mu$ is very small for $\sin^2\theta_W \simeq \frac{1}{4}$. In order to get better constraints at small $\sqrt{s}$ ($\sqrt{s} \simeq 40$ GeV, say) we now turn to A_{FB}. The reason we expect larger deviations from the standard model here is that the first contributing term is from γZ interference and the value of $a_e a_\mu$ is reasonably large. Figure 2 shows A_{FB} for the standard model (SM) as well as for $\Lambda = 0.1$ and 0.2 TeV; the data[13] are in complete agreement with the SM with the theoretical and experimental uncertainties. Note that the experimental values are uncertain by, at least, 10% and hence are still in agreement with the modified prediction even for $\Lambda = 0.1$ TeV. As $\sqrt{s}$ increases and the experimental errors are reduced, we may be able to constrain $\Lambda \gtrsim 0.15$ TeV in the near future but, at present, we have no real constraint from the present A_{FB} data.

Figure 3 shows a comparison of A_{FB} in the SM and with $\Lambda = 0.1$ TeV. There is very little difference in the

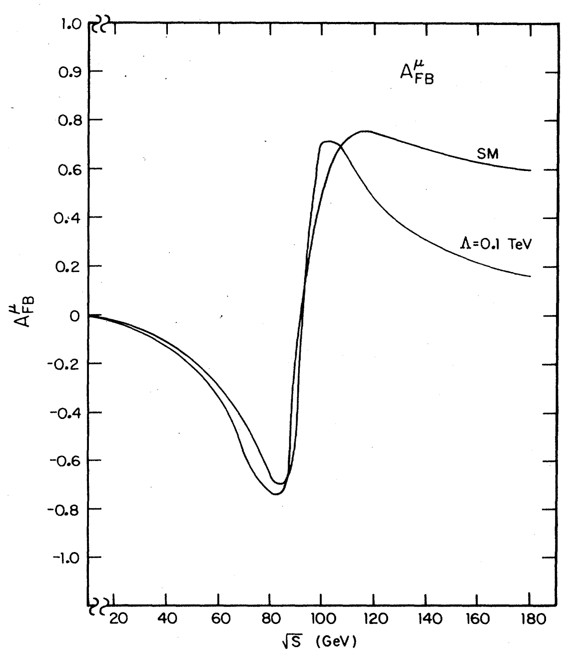

FIG. 3. Same as Fig. 2 but for larger values of $\sqrt{s}$.

curves except for $50 \leq \sqrt{s} \leq 80$ GeV and above the Z pole. Note both models predict almost identical results near the Z pole; this is due to the cancellation of the F factors in the B/A ratio used in defining $A_{\rm FB}$ at $\sqrt{s}=M_Z$. Unfortunately, $A_{\rm FB}$ measurements on the Z pole will not help us to constrain the value of Λ any further.

In addition, we have analyzed the results for A and B for the processes $e^+e^- \to u\bar{u}$ and $d\bar{d}$. We again find the deviation of $\sigma/\sigma_{\rm SM}$ from unity to be tiny; some of this difference if observed will be attributed to a change in the input value of α_s. The values of $A_{\rm FB}$, while larger, have greater errors attached to them than $A_{\rm FB}$ for $\mu^+\mu^-$ production and are, hence, presently useless in restricting the value of Λ. Jet distribution studies of these processes on the Z pole will not help since for $\sqrt{s}=M_Z$, as discussed above, the factor of F cancels in $A_{\rm FB}$. However, the cross section for the two-jet final states on the Z pole will be reduced by the factor $(1+M_Z{}^2/\Lambda^2)^{-2}$ and (as in the case of $e^+e^- \to \mu^+\mu^-$) may be useful in constraining Λ. We again note that our parametrization should not be taken as valid for $\sqrt{s} > \Lambda$ where multiparticle effects would start to dominate.

IV. $e^+e^- \to W^+W^-$

The measurement of the cross section for $e^+e^- \to W^+W^-$ will provide a basic test of the nature of the couplings between gauge bosons. As is well known,[14] the diagrams leading to this reaction (see Fig. 4) would in general lead to a cross section which grows like s as $s \to \infty$. In a gauge theory, however, delicate cancellations occur between the diagrams leading to a $[(1/s)\ln s]$ behavior of $\sigma(e^+e^- \to W^+W^-)$ as $s \to \infty$. If propagators and trilinear couplings are modified from their gauge-model values, the delicate cancellations will not occur and the behavior of $\sigma(e^+e^- \to W^+W^-)$ is substantially altered. This is not true for reactions such as $e^+e^- \to \gamma Z$ or $2Z$ since there are no trilinear couplings or virtual gauge bosons involved. Such reactions would only be sensitive to new couplings and not the modifications suggested here.

We consider modifying the matrix elements determined from the diagrams in Fig. 4 in two ways. As above, the diagram with the Z may have its propagator modified by the factor F, which we now call F_2:

$$\frac{1}{s-M_Z{}^2} \to \frac{1}{s-M_Z{}^2}F_2(s/\Lambda^2)\ . \tag{4.1}$$

In addition, the γW^+W^- vertex defined via

$$-e\Gamma_{\mu\nu\lambda}W^\mu W^\nu A^\lambda\ , \tag{4.2}$$

where $\Gamma_{\mu\nu\lambda}$ is the usual tensor structure[14] gets modified by a factor $F_1(s/\Lambda^2)$, i.e.,

$$\Gamma_{\mu\nu\lambda} \to \Gamma_{\mu\nu\lambda}F_1(s/\Lambda^2)\ . \tag{4.3}$$

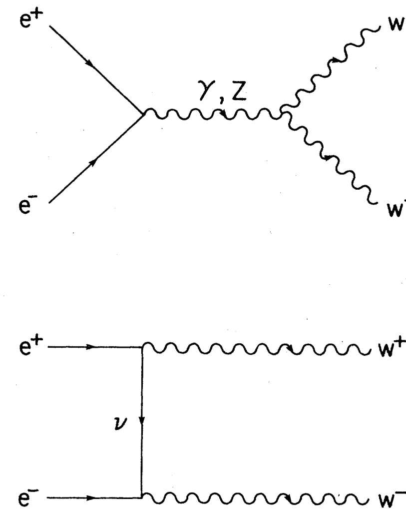

FIG. 4. Feynman diagrams for $e^+e^- \to W^+W^-$.

We will take the functional form of F_1 to be the same as F given above. In the SM $F_1=F_2=1$; we consider all three cases ($F_1 \neq 1$, $F_2=1$; $F_1=1$, $F_2 \neq 1$; $F_1 \neq 1$, $F_2 \neq 1$) of the possible modifications of the cross section. In general, $\Gamma_{\mu\nu\lambda}$ contains charge, magnetic moment, and electric quadruple couplings and *each* may have a form factor such as F_1 with different behavior and different values in the $s \to 0$ limit. This general situation is quite complicated and differences arising from their detailed behaviors become somewhat confused. To simplify this situation we take all these form factors to be identical and the static moments of the W to be the same as in the standard model. Following Alles, Boyer, and Buras[14] the $e^+e^- \to W^+W^-$ total cross section can be written now as

$$\sigma = \frac{\pi\alpha^2}{8x_W{}^2}\beta\frac{1}{s}\sum_{ij}\bar{\sigma}_{ij}\ , \tag{4.4}$$

where

$$\begin{aligned}
&\bar{\sigma}_{\nu\nu}=\sigma_1,\quad \bar{\sigma}_{\gamma\gamma}=x_W{}^2F_1{}^2\sigma_2\ ,\\
&\bar{\sigma}_{ZZ}=(x_W{}^2-\tfrac{1}{2}x_W+\tfrac{1}{8})\frac{s^2}{(s-M_Z{}^2)^2}F_1{}^2F_2{}^2\sigma_2\ ,\\
&\bar{\sigma}_{Z\gamma}=(\tfrac{1}{2}-2x_W)x_W\left[\frac{s}{s-M_Z{}^2}\right]F_1{}^2F_2\sigma_2\ ,\\
&\bar{\sigma}_{\nu Z}=(x_W-\tfrac{1}{2})\left[\frac{s}{s-M_Z{}^2}\right]F_1F_2\sigma_3\ ,\\
&\bar{\sigma}_{\gamma\nu}=-x_WF_1\sigma_3,\quad \beta=(1-4M^2/s)^{1/2}\ ,
\end{aligned} \tag{4.5}$$

and (with M being the W mass)

$$\begin{aligned}
&\sigma_1=2(s/M^2)+\tfrac{1}{12}\beta^2(s/M^2)^2+4[(1-2M^2/s)L/\beta-1]\ ,\\
&\sigma_2=16\beta^2(s/M^2)+\tfrac{2}{3}\beta^2[(s/M^2)^2-4(s/M^2)+12]\ ,\\
&\sigma_3=16-32(M^2/s)L/\beta+8\beta^2(s/M^2)+\tfrac{1}{3}\beta^2(s/M^2)^2(1-2M^2/s)+4(1-2M^2/s)-16(M^2/s)^2L/\beta\ ,
\end{aligned} \tag{4.6}$$

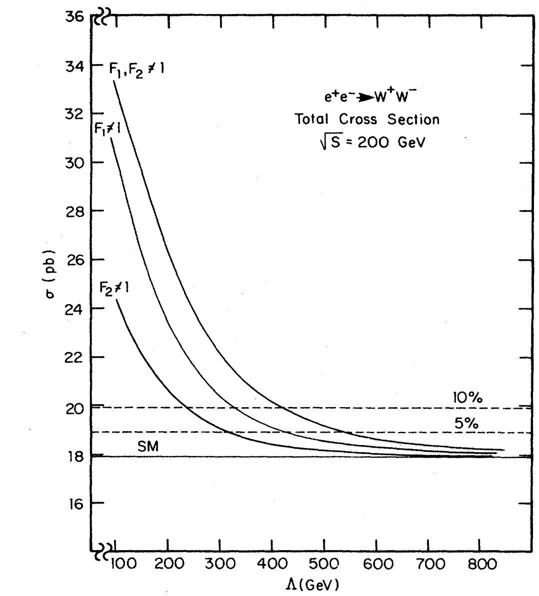

FIG. 5. Cross section for $e^+e^- \to W^+W^-$ at $\sqrt{s} = 200$ GeV as a function of Λ compared to the SM result.

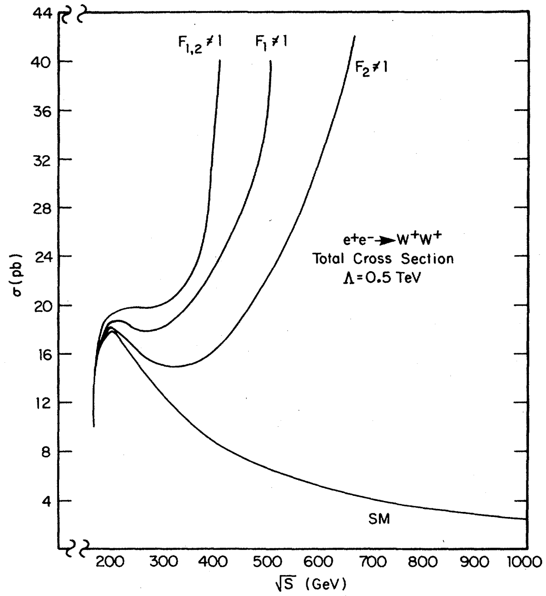

FIG. 7. Cross sections for $e^+e^- \to W^+W^-$ for $\Lambda = 0.5$ TeV as a function of $\sqrt{s}$.

with

$$L = \ln \left| \frac{1+\beta}{1-\beta} \right| .$$

$\bar{\sigma}_{ij}$ are "reduced" cross sections coming from the ij terms in the matrix element.

With LEP II it will be possible to exceed the threshold value for this process at $\sqrt{s} \simeq 165$ GeV so that W^+W^- pair production should be observable. Before examining how our modification alters the SM behavior of σ let us look at $\sigma(\Lambda)$ for $\sqrt{s}$ fixed. Figures 5 and 6 show $\sigma(\Lambda)$ for $\sqrt{s} = 200$ and 300 GeV, respectively, for the SM and

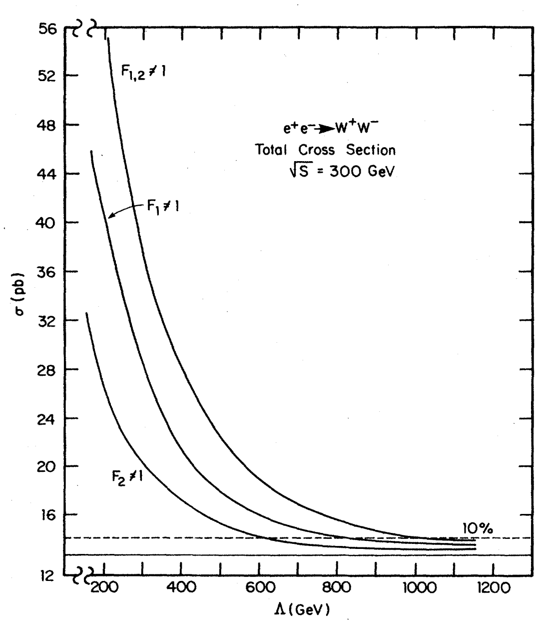

FIG. 6. Same as Fig. 5 but for $\sqrt{s} = 300$ GeV.

different behavior of F_1 and F_2. Note for this energy range $\sigma \simeq 10$—20 pb so that a significantly large number of W^+W^- pairs can be obtained with a reasonable luminosity.[15]

For $\sqrt{s} = 200$ GeV a 10% measurement of the total cross section would significantly constrain the value of Λ (assuming it agrees with the SM); the constraint on Λ depends numerically, however, on the values of $F_{1,2}$. For $F_1 = 1$, $F_2 \neq 1$ we see $\Lambda \gtrsim 0.23$ TeV while for $F_2 = 1$, $F_1 \neq 1$ we have $\Lambda \gtrsim 0.33$ TeV; the strongest constraint occurs when $F_1 = F_2 \neq 1$ wherein we find $\Lambda \gtrsim 0.42$ TeV. Note that all these limits are significantly improved if a measurement of σ at the 5% level can be made; indeed in this case we may be able to push Λ above 0.5 TeV assuming no deviations from the standard model are found.

At $\sqrt{s} = 300$ GeV, just beyond the energy range of LEP II, we see that even a 10% measurement significantly improves the limits on Λ and can push its value up into the 1-TeV region; of course, a more accurate measurement can do even better.

To examine how the total cross section behaves as both a function of $\sqrt{s}$ and Λ we turn to Figs. 7—9. Figure 7 shows the prediction for σ as a function of $\sqrt{s}$ for the SM as well as for $\Lambda = 0.5$ TeV with various assumptions about the nature of $F_{1,2}$. We see that already in the $\sqrt{s} = 200$—300-GeV range the cross section is significantly modified while for $\sqrt{s} > 300$ GeV σ for F_1 and/or $F_2 \neq 1$ rises with increasing $\sqrt{s}$. This rise is sharper than linear and becomes proportional to s for $s/M^2 \gg 1$. Even with $F_{1,2} \neq 1$ which tends to suppress the cross section the $\sigma \sim s$ behavior persists since the ν diagram is left unaltered by the inclusion of the $F_{1,2}$ factors and it alone gives $\sigma \sim s$ for large s. Once the $\sqrt{s}$ values of $\sqrt{s}$ surpasses 500 GeV or so we see that σ in all cases is much larger than the SM value.

Figure 8 shows σ as a function of $\sqrt{s}$ for $\Lambda = 0.8$ TeV;

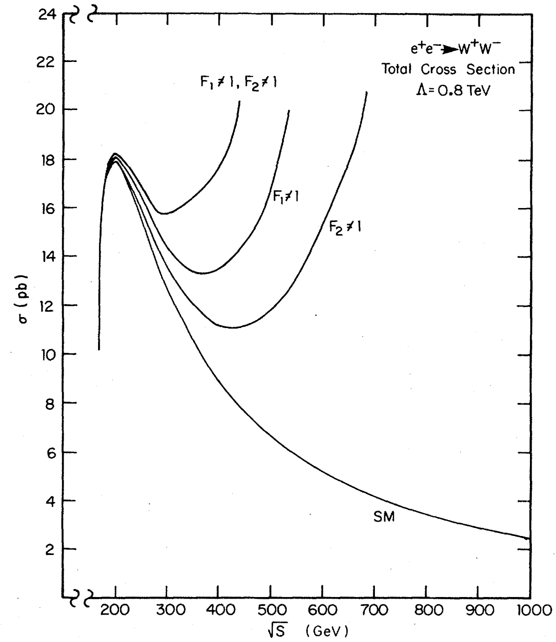

FIG. 8. Same as Fig. 7 with $\Lambda=0.8$ TeV.

the difference between $\Lambda=0.5$ and 0.8 TeV is quite significant. For $\Lambda=0.8$ TeV we see that large deviations from the SM are somewhat delayed in $\sqrt{s}$ compared to the case of $\Lambda=0.5$ TeV. In this case, as in the previous figures, the $F_{1,2}\neq 1$ possibility is most sensitive to finite Λ while the case $F_2\neq 1$ is least sensitive to Λ. For this latter case we only observe a 10% modification of the SM result when $\sqrt{s}$ exceeds $\simeq$350 GeV. In the LEP II energy range ($\sqrt{s}\simeq$200–250 GeV) a highly accurate measurement (say, 5%) is needed to observe any of the predicted deviations. A similar result is observed when we examine the case

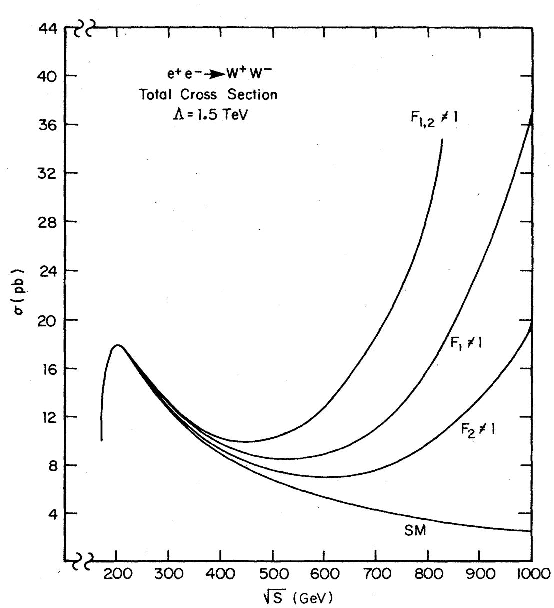

FIG. 9. Same as Fig. 7 with $\Lambda=1.5$ TeV.

$\Lambda=1.5$ TeV as seen in Fig. 9; again the case $F_{1,2}\neq 1$ is most sensitive to the finite Λ value. We see again that the Λ sensitivity is quite poor in the LEP II energy regime and we need to go to substantially large values of $\sqrt{s}$ to observe significant deviations from the SM prediction.

As we have seen, the reaction $e^+e^-\to W^+W^-$ at LEP II energies is very sensitive to the values of Λ, although at higher $\sqrt{s}$ values the increase in sensitivity grows rapidly; this is due to the tendency for σ to grow with s when the sensitive gauge-theory cancellations do not occur.

We have also examined the angular differential cross section for $e^+e^-\to W^+W^-$ in the $\sqrt{s}\simeq$250-GeV region and how it varies with Λ. We have found that $d\sigma/d\cos\theta$ is fairly flat in $\cos\theta$ for all Λ values and is no more sensitive to Λ variations than the total integrated cross section. Since accurate measurements of $d\sigma/d\cos\theta$ are more difficult than σ due to poor statistics, data on $d\sigma/d\cos\theta$ do not really help us to constrain the value of Λ beyond those obtainable directly from σ.

V. OTHER PROCESSES

Another reaction where one might see the modification we propose is in the Z decay $Z\to Hl^+l^-$ which is a source of Higgs bosons (H); the diagram for this process is shown in Fig. 10. Assuming no anomalous coupling (such as a direct $ZH\gamma$ coupling absent in gauge theories[16]) it is the deviation in the Z-boson propagator which is again the signal for compositeness. The decay rate for this process is proportional to the integral

$$\int_{\delta}^{(1/2)(1+\delta^2)} d\chi(\chi^2-\delta^2)^{1/2}\frac{3+2\delta^2-6\chi+\chi^2}{(\delta^2-2\chi)^2+(\Gamma_Z/M_Z)^2}\times\left[1+\lambda\frac{M_Z{}^2(1-2\chi)+M_H{}^2}{\Lambda^2}\right]^{-2}, \quad (5.1)$$

where $\delta=m_H/M_Z$, $\chi=E_H/M_Z$, and Γ_Z is the Z-boson width. The integral (5.1) with $\Lambda\to\infty$ is the usual SM result. Figure 11 shows the ratio $\Gamma(Z\to Hl^+l^-)/\Gamma_{SM}$ for $m_H=20$ and 50 GeV (for both $\lambda=+1$ and -1) as a function of the Z composite scale Λ. Note that the ratio Γ/Γ_{SM} is quite sensitive to m_H and that the deviation from unity decreases as $\delta\to 1$. We also see that, for m_H fixed Γ/Γ_{SM} is more sensitive to finite Λ where $\lambda=-1$; clearly, this is what we would expect from the form (5.1).

Since the branching ratio in the SM for this process is a few $\times 10^{-5}$, over several years of running at LEP, we would expect, perhaps $\simeq$100–200 events of this kind. It

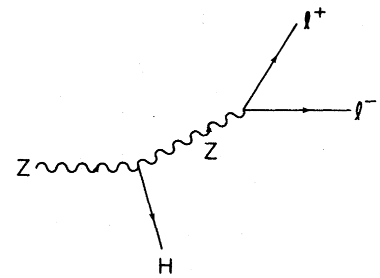

FIG. 10. Feynman diagram for $Z\to Hl^+l^-$.

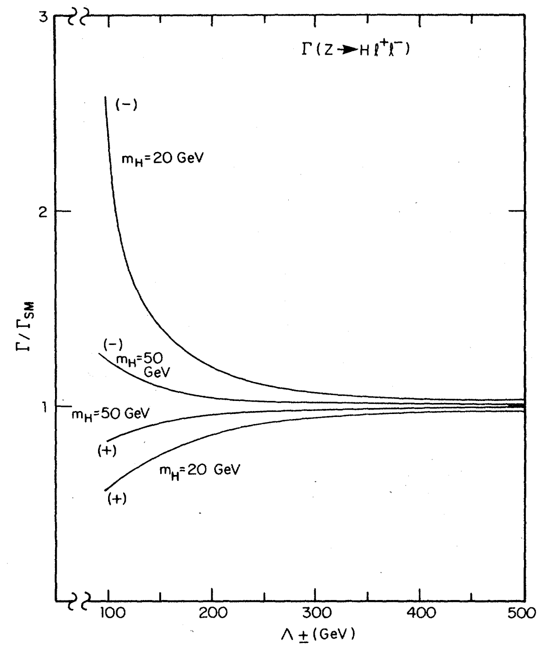

FIG. 11. $\Gamma(Z\to Hl^+l^-)$ relative to its SM prediction for $m_H=20$ or 50 GeV as a function of Λ with $\lambda=\pm 1$.

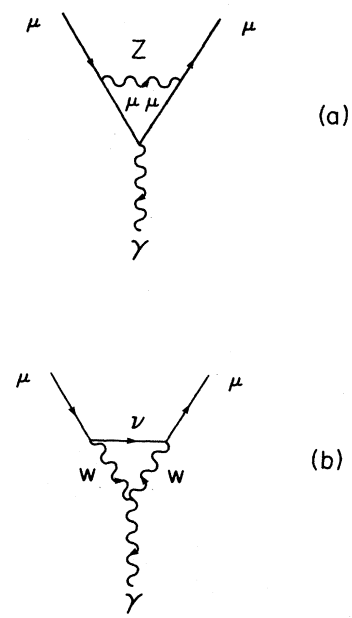

FIG. 12. Feynman diagrams contributing to the muon $g-2$ in the standard model.

does not seem unreasonable then that the decay rate for $Z\to Hl^+l^-$ will be determined at the 10—15 % level. For $\lambda=1$ and $m_H\gtrsim 50$ GeV this would not lead to any further constraint on Λ; for $m_H\simeq 20$ GeV, Λ would be increased to $\simeq 200$ GeV assuming no deviation from the SM were observed. For $\lambda=-1$ we would again find that for reasonably heavy Higgs bosons ($m_H\gtrsim 50$ GeV) there would be no real improvement in the constraint on Λ. However, if $m_H\simeq 20$ GeV or so we could again push Λ above 200 GeV but not much further. It seems unlikely that the statistics could be much improved (to, say, the few percent level) to radically alter these bounds. Apparently, the $Z\to Hl^+l^-$ process is not a good place to look for Z substructure unless one is looking for anomalous couplings.

The last process we will examine is the $g-2$ of the μ; the weak contribution is given by the diagrams of Fig. 12. For $\sin^2\theta_W=0.220$ the contribution of each diagram is given by[17]

$$\mu^{\rm wk}_{g-2}|_{\rm a}=\frac{G_F m_\mu^{\,2}}{6\sqrt{2}\pi^2}(-1-x_W+x_W^{\,2})\simeq -18.22\times 10^{-10}\,,$$
$$\mu^{\rm wk}_{g-2}|_{\rm b}=\frac{5G_F m_\mu^{\,2}}{12\sqrt{2}\pi^2}\simeq 38.87\times 10^{-10}\,, \tag{5.2}$$

which together yield the usual SM result. Present experimental data[6] constrains any extra contribution (Δ) beyond that of the SM to be in the range

$$-46\times 10^{-10}\le \Delta\le 122\times 10^{-10}\,. \tag{5.3}$$

In order to get a rough limit on Λ from the constraint (5.3) we ask how the above propagator and trilinear form factors would modify the SM predictions. A crude estimate can be obtained by cutting off the modified propagator contribution at the respective gauge-boson mass; this would yield (in the notation from above)

$$\Delta=[38.87(F_1F_2^{\,2}-1)_W-18.22(F_2-1)_Z]\times 10^{-10} \tag{5.4}$$

with

$$(F_{1,2})_W=(1+M_W^{\,2}/\Lambda^2)^{-1} \tag{5.5}$$

and corresponding expressions for $(F_{1,2})_Z$. Numerically, for $\Lambda=100$ GeV we find

$$(F_2-1)_Z=\begin{cases}0, & F_2=1,\\ -0.46, & F_2\neq 1\,,\end{cases}$$
$$(F_1F_2^{\,2}-1)_W=\begin{cases}0, & F_1=F_2=1,\\ -0.40, & F_2=1,\ F_1\neq 1,\\ -0.79, & F_{1,2}\neq 1\,.\end{cases} \tag{5.6}$$

Note that in all cases this estimate of Δ gives a value well inside the range (5.3); this is partly due to the cancellation between the two contributions in (5.4). If this estimate is at all correct, it clearly indicates that the present uncertainties in Δ are sufficiently large to allow for a gauge-boson composite scale Λ of under 100 GeV. A fairly significant improvement on the bounds on Δ (by a factor of a few) would be needed before the muon $g-2$ would give any real restriction on the value of Λ. We thus conclude that by an order-of-magnitude estimate finite-Λ effects in muon $g-2$ are not yet visible so that as far as this process is considered $\Lambda=100$ GeV is completely consistent with present data.

IV. CONCLUSIONS

We have examined a large number of processes in order to constrain any possible gauge-boson composite scale Λ; a value of Λ as low as 100 GeV is apparently consistent with all currently available data. This result assumes that there are no new couplings associated with the scale Λ which lead to processes not allowed (at the tree level) in the standard model.

When present experiments are extended to higher-energy regimes—particularly in e^+e^- reactions—it may be possible to either greatly improve the bound on the composite scale or observe some deviations from the SM. We have found that the reaction $e^+e^-\to W^+W^-$ is most sensitive to the kinds of modifications of the SM discussed here since its larger-s behavior is so sensitive to cancellations among the various contributions.

Our last comment concerns the simple choice of the propagator and vertex modification made in our analysis for the composite W and Z. Earlier the modifications of the composite-ζ-meson propagator by Hammer and coworkers[18,19] indicate that a simple, crude approximation to the exact form found by the authors is given by Eq. (2.2). Hence we have some hope that *below* the scale Λ our form-factor modification will yield an estimate of the scale Λ.

Because the scale Λ for gauge bosons is possibly so small compared to the present limits on the fermion composite size, a detailed analysis of the properties of gauge bosons may yield the first signal of compositeness.

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