Low-energy limit of strings

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We investigate terms with four derivatives in the low-energy action of closed Bose strings. Our analysis is consistent with the possibility that the action involves only curvatures of a connection with torsion. The torsion is provided by the antisymmetric tensor gauge field.

I. INTRODUCTION

Over a decade ago, it was suggested[1] that string models could provide a consistent, calculable description of quantum gravity. Recent work[2–5] indicates that it may in fact be possible to incorporate all known particles and interactions within the framework of ten-dimensional superstrings.

In the most commonly studied scenario, compactification of spacetime down to four dimensions occurs at an energy scale somewhat lower than $(\alpha')^{1/2}$, i.e., the scale of massive string excitations. At this relatively low energy scale, one can attempt to approximate the full string theory by a conventional field theory involving only the massless modes of the string. Hence, to the extent that this approximation is valid, the problem of compactification reduces to finding product-space solutions of the classical field equations.

The action for the string low-energy effective theory is actually an infinite sum of terms with increasing number of derivatives (not unlike the low-energy expansion which describes pions); thus far, only the first few terms have been explicitly computed. However, the higher-derivative terms can have an important bearing on physically interesting questions and therefore cannot be categorically ignored. For instance, such terms have been shown[2] to play a crucial role in the cancellation of anomalies. Moreover, it is the presence of a certain curvature-squared term in the low-energy theory that allows[4] compactifications with nontrivial Yang-Mills configurations, thereby evading a no-go theorem.[6]

This curvature-squared term has four derivatives; evidently, the low-energy theory contains many more terms of the same order (i.e., also with four derivatives), whose presence could in principle have equally dramatic consequences. In particular, one could imagine that such terms would allow compactifications with a nontrivial configuration for the antisymmetric tensor gauge field.[7]

This state of affairs has motivated us to seek the complete set of four-derivative terms in the string low-energy theory. In this paper we have restricted our attention primarily to the closed Bose string, whose massless sector consists of a graviton $h_{\mu\nu}$, an antisymmetric tensor $A_{\mu\nu}$, and a scalar ϕ. (Superstrings include an additional massless bosonic field, namely, the Yang-Mills vector.) To lowest order, the string low-energy theory is given by[1]

$$\int d^D x\sqrt{-g}\left[-\frac{1}{16\pi G}R-\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-\tfrac{1}{6}e^{-2m\phi}H_{\mu\nu\rho}H^{\mu\nu\rho}\right], \tag{1.1}$$

where $m\equiv[32\pi G/(D-2)]^{1/2}$, $H_{\mu\nu\rho}$ is the field strength of $A_{\mu\nu}$, and D is the spacetime dimension. [The restriction $D=26$ is needed for unitarity of string loop corrections; however, only tree-level considerations enter into (1.1).] It was noticed by Scherk and Schwarz that the antisymmetric tensor can be incorporated into a connection with torsion. Indeed, consider the connection

$$\Gamma^\rho_{\mu\nu}\equiv\left\{{\rho \atop \mu\nu}\right\}+ae^{-m\phi}H_{\mu\nu}{}^{\rho}+\tfrac{1}{2}p(\delta^\rho_\nu\partial_\mu\phi+\delta^\rho_\mu\partial_\nu\phi-g_{\mu\nu}g^{\rho\rho'}\partial_{\rho'}\phi)\,, \tag{1.2}$$

where $a^2\equiv(8\pi G)/3$, and $p^2\equiv 32\pi G/(D-1)(D-2)$, and $\{{\rho \atop \mu\nu}\}$ is the Riemann-Christoffel connection. The action (1.1) can be reexpressed (up to a surface term) simply as the integral of the scalar curvature density computed with this new connection,[1]

$$\int d^D x\sqrt{-g}\left[-\frac{1}{16\pi G}R(\Gamma)\right]. \tag{1.3}$$

Observe that the torsion tensor does not appear explicitly in the action. One might have dismissed this pretty result as a curious coincidence, and in this way have denied any special significance to the connection (1.2). Here, we determine all the terms of next highest order in the string low-energy theory which contribute to on-shell three-body scattering amplitudes; and we find that these are consistent with

$$\tfrac{1}{2}g_{\rm VS}{}^{-4/(D-2)}(32\pi G)^{-(D-4)/(D-2)}\int d^Dx\sqrt{-g}\,e^{-m\phi}[(k-1)R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\nu\rho\sigma}(\Gamma)-4(k-1)R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\rho\nu\sigma}(\Gamma)+kR^{\mu\nu\rho\sigma}(\Gamma)R_{\rho\sigma\mu\nu}(\Gamma)]\,, \tag{1.4}$$

up to possible additional terms which do not contribute to such amplitudes. Here, g_{VS} is the dimensionless closed string (Virasoro-Shapiro) coupling constant, and k is an undetermined constant.

The appearance of curvature-squared terms was already noted by Candelas, Horowitz, Strominger, and Witten[4] and Zwiebach.[8] The point we would like to emphasize here is that the new terms in the action again involve curvatures of the connection (1.2)—the torsion (due to the antisymmetric tensor) does not appear explicitly. (The scalar field, on the other hand, does appear explicitly.) When there is torsion, there are *a priori* several inequivalent ways of contracting together two Riemann tensors; unless $k=1$, all of the various forms seem to arise. Indeed, a unitarity argument suggests that $k=1$ is the correct value.

It is easy to see that (1.4) implies, in particular, the presence of terms quartic in $H_{\mu\nu\rho}$; as already mentioned, these could possibly allow compactification solutions of the field equations with $H_{\mu\nu\rho}$ nonvanishing.[7] We have verified that, unfortunately, such solutions do not in fact become possible.

This paper is organized as follows: in Sec. II the method of calculating the string low-energy theory is explained, and the result (1.4) is derived. In Sec. III the search for a compactification solution of the field equations with nonvanishing $H_{\mu\nu\rho}$ is described. Conclusions and open problems are presented in Sec. IV. There are two appendixes: the first lists our conventions and collects some useful formulas; the second presents a calculation of the closed string three-point function, with massless external states.

II. STRING LOW-ENERGY LIMIT

As already noted, the massless states of the closed Bose string are described by the fields $h_{\mu\nu}$, $A_{\mu\nu}$, and ϕ. The other string excitations have masses $\sim(\alpha')^{-1/2}$; hence, at energies below this scale, one can attempt to describe physics with a low-energy effective theory involving only the massless fields.

The procedure[9,1,10] for constructing this low-energy theory is relatively straightforward. Essentially, one calculates string tree N-point amplitudes with massless external states, and then one looks for a conventional field-theory action which at the tree level reproduces these amplitudes. (In similar fashion, string loop amplitudes with massless external states are reproduced by corresponding loop amplitudes of the field theory. That such an action can in fact be found is not obvious; but this was proved to be generally possible by Scherk.[9]) In practice, one explicitly calculates gauge couplings only at lowest nontrivial order, since the presence of higher-order couplings can then be inferred from gauge invariance.[11] There does exist a certain ambiguity in the procedure: since at present only on-shell string amplitudes can be calculated, the corresponding field-theory action can be determined only up to terms which do not contribute to such amplitudes.

A massless, on-shell physical state of a closed string is characterized by a momentum vector p^{μ}, and a polarization tensor $\zeta^{\mu\nu}(p)$. These satisfy $p^2=0$, and also $p^{\mu}\zeta_{\mu\nu}=0=p^{\nu}\zeta_{\mu\nu}$. States of definite spin content correspond to various projections of $\zeta^{\mu\nu}(p)$. Indeed, let[1] $\bar{p}^{\mu}$ be a momentum vector such that $p\cdot\bar{p}=1$ and $\bar{p}^2=0$. Then the polarization tensors for the graviton, antisymmetric tensor, and scalar are given by

$$h_{\mu\nu}=\tfrac{1}{2}(\zeta_{\mu\nu}+\zeta_{\nu\mu})-\frac{\zeta_{\alpha}{}^{\alpha}}{(D-2)}(\eta_{\mu\nu}-p_{\mu}\bar{p}_{\nu}-p_{\nu}\bar{p}_{\mu})\ , \qquad (2.1a)$$

$$A_{\mu\nu}=\tfrac{1}{2}(\zeta_{\mu\nu}-\zeta_{\nu\mu})\ , \qquad (2.1b)$$

$$s_{\mu\nu}=\frac{\zeta_{\alpha}{}^{\alpha}}{(D-2)}(\eta_{\mu\nu}-p_{\mu}\bar{p}_{\nu}-p_{\nu}\bar{p}_{\mu})$$
$$=\phi(D-2)^{-1/2}(\eta_{\mu\nu}-p_{\mu}\bar{p}_{\nu}-p_{\nu}\bar{p}_{\mu})\ , \qquad (2.1c)$$

respectively. In particular,

$$\zeta_{\mu\nu}=h_{\mu\nu}+A_{\mu\nu}+s_{\mu\nu}\ . \qquad (2.1d)$$

Notice also that

$$p^{\mu}h_{\mu\nu}=0=p^{\mu}A_{\mu\nu}\ , \qquad (2.2a)$$

which define the so-called "dual gauge," and also

$$h_{\mu}{}^{\mu}=0\ . \qquad (2.2b)$$

In order to construct the trilinear couplings of the low-energy theory, clearly we must examine the string three-point function with massless external states. This is calculated in Appendix B, and is given by (B9). We can now use (2.1) to project out from the string three-point function all the various three-point amplitudes involving $h_{\mu\nu}$, $A_{\mu\nu}$, and ϕ. For instance, the hAA amplitude is proportional to

$$-g_{VS}(\sqrt{\alpha'})^{D/2-1}[(p_2h_1p_2)(A_2A_3)+2(p_3A_2h_1A_3p_2)+2(p_2h_1A_3A_2p_3)+2(p_1A_3A_2h_1p_2)-\alpha'(p_2h_1p_2)(p_1A_2A_3p_1)]\ . \qquad (2.3)$$

Following Ref. 1, here we have used a matrix notation, such that $(A_2A_3)\equiv A_2^{\mu\nu}A_{3\nu\mu}$, and $(p_2h_1p_2)\equiv p_2^{\mu}h_{1\mu\nu}p_2^{\nu}$, etc. We find that all of the three-point amplitudes can be reproduced by the following sum of trilinear couplings:

$$-\tfrac{1}{2}g_{VS}(\sqrt{\alpha'})^{D/2-1}\int d^Dx\{[h^{\mu\mu'}(\partial_{\mu}\partial_{\mu'}h_{\nu\rho}h^{\rho\nu}+2\partial_{\mu}h^{\rho\nu}\partial_{\nu}h_{\mu'\rho})-h^{\mu\mu'}(\partial_{\mu}\partial_{\mu'}A_{\nu\rho}A^{\rho\nu}+4\partial_{\mu}A^{\rho\nu}\partial_{\nu}A_{\mu'\rho}+2\partial_{\nu}A^{\rho}{}_{\mu}\partial_{\rho}A_{\mu'}{}^{\nu})$$
$$-h^{\mu\mu'}\partial_{\mu}\phi\partial_{\mu'}\phi-4(D-2)^{-1/2}\phi\,\partial_{\mu'}A^{\mu}{}_{\rho}\partial_{\mu}A^{\rho\mu'}]$$
$$+\alpha'[\partial_{\mu}\partial_{\rho}h^{\alpha\beta}\partial_{\alpha}\partial_{\beta}h^{\rho}{}_{\nu}h^{\nu\mu}-\partial_{\mu}\partial_{\rho}h^{\alpha\beta}\partial_{\alpha}\partial_{\beta}A^{\rho}{}_{\nu}A^{\nu\mu}+(D-2)^{-1/2}\partial_{\rho}\partial_{\sigma}h^{\alpha\beta}\partial_{\alpha}\partial_{\beta}h^{\rho\sigma}\phi]\}\ . \qquad (2.4)$$

Although in principle there could have been a scalar-graviton-graviton (ϕhh) coupling with two derivatives, such a term does not in fact appear.[1] However, we see that a four-derivative coupling of this type does arise.

The problem now is to find a gauge-invariant action which reduces to (2.4) upon carrying out the following steps:

(a) expanding $g_{\mu\nu}=\eta_{\mu\nu}+\sqrt{32\pi G}\,h_{\mu\nu}$, and retaining only the terms trilinear in $h_{\mu\nu}$, $A_{\mu\nu}$, and/or ϕ;

(b) imposing the dual gauge conditions (2.2a), and dropping terms which vanish on-shell (i.e., $\Box\phi$, $\Box A_{\mu\nu}$, $\Box h_{\mu\nu}$, $h_\mu{}^\mu$); and

(c) dropping terms whose contribution to a three-point amplitude vanishes due to momentum conservation ($\sum_{i=1}^{3}p_i=0$; together with $p_i{}^2=0$, this implies $p_i\cdot p_j=0$).

Such an action is[12]

$$\frac{g_{\rm VS}(\sqrt{\alpha'})^{D/2-1}}{\sqrt{32\pi G}}\int d^Dx\sqrt{-g}\left\{\frac{-1}{16\pi G}R-\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-\tfrac{1}{6}(1-2m\phi)H_{\mu\nu\rho}H^{\mu\nu\rho}\right.$$
$$+\frac{\alpha'}{64\pi G}\left[(1-m\phi)R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}+(32\pi G)\left[-\frac{c_1}{2}\nabla_\lambda H_{\nu\rho\sigma}\nabla^\rho H^{\nu\lambda\sigma}-\frac{c_2}{6}\nabla_\lambda H_{\nu\rho\sigma}\nabla^\lambda H^{\nu\rho\sigma}\right.\right.$$
$$\left.\left.\left.+c_3R^{\sigma\nu\rho\lambda}H_{\nu\rho\alpha}H^{\alpha}{}_{\lambda\sigma}\right]\right]\right\}, \tag{2.5}$$

where

$$c_1+c_2+c_3=1\ , \tag{2.6}$$

and $m=[32\pi G/(D-2)]^{1/2}$. Setting

$$g_{\rm VS}(\sqrt{\alpha'})^{D/2-1}=\sqrt{32\pi G}\ , \tag{2.7}$$

we see that the first three terms agree with (1.1), as expected. (The combination $1-2m\phi$ is assumed to exponentiate to $e^{-2m\phi}$.)

Our analysis, which relies only on the three-point function, cannot determine the individual coefficients (c_i) of the last three terms of (2.5); rather, only the linear combination (2.6) is fixed. However, the following set of coefficients is particularly noteworthy:

$$c_1=-\tfrac{7}{3}+3k,\quad c_2=3-3k,\quad c_3=\tfrac{1}{3}\ , \tag{2.8}$$

where k is an arbitrary constant. With these values for c_i, the higher-derivative terms in (2.5) coincide with those terms in

$$\frac{g_{\rm VS}(\sqrt{\alpha'})^{D/2+1}}{2(32\pi G)^{3/2}}\times\int d^Dx\sqrt{-g}\,e^{-m\phi}[(k-1)R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\nu\rho\sigma}(\Gamma)$$
$$-4(k-1)R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\rho\nu\sigma}(\Gamma)$$
$$+kR^{\mu\nu\rho\sigma}(\Gamma)R_{\rho\sigma\mu\nu}(\Gamma)]\ , \tag{2.9}$$

which contribute to three-point amplitudes. [See Eq. (A4).] This is the result announced in the Introduction.

The way in which indices are contracted in (2.9) should be noted. Since $R_{\mu\nu\rho\sigma}(\Gamma)$ does not satisfy the usual cyclic identity, the quantities

$$R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\nu\rho\sigma}(\Gamma)\quad\text{and}\quad R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\rho\nu\sigma}(\Gamma)$$

are not proportional to each other. Moreover,

$$R_{\mu\nu\rho\sigma}(\Gamma)\neq R_{\rho\sigma\mu\nu}(\Gamma)\ ;$$

hence, $R^{\mu\nu\rho\sigma}(\Gamma)R_{\rho\sigma\mu\nu}(\Gamma)$ is yet another inequivalent way of contracting together two curvature tensors. For $k\neq 1$, all of these forms seem to appear.

This raises an interesting dilemma. It has been conjectured[8] that the string low-energy theory may include additional curvature-squared terms ($R_{\mu\nu}{}^2,R^2$) in such a way as to make up the four-dimensional Euler invariant. (These additional terms do *not* contribute to three-point on-shell scattering amplitudes.) This combination $[R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\nu\rho\sigma}(\Gamma)-4R^{\mu\nu}(\Gamma)R_{\mu\nu}(\Gamma)+R(\Gamma)^2]$ of curvature-squared terms has the desirable feature of not containing the terms $h^{\mu\nu}\Box^2h_{\mu\nu}$, $A^{\mu\nu}\Box^2A_{\mu\nu}$, and $\phi\Box^2\phi$, which would lead to unitarity-violating p^4 propagator corrections.[13] Conversely, unless $k=1$, the action (2.9) also contains $R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\rho\nu\sigma}(\Gamma)$; hence, it is then no longer clear how unitarity difficulties can be avoided.

It is not at all evident why the connection (1.2) should be relevant for string theory. However, in this regard, the following two remarks may be useful. First, let us introduce the variable

$$\psi_{\mu\nu}\equiv b_1h_{\mu\nu}+b_2\eta_{\mu\nu}\phi+b_3A_{\mu\nu}\ , \tag{2.10}$$

where b_i ($i=1,2,3$) are constants not yet specified. One might speculate that the covariant action [e.g. (2.5)] actually depends only on the combination (2.10), and not separately on the various projections $\frac{1}{2}(\psi_{\mu\nu}+\psi_{\nu\mu})$, etc. Remarkably, when linearized ($g_{\mu\nu}=\eta_{\mu\nu}+\sqrt{32\pi G}\,h_{\mu\nu}$), the connection (1.2) is simply

$$\Gamma^\rho_{\mu\nu}=\tfrac{1}{2}\eta^{\rho\rho'}(\partial_\mu\psi_{\rho'\nu}+\partial_\nu\psi_{\mu\rho'}-\partial_{\rho'}\psi_{\mu\nu})+\cdots\ ,$$

with (2.11)

$$b_1=\sqrt{32\pi G}\ ,\quad b_2=p,\quad b_3=-2a\ .$$

This is certainly suggestive; and geometries involving nonsymmetric "metrics" are not new to the literature.[14] However, the action (2.9) involves not only the connection Γ, but also $g_{\mu\nu}$ and ϕ explicitly; hence, it does not seem possible to formulate the theory in terms of the (unprojected) field $\psi_{\mu\nu}$ only.

Another possibility one might entertain instead is that only the gauge-fixed, on-shell action (2.4) depends on the (unprojected) combination $\psi_{\mu\nu}$, for some values of b_i. One can verify that this is also not the case. However, from the expression (B9) for the string three-point function, it is easy to see that the gauge-fixed action (2.4) can be written as

$$-\tfrac{1}{2}g_{VS}(\sqrt{\alpha'})^{D/2-1} \times \int d^D x [\zeta^{\mu\mu'}(\partial_\mu\partial_{\mu'}\zeta_{\nu\rho}\zeta^{\nu\rho}+2\,\partial_\mu\zeta^{\rho\nu}\partial_\nu\zeta_{\rho\mu'}) + \tfrac{1}{2}\alpha'(\partial_\mu\partial_\rho\zeta^{\alpha\beta})(\partial_\alpha\partial_\beta\zeta^{\rho}{}_\nu\zeta^{\mu\nu}+\partial_\alpha\partial_\beta\zeta_\nu{}^\rho\zeta^{\nu\mu})] , \quad (2.12)$$

where

$$\zeta_{\mu\nu}(x)=h_{\mu\nu}(x)+A_{\mu\nu}(x) +(D-2)^{-1/2}(\eta_{\mu\nu}-\partial_\mu\bar{\partial}_\nu-\bar{\partial}_\mu\partial_\nu)\phi(x) , \quad (2.13)$$

and $\bar{\partial}_\mu$ is some (nonunique) operator which obeys $\partial\cdot\bar{\partial}=1$ [cf. (2.1)].

III. COMPACTIFICATION SOLUTIONS

As discussed in the Introduction, it is of considerable physical interest to find compactification solutions of string low-energy theories. The case in which the compact space is a Calabi-Yau manifold[15] has received particular attention.[4,5] That such solutions are possible is easy to see. Indeed, the equations of motion following from the lowest-order action (1.1) are

$$\frac{-1}{16\pi G}(R_{MN}-\tfrac{1}{2}g_{MN}R) =\tfrac{1}{2}[\partial_M\phi\,\partial_N\phi-\tfrac{1}{2}g_{MN}(\partial_P\phi)^2] +\tfrac{1}{2}e^{-2m\phi}[H_{MPQ}H_N{}^{PQ}-\tfrac{1}{6}g_{MN}(H_{PQR}{}^2)] ,$$

$$\nabla^P(e^{-2m\phi}H_{PMN})=0 , \quad (3.1)$$

$$\Box\phi+\frac{m}{3}e^{-2m\phi}H_{PQR}{}^2=0 .$$

Clearly, these are satisfied by the configuration

$$R_{\mu\nu}=0=R_{pq},\ \ H_{MNP}=0,\ \ \phi=\text{const} . \quad (3.2)$$

[In this section, the coordinates x and indices $\mu,\nu,\ldots$ refer to four-dimensional spacetime; the coordinates y and indices $p,q,\ldots$ refer to the $(D-4)$-dimensional compact manifold; and the indices $M,N,\ldots$ range over all D dimensions.] That is, there are compactification solutions for *any* Ricci-flat compact manifold, and thus, in particular for a Calabi-Yau manifold.

Two difficulties with this approach should be evident: there is no apparent dynamical reason for the particular decomposition $M^D\to M^4\times M^{D-4}$; and there is no dynamical explanation for singling out a Calabi-Yau space among all possible Ricci-flat compact manifolds.

A possible way of resolving both of these difficulties in the case $D=10$ was pointed out in Ref. 7. The basic idea would be to have as the vacuum solution a configuration with H nonvanishing [unlike (3.2)]. More precisely, the relevant configuration is one for which the three-form $H_{pqr}(y)dy^p dy^q dy^r$ is proportional to the sum of the Calabi-Yau three-form[15,4] and its complex conjugate. In terms of complex coordinates $Z^\alpha=y^\alpha+iy^{3+\alpha}$, $Z^{\bar\alpha}=y^\alpha-iy^{3+\alpha}$, $\alpha=1,2,3$, this configuration is simply[7]

$$H_{\alpha\beta\gamma}(Z)=\tfrac{1}{2}f(Z)\epsilon_{\alpha\beta\gamma},\ \ H_{\bar\alpha\bar\beta\bar\gamma}(\bar Z)=\tfrac{1}{2}\overline{f(Z)}\epsilon_{\bar\alpha\bar\beta\bar\gamma} . \quad (3.3)$$

Here, $\epsilon_{\alpha\beta\gamma}$ is the totally antisymmetric symbol ($\epsilon_{123}=1$); and $f(Z)$ is a holomorphic function, which transforms under the holomorphic transformation $Z'=Z'(Z)$ according to $f'(Z')=[\det(\partial Z'/\partial Z)]^{-1}f(Z)$. Moreover, $H_{pqr}(y)$ is covariantly constant. It is easy to see that the stress-energy tensor for this configuration is zero; it follows, however, that this configuration is not a solution of (3.1).

Nevertheless, the possibility remains that the higher-derivative terms in the low-energy theory allow this configuration to become a solution. Indeed, if the low-energy theory has $H_{MNP}{}^4$ terms, one can hope that these "balance" with $H_{MNP}{}^2$, in the same way that the term $\frac{1}{30}\text{Tr}F_{MN}{}^2$ balances[4] with $R_{MNPQ}{}^2$.

Unfortunately, this possibility is also not realized. The basic difficulty can be readily seen already in the following simplified model:

$$\int d^{10}x\sqrt{-g}\,e^{\alpha\phi}\left[-\frac{1}{16\pi G}R-\tfrac{1}{6}H^2-\xi H^4\right] , \quad (3.4)$$

where we have introduced the notation $H^2\equiv H_{PQR}H^{PQR}$, and $H^4\equiv H_{PQ}{}^R H_{RST}H^{PQV}H_V{}^{ST}$. Also, α and ξ are constants, which for our purposes need not be specified. The ϕ equation of motion is

$$\frac{1}{16\pi G}R+\tfrac{1}{6}H^2+\xi H^4=0 , \quad (3.5)$$

and the g_{MN} field equations are

$$-\frac{1}{16\pi G}(R_{MN}-\tfrac{1}{2}g_{MN}R)-\tfrac{1}{2}(H_M{}^{PQ}H_{NPQ}-\tfrac{1}{6}g_{MN}H^2) -2\xi(H_{MSR}H_{NPQ}H^{SRT}H_T{}^{PQ} -2H_{MSR}H_N{}^{RT}H^S{}_{PQ}H_T{}^{PQ}-\tfrac{1}{4}g_{MN}H^4)=0 . \quad (3.6)$$

With our *Ansatz* for H_{PQR}, the $g_{\mu\nu}$ equations become

$$-\frac{1}{16\pi G}(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R)+\tfrac{1}{2}g_{\mu\nu}(\tfrac{1}{6}H^2+\xi H^4)=0 . \quad (3.7)$$

Since $R_{pq}=0$, then $R=g^{\mu\nu}R_{\mu\nu}$; hence, taking the trace of (3.7), we find

$$\frac{1}{32\pi G}R+\tfrac{1}{6}H^2+\xi H^4=0 .$$

Comparing with the ϕ equation (3.5), we learn that

$$R=0 . \quad (3.8)$$

Now consider the g_{mn} equations. Tracing, we obtain $H^4=0$; this implies $H^2=0$, and hence,

$$H_{PQR}=0 . \quad (3.9)$$

That is, the model (3.4) does not allow the compactification solution with $H\neq 0$.

Let us assume that the bosonic sector of the $D=10$ superstring low-energy theory is given by (1.3) and (1.4), along with further terms involving the Yang-Mills field. Clearly, this action contains many more terms than the simple model (3.4). Nevertheless, if we adopt the *Ansätze* (3.3) and[4] $A_p{}^{ab}=\omega_p{}^{ab}$ (i.e., the gauge field set equal to the spin connection on the compact space), and also take ϕ=constant, $R_{pq}=0$, and assume maximal spacetime symmetry $R_{\mu\nu\alpha\beta}\sim(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha})$, then the contribution of these additional terms to the field equations can be shown to vanish. [Because the action has a global scale invariance,[16] the ϕ dependence can be brought to the form in (3.4) by a suitable rescaling of the metric, for constant ϕ.] Hence, the analysis of the model (3.4) applies. Under the stated assumptions, the fourth-derivative terms of superstring low-energy theory do not allow the compactification solution with $H\neq 0$.

IV. DISCUSSION

We have seen that the antisymmetric tensor field appears in the low-energy theory of strings only as the torsion part of a connection. Moreover, it is possible that the low-energy action involves only curvatures of this connection, such that the torsion tensor does not appear explicitly. In order to make a definitive statement with regard to terms with four derivatives, it is necessary at the very least to also examine four-point functions. Such an investigation requires considerably more effort than the present three-point function analysis; nevertheless, this work is currently under way, and we hope to report on it soon. If indeed this possibility is confirmed, one would want to try to understand why it happens, and perhaps provide an argument that it should persist to all orders of the low-energy expansion. That geometries with torsion are relevant for Bose strings, if true, would presumably be manifest in a covariant field theory of strings.

We have also pointed out a possible difficulty with unitarity, if the low-energy theory contains the term $\sim R^{\mu\nu\rho\sigma}(\Gamma)R_{\mu\rho\nu\sigma}(\Gamma)$ (with the "unnatural" contraction of indices). However, our analysis is not complete; in particular, we have not discussed string loop effects.

Finally, it is now clear that fourth-derivative terms in the string low-energy theory do not allow compactifications with $H\neq 0$. Thus, the dynamics responsible for compactifying on a six-dimensional Calabi-Yau space, thereby leaving four "large" dimensions for spacetime, remains a mystery.

Note added

As this work was nearing completion, we became aware of a series of papers[17,18] on σ models and their relation to strings. Consider the σ model constructed by coupling a closed Bose string to background fields $g_{\mu\nu}$, $A_{\mu\nu}$, and ϕ. By demanding that the one-loop β functions vanish, one obtains[18] the field equations corresponding to the lowest-order action (1.1). Moreover, higher loops of the σ model correspond to terms in the action with higher derivatives. In short, the σ model provides an alternative approach to finding the string low-energy limit.

Fridling and van de Ven[17] have recently performed the two-loop calculation for this σ model, except without the ϕ coupling. They find that the corrections to the β functions involve certain products of the generalized curvatures. This seems to support our observation (1.4). Moreover, as argued by Sen,[19] this result is also consistent with the conjecture[8] that the string low-energy action includes the four-dimensional Euler invariant. To fully check the conjecture with these methods, one would need to perform high-loop calculations.

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APPENDIX A

We use the following conventions:

$$R^{\sigma}{}_{\nu\rho\lambda}(\Gamma)\equiv\partial_\lambda\Gamma^{\sigma}_{\nu\rho}+\Gamma^{\alpha}_{\nu\rho}\Gamma^{\sigma}_{\alpha\lambda}-(\rho\leftrightarrow\lambda)\,,$$
$$R_{\nu\lambda}(\Gamma)\equiv R^{\rho}{}_{\nu\rho\lambda}(\Gamma),\quad R(\Gamma)\equiv g^{\nu\lambda}R_{\nu\lambda}(\Gamma)\,, \tag{A1}$$
$$H_{\mu\nu\rho}\equiv\partial_\mu A_{\nu\rho}+\partial_\nu A_{\rho\mu}+\partial_\rho A_{\mu\nu}\,.$$

The metric signature is $(-+++\cdots)$.

Following Scherk and Schwarz,[1] define the symmetric connection

$$\overline{\Gamma}{}^{\rho}_{\mu\nu}\equiv\begin{Bmatrix}\rho\\ \mu\nu\end{Bmatrix}+\tfrac{1}{2}p\,(\delta^{\rho}_{\nu}\partial_\mu\phi+\delta^{\rho}_{\mu}\partial_\nu\phi-g_{\mu\nu}g^{\rho\rho'}\partial_{\rho'}\phi)\,, \tag{A2}$$

so that the connection (1.2) is given by

$$\Gamma^{\rho}_{\mu\nu}=\overline{\Gamma}{}^{\rho}_{\mu\nu}+ae^{-m\phi}H_{\mu\nu}{}^{\rho}\,. \tag{A3}$$

Then we find

$$\begin{aligned}R^{\sigma}{}_{\nu\rho\lambda}(\Gamma)=&R^{\sigma}{}_{\nu\rho\lambda}(\overline{\Gamma})\\&+a\,[\overline{\nabla}_\lambda(e^{-m\phi}H_{\nu\rho}{}^{\sigma})-\overline{\nabla}_\rho(e^{-m\phi}H_{\nu\lambda}{}^{\sigma})]\\&+a^2e^{-2m\phi}(H_{\nu\rho}{}^{\alpha}H_{\alpha\lambda}{}^{\sigma}-H_{\nu\lambda}{}^{\alpha}H_{\alpha\rho}{}^{\sigma})\,,\end{aligned} \tag{A4}$$

with

$$\begin{aligned}R^{\sigma}{}_{\nu\rho\lambda}(\overline{\Gamma})=R^{\sigma}{}_{\nu\rho\lambda}+\Bigg\{&\frac{p}{2}[\delta^{\sigma}_{\rho}\nabla_\lambda(\partial_\nu\phi)-g_{\nu\rho}g^{\sigma\alpha}\nabla_\lambda(\partial_\alpha\phi)]\\&+\frac{p^2}{4}[\delta^{\sigma}_{\lambda}(\partial_\rho\phi\partial_\nu\phi-g_{\rho\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi)\\&+g_{\nu\rho}\partial_\lambda\phi g^{\sigma\sigma'}\partial_{\sigma'}\phi]-(\rho\leftrightarrow\lambda)\Bigg\}\,.\end{aligned} \tag{A5}$$

Here and in text, if a curvature tensor is written without specifying a particular connection, then the usual Riemann-Christoffel connection is understood. Covariant differentiation with respect to the Riemann-Christoffel connection and the connection (A2) is denoted by ∇ and $\overline{\nabla}$, respectively.

APPENDIX B

Here we present a calculation of the closed string three-point function, with massless external states. We follow the notations and conventions of Schwarz;[10] in particular, the closed string field $X^\mu(\sigma,\tau)=\frac{1}{2}[\widetilde{X}^\mu_c(\tau_+)+X^\mu_c(\tau_-)]$ has the following expansion in terms of a doubled set of oscillators:

$$\widetilde{X}^\mu_c(\tau_+)=x^\mu+2\alpha' p^\mu \tau_+ + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\widetilde{\alpha}^\mu_n e^{-2in\tau_+} , \quad \text{(B1)}$$

$$X^\mu_c(\tau_-)=x^\mu+2\alpha' p^\mu \tau_- + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in\tau_-} ,$$

where $\tau_\pm \equiv \tau\pm\sigma$, $\alpha^\mu_{-n}\equiv\alpha^{\mu\dagger}_n$, and

$$[x^\mu,p^\nu]=i\eta^{\mu\nu} ,$$
$$[\alpha^\mu_n,\alpha^{\nu\dagger}_m]=n\delta_{mn}\eta^{\mu\nu}=[\widetilde{\alpha}^\mu_n,\widetilde{\alpha}^{\nu\dagger}_m] , \quad n,m>0 , \quad \text{(B2)}$$
$$[\alpha^\mu_n,\widetilde{\alpha}^\nu_m]=0, \text{ etc .}$$

Moreover,

$$\widetilde{P}^\mu_c(\tau_+)\equiv\frac{1}{2\alpha'}\frac{\partial}{\partial\tau_+}\widetilde{X}^\mu_c(\tau_+)=p^\mu+\left[\frac{2}{\alpha'}\right]^{1/2}\sum_{n\neq 0}\widetilde{\alpha}^\mu_n e^{-2in\tau_+} , \quad \text{(B3)}$$

$$P^\mu_c(\tau_-)\equiv\frac{1}{2\alpha'}\frac{\partial}{\partial\tau_-}X^\mu_c(\tau_-)=p^\mu+\left[\frac{2}{\alpha'}\right]^{1/2}\sum_{n\neq 0}\alpha^\mu_n e^{-2in\tau_-} .$$

Let $p^\mu{}_i$, $i=1,2,3$, be the momenta of three massless, on-shell physical states, and let $\zeta^{\mu\nu}_i(p_i)$ be the corresponding polarization tensors. Hence, for each i, we have $(p_i)^2=0$, and also

$$p^\mu_i\zeta_{i\mu\nu}=0=p^\nu_i\zeta_{i\mu\nu} . \quad \text{(B4)}$$

The three-point function is given by

$$(\sqrt{\alpha'})^{(D/2)-1}\int_0^\pi d\sigma\frac{g_{\rm VS}}{\pi}\langle 0,-p_1\,|\,\alpha^\mu_1\widetilde{\alpha}^\nu_1 P^\rho_c(\tau_-)\widetilde{P}^\sigma_c(\tau_+)e^{ip_2\cdot X(\sigma,\tau)}\alpha^{\alpha\dagger}_1\widetilde{\alpha}^{\beta\dagger}_1\,|\,0,p_3\rangle\zeta_{1\mu\nu}\zeta_{2\rho\sigma}\zeta_{3\alpha\beta} ; \quad \text{(B5)}$$

that is, the matrix element consisting of a massless tensor emission vertex[10] sandwiched between two massless tensor states. Here, $g_{\rm VS}$ is the dimensionless closed string (Virasoro-Shapiro) coupling constant, and $|0,p\rangle$ denotes the state with no oscillators excited and momentum p.

The evaluation of (B5) begins with the observation that, since $(p_2)^2=0$, we can write

$$e^{ip_2\cdot X}=B(\alpha^\dagger,\widetilde{\alpha}^\dagger)e^{ip_2\cdot(x+2\alpha' p\tau)}A(\alpha,\widetilde{\alpha}) , \quad \text{(B6)}$$

where

$$A(\alpha,\widetilde{\alpha})\equiv\exp\left[-p_2\cdot\left[\frac{\alpha'}{2}\right]^{1/2}\sum_{n=1}^\infty\frac{1}{n}(\alpha_n e^{-2in\tau_-}+\widetilde{\alpha}_n e^{-2in\tau_+})\right] ,$$

$$B(\alpha^\dagger,\widetilde{\alpha}^\dagger)\equiv\exp\left[p_2\cdot\left[\frac{\alpha'}{2}\right]^{1/2}\sum_{n=1}^\infty\frac{1}{n}(\alpha^\dagger_n e^{2in\tau_-}+\widetilde{\alpha}^\dagger_n e^{2in\tau_+})\right] .$$

Next, we note the relations

$$A(\alpha,\widetilde{\alpha})\alpha^{\alpha\dagger}_1\widetilde{\alpha}^{\beta\dagger}_1=\left[\frac{\alpha'}{2}p^\alpha_2 p^\beta_2 e^{-4i\tau}-\left[\frac{\alpha'}{2}\right]^{1/2}(p^\alpha_2\widetilde{\alpha}^{\beta\dagger}_1 e^{-2i\tau_-}+p^\beta_2\alpha^{\alpha\dagger}_1 e^{-2i\tau_+})+\alpha^{\alpha\dagger}_1\widetilde{\alpha}^{\beta\dagger}_1\right]A(\alpha,\widetilde{\alpha}) , \quad \text{(B7a)}$$

and similarly,

$$\alpha^\mu_1\widetilde{\alpha}^\nu_1 B(\alpha^\dagger,\widetilde{\alpha}^\dagger)=B(\alpha^\dagger,\widetilde{\alpha}^\dagger)\left[\frac{\alpha'}{2}p^\mu_2 p^\nu_2 e^{4i\tau}+\left[\frac{\alpha'}{2}\right]^{1/2}(p^\mu_2\widetilde{\alpha}^\nu_1 e^{2i\tau_-}+p^\nu_2\alpha^\mu_1 e^{2i\tau_+})+\alpha^\mu_1\widetilde{\alpha}^\nu_1\right] . \quad \text{(B7b)}$$

It follows that (B5) is equal to

$$\begin{aligned}(\sqrt{\alpha'})^{D/2-1}\int_0^\pi d\sigma\frac{g_{\rm VS}}{\pi}\Big\langle 0,-p_1\Big|\Big|&\left[\alpha^\mu_1\widetilde{\alpha}^\nu_1+\left[\frac{\alpha'}{2}\right]^{1/2}(p^\mu_2\widetilde{\alpha}^\nu_1 e^{2i\tau_-}+p^\nu_2\alpha^\mu_1 e^{2i\tau_+})+\frac{\alpha'}{2}p^\mu_2 p^\nu_2 e^{4i\tau}\right]\\ &\times\left[-p^\rho_1+\left[\frac{2}{\alpha'}\right]^{1/2}(\alpha^\rho_1 e^{-2i\tau_-}+\alpha^{\rho\dagger}_1 e^{2i\tau_-})\right]\\ &\times\left[-p^\sigma_1+\left[\frac{2}{\alpha'}\right]^{1/2}(\widetilde{\alpha}^\sigma_1 e^{-2i\tau_+}+\widetilde{\alpha}^{\sigma\dagger}_1 e^{2i\tau_+})\right]\\ &\times\left[\alpha^{\alpha\dagger}_1\widetilde{\alpha}^{\beta\dagger}_1-\left[\frac{\alpha'}{2}\right]^{1/2}(p^\alpha_2\widetilde{\alpha}^{\beta\dagger}_1 e^{-2i\tau_-}+p^\beta_2\alpha^{\alpha\dagger}_1 e^{-2i\tau_+})\right.\\ &\left.+\frac{\alpha'}{2}p^\alpha_2 p^\beta_2 e^{-4i\tau}\right]\Big|\Big|0,p_2+p_3\Big\rangle\zeta_{1\mu\nu}\zeta_{2\rho\sigma}\zeta_{3\alpha\beta} .\end{aligned} \quad \text{(B8)}$$

This expression is readily evaluated using again the commutation relations (B2), yielding finally

$$
\begin{aligned}
(\sqrt{\alpha'})^{(D/2)-1} g_{\mathrm{VS}}\,\delta\left[\sum_i p_i\right]\{ & (p_3^\rho \zeta_{2\rho\sigma} p_3^\sigma \bar{\zeta}_1^{\mu\nu}\zeta_{3\mu\nu} + p_1^\alpha \zeta_{3\alpha\beta} p_1^\beta \bar{\zeta}_1^{\mu\nu}\zeta_{2\mu\nu} + p_2^\mu \zeta_{1\mu\nu} p_2^\nu \bar{\zeta}_2^{\rho\sigma}\zeta_{3\rho\sigma} \\
& + p_3^\rho \zeta_{2\rho\nu}\bar{\zeta}_1^{\mu\nu}\zeta_{3\mu\beta} p_1^\beta + p_3^\sigma \zeta_{2\mu\sigma}\bar{\zeta}_1^{\mu\nu}\zeta_{3\alpha\nu} p_1^\alpha + p_2^\mu \zeta_{1\mu\nu}\bar{\zeta}_3^{\alpha\nu}\zeta_{2\alpha\sigma} p_3^\sigma \\
& + p_2^\nu \zeta_{1\mu\nu}\bar{\zeta}_3^{\mu\beta}\zeta_{2\rho\beta} p_3^\rho + p_1^\beta \zeta_{3\rho\beta}\bar{\zeta}_2^{\rho\nu}\zeta_{1\mu\nu} p_2^\mu + p_1^\alpha \zeta_{3\alpha\sigma}\bar{\zeta}_2^{\mu\sigma}\zeta_{1\mu\nu} p_2^\nu) \\
& - \tfrac{1}{2}\alpha'[p_3^\rho \zeta_{2\rho\sigma} p_3^\sigma (p_2^\alpha \zeta_{3\alpha\nu}\bar{\zeta}_1^{\mu\nu} p_{2\mu} + p_2^\beta \zeta_{3\mu\beta}\bar{\zeta}_1^{\mu\nu} p_{2\nu}) + p_1^\alpha \zeta_{3\alpha\beta} p_1^\beta (p_3^\mu \zeta_{1\mu\nu}\bar{\zeta}_2^{\rho\nu} p_{3\rho} + p_3^\nu \zeta_{1\mu\nu}\bar{\zeta}_2^{\mu\sigma} p_{3\sigma}) \\
& + p_2^\mu \zeta_{1\mu\nu} p_2^\nu (p_1^\rho \zeta_{2\rho\sigma}\bar{\zeta}_3^{\alpha\sigma} p_{1\alpha} + p_1^\sigma \zeta_{2\rho\sigma}\bar{\zeta}_3^{\rho\beta} p_{1\beta})] + \tfrac{1}{4}(\alpha')^2 p_2^\mu \zeta_{1\mu\nu} p_2^\nu p_3^\rho \zeta_{2\rho\sigma} p_3^\sigma p_1^\alpha \zeta_{3\alpha\beta} p_1^\beta\}\ .
\end{aligned}
\tag{B9}
$$

In deriving this result, we have made repeated use of (B4) and momentum conservation ($\sum_i p_i = 0$). Zwiebach[8] considered only the case of $\bar{\zeta}_i^{\mu\nu}$ symmetric; in this case, our expression for the three-point function is proportional to his, up to a rescaling of α' by a factor $\frac{1}{2}$. Finally, we remark that closed-string N-point functions have the property of nonplanar duality (symmetry under the interchange of ζ_i, p_i with ζ_j, p_j); in contrast with the case of open strings (which have only planar duality), a sum over noncyclic permutations of the external states is not required in order to obtain the full scattering amplitude.

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[12]We note that the scalar curvature R contributes $\frac{1}{4}(\sqrt{32\pi G})^3 h^{\mu\mu'}[(\partial_\mu\partial_{\mu'}h_{\nu\rho})h^{\rho\nu}+2\partial_\mu h^{\rho\nu}\partial_\nu h_{\mu'\rho}]$; this is smaller by a factor of 3 from the work of Scherk and Schwarz (Ref. 1), who miss the h^3 contribution from $R_{\mu\nu}$.

[13]This property is also shared by the combination $R^{\mu\nu\rho\sigma}(\Gamma)R_{\rho\sigma\mu\nu}(\Gamma)-4R^{\mu\nu}(\Gamma)R_{\nu\mu}(\Gamma)+R(\Gamma)^2$. Hence, for $k=1$, there is no problem with unitarity.

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