Mass spectrum of chiral ten-dimensional $N=2$ supergravity on S^5

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We discuss the spontaneous compactification of chiral $N=2$ ten-dimensional supergravity from ten to five dimensions on S^5. Harmonic analysis on S^5 is used to compute the complete mass spectrum. Our results indicate that scalars and spinors in different SO(6) multiplets have different masses, even within the "massless" supermultiplet. We show that the conformal diffeomorphisms, which remain after imposing certain covariant gauge conditions for the general coordinate invariance, can be used to gauge away twice as many modes as there are gauge parameters. A doubleton multiplet of pure gauge modes is identified, and all modes in the massless supermultiplet lie at the beginning of infinite towers of modes.

I. INTRODUCTION

Chiral $N=2$ supergravity in ten dimensions[1,2] is an interesting alternative to the eleven-dimensional supergravity theory. Both are fundamental maximal supergravities in the sense that neither is derivable from a higher-dimensional theory (in any known way). All other maximal supergravities, gauged or ungauged, are believed to be derivable from one or both of these theories by a process of compactification and truncation.

Unlike the $d=11$ theory, this ten-dimensional theory, along with the nonchiral $N=2$ and various (chiral) $N=1$ models in ten dimensions, is obtainable as the zero-slope limit of a superstring theory.[3] It seems likely that this model, as well as its compactifications, will play a role in the study of the connections between string theories and conventional field theories.

Moreover, this model has the virtue of being a chiral theory without gravitational anomalies.[4] The absence of minimal coupling in the ten-dimensional theory is not encouraging for the emergence of chiral fermions in the $d=4$ spectrum,[5] even if a compactification to $d=4$ were to be found, although topologically nontrivial configurations of the tensor field might lead to surprises (see, e.g., Refs. 5 and 6).

The chiral ten-dimensional theory includes the graviton, a complex scalar B, a complex two-index antisymmetric tensor $A_{\hat{\mu}\hat{\nu}}$, and a real four-index antisymmetric tensor $A_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ whose five-index field strength is self-dual. The fermionic sector consists of a chiral complex gravitino $\hat{\psi}_{\hat{\mu}}$ and a chiral complex spinor $\hat{\lambda}$ of opposite chirality. Carets denote ten-dimensional quantities. For further conventions, see Ref. 7.

It was noted in Ref. 1 that one can compactify this model to $d=5$ using the $d=10$ analog of the $d=11$ Freund-Rubin ansatz,[8]

$$F_{\mu\nu\rho\sigma\tau}=e\epsilon_{\mu\nu\rho\sigma\tau},\quad F_{\alpha\beta\gamma\delta\epsilon}=e\epsilon_{\alpha\beta\gamma\delta\epsilon}\ , \tag{1.1}$$

where the parameter e is an arbitrary overall mass scale for the compactification.

Assuming that only the four-index tensor and the metric are nonvanishing in the background, the Einstein equations read

$$R_{\mu\nu}=4e^2g_{\mu\nu},\quad R_{\alpha\beta}=-4e^2g_{\alpha\beta} \tag{1.2}$$

while all other field equations are automatically satisfied. In this paper we are concerned with the maximally symmetric solution of (1.2), in which $g_{\mu\nu}$ describes a five-dimensional anti-de Sitter spacetime (AdS^5) and $g_{\alpha\beta}$ describes the five-sphere S^5, both of radius e^{-1}:

$$\begin{aligned}R_{\mu\nu\rho\sigma}&=-e^2(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})\ ,\\ R_{\alpha\beta\gamma\delta}&=e^2(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma})\ .\end{aligned} \tag{1.3}$$

Other compactifications are discussed in Ref. 9.

The model raises some interesting questions. The first question concerns the vector fields. The ungauged maximal ($N=8$) $d=5$ model[10] has 27 massless Abliean vectors while compactification on S^5 is expected to yield only 15 SO(6) Yang-Mills fields. Where have the remaining 12 vectors gone? As we shall see, they have been replaced by a complex sextet of antisymmetric tensor fields satisfying field equations of self-dual type.[11] Another question concerns the $d=4$ singleton[12] and $d=7$ doubleton[13] supermultiplets. These multiplets form unitary irreducible representations of the pertinent superalgebras, but there are no corresponding propagating modes in the $d=4$ and $d=7$ theories. Does a similar phenomenon occur for the $d=5$ doubleton[14] in this model?

Perhaps most interesting is the question of masslessness. We recall that in $d=4$ the scalars and spinors in the "massless" supermultiplet (the supermultiplet containing the massless graviton, gravitinos, and vectors, i.e., the gauge fields) satisfy a conformally invariant field equation, which has often been "explained" by asserting that they are massless and thus must propagate on the light cone. In $d=7$, however, it was found in several models[15,16] that the scalars in the same supermultiplet as the massless graviton do not have a conformal field equation, although for different models these field equations are still the same (to linear order). Here we will find a further surprise: scalars and spinors in different SO(6) representations, but all within the massless supermultiplet, have different mass terms.

Closely related to the doubleton issue is a subtlety concerning the removal of modes by fixing the general coordinate gauges. After imposing de Donder-type gauge choices, one is still left with a residual gauge symmetry whose spherical harmonics are conformal scalars[17] (scalar harmonics Y on spheres for which $D_\alpha D_\beta Y$ is proportional to $g_{\alpha\beta}Y$). In the sector of these conformal scalars one can either algebraically eliminate certain nonpropagating fields, or remove them by a conformal gauge choice. Counting seems to present a problem, because in the latter procedure one ends up with more modes than in the former. The resolution we will discuss is that one must use the gauge freedom twice to eliminate fields, as in electromagnetism.

Based on the results of this article and of Refs. 16 and 14, the maximal $d=5$ gauged supergravity has been constructed in the meantime.[18,19] This allows one to study its potential and its critical points.

Our work makes contact with the group-theoretical analysis of Ref. 14 insofar as the entire set of modes we find fit precisely into the supermultiplets of Ref. 14. In addition, however, we have obtained the spectrum of masses.

The article is organized as follows. In Sec. II we determine the bosonic mass spectrum while in Sec. III the fermionic mass spectrum is deduced. In four figures some of the results are given, while our final results are summarized in Table III.

II. THE BOSONIC MASS SPECTRUM

In this section we will determine the bosonic modes. We shall first determine the bosonic field equations linearized in excitations. Then we shall expand the bosonic excitations into spherical harmonics, and choose covariant gauge conditions which will reduce these harmonic expansions to a very simple form. Next we shall insert these simple harmonic expansions into the linearized $d=10$ field equations, thus obtaining a set of coupled $d=5$ field equations. After diagonalizing these, we will end up with the bosonic spectrum.

An important aspect concerns the remaining gauge freedom which is still allowed by the gauge conditions. We shall elucidate the connection between the remaining conformal invariance of Refs. 17 and 20, and the disappearance of the scalar modes which are present in the singleton and doubleton representations of the relevant superalgebras.[12,13]

We shall begin by treating the metric and the self-dual tensor, since their modes are decoupled from the other modes. Afterwards we shall analyze the modes contained in $A_{\mu\nu}$ and B, which are decoupled from any other modes.

The bosonic field equations, linearized in excitations, read[1]

$$R_{\hat\mu\hat\nu}=-\tfrac{1}{6}F_{\hat\mu\hat\rho\hat\sigma\hat\tau\hat\kappa}F_{\hat\nu}{}^{\hat\rho\hat\sigma\hat\tau\hat\kappa}\,, \tag{2.1}$$

$$F_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau}=\frac{1}{5!}\epsilon_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau\hat\mu'\hat\nu'\hat\rho'\hat\sigma'\hat\tau'}F^{\hat\mu'\hat\nu'\hat\rho'\hat\sigma'\hat\tau'}\,, \tag{2.2}$$

$$D^{\hat\mu}\partial_{[\hat\mu}A_{\hat\nu\hat\rho]}=-\frac{2i}{3}\dot F_{\hat\nu\hat\rho\hat\sigma\hat\tau\hat\kappa}D^{\hat\sigma}A^{\hat\tau\hat\kappa}\,, \tag{2.3}$$

$$D^{\hat\mu}\partial_{\hat\mu}B=0\,. \tag{2.4}$$

The covariant derivatives contain only the background metric, the curl F has strength five, and we have put the gravitational coupling constant κ equal to one. The square brackets denote antisymmetrization with strength one. After choosing the scale of F such that (2.1) holds, the only nontrivial constant is the $2i/3$ in (2.3).

Let us begin with the Einstein equations. There are three cases to be considered, namely, the cases with $R_{\mu\nu}$, $R_{\mu\alpha}$, and $R_{\alpha\beta}$. We shall denote the corresponding equations by (E1), (E2), and (E3), respectively. The fluctuations of the gravitational field $g_{\hat\mu\hat\nu}$ are parametrized as follows:

$$\begin{aligned} &g_{\mu\nu}=\dot g_{\mu\nu}+h_{\mu\nu},\quad g_{\mu\alpha}=h_{\mu\alpha}\,,\\ &g_{\alpha\beta}=\dot g_{\alpha\beta}+h_{\alpha\beta}\,,\\ &h_{\mu\nu}=h'_{\mu\nu}-\tfrac{1}{3}\dot g_{\mu\nu}h^\alpha{}_\alpha\,. \end{aligned} \tag{2.5}$$

The fields $\dot g_{\mu\nu}$ and $\dot g_{\alpha\beta}$ are the background fields, and $h^\alpha{}_\alpha$ is equal to $h_{\alpha\beta}\dot g^{\alpha\beta}$, while $h'_{\mu\nu}$ is related to $h_{\mu\nu}$ by a linearized $d=5$ Weyl shift. [Note that for d spacetime dimensions, the Weyl shift is proportional to $(d-2)^{-1}$.] We will need the expansion of the ten-dimensional Ricci tensor in terms of fluctuations. It reads

$$\begin{aligned} R_{\hat\mu\hat\nu}={}&\dot R_{\hat\mu\hat\nu}+\tfrac{1}{2}(\Box_x+\Box_y)h_{\hat\mu\hat\nu}\\ &+\tfrac{1}{2}D_{\hat\mu}D_{\hat\nu}h_{\hat\rho}{}^{\hat\rho}-\tfrac{1}{2}D_{\hat\mu}D^{\hat\rho}h_{\hat\rho\hat\nu}-\tfrac{1}{2}D_{\hat\nu}D^{\hat\rho}h_{\hat\rho\hat\mu}\\ &-R_{\hat\mu\hat\rho\hat\sigma\hat\nu}h^{\hat\rho\hat\sigma}+\tfrac{1}{2}R_{\hat\mu}{}^{\hat\rho}h_{\hat\nu\hat\rho}+\tfrac{1}{2}R_{\hat\nu}{}^{\hat\rho}h_{\hat\mu\hat\rho}\,. \end{aligned} \tag{2.6}$$

Inserting (2.6) and (2.5) into (2.1) gives the linearized Einstein field equations. They are given in Table I.

Next we expand the fields $h'_{\mu\nu}(x,y)$, $h_{\mu\alpha}(x,y)$, and $h_{\alpha\beta}(x,y)$ into a complete set of spherical harmonics as follows:

$$h'_{\mu\nu}=\sum H_{\mu\nu}^{I_1}(x)Y^{I_1}(y)\,, \tag{2.7}$$

$$h_{\mu\alpha}=\sum\,[B_\mu^{I_5}(x)Y_\alpha^{I_5}(y)+B_\mu^{I_1}(x)D_\alpha Y^{I_1}(y)]\,, \tag{2.8}$$

$$\begin{aligned} h_{(\alpha\beta)}=\sum\,[&\phi^{I_{14}}(x)Y_{(\alpha\beta)}^{I_{14}}(y)+\phi^{I_5}(x)D_{(\alpha}Y_{\beta)}^{I_5}(y)\\ &+\phi^{I_1}(x)D_{(\alpha}D_{\beta)}Y^{I_1}(y)]\,, \end{aligned} \tag{2.9}$$

$$h_\alpha{}^\alpha=\sum\pi^{I_1}(x)Y^{I_1}(y)\,. \tag{2.10}$$

The symbol $(\alpha\beta)$ means that this index pair is sym-

TABLE I. Linearized field equations for $g_{\hat{\mu}\hat{\nu}}$ and $A_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$.

$$\tfrac{1}{2}(\Box_x+\Box_y+2e^2)h'_{\mu\nu}+3e^2\dot{g}_{\mu\nu}h'^{\sigma}_{\sigma}-\tfrac{1}{2}\nabla_\mu\nabla^\rho h'_{\nu\rho}-\tfrac{1}{2}\nabla_\nu\nabla^\rho h'_{\mu\rho}+\tfrac{1}{2}\nabla_\mu\nabla_\nu h'^{\sigma}_{\sigma}$$
$$-\tfrac{1}{2}\nabla_\mu\nabla^\alpha h_{\nu\alpha}-\tfrac{1}{2}\nabla_\nu\nabla^\alpha h_{\mu\alpha}-\tfrac{1}{6}\dot{g}_{\mu\nu}(\Box_x+\Box_y)h^{\gamma}_{\gamma}-\tfrac{16}{3}e^2\dot{g}_{\mu\nu}h^{\gamma}_{\gamma}=-\tfrac{1}{3}e\dot{g}_{\mu\nu}\epsilon^{\mu'\nu'\rho\sigma\tau}\partial_{\mu'}a_{\nu'\rho\sigma\tau} \quad \text{(E1)}$$

$$\tfrac{1}{2}(\Box_x+\Box_y)h_{\mu\alpha}-\tfrac{1}{2}\nabla_\mu\nabla^\rho h_{\alpha\rho}-\tfrac{1}{2}\nabla_\alpha\nabla^\rho h'_{\mu\rho}-\tfrac{4}{15}\nabla_\mu\nabla_\alpha h^{\gamma}_{\gamma}+\tfrac{1}{2}\nabla_\mu\nabla_\alpha h'^{\sigma}_{\sigma}-\tfrac{1}{2}\nabla_\mu\nabla^\beta h_{(\alpha\beta)}-\tfrac{1}{2}\ \nabla_\alpha\nabla^\beta h_{\mu\beta}$$
$$=-\frac{e}{6}\epsilon_\mu{}^{\nu\rho\sigma\tau}(\partial_\alpha a_{\nu\rho\sigma\tau}-4\partial_\nu a_{\alpha\rho\sigma\tau})-\frac{e}{6}\epsilon_\alpha{}^{\beta\gamma\delta\epsilon}(\partial_\mu a_{\beta\gamma\delta\epsilon}-4\partial_\beta a_{\mu\gamma\delta\epsilon}) \quad \text{(E2)}$$

$$\tfrac{1}{2}(\Box_x+\Box_y-2e^2)h_{(\alpha\beta)}+\tfrac{1}{10}\dot{g}_{\alpha\beta}(\Box_x+\Box_y-32e^2)h^{\gamma}_{\gamma}-\tfrac{8}{15}\nabla_\alpha\nabla_\beta h^{\gamma}_{\gamma}+\tfrac{1}{2}\nabla_\alpha\nabla_\beta h'^{\sigma}_{\sigma}$$
$$-\tfrac{1}{2}\nabla_\alpha\nabla^\rho h_{\beta\rho}-\tfrac{1}{2}\nabla_\beta\nabla^\rho h_{\alpha\rho}-\tfrac{1}{2}\nabla_\alpha\nabla^\gamma h_{(\beta\gamma)}-\tfrac{1}{2}\nabla_\beta\nabla^\gamma h_{(\alpha\gamma)}=-\tfrac{1}{3}e\dot{g}_{\alpha\beta}\epsilon^{\alpha'\beta'\gamma\delta\epsilon}\partial_{\alpha'}a_{\beta'\gamma\delta\epsilon} \quad \text{(E3)}$$

$$5\partial_{[\mu}a_{\nu\rho\sigma\tau]}=\frac{5}{5!}\epsilon_{\mu\nu\rho\sigma\tau}{}^{\alpha\beta\gamma\delta\epsilon}\partial_\alpha a_{\beta\gamma\delta\epsilon}+\frac{e}{2}(h'^{\sigma}_{\sigma}-\tfrac{8}{3}h^{\gamma}_{\gamma})\epsilon_{\mu\nu\rho\sigma\tau} \quad \text{(M1)}$$

$$\partial_\alpha a_{\mu\nu\rho\sigma}+4\partial_{[\mu}a_{\nu\rho\sigma]\alpha}=\frac{5}{5!}\epsilon_{\mu\nu\rho\sigma\alpha}{}^{\tau\beta\gamma\delta\epsilon}(\partial_\tau a_{\beta\gamma\delta\epsilon}+4\partial_\beta a_{\gamma\delta\epsilon\tau})+e\,\epsilon_{\mu\nu\rho\sigma}{}^{\tau}h_{\alpha\tau} \quad \text{(M2)}$$

$$3\partial_{[\mu}a_{\nu\rho]\alpha\beta}+2\partial_{[\alpha}a_{\beta]\mu\nu\rho}=\frac{10}{5!}\epsilon_{\mu\nu\rho\alpha\beta}{}^{\sigma\tau\gamma\delta\epsilon}(3\partial_\gamma a_{\delta\epsilon\sigma\tau}+2\partial_\sigma a_{\tau\gamma\delta\epsilon}) \quad \text{(M3)}$$

metrized with the trace removed. Let us now impose the following de Donder and Lorentz-type gauge conditions:

$$D^\alpha h_{(\alpha\beta)}=0,\quad D^\alpha h_{\alpha\mu}=0\ . \tag{2.11}$$

Under diffeomorphisms, one has $\delta h_{\hat{\mu}\hat{\nu}}=D_{\hat{\mu}}\xi_{\hat{\nu}}+D_{\hat{\nu}}\xi_{\hat{\mu}}$, and by expanding $\xi_{\hat{\mu}}$ into spherical harmonics, it becomes clear that one can gauge away all x-space fields which correspond to gradients of spherical harmonics in (2.8) and (2.9). This yields

$$h'_{\mu\nu}=\sum H^{I_1}_{\mu\nu}(x)Y^{I_1}(y),\quad h_{\mu\alpha}=\sum B^{I_5}_{\mu}(x)Y^{I_5}_{\alpha}(y)\ ,$$
$$h_{(\alpha\beta)}=\sum\phi^{I_{14}}(x)Y^{I_{14}}_{(\alpha\beta)}(y),\quad h^{\alpha}{}_{\alpha}=\sum\pi^{I_1}(x)Y^{I_1}(y)\ . \tag{2.12}$$

Those diffeomorphisms which respect (2.11) are given by

$$D_{(\alpha}\xi_{\beta)}+D_{(\beta}\xi_{\alpha)}=0\ \text{ and }\ D^\alpha(D_\alpha\xi_\mu+D_\mu\xi_\alpha)=0\ . \tag{2.13}$$

They consist of (i) ordinary Yang-Mills symmetries for which $\xi_\beta=\lambda^I(x)Y^I_\beta$ with Y^I_β equal to Killing vectors, (ii) ordinary diffeomorphisms for which $\xi_\mu=\xi_\mu(x)$, and finally (iii) what have been called conformal diffeomorphisms[17]

$$\xi_\alpha=k^I(x)D_\alpha Y^I(y),\quad \xi_\mu=-\partial_\mu k^I(x)Y^I(y)\ , \tag{2.14}$$

where $Y^I(y)$ are the $k=1$ scalars which satisfy $D_{(\alpha}D_{\beta)}Y^I=0$. These three classes of diffeomorphisms also respect the form of (2.12) although the x-space coefficient fields will, in general, be transformed. The appearance of the extra conformal diffeomorphisms is not surprising, because in (2.9) the terms with $D_{(\alpha}D_{\beta)}Y^{I_1}$ cancel when Y^{I_1} is a $k=1$ scalar harmonic, so that no gauge parameter need be fixed to eliminate these modes. We shall come back to the role of these conformal diffeomorphisms later.

We now repeat the above analysis for the antisymmetric tensor $A_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$. We decompose it into background values $\dot{A}_{\mu\nu\rho\sigma}(x)$ and $\dot{A}_{\alpha\beta\gamma\delta}(y)$ and fluctuations $a_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}(x,y)$. We choose the following covariant gauge conditions

$$D^\alpha a_{\alpha\beta\gamma\delta}=D^\alpha a_{\alpha\beta\gamma\mu}=D^\alpha a_{\alpha\beta\mu\nu}=D^\alpha a_{\alpha\mu\nu\rho}=0\ . \tag{2.15}$$

Again, these conditions remove terms with gradients from the spherical harmonics of the various fluctuations. However, on S^5 the transverse-traceless $Y_{\alpha\beta\gamma}$ and $Y_{\alpha\beta\gamma\delta}$ can be expressed in terms of $D_\delta Y_\epsilon$ and $D_\epsilon Y$, respectively, and one ends up with

$$\begin{aligned}
a_{\mu\nu\rho\sigma}&=\sum b^{I_1}_{\mu\nu\rho\sigma}(x)Y^{I_1}(y)\ ,\\
a_{\mu\nu\rho\alpha}&=\sum b^{I_5}_{\mu\nu\rho}(x)Y^{I_5}_{\alpha}(y)\ ,\\
a_{\mu\nu\alpha\beta}&=\sum b^{I_{10}}_{\mu\nu}(x)Y^{I_{10}}_{[\alpha\beta]}(y)\ ,\\
a_{\mu\alpha\beta\gamma}&=\sum\phi^{I_5}_{\mu}(x)\epsilon_{\alpha\beta\gamma}{}^{\delta\epsilon}D_\delta Y^{I_5}_{\epsilon}(y)\ ,\\
a_{\alpha\beta\gamma\delta}&=\sum b^{I_1}(x)\epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon}D_\epsilon Y^{I_1}(y)\ .
\end{aligned} \tag{2.16}$$

The Y and Y_α harmonics are eigenfunctions of the Hodge-de Rham operator Δ, but $Y_{[\alpha\beta]}$ are, in addition, eigenfunctions of $\epsilon_{\alpha\beta}{}^{\gamma\delta\epsilon}D_\gamma Y_{[\delta\epsilon]}$.

The self-duality equations in (2.2) split into three pairs of equations, where the two equations of a given pair are equivalent. Hence, one obtains from (2.2) three independent field equations, which we will call (M1), (M2), and (M3), respectively, and which appear in Table I also.

Having obtained the linearized field equations, we substitute the expansions of the graviton and four-index tensor, and collect in each field equation the coefficients of a given spherical harmonic. In this way one obtains the results in Table II where we have denoted those equations which follow from (E1) by (E1.1), (E1.2), etc. Let us first consider the M equations in Table II. From (M2.2) and (M3.2) we *algebraically* eliminate $b^{I_1}_{\mu\nu\rho\sigma}(x)$ and $b^{I_5}_{\mu\nu\rho}(x)$, respectively. Substituting the results into (M1) and (M2.1), respectively, one finds the following three equations which summarize the content of (2.2):

$$\left[(\Box_x+\Box_y)b^{I_1}+\frac{e}{2}H^{I_1}_{\sigma\sigma}-\frac{4e}{3}\pi^{I_1}\right]Y^{I_1}=0\quad(I_1\neq0)\ , \tag{2.17}$$

$$[(\mathrm{Max}+\Delta_y)\phi^{I_5}_{\mu}-eB^{I_5}_{\mu}]Y^{I_5}_{\alpha}=0\ , \tag{2.18}$$

$$\left[3D_{[\mu}b^{I_{10},\pm}_{\nu\rho]}\mp\frac{i}{2}\epsilon_{\mu\nu\rho}{}^{\sigma\tau}b^{I_{10},\pm}_{\sigma\tau}\sqrt{-\Delta_y}\right]Y^{I_{10},\pm}_{[\alpha\beta]}=0\ . \tag{2.19}$$

TABLE II. Linearized field equations for various fields from $g_{\hat{\mu}\hat{\nu}}$ and $A_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$.

$[\frac{1}{2}(\Box_x+\Box_y)H^{I_1}_{(\mu\nu)}-D_{(\mu}D^{\lambda}H^{I_1}_{\nu)\lambda}+\frac{1}{2}D_{(\mu}D_{\nu)}H^{I_1}_{\lambda\lambda}+e^2H^{I_1}_{(\mu\nu)}]Y^{I_1}=0$	(E1.1)
$\left[-\frac{1}{5}D^{\rho}D^{\sigma}H^{I_1}_{\rho\sigma}+\frac{1}{10}(2\Box_x+\Box_y+2e^2)H^{I_1}_{\lambda\lambda}+3e^2H^{I_1}_{\lambda\lambda}-\frac{1}{6}(\Box_x+\Box_y+32e^2)\pi^{I_1}+\frac{e}{3}\epsilon^{\mu\nu\rho\sigma\tau}\partial_{\mu}b^{I_1}_{\nu\rho\sigma\tau}\right]Y^{I_1}=0$	(E1.2)
$\left[\frac{1}{2}(\mathrm{Max}+\Delta_y)B^{I_5}_{\mu}+\frac{2e}{3}\epsilon_{\mu}{}^{\nu\rho\sigma\tau}\partial_{\nu}b^{I_5}_{\rho\sigma\tau}+4e\Delta_y\phi^{I_5}_{\mu}\right]Y^{I_5}_{\alpha}=0$	(E2.1)
$\left[-\frac{1}{2}D^{\rho}H^{I_1}_{\rho\mu}+\frac{1}{2}D_{\mu}H^{I_1}_{\rho\rho}-\frac{4}{15}D_{\mu}\pi^{I_1}+4eD_{\mu}b^{I_1}+\frac{e}{6}\epsilon_{\mu}{}^{\nu\rho\sigma\tau}b^{I_1}_{\nu\rho\sigma\tau}\right]D_{\alpha}Y^{I_1}=0$	(E2.2)
$\frac{1}{2}(\Box_x+\Box_y-2e^2)\phi^{I_{14}}Y^{I_{14}}_{(\alpha\beta)}=0$	(E3.1)
$(\frac{1}{2}H^{I_1}_{\mu\mu}-\frac{8}{15}\pi^{I_1})D_{(\alpha}D_{\beta)}Y^{I_1}=0$	(E3.2)
$(D^{\mu}B^{I_5}_{\mu})D_{(\alpha}Y^{I_5}_{\beta)}=0$	(E3.3)
$[(\frac{1}{10}(\Box_x+\Box_y)-\frac{16}{5}e^2-\frac{8}{75}\Box_y)\pi^{I_1}+\frac{1}{10}\Box_yH^{I_1}_{\mu\mu}+8e\Box_y b^{I_1}]Y^{I_1}=0$	(E3.4)
$\left[5\partial_{[\mu}b^{I_1}_{\nu\rho\sigma\tau]}-\epsilon_{\mu\nu\rho\sigma\tau}\left[\frac{e}{2}H^{I_1}_{\lambda\lambda}-\frac{4e}{3}\pi^{I_1}+\Box_y b^{I_1}\right]\right]Y^{I_1}=0$	(M1)
$[4\partial_{[\mu}b^{I_5}_{\nu\rho\sigma]}+\epsilon_{\mu\nu\rho\sigma}{}^{\tau}(\Delta_y\phi^{I_5}_{\tau}-eB^{I_5}_{\tau})]Y^{I_5}_{\alpha}=0$	(M2.1)
$[b^{I_1}_{\mu\nu\rho\sigma}+\epsilon_{\mu\nu\rho\sigma}{}^{\tau}D_{\tau}b^{I_1}]D_{\alpha}Y^{I_1}=0$	(M2.2)
$3\partial_{[\mu}b^{I_{10}}_{\nu\rho]}Y^{I_{10}}_{[\alpha\beta]}-\frac{1}{4}\epsilon_{\mu\nu\rho}{}^{\sigma\tau}b^{I_{10}}_{\sigma\tau}\epsilon_{\alpha\beta}{}^{\gamma\delta\epsilon}D_{\gamma}Y^{I_{10}}_{[\delta\epsilon]}=0$	(M3.1)
$[b^{I_5}_{\mu\nu\rho}+\epsilon_{\mu\nu\rho}{}^{\sigma\tau}D_{\sigma}\phi^{I_5}_{\tau}]D_{[\alpha}Y^{I_5}_{\beta]}=0$	(M3.2)

The expression Max denotes the Maxwell operator, which is normalized such that it starts with $\Box_x$. The symbol Δ denotes the Hodge-de Rham operator, and satisfies

$$\Delta Y^k_{[\alpha\beta]}=(\Box-6e^2)Y^k_{[\alpha\beta]}=-e^2(k+2)^2Y^k_{[\alpha\beta]}\ ,\quad k=1,2,\ldots$$
$$\Delta Y^k_{\alpha}=(\Box-4e^2)Y^k_{\alpha}=-e^2(k+1)(k+3)Y^k_{\alpha}\ ,\quad k=1,2,\ldots \tag{2.20}$$
$$\Delta Y^k=\Box Y^k=-e^2k(k+4)Y^k,\quad k=0,1,2,\ldots$$
$$\Delta Y^k_{(\alpha\beta)}=(\Box-10e^2)Y^k_{(\alpha\beta)}=-e^2(k^2+4k+8)Y^k_{(\alpha\beta)}\ ,\quad k=2,3,\ldots .$$

There is one special case to be discussed separately: when $Y^{I_1}(y)$ is constant, the (M2.2) equation disappears and (2.17) must be replaced by

$$5\partial_{[\mu}b^{k=0}_{\nu\rho\sigma\tau]}=\epsilon_{\mu\nu\rho\sigma\tau}\left[\frac{e}{2}H^{k=0}_{\lambda\lambda}-\frac{4e}{3}\pi^{k=0}\right] . \tag{2.21}$$

The Einstein equations yield, in a similar way, equations which are denoted by (E1.1), (E1.2), (E2.1), (E2.2), and (E3.1)—(E3.4), where (E1.1) contains the traceless part of the $R_{\mu\nu}$ equation (E1) while (E1.2) contains its trace. These equations are given in Table II also. We now divide all E and M equations into three classes: (i) Maxwell-Proca equations, (ii) coupled scalar equations, and (iii) diagonal equations. We shall discuss them separately.

Maxwell-Proca equations. We list these equations as well as their origin:

$$[\tfrac{1}{2}(\mathrm{Max}+\Delta_y)B^{I_5}_{\mu}-4e(\mathrm{Max}-\Delta_y)\phi^{I_5}_{\mu}]Y^{I_5}_{\alpha}=0 \quad [\text{from (E2.1) and (M3.2)}]\ , \tag{2.22}$$

$$[(\mathrm{Max}+\Delta_y)\phi^{I_5}_{\mu}-eB^{I_5}_{\mu}]Y^{I_5}_{\alpha}=0\quad [\text{from (2.18)}]\ , \tag{2.23}$$

$$(D^{\mu}B^{I_5}_{\mu})(D_{(\alpha}Y^{I_5}_{\beta)})=0\quad [\text{from (E3.3)}]\ , \tag{2.24}$$

$$(D^{\mu}\phi^{I_5}_{\mu}\Delta_y-eD^{\mu}B^{I_5}_{\mu})Y^{I_5}_{\alpha}=0\quad [\text{from (2.23)}]\ . \tag{2.25}$$

Thus, we must diagonalize the following 2×2 system:

$$\mathrm{Max}\begin{bmatrix}B_{\mu}\\ \phi_{\mu}\end{bmatrix}+\begin{bmatrix}\Delta_y-8e^2 & 16e\Delta_y\\ -e & \Delta_y\end{bmatrix}\begin{bmatrix}B_{\mu}\\ \phi_{\mu}\end{bmatrix}=0\ . \tag{2.26}$$

The eigenvalues form two branches

$$M^2=(k^2-1)e^2,\quad M^2=(k+3)(k+5)e^2\quad (k\geq1) \tag{2.27}$$

and the corresponding eigenvectors are

$$B^k_{\mu}-4e(k+3)\phi^k_{\mu},\quad B^k_{\mu}+4e(k+1)\phi^k_{\mu}\ . \tag{2.28}$$

Clearly, the $k=1$ modes in the first branch form the SO(6) gauge fields since they are massless. The transversality conditions confirms this: (2.25) states that $B^{k=1}_{\mu}+8e\phi^{k=1}_{\mu}$ is transversal, while (2.24) vanishes for $k=1$, since in that case the Y_{α} are Killing vectors. We summarize the results in Fig. 1.

Coupled scalar equations. There are five equations involving the three scalars $H^{I_1}_{\mu\mu}$, π^{I_1}, and b^{I_1}:

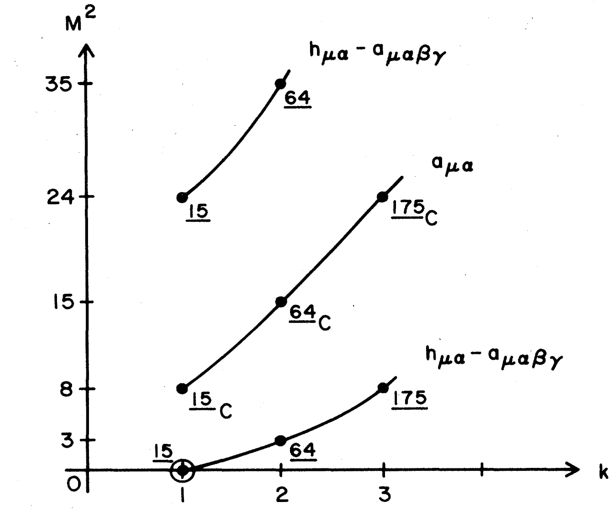

FIG. 1. Mass spectrum of vectors.

$$(H^{I_1}_{\mu\mu}-\tfrac{16}{15}\pi^{I_1})D_{(\alpha}D_{\beta)}Y^{I_1}=0 \quad [\text{from (E3.2)}]\ , \tag{2.29}$$

$$[D_\rho H^{I_1}_{\rho\mu}-D_\mu(H^{I_1}_{\rho\rho}-\tfrac{8}{15}\pi^{I_1}+16eb^{I_1})]D_\alpha Y^{I_1}=0 \quad [\text{from (E2.2)}]\ , \tag{2.30}$$

$$[(\Box_x+\Box_y-32e^2)\pi^{I_1}+80e\Box_y b^{I_1} +\Box_y(H^{I_1}_{\mu\mu}-\tfrac{16}{15}\pi^{I_1})]Y^{I_1}=0 \quad [\text{from (E3.4)}]\ , \tag{2.31}$$

$$\left[(\Box_x+\Box_y)b^{I_1}+\left[\frac{e}{2}H^{I_1}_{\sigma\sigma}-\frac{4e}{3}\pi^{I_1}\right]\right]Y^{I_1}=0 \quad [\text{from (2.17)}]\ . \tag{2.32}$$

The fifth scalar equation is (E1.2), but since it is linearly dependent on the previous four equations, we shall drop it. We shall first consider the case that $D_\alpha Y^{I_1}$ and $D_{(\alpha}D_{\beta)}Y^{I_1}$ are nonvanishing, i.e., the case $k\geq 2$. In this case one obtains a simple 2×2 coupled system because one can eliminate $H^{I_1}_{\mu\mu}$ from (2.29), so that (2.31) and (2.32) yield

$$\Box_x\begin{bmatrix}\pi\\ b\end{bmatrix}+\begin{bmatrix}\Box_y-32e^2 & 80e\Box_y\\ -\tfrac{4}{5}e & \Box_y\end{bmatrix}\begin{bmatrix}\pi\\ b\end{bmatrix}=0\ . \tag{2.33}$$

The mass eigenvalues are

$$M^2=k(k-4)e^2 \quad (k\geq 2)\ ,$$
$$M^2=(k+4)(k+8)e^2 \quad (k\geq 0)\ , \tag{2.34}$$

where, in the absence of an unambiguous definition of mass, M^2 is simply taken to be the eigenvalue of $\Box_x$. We have indicated by $(k\geq 0)$ in (2.34) that the second branch also contains modes for $k=0,1$ (see below).

When $D_{(\alpha}D_{\beta)}Y^{I_1}=0$ and $D_\alpha Y^{I_1}\neq 0$, one is in the $k=1$ scalar sector. In this case (2.29) no longer holds, but one could, in principle, still use the remaining conformal diffeormorphisms to obtain $H^{k=1}_{\mu\mu}=\frac{16}{15}\pi^{k=1}$. However, one could directly eliminate $H^{k=1}_{\mu\mu}$ from (2.31) and insert the result into (2.32). In the latter case one would find only one field equation, and for one field only, namely,

$$\Box_x(\pi+10eb)-45e^2(\pi+10eb)=0\ . \tag{2.35}$$

Under the conformal diffeormorphisms, the scalars transform as follows:

$$\delta H_{\mu\mu}=2\Box_x\lambda+\tfrac{50}{3}\lambda,\quad \delta\pi=10\lambda,\quad \delta(eb)=-\lambda\ , \tag{2.36}$$

where the $\frac{50}{3}$ comes from the Weyl rescaling.[21]

With these preliminaries we now will demonstrate that fixing the conformal gauge will lead to the same result as direct elimination of $H_{\lambda\lambda}$. Consider as a simplified model the equation with gauge invariance:

$$\Box A+B=0,\quad \delta A=\lambda,\quad \delta B=-\Box\lambda\ . \tag{2.37}$$

One can either directly eliminate B by $B=-\Box A$, in which case there are no surviving propagating modes, or one can use the local symmetry to set $B=\alpha A$. In the latter case, one obtains $\Box A+\alpha A=0$, which seems to indicate a propagating mode with a gauge-dependent mass. However, we can use the local symmetry once more to gauge A away. To see this, note that we can still make gauge transformations which respect $B=\alpha A$ by using a parameter λ which satisfies $\Box\lambda+\alpha\lambda=0$. Thus, "the gauge shoots twice," and no modes remain. Returning to our original model, we conclude that in the $k=1$ sector there is only one mode propagating, namely the mode in (2.35). This is also the $k=1$ mode in the second branch in (2.34). Thus, although in the spherical harmonic expansion one does find a second mode at $k=1$, it drops from the theory. In the conclusions we shall argue that this **6** of scalars is part of the doubleton multiplet.

The necessity of utilizing a residual gauge invariance for a correct identification of physical modes is familiar in the context of Maxwell theory. There, the field equation (in flat $d=4$ spacetime) reads

$$\partial^\rho F_{\rho\mu}=\Box A_\mu-\partial_\mu\partial\cdot A=0\ . \tag{2.38}$$

One may use the gauge invariance $\delta A_\mu=\partial_\mu\Lambda$ to gauge $\partial\cdot A=0$ thus reducing the number of degrees of freedom carried by A_μ from four to three; the wave equation for these modes is then

$$\Box A_\mu=0\ . \tag{2.39}$$

In order to restrict A_μ to the two physical polarizations, one must utilize the remaining invariance of (2.39) under gauge transformations parameterized by Λ satisfying $\Box\Lambda=0$.

When Y^{I_1} is a constant, one is in the $k=0$ sector. In this case, there is no b term in the expansion of $a_{\alpha\beta\gamma\delta}$ in (2.16), while (2.32) is replaced by (2.21). Equation (2.31) reduces to

$$(\Box_x-32e^2)\pi^{k=0}=0 \tag{2.40}$$

which describes the dilatational mode of the internal metric. Again, this mode can be found in the second

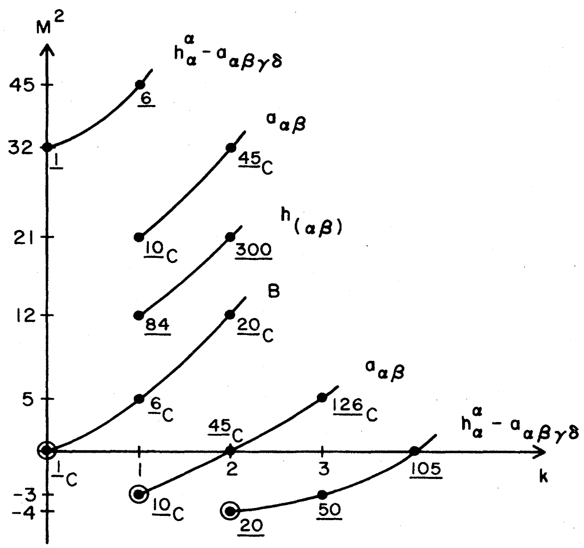

FIG. 2. Mass spectrum of scalars.

branch of (2.34), namely at $k=0$. We summarize the results of all scalar modes in Fig. 2.

Diagonal equations. The remaining fields, $b_{\mu\nu}$ in $a_{\mu\nu\alpha\beta}$ and $\phi^{I_{14}}$ in $h_{(\alpha\beta)}$ as well as $H_{(\mu\nu)}$ in $h'_{\mu\nu}$, have diagonal field equations which read

$$(\mathrm{Max}+\Delta_y)b_{\nu\rho}^{I_{10},\pm}Y_{[\alpha\beta]}^{I_{10},\pm}=0 \quad [\text{from } (2.19)] , \tag{2.41}$$

$$(\Box_x+\Box_y-2e^2)\phi^{I_{14}}Y^{I_{14}}_{(\alpha\beta)}=0 \quad [\text{from } (\mathrm{E}3.1)] , \tag{2.42}$$

$$[\tfrac{1}{2}(\Box_x+\Box_y)H^{I_1}_{(\mu\nu)}+e^2H^{I_1}_{(\mu\nu)}-D_{(\mu}D^{\lambda}H^{I_1}_{\nu)\lambda} + \tfrac{1}{2}D_{(\mu}D_{\nu)}H^{I_1}_{\lambda\lambda}]Y^{I_1}=0 \quad [\text{from } (\mathrm{E}1.1)] . \tag{2.43}$$

The last equation can be diagonalized for $k\geq 1$ by

$$H_{(\mu\nu)}=\phi_{(\mu\nu)}+D_{(\mu}D_{\nu)}(\tfrac{2}{5}\pi-12eb)/[(k+1)(k+3)] . \tag{2.44}$$

The traceless field $\phi_{(\mu\nu)}$ is then transversal on-shell from (2.30) and satisfies the Einstein equation

$$[\mathrm{Ein}-k(k+4)e^2]\phi_{(\mu\nu)}=0 , \tag{2.45}$$

where Ein stands for the Einstein operator

$$2R^{(5)\mathrm{lin}}_{\mu\nu}(\phi_{(\mu\nu)})-8e^2\phi_{(\mu\nu)}=(\Box_x+2e^2)\phi_{(\mu\nu)} .$$

Here $R^{(5)}_{\mu\nu}$ is the Ricci tensor of five-dimensional spacetime. One should not be confused with $R^{(5)}_{\mu\nu}$ and the orginial $R_{\mu\nu}$. Recall that $R_{\mu\nu}$ is the $\mu\nu$ component of the full Ricci tensor in ten dimensions. For $k=0$, the (E1) equation, together with (2.21) and (2.40) yields

$$R^{(5)}_{\mu\nu}(\dot{g}_{\mu\nu}+h'_{\mu\nu})-4e^2(\dot{g}_{\mu\nu}+h'_{\mu\nu})=0 . \tag{2.46}$$

This clearly demonstrates that $h'_{\mu\nu}$ is the massless graviton, as expected.

The real scalars $\phi^{I_{14}}$ in (2.42) have masses

$$-(\Box_y-2e^2)Y_{(\alpha\beta)}=e^2k(k+4)Y_{(\alpha\beta)}, \quad k=2,3,\cdots \tag{2.47}$$

while the two complex fields $b_{\mu\nu}^{I_{10},+}$ and $b_{\mu\nu}^{I_{10},-}$ in (2.41) have masses $e^2(k+2)^2$. The field $b_{\mu\nu}^{I_{10},+}$ is the complex conjugate of $b_{\mu\nu}^{I_{10},-}$ because the four-index antisymmetric tensor is real.

We now discuss the modes contained in the fields $A_{\hat{\mu}\hat{\nu}}$ and B. These fields are purely fluctuations and contain no background parts. We expand them into spherical harmonics as follows:

$$\begin{aligned} A_{\mu\nu}&=\sum a^{I_1}_{\mu\nu}(x)Y^{I_1}(y) , \\ A_{\mu\alpha}&=\sum[a_{\mu}^{I_5}(x)Y_{\alpha}^{I_5}(y)+a_{\mu}^{I_1}(x)D_{\alpha}Y^{I_1}(y)] , \\ A_{\alpha\beta}&=\sum[a^{I_{10}}(x)Y^{I_{10}}_{[\alpha\beta]}(y)+a^{I_5}(x)D_{[\alpha}Y^{I_5}_{\beta]}(y)] , \\ B&=\sum B^{I_1}(x)Y^{I_1}(y) . \end{aligned} \tag{2.48}$$

We choose the Lorentz-type gauges

$$D^{\alpha}A_{\alpha\beta}=0, \quad D^{\alpha}A_{\alpha\mu}=0 \tag{2.49}$$

which can be implemented by first fixing the transversal part of Λ_α in $\delta A_{\alpha\beta}=D_\alpha\Lambda_\beta-D_\beta\Lambda_\alpha$ to gauge $a^{I_5}=0$, and then fixing the $D_\alpha\Lambda_\mu$ part of $\delta A_{\alpha\mu}=D_\alpha\Lambda_\mu-D_\mu\Lambda_\alpha$ to set $a_{\mu}^{I_1}=0$. The only gauge transformations which respect these gauges have y-independent $\Lambda_\mu(x)$, which are the usual gauge parameters for $a^{k=0}_{\mu\nu}(x)$. Thus we may use the expansion in (2.48) with $a_{\mu}^{I_1}=a^{I_5}=0$. Substituting these expansions into the field equations yields

$$[(\mathrm{Max}+\Box_y)a^{I_1}_{\nu\rho}+2ie\epsilon_{\nu\rho}{}^{\sigma\tau\kappa}\partial_\sigma a^{I_1}_{\tau\kappa}]Y^{I_1}=0 , \tag{2.50}$$

$$(\Box_x+\Box_y-6e^2)a^{I_{10}}Y^{I_{10}}_{[\alpha\beta]}+2iea^{I_{10}}\epsilon_{\alpha\beta}{}^{\gamma\delta\epsilon}D_\gamma Y^{I_{10}}_{[\delta\epsilon]} -2(D^{\mu}a_{\mu}^{I_5})(D_{[\alpha}Y^{I_5}_{\beta]})=0 , \tag{2.51}$$

$$(\mathrm{Max}+\Box_y-4e^2)a_{\nu}^{I_5}Y_{\alpha}^{I_5}+(D^{\mu}a^{I_1}_{\mu\nu})(D_\alpha Y^{I_1})=0 , \tag{2.52}$$

$$(\Box_x+\Box_y)B^{I_1}Y^{I_1}=0 . \tag{2.53}$$

We recall that the spherical harmonics $Y^{I_{10}}_{[\alpha\beta]}$ are not only eigenfunctions of Δ, but also of the operator

$$({}^*D)Y_{[\alpha\beta]}\equiv\epsilon_{\alpha\beta}{}^{\gamma\delta\epsilon}D_\gamma Y_{[\delta\epsilon]} .$$

Since $({}^*D)({}^*D)=4(\Box_y-6e^2)$, we can divide the $Y_{[\alpha\beta]}$ into $Y^{+}_{[\alpha\beta]}$ and $Y^{-}_{[\alpha\beta]}$, where

$$({}^*D)Y^{k,\pm}_{[\alpha\beta]}=\pm 2i(-\Box_y+6e^2)^{1/2}Y^{k,\pm}_{[\alpha\beta]} . \tag{2.54}$$

Since

$$(-\Box_y+6e^2)Y^{k,\pm}_{[\alpha\beta]}=-\Delta_yY^{k,\pm}_{[\alpha\beta]}=e^2(k+2)^2Y^{k,\pm}_{[\alpha\beta]}$$

we thus have

$$({}^*D)Y^{k,\pm}_{[\alpha\beta]}=\pm 2ie(k+2)Y^{k,\pm}_{[\alpha\beta]} . \tag{2.55}$$

Collecting all terms with a given spherical harmonic, one gets the $d=5$ field equations

$$[\mathrm{Max}-e^2k(k+4)]a^k_{\nu\rho}+2ie\epsilon_{\nu\rho}{}^{\sigma\tau\lambda}\partial_\sigma a^k_{\tau\lambda}=0,\quad k\geq 0 \tag{2.56}$$

$$[\Box_x-e^2(k+2)^2\mp 4e^2(k+2)]a^{k,\pm}=0,\quad k\geq 1 \tag{2.57}$$

$$(\mathrm{Max}+\Delta_y)a^k_\nu=[\mathrm{Max}-e^2(k+1)(k+3)]a^k_\nu=0\ ,\quad k\geq 1 \tag{2.58}$$

$$D^\mu a^k_\mu=0,\quad k\geq 1,\quad D^\mu a^k_{\mu\nu}=0,\quad k\geq 1 \tag{2.59}$$

$$[\Box_x-e^2k(k+4)]B^k=0,\quad k\geq 0\ . \tag{2.60}$$

Thus, all vectors a^k_μ, and all tensors $a^k_{\mu\nu}$, except $a^{k=0}_{\mu\nu}$, are massive.

The $a^k_{\mu\nu}$ fields in (2.56) satisfy a field equation containing both an explicit mass term $\Box_y=-e^2k(k+4)$, and a topological mass term $+2ie\epsilon_{\nu\rho}{}^{\sigma\tau\lambda}\partial_\sigma a_{\tau\lambda}$. For $k=0$ there is only a toplogical mass, while there is also the gauge invariance

$$\delta a_{\mu\nu}=\partial_\mu\Lambda_\nu(x)-\partial_\nu\Lambda_\mu(x)\ .$$

This suggests that the self-duality mechanism of Ref. 11 can be applied. Indeed, the field equation factorizes into

$$(2ekI+i^*D)[2e(k+4)I-i^*D]a_{\mu\nu}=0 \tag{2.61}$$

so that there are two sets of field equations

$$(2ekI+i^*D)a^{1,k}_{\mu\nu}=0,\quad [2e(k+4)I-i^*D]a^{2,k}_{\mu\nu}=0\ . \tag{2.62}$$

Iterating these field equations one obtains two massive Proca field equations

$$(\mathrm{Max}-k^2e^2)a^{1,k}_{\mu\nu}=0,\quad k\geq 0 \tag{2.63}$$

$$[\mathrm{Max}-(k+4)^2e^2]a^{2,k}_{\mu\nu}=0,\quad k\geq 0\ . \tag{2.64}$$

However, the $k=0$ mode of $a^1_{\mu\nu}$ can be gauged away, since its field equation states that it is a pure gauge. These results are analogous to those in seven dimensions.[17] We summarize these results in Fig. 3.

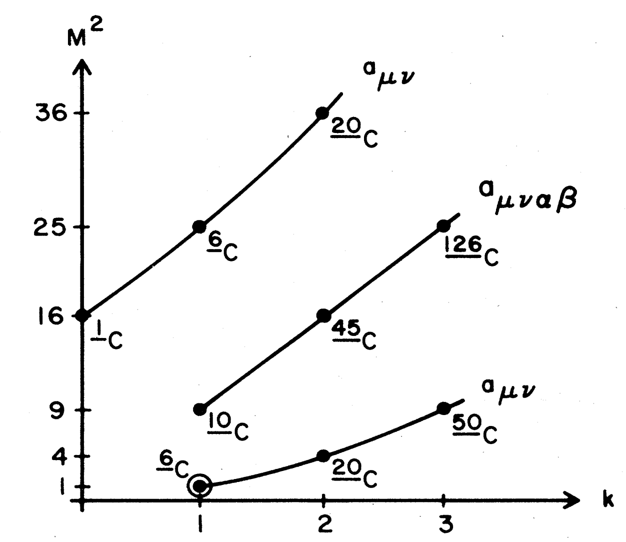

FIG. 3. Mass spectrum of antisymmetric tensors.

III. THE FERMIONIC MASS SPECTRUM

The gravitino and spinor field equations in $d=10$, linearized in fermion fields, read[1]

$$\Gamma^{\hat\mu\hat\nu\hat\rho}\left[D_{\hat\nu}\hat\psi_{\hat\rho}+\frac{i}{480}\Gamma^{\hat\mu'\hat\nu'\hat\rho'\hat\sigma'\hat\tau'}F_{\hat\mu'\hat\nu'\hat\rho'\hat\sigma'\hat\tau'}\Gamma_{\hat\nu}\hat\psi_{\hat\rho}\right]=0\ , \tag{3.1}$$

$$\not{D}\hat\lambda=\frac{iA}{5!}\Gamma^{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau}F_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau}\hat\lambda\quad (A=\tfrac{1}{2})\ . \tag{3.2}$$

The gravitino field equation can be derived in an elementary way, once one assumes that a supersymmetric $d=10$ theory exists, by requiring that it can be written as $\Gamma^{\hat\mu\hat\nu\hat\rho}\overline{D}_{\hat\nu}\hat\psi_{\hat\rho}=0$ where $\overline{D}_{\hat\nu}$ is in the background given by

$$\overline{D}_{\hat\nu}\hat\epsilon=\left[D_{\hat\nu}+\frac{ie}{2}\Gamma_{\hat\nu}\right]\hat\epsilon\ .$$

In order that the variation of the gravitino field equation vanish in the background, the commutator of two $\overline{D}$ derivatives should vanish, and using that $R_{\mu\nu}=4e^2g_{\mu\nu}$ and $R_{\alpha\beta}=-4e^2g_{\alpha\beta}$, this fixes the factor $\frac{1}{480}$ in the gravitino field equation. (The sign in front of $\frac{1}{480}$ can be fixed by choosing the sign $F_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau}$.) The value $A=\frac{1}{2}$ comes out of a lengthy analysis given in Ref. 1.

We choose the following representation of the $d=10$ Dirac matrices

$$\Gamma^\mu=\gamma^\mu\otimes \mathbb{I}_4\otimes\sigma^1,\quad \Gamma^\alpha=\mathbb{I}_4\otimes\tau^\alpha\otimes(-\sigma^2)\ , \tag{3.3}$$

where σ^i are Pauli matrices, and γ^μ (τ^α) are $d=5$ ($d=5$) Dirac matrices satisfying

$$\{\Gamma_{\hat m},\Gamma_{\hat n}\}=2\eta_{\hat m\hat n},\quad \{\gamma_m,\gamma_n\}=2\eta_{mn},\quad \{\tau_a,\tau_b\}=2\delta_{ab}\ . \tag{3.4}$$

The γ_5 matrix in $d=10$ will be called Γ_{11} with $\Gamma_{11}{}^2=+1$. Further, the gravitino (spinor) is left (right) handed,

$$\Gamma_{11}=\Gamma^0\cdots\Gamma^9=\begin{bmatrix}\mathbb{I}_{16} & 0\\ 0 & -\mathbb{I}_{16}\end{bmatrix}\ ,$$

$$\hat\psi_{\hat\mu}=\begin{bmatrix}\psi_{\hat\mu}\\ 0\end{bmatrix},\quad \hat\lambda=\begin{bmatrix}0\\ \lambda\end{bmatrix},\quad \hat\epsilon=\begin{bmatrix}\epsilon\\ 0\end{bmatrix}\ . \tag{3.5}$$

Substituting the background values for $F_{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\tau}$ into the λ-field equation, we obtain

$$\not{D}\begin{bmatrix}0\\ \lambda\end{bmatrix}=iA\begin{bmatrix}0 & 2i\\ 0 & 0\end{bmatrix}\begin{bmatrix}0\\ \lambda\end{bmatrix}\ ,$$

$$(\not{D}_x+i\not{D}_y)\lambda=-2A\lambda\ . \tag{3.6}$$

Here, $\not{D}_x=\gamma^\mu D_\mu\otimes\mathbb{I}_4$ and $\not{D}_y=\mathbb{I}_4\otimes\tau^\alpha D_\alpha$, and λ is a 16-component spinor. We decompose $\lambda(x,y)$ into spherical harmonics

$$\lambda(x,y)=\sum\lambda^{I_L}(x)\Xi^{I_L}(y)\ , \tag{3.7}$$

where both λ^{I_L} and Ξ^{I_L} are four-component spinors, and Ξ^{I_L} are eigenfunctions of $\not{D}_y$.

These eigenspinors can be expressed in terms of Killing spinors η^+ and η^-, and bosonic spherical harmonics as follows. On S^n, with unit "radius," and with $D_\alpha\eta^\pm \pm (i/2)\tau_\alpha\eta^\pm = 0$, one has for the spinor spherical harmonics

$$\not{D}_y \Xi^{I_L,\pm} = m^{I_L,\pm}\Xi^{I_L,\pm} = \mp i\left[k+\frac{n}{2}\right]\Xi^{k,\pm}, \quad k\geq 0 \tag{3.8}$$

where $I_L = k$ satisfies $k \geq 0$ and

$$\Xi^{k,+} = [(k+n-1+i\not{D}_y)Y^k]\eta^+, \quad k\geq 0 \tag{3.9}$$

$$\Xi^{k,-} = [(k+n-1+i\not{D}_y)Y^k]\eta^-, \quad k\geq 0 \ . \tag{3.10}$$

An alternative expression for $\Xi^{k,-}$ in terms of η^+ can also be obtained; it is given by

$$\Xi^{k,-} = (+k+1+i\not{D}_y)Y^{k+1}\eta^+, \quad k\geq 0 \ .$$

Similarly, the vector-spinor spherical harmonics satisfy

$$\not{D}_y \Xi^{I_T}_\alpha = m^{I_T}\Xi^{I_T}_\alpha = \mp i\left[k+\frac{n}{2}\right]\Xi^k_\alpha, \quad k\geq 1 \tag{3.11}$$

and can be expressed in terms of bosonic vector spherical harmonics and Killing spinors as follows:

$$\begin{aligned}\Xi^{k,\pm}_\alpha = \{&(n-2)g_{\alpha\gamma}[(n+k)(n+k-2)\mp(n+k-1)i\not{D}]\\ &-(n+k-1)(n+k-2)\tau_{\alpha\gamma}\mp(n+k)i\tau_{\alpha\beta\gamma}D^\beta\\ &\pm(n-2)i\tau_\gamma D_\alpha - \tau_{\beta\gamma}D_\alpha D^\beta\}Y^{\gamma,k}\eta^\pm \ .\end{aligned} \tag{3.12}$$

Again one can express the $\Xi^{k,-}_\alpha$ in terms of η^+

$$\begin{aligned}\Xi^{k,-}_\alpha = \{&(n-2)g_{\alpha\gamma}[k(k+2)+(k+1)i\not{D}]\\ &-(k+1)(k+2)\tau_{\alpha\gamma}+ki\tau_{\alpha\beta\gamma}D^\beta\\ &+(n-2)i\tau_\gamma D_\alpha - \tau_{\beta\gamma}D_\alpha D^\beta\}Y^{\gamma,k}\eta^+ \ .\end{aligned} \tag{3.13}$$

In all these equations, the derivatives do not act on the Killing spinors but only on the Y harmonics. The basic relations to obtain the fermionic spectrum are given in (3.8) and (3.12).

Returning to (3.7), we see that the spinors λ^{I_L} have masses given by

$$(\not{D}_x + i\not{D}_y + 2A)\lambda^{k,\pm} = \begin{bmatrix}\not{D}_x + e(\frac{7}{2}+k)\\ \not{D}_x - e(\frac{3}{2}+k)\end{bmatrix}\lambda^{I_L,\pm} = 0 \ . \tag{3.14}$$

We now turn to the gravitino field equation. There are two cases: $\hat{\mu}=\mu$ and $\hat{\mu}=\alpha$. They yield, respectively,

$$\begin{aligned}&\gamma^{\mu\rho\sigma}D_\rho\psi_\sigma - i\gamma^{\mu\rho}D_\rho(\tau^\alpha\psi_\alpha)+i\gamma^{\mu\rho}\not{D}_y\psi_\rho\\ &\qquad +\gamma^\mu(\tau^{\alpha\beta}D_\alpha\psi_\beta)-\gamma^{\mu\rho}\psi_\rho = 0 \ ,\end{aligned} \tag{3.15}$$

$$\begin{aligned}&\tau^{\alpha\beta\gamma}D_\beta\psi_\gamma + \tau^\alpha\gamma^{\rho\sigma}D_\rho\psi_\sigma + i\tau^{\alpha\beta}D_\beta(\gamma^\sigma\psi_\sigma)\\ &\qquad - i\tau^{\alpha\beta}\not{D}_x\psi_\beta + i\tau^{\alpha\beta}\psi_\beta = 0 \ .\end{aligned} \tag{3.16}$$

We fix the $d=10$ local supersymmetries by trying to achieve $\Gamma^\alpha\hat{\psi}_\alpha = 0$. However, with

$$\delta\hat{\psi}_\alpha = \bar{D}_\alpha\hat{\epsilon} = D_\alpha\hat{\epsilon} + \frac{ie}{2}\mathbb{I}_4\otimes\tau_\alpha\otimes\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\hat{\epsilon} \ ,$$

and decomposing

$$\epsilon(x,y) = \Sigma\epsilon^{I_L}(x)\Xi^{I_L}(y) \ ,$$

it is clear that there are modes in $\Gamma^\alpha\hat{\psi}_\alpha$ which are invariant, namely, those proportional to η^+. Thus the nearest one can come to $\Gamma^\alpha\hat{\psi}_\alpha = 0$ is

$$\begin{aligned}&\psi_\alpha = \psi_{(\alpha)} + \chi^I(x)\tau_\alpha\eta^{I,+}(y) \ ,\\ &\left[D_\alpha + \frac{ie}{2}\tau_\alpha\right]\eta^{I,+} = 0\end{aligned} \tag{3.17}$$

where (α) indicates that $\tau^\alpha\psi_{(\alpha)} = 0$. Note that the other Killing spinors, η^- with

$$\left[D_\alpha - \frac{ie}{2}\tau_\alpha\right]\eta^- = 0 \ ,$$

can be gauged away from ψ_α. We expand the gravitino fields as follows

$$\psi_\mu = \sum\psi^{I_L}_\mu\Xi^{I_L} \ , \tag{3.18}$$

$$\psi_{(\alpha)} = \sum(\psi^{I_T}\Xi^{I_T}_\alpha + \psi^{I_L}D_{(\alpha)}\Xi^{I_L}) \ . \tag{3.19}$$

We will now first analyze the gravitino field equations in the sectors without η^+ (but with η^-) and later we will separately analyze the sector with η^+.

From (3.15) we find

$$\begin{aligned}\{&\gamma^{\mu\nu\rho}D_\nu\psi^{I_L}_\rho + (im^{I_L}-e)\gamma^{\mu\nu}\psi^{I_L}_\nu\\ &-[5e^2+\tfrac{4}{5}(m^{I_L})^2]\gamma_\mu\psi^{I_L}\}\Xi^{I_L} = 0 \ ,\end{aligned} \tag{3.20}$$

where the last term comes from $\gamma_\mu D^\alpha\psi_{(\alpha)}$. From (3.16) one finds

$$[\not{D}_x\psi^{I_T} - (e+im^{I_T})\psi^{I_T}]\Xi^{I_T}_\alpha = 0 \ , \tag{3.21}$$

$$\begin{aligned}\{&(-3)[5e^2+\tfrac{4}{5}(m^{I_L})^2]\psi^{I_L} + 5\gamma^{\rho\sigma}D_\rho\psi^{I_L}_\sigma\\ &+4im^L\gamma^\sigma\psi^{I_L}_\sigma\}\Xi^{I_L} = 0 \ ,\end{aligned} \tag{3.22}$$

$$[(\not{D}_x - e - \tfrac{3}{5}im^{I_L})\psi^{I_L} - \gamma^\sigma\psi^{I_L}_\sigma]D_{(\alpha)}\Xi^{I_L} = 0 \ . \tag{3.23}$$

Here, (3.22) is obtained by contracting (3.16) with τ^α, while (3.23) is the τ_α-transverse part.

Eliminating $\gamma^\sigma\psi^{I_L}_\sigma$ in terms of ψ^{I_L} from (3.20) and (3.22) yields

$$\gamma^\sigma\psi^{I_L}_\sigma = -\tfrac{4}{5}(5e+2im^{I_L})\psi^{I_L} \tag{3.24}$$

and inserting into (3.23) yields

$$\not{D}_x\psi^{I_L} + (3e+im^{I_L})\psi^{I_L} = 0 \ . \tag{3.25}$$

Further, (3.20) yields, after eliminating ψ^{I_L},

$$\begin{aligned}&\gamma^{\mu\nu\rho}D_\nu\psi^{I_L}_\rho + (im^{I_L}-e)\gamma^{\mu\rho}\psi^{I_L}_\rho\\ &\qquad + \tfrac{1}{4}(5e-2im^{I_L})\gamma^\mu(\gamma^\sigma\psi^{I_L}_\sigma) = 0 \ .\end{aligned} \tag{3.26}$$

In order to find the physical gravitino modes, we redefine

$$\psi^{I_L}_\mu = \varphi^{I_L}_{(\mu)} + \tfrac{1}{5}\gamma_\mu\gamma\cdot\psi^{I_L} + \alpha D_{(\mu)}\gamma\cdot\psi^{I_L} \ . \tag{3.27}$$

Requiring that $D^\mu\varphi^{I_L}_{(\mu)}=0$ on-shell fixes α,

$$\alpha=-\frac{3}{4}\frac{1}{2im^{I_L}+e} . \tag{3.28}$$

Then,

$$\gamma^{\mu\nu\rho}D_\nu\varphi^{I_L}_{(\rho)}+(im^{I_L}-e)\varphi^{(\mu)I_L}=0 . \tag{3.29}$$

Thus the spectrum is given by

$$\gamma^{\mu\nu\rho}D_\nu\varphi^{I_L}_{(\rho)}+(im^{I_L}-e)\varphi^{(\mu)I_L}=0,\quad k\geq 0 \tag{3.30}$$

$$\not{D}_x\psi^{I_L}+(im^{I_L}+3e)\psi^{I_L}=0,\quad k\geq 1 \tag{3.31}$$

$$\not{D}_x\psi^{I_T}-(im^{I_T}+e)\psi^{I_T}=0,\quad k\geq 0 \tag{3.32}$$

where

$$im^{I_L}=\mp e(\tfrac{5}{2}+k),\quad im^{I_T}=\mp e(\tfrac{7}{2}+k) . \tag{3.33}$$

Note that the $k=0$ positions in the ψ^{I_L} series are unoccupied at this point.

We must now analyze the η^+ modes. One finds the following two equations

$$\gamma^{\mu\nu\rho}D_\nu\psi_\rho+(im^{I_L}-e)\gamma^{\mu\nu}\psi_\nu+5i\gamma^{\mu\nu}D_\nu\chi-4m^{I_L}\gamma^\mu\chi=0 , \tag{3.34}$$

$$\tau^\alpha(5\gamma^{\mu\nu}D_\mu\psi_\nu+10e\gamma^\mu\psi_\mu+20i\not{D}_x\chi+10ie\chi)=0 . \tag{3.35}$$

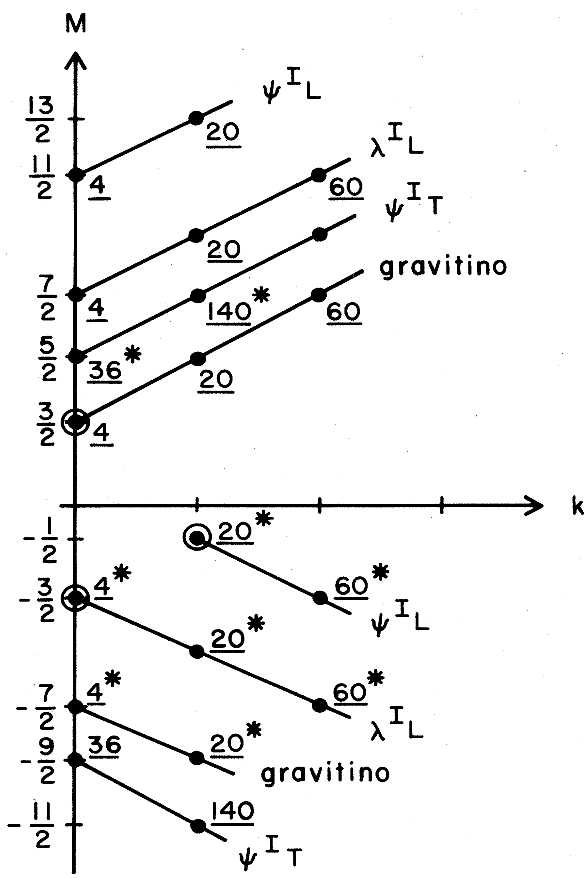

FIG. 4. Mass spectrum of fermions.

Tracing the first equation and combining with the second, the ψ terms are the same, and hence one finds a diagonal result for χ:

$$\not{D}_x\chi+\tfrac{11}{2}e\chi=0 . \tag{3.36}$$

The shift

$$\psi_\mu=\varphi_\mu-\frac{5i}{3}\gamma_\mu\chi \tag{3.37}$$

removes then χ from these equations, giving

$$\gamma^{\mu\nu\rho}D_\nu\varphi_\rho+\tfrac{3}{2}e\gamma^{\mu\nu}\varphi_\nu=0 . \tag{3.38}$$

This is the massless gravitino field equation; since φ_μ is complex, there are 8 gravitinos, as expected.

The mode χ has the correct mass to fill the $k=0$ position of the ψ^{I_L} series with

$$im^{I_L}=+e(\tfrac{5}{2}+k) .$$

Hence, one might identify χ with $\psi^{k=0}(im^{I_L}>0)$. The other $k=0$ slot remains unoccupied. There is a group-theoretical explanation of these results which we will discuss in the conclusions. The mass spectrum of the fermions is summarized in Fig. 4.

IV. DISCUSSION

We have obtained the complete mass spectrum for the compactification of chiral $N=2,d=10$ supergravity on S^5. Our results are summarized in Table III, as well as in the four figures.

The identification of the massless modes is achieved either by using the fact that one knows from the group theory the content of the graviton supermultiplet,[14] or by looking up the field content of the gauged $N=8,d=5$ supergravity.[18,19] We have encircled these massless modes in the four figures.

All massless modes are found at the lowest k mode of their respective branches. This seems to be a general rule and may be helpful in future compactifications to select in advance the massless modes.

From these results we see that the field equations for the scalars in the massless graviton supermultiplet are not all equal. In fact, one has

$$(\Box_x+4e^2)\pi^{k=2}(\mathbf{20})=0 ,$$

$$\Box_x B^{k=0}(1_c)=0 ,$$

$$(\Box_x+3e^2)a^{k=1}(\mathbf{10}_c)=0 .$$

These scalars lie inside or on the boundary of the stability region for which the field equation reads $(\Box+4e^2)\phi=0$.[22] Note that the conformal value $(\Box+\frac{15}{4}e^2)\phi=0$ lies inside this stability region, and that none of the scalars has a conformal field equation.

This raises the possibility that the scalars in the massless supermultiplet may not have any particular properties which determine their field equations, except supersymmetry.

The massless modes consist of the 42 scalars above, and further the graviton, a complex quartet of gravitinos, 15 Yang-Mills fields for SO(6), and, most interesting, a com-

TABLE III. Complete mass spectrum.

Spin	Field	Masses on S^5		Irred. reps.
2	$h'_{\mu\nu}=H^{I_1}_{\mu\nu}Y^{I_1}$	$M^2=e^2k(k+4)$	$(k\geq 0)$	**1,6,20,...**
1	$h_{\alpha\mu}=B^{I_5}_{\mu}Y^{I_5}_{\alpha}$ $a_{\mu\alpha\beta\gamma}=\phi^{I_5}_{\mu}\epsilon_{\alpha\beta\gamma}{}^{\delta\epsilon}D_\delta Y^{I_5}_{\epsilon}$	$M^2=e^2(k-1)(k+1)$ $M^2=e^2(k+3)(k+5)$	$(k\geq 1)$ $(k\geq 1)$	**15,64,175,...** **15,64,175,...**
0	$h^{\alpha}_{\alpha}=\pi^{I_1}Y^{I_1}$ $a_{\alpha\beta\gamma\delta}=b^{I_1}\epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon}D_\epsilon Y^{I_1}$	$M^2=e^2k(k-4)$ $M^2=e^2(k+4)(k+8)$	$(k\geq 2)$ $(k\geq 0)$	**20,50,...** **1,6,20,...**
0	$h_{(\alpha\beta)}=\phi^{I_{14}}Y^{I_{14}}_{(\alpha\beta)}$	$M^2=e^2k(k+4)$	$(k\geq 2)$	**84,300,...**
0	$B=B^{I_1}Y^{I_1}$	$M^2=e^2k(k+4)$	$(k\geq 0)$	$\mathbf{1}_c,\mathbf{6}_c,\mathbf{20}_c,\ldots$
ant	$a_{\mu\nu\alpha\beta}=b^{I_{10,\pm}}_{\mu\nu}Y^{I_{10,\pm}}_{[\alpha\beta]}$	$M^2=e^2(k+2)^2$	$(k\geq 1)$	$\mathbf{10}_c,\mathbf{45}_c,\ldots$
ant	$A_{\mu\nu}=a^{I_1}_{\mu\nu}Y^{I_1}$	$M^2=e^2k^2$ $M^2=e^2(k+4)^2$	$(k\geq 1)$ $(k\geq 0)$	$\mathbf{6}_c,\mathbf{20}_c,\ldots$ $\mathbf{1}_c,\mathbf{6}_c,\ldots$
1	$A_{\mu\alpha}=a^{I_5}_{\mu}Y^{I_5}_{\alpha}$	$M^2=e^2(k+1)(k+3)$	$(k\geq 1)$	$\mathbf{15}_c,\mathbf{64}_c,\ldots$
0	$A_{\alpha\beta}=a^{I_{10,\pm}}Y^{I_{10,\pm}}_{[\alpha\beta]}$	$M^2=e^2(k-2)(k+2)$ $M^2=e^2(k+2)(k+6)$	$(k\geq 1)$ $(k\geq 1)$	$\mathbf{10}_c,\mathbf{45}_c,\ldots$ $\mathbf{10}_c,\mathbf{45}_c,\ldots$
$\frac{3}{2}$	$\psi_\mu=\psi^{I_L}_{\mu}\Xi^{I_L}$	$M=ek$ $M=-e(k+\frac{10}{2})$	$(k\geq 0)$ $(k\geq 0)$	**4,20,...** **4*,20*,...**
$\frac{1}{2}$	$\psi_{(\alpha)}=\psi^{I_T}\Xi^{I_T}_{\alpha}$	$M=e(k+\frac{5}{2})$ $M=-e(k+\frac{9}{2})$	$(k\geq 0)$ $(k\geq 0)$	**36*,140*,...** **36,140,...**
$\frac{1}{2}$	$\psi_{(\alpha)}=\psi^{I_L}D_{(\alpha)}\Xi^{I_L}+\chi\tau_\alpha\eta^+$	$M=e(k+\frac{11}{2})$ $M=-e(k-\frac{1}{2})$	$(k\geq 0)$ $(k\geq 1)$	**4,20,...** **20*,...**
$\frac{1}{2}$	$\lambda=\lambda^{I_L}\Xi^{I_L}$	$M=e(k+\frac{7}{2})$ $M=-e(k+\frac{3}{2})$	$(k\geq 0)$ $(k\geq 0)$	**4,20,...** **4*,20*,...**

plex sextext of antisymmetric tensors. Their linearized field equations read

$$D^\mu F^{IJ}_{\mu\nu}=0\ ,$$

$$i\epsilon_{\mu\nu}{}^{\rho\sigma\tau}D_\rho a^{k=1}_{\sigma\tau}+2ea^{k=1}_{\mu\nu}=0\ .$$

The 48 spinors have the field equations

$$(\not{D}_x-\tfrac{1}{2}e)\psi^{I_L}(\mathbf{20}^*_c)=0\ ,$$

$$(\not{D}_x-\tfrac{3}{2}e)\lambda^{I_L}(\mathbf{4}^*_c)=0\ .$$

Thus, also the spinors in different SO(6) multiplets have different masses.

Among the unitary infinite-dimensional irreducible representations of the superalgebra for the $d=5,N=8$ model, SU(2,2 | 4), there is one irreducible representation which is even smaller than the massless supermultiplet.[14] It is called the doubleton multiplet and contains one complex antisymmetric tensor, six real scalars, and four complex spinors.

These fields we identify as follows: (i) With those six scalars in the $k=1$ sector of π^I and b^I, which would have been present if we would not have had the extra conformal invariance (or, equivalently, which at once disappears after algebraically eliminating $H_{\lambda\lambda}$); (ii) with that $k=0$ component of $a_{\mu\nu}$ which became pure gauge on-shell; and (iii) with the $\Xi^{I_L}=\eta^+$ terms in $\psi^{I_L}D_{(\alpha)}\Xi^{I_L}$ which are also absent because $D_{(\alpha)}\Xi^{I_L}=0$ in this case.

A technical point we solved has to do with the conformal diffeomorphisms which remain after imposing the de Donder conditions $D^\alpha h_{(\alpha\beta)}=D^\alpha h_{\alpha\mu}=0$. We showed that these symmetries must in general be used twice to eliminate redundant modes: once using the inhomogeneous equation for the gauge parameter, and once more using the homogeneous equation. Thus, there is complete agreement between gauge fixing and direct elimination of nonprogating fields.

ACKNOWLEDGMENTS

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