Gauge-invariant description of massive higher-spin particles by dimensional reduction

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We study the dimensional reduction of free massless higher-spin theories from five dimensions to four-dimensional space-time. The fifth dimension is assumed to be compactified to a circle of radius $1/m$. For spin s, the resulting four-dimensional theory describes massless fields of spin s, $s-1$, $s-2, \ldots$, together with an infinite number of massive fields of spin s. These correspond to a member of a class of theories given by Zinoviev which are gauge-invariant descriptions of massive higher-spin particles. We show that in our theory massive modes have gauge invariance associated with them. We also show that Schwinger's source constraints are obtained as source constraints due to gauge invariance. These results have been shown for spins $s=1$, $\frac{3}{2}$, 2, and $\frac{5}{2}$. We also discuss in the context of our work interacting higher-spin theories and Fronsdal's source constraints for a smooth massless limit.

There exist satisfactory constructions for relativistic wave equations for free massless as well as massive particles of all spins.[1] But in the presence of interaction with external fields, higher-spin ($s \geq 1$) theories exhibit well-known problems like noncausal wave propagation, change in the number of degrees of freedom, the presence of complex energy modes in the presence of a homogeneous magnetic field, etc.[2] Moreover, higher-spin theories do not have a smooth massless limit.[3] However, a consistent massless spin-$\frac{3}{2}$ theory was found in supergravity. This was later extended to the massive case by means of the super-Higgs mechanism.[4] The problems mentioned above still exist for other spins and await satisfactory resolution. Thus, although for free massive particles the Singh-Hagen equations are satisfactory, it is likely that the interacting theory should have an underlying gauge invariance for it to be consistent. With this possibility in mind, we seek a gauge-invariant description of free massive higher-spin particles. We show that dimensional reduction of massless gauge-invariant theories from five dimensions gives rise to the desired gauge-invariant massive theories in four dimensions.

The study of spontaneously broken gauge theories and supergravity can point to the specific properties of massive gauge-invariant theories useful in the formulation of theories of higher spins. These have been summarized by Zinoviev.[5] The first property is the appearance of Goldstone fields which transform inhomogeneously under gauge transformations and which provide the necessary degrees of freedom for a massless particle to become massive. The second is that, as the mass of the gauge field goes to zero, the Lagrangian for a spin-1 field breaks up into Lagrangians corresponding to massless spin-1 and spin-0 fields. Hence, in order to have a gauge-invariant description of massive high-spin particles, it is essential that in the $m \to 0$ limit the massive spin-s Lagrangian decompose into those of massless helicities $\pm s$, $\pm(s-1), \ldots$.

It was first suggested by Schwinger[6] that a description of all massless particles with helicities $\pm s$, $\pm(s-1), \ldots$ should be contained in the account of a massive particle with spin s, which forces the $m \to 0$ limit to have the nature discussed earlier, and he applied it to the case of spin-1 and spin-2 particles.

This approach has been incorporated by Zinoviev[5] in gauge-invariant massive theories of higher spin up to spin 3. He considered the most general Lagrangian for a massive high-spin field along with all lower-spin fields with arbitrary parameters. By demanding invariance of the Lagrangian under a transformation in which the massive high-spin field undergoes a gauge transformation and other lower-spin fields inhomogeneous transformations like those of Goldstone fields, values for the arbitrary parameters are obtained to the extent possible.

In this paper, we obtain the same results by dimensional reduction of the massless theory from five dimensions and show that the resulting theory has the following features.

(i) Massive modes have gauge invariance associated with them.

(ii) They have the appropriate massless limit discussed earlier, as required by Schwinger.

(iii) Source constraints which we obtain from gauge invariance are the ones obtained by Schwinger by demanding that in the zero-mass limit the amplitude for the exchange of a massive particle of a certain spin should decompose into amplitudes for the exchange of massless particles with that spin and also lower spins. Thus, precisely the features discussed by Zinoviev are present. In contrast with Zinoviev's procedure, our method has no arbitrariness, and once we start with a gauge-invariant theory from five dimensions, dimensional reduction automatically ensures that the massive theory obtained has all the features mentioned above. Of course, we are led to one member of the class of theories obtained by Zinoviev, as our method has no arbitrary parameters.

We next describe the procedure of dimensional reduction which has been applied in Kaluza-Klein theories, in which there has been a revival of interest in recent times.

Though we do not consider a general covariant theory in five dimensions, the process of dimensional reduction is of the same nature as in the Kaluza-Klein theory.[7] We start with five-dimensional space-time in which one dimension is assumed to be compactified to a circle of radius $1/m$. Fields are defined on the product manifold $M(4)\times S(1)$, where $M(4)$ is four-dimensional Minkowski space-time and $S(1)$ is a circle, which inherits the natural product metric. Fields are expanded in Fourier series where the expansion coefficients are fields depending only on coordinates of $M(4)$. The extra coordinate is integrated over in the action, and in the resulting theory the mass scale for the fields is inversely proportional to the radius $(1/m)$ of $S(1)$. When the massless theory defined in five dimensions has gauge invariance, the four-dimensional massive theory which results from dimensional reduction also has gauge invariance. The zero mode ($n=0$) in the Fourier series of the theory for spin s consists of massless particles of all helicities of magnitude $\leq s$, in integral steps. For each nonzero mode ($n\neq 0$), when the gauge is fixed suitably, only a massive spin-s theory is obtained.

We apply this procedure for spins $s=1$, 2, $\frac{3}{2}$, and $\frac{5}{2}$. We see no difficulty in extending this procedure to other higher spins.[8] This paper is organized as follows. Section I deals with spin-1 and spin-2 theories. Section II deals with spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ theories. We conclude with a discussion of our results.

We use the metric $\eta^{AB}=(+1,-1,-1,-1,-1)$ in five dimensions. Our notation is that upper case latin indices take values in the five-dimensional space-time, and greek indices in four-dimensional space-time. In four-dimensional space-time our metric and notation are those of Bjorken and Drell.[9]

I. INTEGRAL SPIN

A. Spin-1 fields

Consider the action for a massless vector field in five-dimensional space-time,

$$I=\int d^5x\, \mathcal{L}^{\text{kin}}_{(s=1)} \equiv -\tfrac{1}{4}\int d^5x\, F_{AB}F^{AB} \quad (A,B=0,1,2,3,5)\ , \tag{1.1}$$

where $F_{AB}=\partial_A A_B-\partial_B A_A$. We define A_A on a product manifold $M(4)\times S(1)$, where $S(1)$ is a circle of radius $1/m$. We perform a Fourier decomposition of the field,

$$A_A(x^\mu,x^5)=\left[\frac{m}{2\pi}\right]^{1/2}\sum_{n=-\infty}^{\infty}A_A^{(n)}(x^\mu)\exp(imnx^5) \quad (\mu=0,\ldots,3)\ , \tag{1.2}$$

where the infinite set of Fourier coefficients $A_A^{(n)}$ depend only on the coordinates of $M(4)$. Substituting (1.2) in the action, we get

$$I=-\tfrac{1}{4}\int d^4x\int_0^{2\pi/m}dx^5(m/2\pi)\times\sum_{n,s=-\infty}^{\infty}(F_{\mu\nu}^{(n)}F^{\mu\nu(s)}+2F_{\mu5}^{(n)}F^{\mu5(s)})\times\exp[im(n+s)x^5]\ , \tag{1.3}$$

where

$$F_{\mu\nu}^{(n)}=\partial_\mu A_\nu^{(n)}-\partial_\nu A_\mu^{(n)}\ ,$$
$$F_{\mu5}^{(n)}=\partial_\mu A_5^{(n)}-imnA_\mu^{(n)}\ .$$

Integrating over the fifth coordinate we get

$$I=-\tfrac{1}{4}\int d^4x\sum_{n=-\infty}^{\infty}(F_{\mu\nu}^{(-n)}F^{\mu\nu(n)}+2F_{\mu5}^{(-n)}F^{\mu5(n)})\ . \tag{1.4}$$

From (1.2), because of the reality of fields in five dimensions we get $A_B^{(-n)}=A_B^{*(n)}$. The action is then

$$I=\int d^4x\sum_{n=0}^{\infty}(-\tfrac{1}{2}F_{\mu\nu}^{*(n)}F_{\mu\nu}^{(n)}+\partial_\mu\phi^{*(n)}\partial^\mu\phi^{(n)}+m^2n^2A_\mu^{*(n)}A^{\mu(n)}+imnA_\mu^{*(n)}\partial^\mu\phi^{(n)}-imnA_\mu^{(n)}\partial^\mu\phi^{*(n)})\ , \tag{1.5}$$

where $A_5^{(n)}=-A^{5(n)}=\phi^{(n)}$. This action is similar to the one occurring in Stueckelberg's formulation and was also considered by Schwinger.[10] The action has an invariance under the transformation

$$\delta A_\mu^{(n)}=\partial_\mu\Lambda^{(n)}\ ,\qquad \delta\phi^{(n)}=imn\Lambda^{(n)} \tag{1.6}$$

which is, of course, obtained from the gauge symmetry of the five-dimensional theory. Thus we have, for each mode with $n\neq 0$, a free massive vector action, which is nevertheless invariant under gauge transformations (1.6). That the action (1.5) does describe a free massive vector field for each of the nonzero modes can be seen by choosing a gauge $\phi^{(n)}=0$ ($n\neq 0$), whereupon the action reduces to that of the Proca field.

When the five-dimensional theory is coupled to a source J_A, the constraint on the source due to gauge invariance obtained by dimensional reduction is

$$\partial_\mu J^{\mu(n)}=imnJ^{(n)}\ , \tag{1.7}$$

where $J^{\mu(n)}$ is the source of $A^{\mu(n)}$ and $J_5^{(n)}\equiv J^{(n)}$ is the source for $\phi^{(n)}$. As $m\to 0$, $J^{\mu(n)}$ and $J^{(n)}$ are independent sources. This is the source constraint obtained by Schwinger.[11] The massless limit ($m\to 0$) of the theory for each n is as demanded by Schwinger; i.e., it comprises only a massless vector and a massless scalar field. Hereafter we shall refer to the massless limit required by Schwinger as the "correct" massless limit.

B. Spin-2 fields

We start with the action for a massless second-rank symmetric tensor field in five dimensions,

$$I=\int d^5x\, \mathcal{L}^{\text{kin}}_{(s=2)}\equiv d^5x(\tfrac{1}{2}\partial_A h_{BC}\partial^A h^{BC}-\partial_A h^{AB}\partial^C h_{CB}+\partial_A h^{AB}\partial_B\hat{h}-\tfrac{1}{2}\partial_A\hat{h}\partial^A\hat{h})\ , \tag{1.8}$$

where $\hat{h}=h^A{}_A$. The action in (1.8) is invariant under the gauge transformation

$$\delta h_{AB}=\tfrac{1}{2}(\partial_A\Lambda_B+\partial_B\Lambda_A)\ .$$

As before, we make a Fourier decomposition

$$h_{AB}(x^\mu,x^5)=\left[\frac{m}{2\pi}\right]^{1/2}\sum_{n=-\infty}^{\infty} h_{AB}(x^\mu)\exp(imnx^5)\ . \tag{1.9}$$

Using (1.9) in (1.8) and integrating over x^5, we get

$$\begin{aligned}I=\int d^4x \sum_{n=0}^{\infty}\{&\partial_\mu h^{*(n)}_{\nu\lambda}\partial^\mu h^{\nu\lambda(n)}-2\partial_\nu h^{\nu\mu*(n)}\partial^\rho h^{(n)}_{\rho\mu}+(\partial_\mu h^{\mu\nu*(n)}\partial_\nu h^{(n)}+\text{c.c.})-\partial_\mu h^{*(n)}\partial^\mu h^{(n)}\\&+2\partial_\nu h_5^{\nu(n)*}\partial^\rho h^{(n)}_{\rho 5}-2\partial_\mu h^{*(n)}_{\nu 5}\partial^\mu h^{\nu(n)}_5-[\partial_\nu h^{*(n)}_{55}(\partial_\mu h^{\mu\nu(n)}-\partial^\nu h^{(n)})+\text{c.c.}]\\&-[2imn(h_5^{\mu*(n)}\partial^\rho h^{(n)}_{\rho\mu}+h_5^{\nu*(n)}\partial_\nu h^{(n)}_{55}+\partial_\mu h_5^{\mu*(n)}h^{(n)})+\text{c.c.}]-m^2n^2(h^{*(n)}_{\nu\lambda}h^{\nu\lambda(n)}-h^{*(n)}h^{(n)})\}\ ,\end{aligned} \tag{1.10}$$

where $h^{(n)}=h_\mu{}^{\mu(n)}$. Upon taking the $m\to 0$ limit in the above action it is obvious that the correct massless limit is not present, as there is no kinetic energy term for the scalar field h_{55}, and also terms off-diagonal in fields are present. But by redefining the fields as

$$h^{(n)}_{\mu\nu}\to h'^{(n)}_{\mu\nu}=h^{(n)}_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}h^{(n)}_{55}\ , \tag{1.11}$$

and identifying

$$h^{(n)}_{\nu 5}=iA^{(n)}_\nu,\quad h^{(n)}_{55}=\phi^{(n)}\ ,$$

we obtain the Lagrangian (after omitting primes)

$$\begin{aligned}\mathcal{L}=\sum_{n=0}^{\infty}[&\partial_\mu h^{*(n)}_{\nu\lambda}\partial^\mu h^{\nu\lambda(n)}-2\partial_\nu h^{\nu\mu*(n)}\partial^\rho h^{(n)}_{\rho\mu}+(\partial_\mu h^{\mu\nu*(n)}\partial_\nu h^{(n)}+\text{c.c.})-\partial_\mu h^{*(n)}\partial^\mu h^{(n)}\\&-F^{*(n)}_{\mu\nu}F^{\mu\nu(n)}+\tfrac{3}{2}\partial_\mu\phi^{*(n)}\partial^\mu\phi^{(n)}+mn(-2A^{\mu*(n)}\partial^\rho h^{(n)}_{\rho\mu}+3A^{\nu*(n)}\partial_\nu\phi^{(n)}-2h^{*(n)}\partial_\mu A^{\mu(n)}+\text{c.c.})\\&+m^2n^2(-h^{*(n)}_{\mu\nu}h^{\mu\nu(n)}+h^{*(n)}h^{(n)}+3\phi^{*(n)}\phi^{(n)}+\tfrac{3}{2}h^{*(n)}\phi^{(n)}+\tfrac{3}{2}\phi^{*(n)}h^{(n)})]\ .\end{aligned} \tag{1.12}$$

The above Lagrangian has manifestly the correct massless limit, and it has gauge invariance under

$$\begin{aligned}&\delta h^{(n)}_{\mu\nu}=\tfrac{1}{2}(\partial_\mu\Lambda^{(n)}_\mu+\partial_\nu\Lambda^{(n)}_\mu)-\frac{im}{2}\eta_{\mu\nu}\Lambda^{(n)}\ ,\\&\delta A^{(n)}_\mu=-\frac{i}{2}(\partial_\mu\Lambda^{(n)}+imn\Lambda^{(n)}_\mu)\ ,\\&\delta\phi^{(n)}=imn\Lambda^{(n)}\ ,\end{aligned} \tag{1.13}$$

where $\Lambda^{(n)}\equiv\Lambda^{(n)}_5$.

When we couple $h^{(n)}_{\mu\nu}$, $A^{(n)}_\mu$, and $\phi^{(n)}$ to external sources $T^{\mu\nu(n)}$, $T^{\mu(n)}$, and $T^{(n)}$, respectively, and demand invariance of the action under (1.13), the constraints on the sources obtained are

$$\begin{aligned}&\partial_\mu T^{\mu\nu(n)}=\tfrac{1}{2}mnT^{\nu(n)}\ ,\\&\partial_\mu T^{\mu(n)}=-\tfrac{1}{2}mnT^\mu{}_\mu{}^{(n)}-mnT^{(n)}\ .\end{aligned} \tag{1.14}$$

These are the source constraints demanded by Schwinger[12] for the correct massless limit.

II. HALF-INTEGER SPIN

A. Spin-$\frac{3}{2}$ fields

Let us consider the massless Rarita-Schwinger action in five dimensions:

$$I=\int d^5x\,\mathcal{L}^{\text{kin}}_{(s=3/2)}\equiv\int d^5x\, i[\bar\psi^A\gamma\cdot\partial\psi_A-(\bar\psi\cdot\gamma)(\partial\cdot\psi)-(\bar\psi\cdot\partial)(\gamma\cdot\psi)+(\bar\psi\cdot\gamma)\gamma\cdot\partial(\gamma\cdot\psi)]\ , \tag{2.1}$$

which has gauge invariance under

$$\delta\psi_A=\partial_A\epsilon\ .$$

In (2.1) dot products denote five-dimensional scalar products, and $\gamma^A \equiv (\gamma^\mu, i\gamma^5)$. We make a Fourier expansion,

$$\psi_A(x^\mu, x^5) = (m/2\pi)^{1/2} \sum_{n=-\infty}^{\infty} \psi_A^{(n)}(x^\mu)\exp(imnx^5) . \tag{2.2}$$

Substituting this in (2.1) and integrating over x^5, we get, now in four dimensions,

$$I = \sum_{n=-\infty}^{\infty} \int d^4x [\bar{\psi}^{\mu(n)}(i\partial\!\!\!/ - imn\gamma^5)\psi_\mu^{(n)} - i(\bar{\psi}^{(n)}\cdot\gamma\,\partial\cdot\psi^{(n)} + \bar{\psi}^{(n)}\cdot\partial\gamma\cdot\psi^{(n)} + i\bar{\psi}_5^{(n)}\gamma^5\partial\cdot\psi^{(n)} + i\bar{\psi}^{(n)}\cdot\partial\gamma^5\psi_5^{(n)})$$
$$+ i(\bar{\psi}^{(n)}\cdot\gamma\,\partial\!\!\!/\gamma\cdot\psi^{(n)} + i\bar{\psi}^{(n)}\cdot\gamma\,\partial\!\!\!/\gamma^5\psi_5^{(n)} + i\bar{\psi}_5^{(n)}\gamma^5\partial\!\!\!/\gamma\cdot\psi^{(n)} - mn\bar{\psi}^{(n)}\cdot\gamma\gamma^5\gamma\cdot\psi^{(n)})] , \tag{2.3}$$

where γ^5 is the usual four-dimensional γ^5, satisfying $(\gamma^5)^2 = 1$ and $\gamma^5 = \gamma_5$. Defining $\psi_5^{(n)} = i\phi^{(n)} = -\psi^{5(n)}$, and by making a chiral rotation

$$\psi_\mu^{(n)} = \exp\left[-i\frac{\pi}{4}\gamma^5\right]\psi_\mu'^{(n)}, \quad \phi^{(n)} = \exp\left[-i\frac{\pi}{4}\gamma^5\right]\phi'^{(n)} ,$$

the Lagrangian from (2.3) is (after dropping primes)

$$\mathcal{L} = \sum_{n=-\infty}^{\infty} [\bar{\psi}^{\mu(n)}(i\partial\!\!\!/ - mn)\psi_\mu^{(n)} - (i\bar{\psi}^{(n)}\cdot\gamma\,\partial\cdot\psi^{(n)} + \text{H.c.}) + \bar{\psi}^{(n)}\cdot\gamma(i\partial\!\!\!/ + mn)\gamma\cdot\psi^{(n)}$$
$$+ (\bar{\psi}^{(n)}\cdot\partial\,\phi^{(n)} + \text{H.c.}) + (\bar{\phi}^{(n)}\partial\!\!\!/\,\gamma\cdot\psi^{(n)} + \text{H.c.})] . \tag{2.4}$$

This Lagrangian has an invariance under the transformations

$$\delta\psi_\mu^{(n)} = \partial_\mu\epsilon^{(n)}, \quad \delta\phi^{(n)} = mn\epsilon^{(n)} , \tag{2.5}$$

which is obtained from gauge invariance in five dimensions. Equations (2.4) and (2.5) are identical to those given by Zinoviev. Thus we have for each $n \neq 0$, a Lagrangian for a massive spin-$\frac{3}{2}$ field which is gauge invariant. When the gauge is fixed so that $\phi^{(n)} = 0$ for $n \neq 0$, the Lagrangian in (2.4) reduces to that of the usual massive spin-$\frac{3}{2}$ theory. Yet, when we consider the $m \to 0$ limit, the Lagrangian does not have the correct massless limit, there is no kinetic energy term for $\phi^{(n)}$ and also there are off-diagonal terms in the kinetic energy. So, we make the following redefinition which diagonalizes the kinetic energy terms:

$$\psi_\mu^{(n)} \to \psi_\mu'^{(n)} = \psi_\mu^{(n)} + \frac{i}{2}\gamma_\mu\phi^{(n)} ,$$
$$\phi^{(n)} \to \phi'^{(n)} = \phi^{(n)} . \tag{2.6}$$

After this redefinition, the Lagrangian (2.4) becomes (after omitting primes)

$$\mathcal{L} = \sum_{n=-\infty}^{\infty} [\mathcal{L}^{\text{kin}}_{(s=3/2)}(\psi_\mu^{(n)}) - \tfrac{3}{2}i\bar{\phi}^{(n)}\partial\!\!\!/\phi^{(n)} + mn(-\bar{\psi}^{\mu(n)}\psi_\mu^{(n)} + \bar{\psi}^{(n)}\cdot\gamma\gamma\cdot\psi^{(n)} + 3\bar{\phi}^{(n)}\phi^{(n)}) + (-\tfrac{3}{2}imn\bar{\psi}^{(n)}\cdot\gamma\phi^{(n)} + \text{H.c.})] . \tag{2.7}$$

Gauge transformations for the redefined fields are

$$\delta\psi_\mu^{(n)} = (\partial_\mu + \tfrac{1}{2}imn\gamma_\mu)\epsilon^{(n)} ,$$
$$\delta\phi^{(n)} = mn\epsilon^{(n)} . \tag{2.8}$$

On coupling $\psi_\mu^{(n)}$ and $\phi^{(n)}$ with external sources $J^{\mu(n)}$ and $J^{(n)}$, respectively, invariance of (2.7) under (2.8) gives the source constraint

$$\partial_\mu J^{\mu(n)} + \frac{imn}{2}\gamma_\mu J^{\mu(n)} = mnJ^{(n)} . \tag{2.9}$$

B. Spin-$\frac{5}{2}$ fields

We generalize the Lagrangian for the massless $s = \frac{5}{2}$ theory[1] to five dimensions,

$$\mathcal{L}^{\text{kin}}_{(s=5/2)} = i(\bar{\phi}_{AB}\gamma\cdot\partial\,\phi^{AB} - 2\bar{\phi}^{AB}\gamma_B\partial^C\phi_{CA} - 2\bar{\phi}_{AB}\partial^A\gamma_C\phi^{BC} + 2\bar{\phi}^{AB}\gamma_B\gamma\cdot\partial\,\gamma^C\phi_{CA} + \bar{\phi}^{AB}\gamma_A\partial_B\phi$$
$$+ \bar{\phi}\gamma_A\partial_B\phi^{AB} - \tfrac{1}{2}\bar{\phi}\gamma\cdot\partial\,\phi) , \tag{2.10}$$

where ϕ_{AB} is a symmetric tensor-spinor, and $\phi = \phi_A{}^A$. The Lagrangian in Eq. (2.10) has gauge invariance under

$$\delta\phi_{AB}=\tfrac{1}{2}(\partial_A\xi_B+\partial_B\xi_A)$$

with $\gamma_A\xi^A=0$.

As was done earlier, we make a Fourier decomposition of the field

$$\phi_{AB}(x^\mu,x^5)=(m/2\pi)^{1/2}\sum_{n=-\infty}^{\infty}\phi_{AB}^{(n)}(x^\mu)\exp(imnx^5)\ . \tag{2.11}$$

After doing the usual reduction to four dimensions as in earlier sections and making a field redefinition

$$\phi_{AB}\to e^{-i(\pi/4)\gamma_5}\phi_{AB}\ ,$$

we obtain the Lagrangian

$$\begin{aligned}\mathscr{L}=\sum_{n=-\infty}^{\infty}\{&\mathscr{L}^{\text{kin}}_{(s=5/2)}(\phi^{(n)}_{\mu\nu})-\tfrac{3}{2}i\bar\psi^{(n)}\partial\!\!\!/\psi^{(n)}-2i\bar\phi^{(n)}\cdot\gamma\,\partial\!\!\!/\,\gamma\cdot\phi^{(n)}\\&+[i(2\bar\phi^{(n)}\cdot\partial\,\gamma\cdot\phi^{(n)}-2\bar\phi^{\mu(n)}\partial^\rho\phi^{(n)}_{\rho\mu}+2\bar\phi^{\mu(n)}\partial\!\!\!/\gamma^\rho\phi^{(n)}_{\rho\mu}-\bar\phi^{\mu\nu(n)}\gamma_\mu\partial_\nu\psi^{(n)}\\&+\bar\phi^{\mu(n)}\partial_\mu\phi^{(n)}+\bar\phi^{\mu(n)}\partial_\mu\psi^{(n)}-2\bar\phi^{\nu(n)}\gamma_\nu\partial\!\!\!/\psi^{(n)}-\tfrac{1}{2}\bar\psi^{(n)}\partial\!\!\!/\phi^{(n)})+\text{H.c.}]\\&+mn[-\bar\phi^{(n)}_{\mu\nu}\phi^{\mu\nu(n)}+2\bar\phi^{\mu\nu(n)}\gamma_\nu\gamma^\rho\phi^{(n)}_{\rho\mu}+\tfrac{1}{2}\bar\phi^{(n)}\phi^{(n)}+2\bar\phi^{(n)}\cdot\gamma\bar\gamma\cdot\phi^{(n)}-\tfrac{1}{2}\bar\phi^{(n)}\phi^{(n)}\\&+(\tfrac{1}{2}\bar\phi^{(n)}\psi^{(n)})+\bar\phi^{(n)}\cdot\gamma\phi^{(n)}-\bar\phi^{(n)}\cdot\gamma\psi^{(n)}+\text{H.c.})]\}\ ,\end{aligned} \tag{2.12}$$

where $\phi^{(n)}_\mu\equiv\phi^{(n)}_{\mu5}=-\phi^{5(n)}_\mu$, $\psi^{(n)}\equiv\phi^{(n)}_{55}$, $\gamma\cdot\phi^{(n)}=\gamma^\mu\phi^{(n)}_\mu$, and $\phi^{(n)}=\phi^{\mu(n)}_\mu$. Equation (2.12) has gauge invariance under the transformations

$$\begin{aligned}\delta\phi^{(n)}_{\mu\nu}&=\tfrac{1}{2}(\partial_\mu\xi^{(n)}_\nu+\partial_\nu\xi^{(n)}_\mu)\ ,\\ \delta\phi^{(n)}_\mu&=\tfrac{1}{2}(\partial_\mu\xi^{(n)}+imn\xi^{(n)}_\mu)\ ,\\ \delta\psi^{(n)}&=imn\xi^{(n)}\ ,\end{aligned} \tag{2.13}$$

with $\gamma\cdot\xi^{(n)}=-\xi^{(n)}$, which are obtained by dimensional reduction of the gauge transformation for ϕ_{AB}.

Again, the Lagrangian (2.12) does not have the correct massless limit as the spin-$\frac{3}{2}$ fields do not have the complete kinetic energy terms and also there are off-diagonal terms in the limit $m\to0$. So, as was done earlier, we make a field redefinition,

$$\begin{aligned}\phi^{(n)}_{\mu\nu}&=\phi'^{(n)}_{\mu\nu}-\tfrac{1}{4}\gamma_\mu\phi'^{(n)}_\nu-\tfrac{1}{4}\gamma_\nu\phi'^{(n)}_\mu+\tfrac{1}{2}\eta_{\mu\nu}\psi'^{(n)}\ ,\\ \phi^{(n)}_\mu&=\phi'^{(n)}_\mu-\tfrac{1}{2}\gamma_\mu\psi'^{(n)}\ ,\\ \psi^{(n)}&=\psi'^{(n)}\ ,\end{aligned} \tag{2.14}$$

under which the Lagrangian becomes (after omitting primes)

$$\begin{aligned}\mathscr{L}=\sum_{n=-\infty}^{\infty}\{&\mathscr{L}^{\text{kin}}_{(s=5/2)}(\phi^{(n)}_{\mu\nu})-\tfrac{5}{2}\mathscr{L}^{\text{kin}}_{(s=3/2)}(\phi^{(n)}_\mu)+\tfrac{5}{2}\mathscr{L}^{\text{kin}}_{(s=1/2)}(\psi^{(n)})\\&+mn[-\bar\phi^{(n)}_{\mu\nu}\phi^{\mu\nu(n)}+2\bar\phi^{(n)}_{\mu\nu}\gamma^\nu\gamma_\rho\phi^{\rho\mu(n)}+\tfrac{1}{2}\bar\phi^{(n)}\phi^{(n)}-\tfrac{15}{4}(\bar\phi^{(n)}\cdot\gamma)(\gamma\cdot\phi^{(n)})\\&+\tfrac{15}{4}\bar\phi^{\mu(n)}\phi^{(n)}_\mu+(5\bar\phi^{(n)}\cdot\gamma\psi^{(n)}-\tfrac{5}{2}\bar\phi^{(n)}_{\mu\nu}\gamma^\mu\phi^{\nu(n)}+\tfrac{5}{4}\bar\phi^{(n)}\gamma\cdot\phi^{(n)}+\text{H.c.})-\tfrac{15}{2}\bar\psi^{(n)}\psi^{(n)}]\}\ .\end{aligned} \tag{2.15}$$

This evidently has the correct massless limit. It is invariant under the transformations

$$\begin{aligned}\delta\phi^{(n)}_{\mu\nu}&=(\tfrac{1}{2}\partial_\mu\xi^{(n)}_\nu+\tfrac{1}{8}\gamma_\mu\partial_\nu\xi^{(n)}+\tfrac{1}{8}imn\gamma_\mu\xi^{(n)}_\nu+\mu\leftrightarrow\nu)-\tfrac{1}{4}imn\eta_{\mu\nu}\xi^{(n)}\ ,\\ \delta\phi^{(n)}_\mu&=\tfrac{1}{2}(\partial_\mu+imn\gamma_\mu)\xi^{(n)}+\tfrac{1}{2}imn\xi^{(n)}_\mu\ ,\\ \delta\psi^{(n)}&=imn\xi^{(n)}\ ,\end{aligned} \tag{2.16}$$

with $\gamma\cdot\xi^{(n)}=-\xi^{(n)}$. After choosing the gauge in which $\psi^{(n)}=0$ and $\phi^{(n)}_\mu-\frac{1}{4}\gamma_\mu\gamma\cdot\phi^{(n)}=0$ for $n\neq0$, the Lagrangian is

$$\begin{aligned}\mathscr{L}=\sum_{n=-\infty}^{\infty}[&\mathscr{L}^{\text{kin}}_{(s=5/2)}(\phi^{(n)}_{\mu\nu})-\tfrac{15}{16}i\bar\chi^{(n)}\partial\!\!\!/\chi^{(n)}+mn(-\bar\phi^{(n)}_{\mu\nu}\phi^{\mu\nu(n)}+2\bar\phi^{\mu\nu(n)}\gamma_\nu\gamma^\rho\phi^{(n)}_{\rho\mu}+\tfrac{1}{2}\bar\phi^{(n)}\phi^{(n)}\\&-\tfrac{45}{16}\bar\chi^{(n)}\chi^{(n)}+\tfrac{5}{8}\bar\phi^{(n)}\chi^{(n)}+\tfrac{5}{8}\bar\chi^{(n)}\phi^{(n)})]\ ,\end{aligned} \tag{2.17}$$

where $\chi^{(n)}=\gamma\cdot\phi^{(m)}$. The Lagrangian for any $n\neq 0$ in (2.17) is the Lagrangian given by Zinoviev and it corresponds to the usual Lagrangian for a massive spin-$\frac{5}{2}$ field.

DISCUSSION

(i) The mechanism of mass generation in high-spin theories obtained by dimensional reduction of massless, gauge-invariant five-dimensional theories can be understood as utilizing a generalization to higher-spin fields of a feature of the Higgs mechanism occurring in the case of spin 1. Just as in the Higgs mechanism, where a scalar field corresponding to the Goldstone mode provides the necessary extra degree of freedom to make the vector field massive, in the case of a higher spin s, all fields of lower helicities $\pm(s-1)$, $\pm(s-2)$,... provide the necessary extra degrees of freedom.

(ii) Massive higher-spin theories developed in this paper also have the correct massless limit as stipulated by Schwinger. Source constraints due to gauge invariance for $s=1$ and $s=2$ are the ones obtained by Schwinger for the zero-mass limit of the massive theory. Fronsdal[12] on the other hand, has obtained conditions on the sources for a smooth zero-mass limit of field theories. In contrast with Schwinger, who demands that in the zero-mass limit the amplitude for the exchange of a massive spin-2 particle, for example, should decompose into those of massless fields with helicities ± 2, ± 1, and 0, Fronsdal demands that the amplitude for the exchange of a massive $s=2$ particle should reduce in the limit $m\rightarrow 0$, to that of massless spin 2 alone. If in our approach we choose $T_\mu=0$, $T=0$ in the $m\rightarrow 0$ limit of the spin-2 theory (and analogously sources corresponding to all lower helicities for other spins to vanish in the massless limit), we find that Fronsdal's conditions on the sources are automatically met.

(iii) Interacting theories of massive high-spin fields obtained by a smooth deformation of theories with massless fields interacting in a gauge-invariant way with external fields may be devoid of the usual problems of the high-spin theories. This possibility, in fact, is the main motivation of our work. In this context mention should be made of the consistent theory for massive charged spin-2 particles obtained by dimensional reduction of the original Kaluza-Klein theory.[13] It will be interesting to seek consistent theories for massive high-spin particles by dimensional reduction of massless theories having gauge-invariant interactions. Work along these lines is in progress.

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