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"Isothermal" density perturbations in an axion-dominated inflationary universe

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In inflationary models of the Universe which are axion dominated both adiabatic and isothermal density perturbations arise. We point out that the isothermal perturbations can be more important than the adiabatic perturbations and discuss a model for which this is the case. With isothermal perturbations the spectrum of density perturbations when structure formation begins is flatter and we briefly discuss the implications of this fact. That the amplitude of isothermal fluctuations not be too large provides yet another constraint on models of inflation.

INTRODUCTION

The hot-big-bang model provides a general picture of how the observed structure in the Universe developed—small density inhomogeneities present early on grew via the Jeans instability into the highly nonlinear structures we see today.[1] A more detailed picture of this process requires knowledge of the appropriate "initial data" for this problem: the quantity and composition of the matter in the Universe today, and the type and spectrum of density perturbations present initially.

The study of the very early Universe has given us some "important clues" as to what the initial data might be. For example, the inflationary scenario[2–4] predicts $\Omega(\equiv\rho/\rho_{\rm crit})=1.0$ and the Harrison-Zel'dovich[5] spectrum of adiabatic density perturbations;[6] primordial nucleosynthesis constrains $\Omega_{\rm baryon}$ to be ≤ 0.15 (Ref. 7), suggesting that the bulk of the matter in the Universe is nonbaryonic; baryogenesis all but precludes the existence of "isothermal" perturbations in the baryon component, i.e., spatial fluctuations in the baryon-to-photon ratio;[8] and finally, there are numerous species which are candidates for the "dark matter," including the invisible axion.[9–12]

In the case of an axion-dominated Universe,[12] inflation also predicts the existence of isothermal[13,14] (more precisely, isocurvature[1]) axion density perturbations. Physically, very early on these perturbations correspond to local variations in the number density of axions, but not in the total energy density of the Universe. In realistic inflationary models these isothermal perturbations were believed to be significantly less important than their adiabatic counterparts.[13,14] In this paper we show that they need not be subdominant in models where the Peccei-Quinn (PQ) symmetry breaks before or during inflation. (In order for this to occur, both the reheat temperature, $T_{\rm RH}$, and the expansion rate during inflation must be less than the temperature at which the PQ symmetry is restored.) As an example, we carefully calculate both the adiabatic and isothermal spectrum for Pi's inflationary scenario[15] and show that isothermal fluctuations actually dominate. If isothermal axion perturbations dominate the adiabatic perturbations and have the correct amplitude, they will determine how structure formation proceeds. We briefly comment on the differences in how structure formation proceeds in the case of isothermal axion perturbations. Finally we emphasize that the amplitude of the isothermal axion perturbations places a new constraint on models of inflation.

ISOTHERMAL AXION PERTURBATIONS

Let $\phi=\phi\, e^{i\theta}$ be the complex scalar field whose vacuum expectation value $\langle\phi\rangle\equiv f_a$ spontaneously breaks the Peccei-Quinn U(1) quasisymmetry.[9] The axion,[10] a, is the Nambu-Goldstone boson associated with the spontaneous breaking of this global U(1) symmetry and corresponds to the θ degree of freedom, $a=\theta f_a$. At high temperatures, i.e., from PQ symmetry breaking, $T\simeq O(f_a)$, to $T\simeq O(\Lambda_{\rm QCD})$, the axion is very nearly massless—corresponding to $V(\phi)$ being flat in the θ direction. At temperatures below $O(\Lambda_{\rm QCD})$, SU(3) instanton effects break the $\mathrm{U}(1)_{\rm PQ}$, giving rise to minima in the potential at $\theta_0=-\theta_{\rm QCD}+n(2\pi/N)$, where $\theta_{\rm QCD}$ is defined in the bare QCD Lagrangian

$$\mathscr{L}_{\rm QCD}=\cdots+\frac{\theta_{\rm QCD}g^2}{32\pi^2}G^{a\mu\nu}\tilde{G}_{a\mu\nu}\,,$$

$n=1,2,\ldots,N$ and N is a positive integer whose value depends upon the Peccei-Quinn charges of the quarks; for the simplest models $N=6$. (Both PQ symmetry breaking and instanton effects leave a Z_N symmetry unbroken.[16]) When θ is anchored in a minimum of the potential the effective Lagrangian is CP-conserving. Throughout this paper we will take θ to be the deviation from θ_0.

At the time of PQ symmetry breaking no particular value of θ is singled out; thus when the instanton effects lead to the axion developing a potential whose minimum is at $\theta=0$, the initial value of θ, θ_i, will in general be misaligned: $\theta_i\neq 0$. Due to this initial misalignment the axion field will eventually begin to oscillate.[12] The energy density associated with these coherent field oscillations behaves like nonrelativistic matter, a condensate of very cold axions, and contributes a mass density today[14]

$$\Omega_a \simeq 1.0\alpha(f_a/10^{12}\text{ GeV})^{1.18}\theta_1^2 \quad [f_a \lesssim 1.6\times 10^{18}\text{ GeV}(N/6)\Lambda_{200}^{-2.3}]$$
$$\simeq 3.5\times 10^6\beta(f_a/10^{18}\text{ GeV})^{1.5}\theta_1^2, \quad [f_a \gtrsim 1.6\times 10^{18}\text{ GeV}(N/6)\Lambda_{200}^{-2.3}]\,, \tag{1}$$

where $\alpha \equiv T_{2.7}{}^3(N/6)^{0.83}h^{-2}\gamma^{-1}\Lambda_{200}{}^{-0.7}$ and $\beta \equiv T_{2.7}{}^3(N/6)^{1/2}h^{-2}\gamma^{-1}$ are numerical factors of order unity, $\Omega_a \equiv \rho_a/\rho_{\text{crit}}$ is the fraction of critical density contributed by axions today, $\rho_{\text{crit}} \simeq 1.88\times 10^{-29}h^2\text{g cm}^{-3}$ is the critical density, $H_0 = 100h$ km sec^{-1}Mpc^{-1} is the present value of the Hubble parameter ($\frac{1}{2} \lesssim h \lesssim 1$), $\Lambda_{\text{QCD}} = \Lambda_{200}$ 200 MeV, $T_{2.7}$2.7 K is the present temperature of the microwave background radiation, θ_1 is the rms value of $\theta_i(x)$ within the observable Universe, and γ is the ratio of the entropy per comoving volume today to that when $T \simeq \Lambda_{\text{QCD}}$. ($\gamma$ measures the entropy production since the coherent axion oscillations began. Any entropy production since then dilutes the axions and reduces Ω_a; see Ref. 14 for details.)

In the absence of dynamics to specify θ_i, it has generally been assumed that θ_1 is of order unity. [More precisely, $-\pi/N \lesssim \theta_1 \lesssim \pi/N$. In the noninflationary case the rms value of θ_1 should just be $(\pi/N)/\sqrt{3}$.] As we will be restricting our analysis to inflationary models, we will adopt the point of view advocated by Pi[15]—that θ_1 takes on the value required to have $\Omega_a = 1$, the rationale being that θ_1 takes on different values in different bubbles (or fluctuation regions) so that all values of θ_1 occur in some finite fraction of the bubbles. The rms average of θ_1 over *all bubbles* is of course $(\pi/N)/\sqrt{3}$; however, that is of little relevance to us as we live in but one bubble. Then, according to this point of view, determining f_a, H_0, $T_{2.7}$, and γ serves to measure θ_1. Adopting this philosophy we can use Eq. (1) to solve for θ_1:

$$\theta_1 \simeq 1.0\alpha^{-1/2}(f_a/10^{12}\text{ GeV})^{-0.59} \quad [f_a \lesssim 1.6\times 10^{18}\text{ GeV}(N/6)\Lambda_{200}^{-2.3}]$$
$$\simeq 5.4\times 10^{-4}\beta^{-1/2}(f_a/10^{18}\text{ GeV})^{-0.75} \quad [f_a \gtrsim 1.6\times 10^{18}\text{ GeV}(N/6)\Lambda_{200}^{-2.3}]\,. \tag{2}$$

Inflation ensures that $\theta_i(x)$ is nearly constant over the whole of our observable Universe. However, it is well known that quantum fluctuations are induced in scalar fields by de Sitter expansion.[17] As a result there will be spatial fluctuations in the misalignment angle, $\theta_i(x) = \theta_1 + \delta\theta(x)$, which will manifest themselves as isothermal axion density perturbations when $T \simeq \Lambda_{\text{QCD}}$.

In order to discuss the axion density perturbations quantitatively it is convenient to Fourier expand $\delta_\theta(x) \equiv (\theta_i - \theta_1)/\theta_1$, the fractional fluctuation in θ_i:

$$\delta_\theta(x) = (2\pi)^{-3}\int d^3k\, \delta_\theta(k) e^{-ikx}\,,$$
$$\delta_\theta(k) = \int d^3x\, \delta_\theta(x) e^{ikx}\,,$$

where k is the comoving wave number, x_i $(i = 1-3)$ are comoving coordinates, and we have normalized to unit comoving volume. At low temperatures ($T \ll \Lambda_{\text{QCD}}$), the local mass density in axions $\rho_a(x) \propto \theta^2(x)$. Since $\delta_\theta(x)$ is small,

$$\delta_a(x) \equiv \rho_a(x)/\bar{\rho}_a \simeq 2\delta_\theta(x)$$

and

$$\delta_a(k) \equiv \int d^3x\, \delta_a(x) e^{ikx} \simeq 2\delta_\theta(k)\,.$$

A useful quantity for studying the formation of structure is the rms mass fluctuation (or power) on the scale k (usually referred to as "$\delta\rho/\rho$ on the scale k")

$$\langle(\delta M_a/M_a)^2\rangle_k \simeq \Delta_k{}^2 \equiv (2\pi)^{-3}k^3\,|\,\delta_a(k)\,|^2$$
$$\simeq 4(2\pi)^{-3}k^3\,|\,\delta_\theta(k)\,|^2\,, \tag{3}$$

where M_a is the mass in axions associated with the scale k (Ref. 18). When the rms mass fluctuation on a given scale grows to order unity, we expect bound structures of this mass to start forming.

Now we will calculate $\delta_\theta(k)$. Recall that we are assuming that PQ symmetry breaking occurs before or during the inflationary phase. Because the potential $V(\phi)$ is flat in the θ direction, the axion degree of freedom behaves like a massless scalar field, $a = \theta f_a$. The spectrum of quantum fluctuations for a massless scalar field in a de Sitter background is given by[17]

$$|\,a(k)\,|^2 = H^2/2k^3\,,$$

where H is the Hubble parameter during inflation, and the cosmic scale factor $R \propto \exp(Ht)$. This result implies that

$$|\,\delta_\theta(k)\,|^2 = H^2/(2\theta_1{}^2 f_a{}^2 k^3)\,, \tag{4}$$

at the end of the inflationary epoch.

The classical equation of motion for $\delta_\theta(k)$ is

$$\ddot{\delta}_\theta(k) + (3H + 2\dot{\theta}_1/\theta_1)\dot{\delta}_\theta(k) + k^2\delta_\theta(k)/R^2 = 0\,. \tag{5}$$

For modes whose physical wavelength ($\equiv R2\pi/k$) is larger than the horizon ($=H^{-1}$), i.e., $k/RH \ll 1$, the solution to Eq. (5) has $\dot{\delta}_\theta(k) \to 0$. That means that the amplitude for mode k remains constant until it crosses back inside the horizon during the postinflation era.[19]

Once inside the horizon axion fluctuations remain approximately constant until the Universe becomes axion dominated [$T \simeq 6.8(\Omega_a h^2/T_{2.7}{}^3)$eV, $t \simeq 3\times 10^{10}(\Omega_a h^2/T_{2.7}{}^3)^{-2}$ sec]. After this we must include gravitational effects in the evolution equation for $\delta_\theta(k)$. As a result the density fluctuations within the horizon will grow, $\delta_\theta(k) \propto t^{2/3}$, and structure begins to evolve.

From Eqs. (2)—(4) it then follows that the rms mass fluctuation in isocurvature fluctuations is

$$\Delta_{\rm iso} \simeq H/(2\pi^{3/2} f_{\rm gal}\theta_1) \tag{6a}$$

$$\simeq 9\times10^{-2}\alpha^{1/2}(H/f_a)(f_a/f_{\rm gal})(f_a/10^{12}\ {\rm GeV})^{0.59}\quad [f_a \lesssim 1.6\times10^{18}\ {\rm GeV}(N/6)\Lambda_{200}{}^{-2.3}] \tag{6b}$$

$$\simeq 1.7\times10^{2}\beta^{1/2}(H/f_a)(f_a/f_{\rm gal})(f_a/10^{18}\ {\rm GeV})^{0.75}\quad [f_a \gtrsim 1.6\times10^{18}\ {\rm GeV}(N/6)\Lambda_{200}{}^{-2.3}]\ , \tag{6c}$$

where $f_{\rm gal}$ is the value of ϕ when the scales of astrophysical interest cross outside the horizon. For many models $f_{\rm gal}=f_a$, however, if PQ symmetry breaking occurs during inflation (as in Pi's model) then $f_{\rm gal}$ can be $<f_a$.

In order to be important for galaxy formation $\Delta_{\rm iso}$ must be $\simeq 10^{-4}$. If rapid reheating occurs after inflation,[20] then $T_{\rm RH}{}^2 \simeq H m_{\rm Pl}$ and $T_{\rm RH} < f_a$ implies $H/f_a \lesssim f_a/m_{\rm Pl}$. Rapid reheating and $f_a \sim 10^{12}-10^{13}$ GeV results in $\Delta_{\rm iso} \sim 10^{-7}$, a value which is too small to be of interest for galaxy formation.[13,14] However, from Eqs. (6a)—(6c) we see that $\Delta_{\rm iso} \simeq 10^{-4}$ is easily achieved by letting f_a be much larger than 10^{12} GeV (which requires θ_1 to be small), or by letting $H/f_a \sim 10^{-3}$ (which implies slow reheating, $T_{\rm RH}{}^2 \ll H m_{\rm Pl}$). Here $m_{\rm Pl} \equiv G^{-1/2} \simeq 1.22 \times 10^{19}$ GeV.

For reference, the analogous amplitude of adiabatic perturbations $\Delta_{\rm ad}$ is[6,21]

$$\Delta_{\rm ad} \simeq H^2/(\pi^{3/2}\dot{\psi})\ , \tag{7}$$

where ψ is the scalar field which is evolving toward its symmetry-breaking minimum, and whose vacuum energy is driving inflation (with kinetic term normalized to be $\frac{1}{2}\partial_\mu\psi\partial^\mu\psi$). The relationship between the adiabatic and isothermal modes is shown in Fig. 1.

AXION PERTURBATIONS IN PI'S MODEL

Shafi and Vilenkin[22] proposed a grand unified theory (GUT) model of inflation where the field which drives inflation is a very weakly coupled, gauge-singlet scalar field with a potential of the Coleman-Weinberg form.[23] Pi[15] went one step further and used the scalar field which drives inflation to also break a PQ symmetry. Thus her model will have both adiabatic and isothermal axion perturbations and we will analyze them here.

In her model the one-loop effective potential is given by

$$V = V_1 + V_2\ , \tag{8a}$$

$$V_1 = B[\phi^4 \ln(\phi^2/f_a{}^2) + \tfrac{1}{2}(f_a{}^4 - \phi^4)]/4\ , \tag{8b}$$

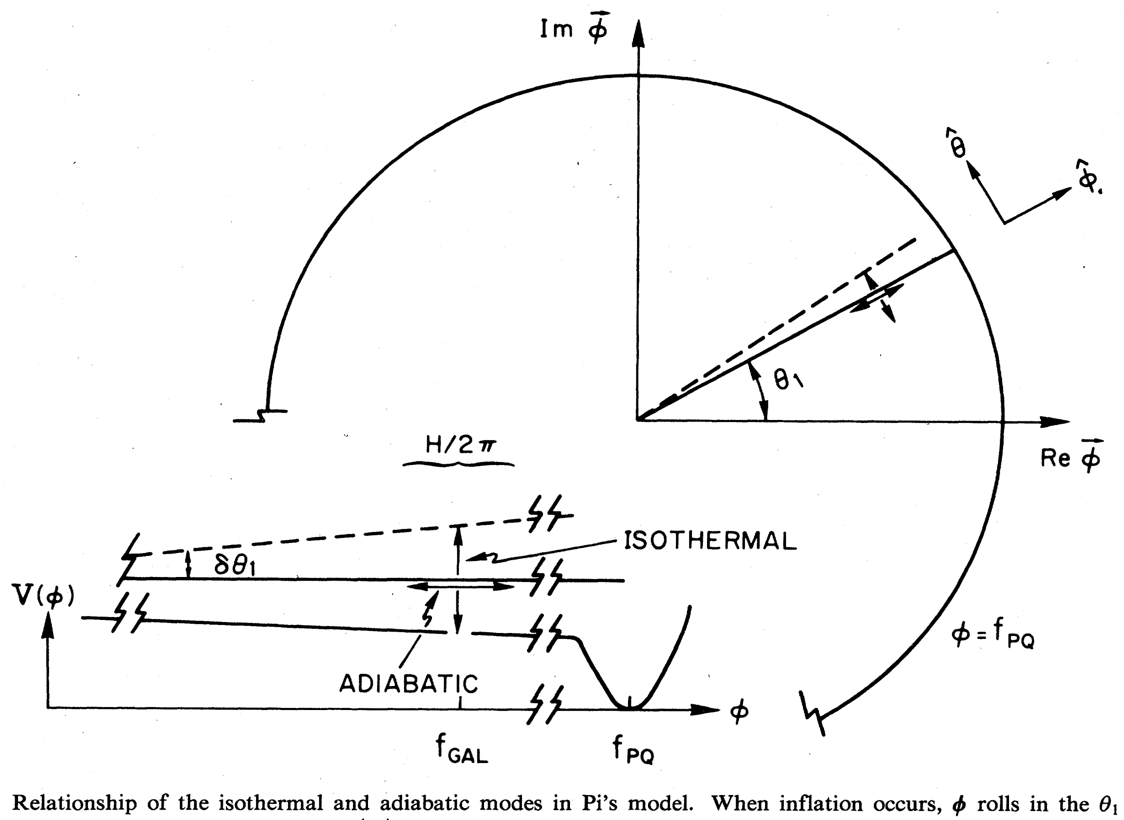

FIG. 1. Relationship of the isothermal and adiabatic modes in Pi's model. When inflation occurs, ϕ rolls in the θ_1 direction. Galactic-sized scales leave the horizon when $\langle\phi\rangle = f_{\rm GAL}$. The de Sitter-space produced quantum fluctuations in ϕ can be resolved into the $\hat{\phi}$ and $\hat{\theta}$ directions. The fluctuations in the $\hat{\phi}$ direction result in adiabatic axion perturbations and the fluctuations in the $\hat{\theta}$ direction result in isothermal axion perturbations. The orthogonality of the two modes is manifest. Note, in this figure f_a is denoted as $f_{\rm PQ}$.

and V_2 describes the coupling of ϕ to the SU(5) **24** whose vacuum expectation value is responsible for $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ symmetry breaking, but is not relevant for our purposes ($V_2 \ll V_1$). B is determined by the self-coupling of ϕ and its couplings to the other fields in the theory. The semiclassical equations of motion for ϕ can be written as

$$\ddot{\phi}+3H\dot{\phi}-\phi\dot{\theta}^2+\partial V/\partial\phi=0 , \tag{9a}$$

$$\ddot{\theta}+(3H+2\dot{\phi}/\phi)\dot{\theta}=0 , \tag{9b}$$

where as before $\phi=\phi e^{i\theta}$, and

$$H=[8\pi V(\phi)/(3m_{\rm Pl}^2)]^{1/2}\simeq(\pi B/3)^{1/2}f_a^2/m_{\rm Pl}$$

is the Hubble constant during inflation. For simplicity in Eq. (9a) we have left out the $\Gamma\dot{\phi}$ term which accounts for the decay of the coherent field oscillations and the reheating of the Universe (see Ref. 20).

During inflation, when $\dot{\phi}/\phi<H$, the solution to Eq. (9b) is $\dot{\theta}\propto\exp(-3Ht)$, implying that $\theta\simeq$const. During inflation the $\ddot{\phi}$ and $\phi\dot{\theta}^2$ terms can be neglected in the ϕ equation of motion[20] so that

$$\dot{\phi}\simeq-V'/3H\simeq-B\phi^3\ln(\phi^2/f_a^2)/3H$$

whose solution is

$$\phi^{-2}\simeq[2B\ln(f_a^2/\phi^2)/3H^2]H(t_*-t)$$
$$\simeq(2/\pi)\ln(f_a^2/\phi^2)(m_{\rm Pl}^2/f_a^4)H(t_*-t) , \tag{10}$$

where the slow logarithmic variation of V_1 has been ignored, and t_* is the time when ϕ reaches its symmetry-breaking minimum ($\phi=f_a$) and inflation ends. The scales of astrophysical interest cross the horizon 50 or so e-folds before the end of inflation, i.e., $H(t_*-t)\simeq 50$. This means that

$$(f_{\rm gal}/f_a)\simeq\frac{f_a}{m_{\rm Pl}}\left[\frac{\pi}{100\ln(f_a^2/f_{\rm gal}^2)}\right]^{1/2} , \tag{11}$$

and for $f_a\simeq 10^{18}$ GeV, the value required in Pi's model to give the correct SU(5) symmetry-breaking scale, $f_{\rm gal}\simeq f_a/230$. That is, the scales of interest cross outside the horizon when ϕ is much less than f_a.

Having computed $f_{\rm gal}$ we can use Eqs. (6) and (7) to calculate $\Delta_{\rm iso}$ and $\Delta_{\rm ad}$ for Pi's model

$$\Delta_{\rm iso}\simeq 3400\beta^{1/2}B^{1/2}f_{18}^{0.75} , \tag{12a}$$

$$\Delta_{\rm ad}\simeq 340B^{1/2} , \tag{12b}$$

$$\Delta_{\rm iso}/\Delta_{\rm ad}\simeq 10\beta^{1/2}f_{18}^{0.75} , \tag{12c}$$

where $f_{18}=f_a/10^{18}$ GeV and we have taken $\ln(f_a^2/f_{\rm gal}^2)\simeq 11$, cf. Eq. (11). Note, that independent of B, for $f_a\gtrsim 10^{18}$ GeV the isothermal axion perturbations are dominant. Normalizing $\Delta_{\rm iso}$ to be $\simeq\delta\times 10^{-4}$, where δ is of order unity, we can solve for B:

$$B\simeq 10^{-15}\delta^2\beta^{-1}f_{18}^{-1.5} . \tag{13}$$

Note, the value of B chosen by Pi,[15] $B\simeq 10^{-12}$, would result in $\Delta_{\rm iso}\simeq 3\times 10^{-3}\beta^{1/2}f_{18}^{0.75}$, which is almost certainly precluded by the isotropy of the microwave background (see below).

CONCLUDING REMARKS

Given the spectrum of density perturbations at the beginning of the epoch of matter domination, one can, in principle, evolve the Universe forward to the present epoch by numerical simulation. To be phenomenologically acceptable, an initial spectrum must result in structure which is consistent with what we observe today, e.g., the galaxy-galaxy correlation function[1] implies that $\delta\rho/\rho$ is of order unity today on the scale of $\lambda_c\simeq 7h^{-1}$ Mpc. The spectrum must also predict microwave anisotropies which are consistent with the measured isotropy on both large ($\gg 1°$) and small ($\ll 1°$) angular scales.[24,25]

In Fig. 2 we show the spectra of density perturbations (adiabatic[26] and isothermal[27]) predicted in axion-dominated models at the time the Universe becomes matter dominated. The two spectra have been normalized to have the same amplitude on the scale $\lambda_c\simeq 7h^{-1}$ Mpc. Several features are apparent. First, galaxies should form slightly later with an isothermal spectrum as $k^{3/2}|\delta_a(k)|$ is a factor of 2 or so smaller on galactic scales in the isothermal case.[21] The isothermal spectrum is slightly flatter, which means that structures will form on a wide range of mass scales almost simultaneously. The most restrictive measurement of small-scale anisotropy is that of Uson and Wilkinson[24] on the scale of $4.5'$ ($\delta T/T \lesssim 3\times 10^{-5}$); the predicted anisotropy on this scale is proportional to $\delta\rho/\rho$ on scales around $\lambda_{4.5}\simeq 8.2h^{-1}$ Mpc (Ref. 28). Since this scale is so close to λ_c, the scale on which both spectra have been normalized, the predicted anisotropies should be very nearly equal. On the other hand, the predicted anisotropy on large angular scales, e.g., the quadrupole anisotropy, should be almost a factor of 10 larger in the isothermal case since $(\delta\rho/\rho)_{\rm iso}\simeq 10(\delta\rho/\rho)_{\rm ad}$ for $\lambda\gg\lambda_{\rm eq}$, the horizon scale at matter radiation equality. This may be problematic for the isothermal spectrum,[29] and certainly constrains $\Delta_{\rm iso}$ to be less than 10^{-3}.

In sum, following Pi's philosophy, we have emphasized that since we have no direct knowledge of θ_1, the initial misalignment angle, measurements/knowledge of Ω_a, h, $T_{2.7}$, and γ serve only to determine θ_1 in terms of f_a, cf. Eq. (2). This point of view has several implications; first, PQ symmetry-breaking scales $f_a\gtrsim 10^{12}$ GeV are not *a priori* cosmologically unacceptable in models which inflate after or during PQ symmetry breaking. This fact is of particular significance to superstring theories in which PQ symmetry breaking appears to occur at a scale of order $10^{18}-10^{19}$ GeV (Ref. 30). Second, isothermal axion perturbations whose amplitude we have calculated[30] to be $\Delta_{\rm iso}\simeq H/(2\pi^{3/2}f_{\rm gal}\theta_1)$, may be important for galaxy formation (if $\Delta_{\rm iso}\simeq 10^{-4}$), and are actually the dominant mode for Pi's model. Even for $f_a\simeq 10^{12}-10^{13}$ GeV and $\theta_1\sim 1$, isothermal axion perturbations may be important if reheating is slow and $H/f_a\gtrsim 10^{-3}$. In any case the

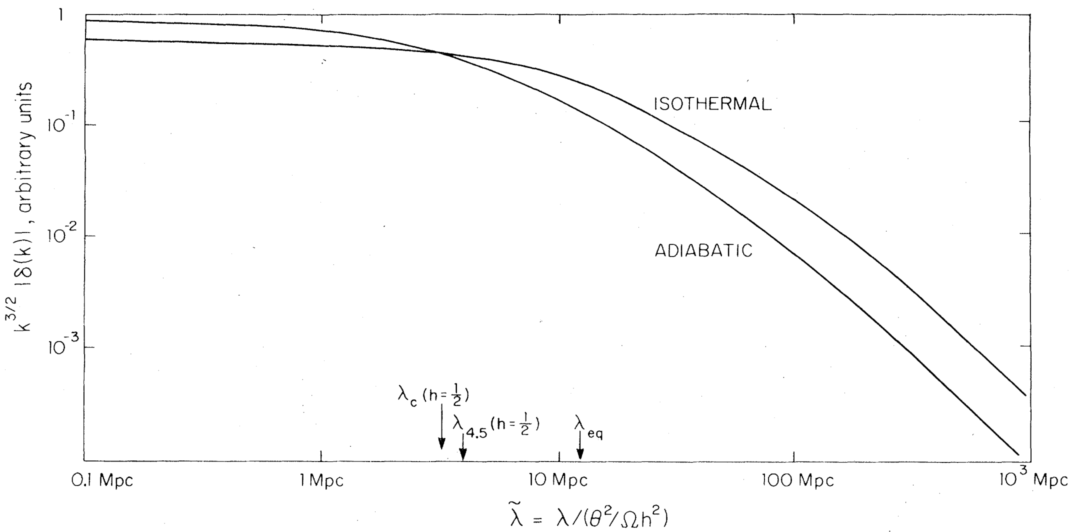

FIG. 2. The spectrum of density perturbations $k^{3/2}|\delta_a(k)|$ at the time of matter domination [$t_{eq}\simeq 3\times 10^{10}(\Omega_a h^2/T_{2.7}{}^3)^{-2}$ sec; $T_{eq}\simeq 6.8(\Omega_a h^2/T_{2.7}{}^3)$ eV] as a function of $\tilde{\lambda}=\lambda(\Omega h^2/\theta^2)$ for adiabatic (Ref. 26) and isothermal (Ref. 27) axion perturbations. Here $\theta = T/2.7$ K $=T_{2.7}$. (Note, the spectra, up to an overall normalization, are only a function of $\tilde{\lambda}\propto\lambda/\lambda_{eq}$; where $\lambda_{eq}\simeq 13h^{-2}T_{2.7}{}^2$ Mpc is the scale which is just entering the horizon when the Universe becomes matter dominated.) The scales $\lambda_c\simeq 7h^{-1}$ Mpc and $\lambda_{4.5}\simeq 8.2h^{-1}$ Mpc are indicated for $h=\frac{1}{2}$, and the two spectra are normalized to the same value on the scale λ_c.

isotropy of the microwave background restricts Δ_{iso} to be not too much larger than 10^{-4} (Ref. 29), and so our result represents yet another constraint on models of inflation.

Note added in proof. After this work was accepted for publication we learned of similar work by Linde (A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 496 (1984) [JETP Lett. **40**, 1333 (1984)]; Phys. Lett. **158B**, 375 (1985)).

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[18]More precisely, the rms axion mass fluctuation on a scale k is

$$\langle(\delta M_a/M_a)^2\rangle_k \equiv \left\langle \left[\int \delta_a(\mathbf{x}+\mathbf{x}')W_k(\mathbf{x}')d^3x'\right]^2\right\rangle \Big/ V_w{}^2 ,$$

where $\langle\ \rangle$ indicates the average over all space and the window function $W_k(\mathbf{x}')$ smoothly defines the volume V_w (and mass M_a) associated with the scale k:

and $V_w = \int \mathbf{W}_k(\mathbf{x}')d^3x'$. For reference, the axion mass contained in a sphere of radius $\lambda/2(\equiv\pi/k)$ is $1.5\Omega_a h^2 \times 10^{11} M_\odot (\lambda/\text{Mpc})^3$, when W_k is taken to be a step function. Expanding $W_k(\mathbf{x}')$ and $\delta_a(\mathbf{x})$ in their Fourier components, it follows that

$$\langle(\delta M_a/M_a)^2\rangle \equiv (2\pi)^{-3}\int d^3k' \,|\,\delta_a(k')\,|^2\,|\,W_k(k')\,|^2/V_w^2 .$$

For a simple window function like

$$W_k(\mathbf{x}) = \exp(-k^2|\,x\,|^2/2) ,$$

$$W_k(k') = (2\pi)^{3/2}k^{-3}\exp(-k'^2/2k^2) ,$$

and

$$\langle(\delta M_a/M_a)^2\rangle \cong (2\pi)^{-3}\int_0^k d^3k'\,|\,\delta_a(k')\,|^2$$
$$\cong (2\pi)^{-3}k^3\,|\,\delta_a(k)\,|^2 \equiv \Delta_k^2 ,$$

where the approximation that the integral is dominated by the contribution of scales with $k' \approx k$ is valid so long as $|\,\delta_a(k)\,|^2$ increases at least as fast as k^{-3}.

[19]This is true for perturbations that reenter the horizon during the radiation-dominated phase. Scales that reenter the horizon during the more recent matter-dominated phase will have order unity corrections to $\delta_\theta(k)$ as the axion number density perturbation turns into a pressure perturbation. Since scales of interest for galaxy formation reenter the horizon during the radiation epoch we will not worry about these corrections here. For further discussion of this issue see J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980); or W. Press and E. T. Vishniac, Astrophys. J. **239**, 1 (1980).

[20]For a detailed discussion of inflation and reheating see P. J. Steinhardt and M. S. Turner, Phys. Rev. D **29**, 2162 (1984).

[21]To be precise, for scales which reenter the horizon while the Universe is still radiation dominated ($\lambda \lesssim \lambda_{\text{eq}} \simeq 13h^{-2}T_{2.7}{}^2$ Mpc), the Fourier amplitudes of the acoustic wave in the baryon-photon fluid are given by

$$k^{3/2}\,|\,\delta_k\,|\,/(2\pi)^{3/2} \cong H^2/(\pi^{3/2}\dot{\psi}) .$$

Adiabatic perturbations in the axion (and other noninteracting components) will grow slowly ($\propto \ln R$) due to the velocity ($\dot{\delta}_a \neq 0$) they had when reentering the horizon, and by the time the Universe becomes matter dominated, their amplitude will be

$$k^{3/2}\,|\,\delta_a(k)\,|\,/(2\pi)^{3/2} \simeq 2H^2/(\pi^{3/2}\dot{\psi}) .$$

Formation of the observed structure requires $\Delta_{\text{ad}} \simeq 10^{-4}$ on the scales of galaxies. Scales which enter the horizon when the Universe is matter dominated, do so with Fourier amplitudes of

$$k^{3/2}\,|\,\delta_k\,|\,/(2\pi)^{3/2} \simeq (H^2/10)/(\pi^{3/2}\dot{\psi}) .$$

[22]Q. Shafi and A. Vilenkin, Phys. Rev. Lett. **52**, 691 (1984).

[23]The original models of new inflation (Refs. 3 and 4) were also based upon potentials of the Coleman-Weinberg form. However, the scalar field responsible for inflation, was not a gauge singlet and the coefficient B was set by the gauge coupling ($B = 25\alpha_{\text{GUT}}{}^2/4 \simeq 10^{-2}$) and resulted in adiabatic perturbations of amplitude much greater than order unity.

[24]J. Uson and D. Wilkinson, Astrophys. J. **277**, L1 (1984).

[25]D. Wilkinson, in *Proceedings of the Inner Space/Outer Space Conference, Fermilab,* edited by E. W. Kolb, M. S. Turner, D. Lindley, K. Olive, and D. Seckel (University of Chicago Press, Chicago, 1985).

[26]P. J. E. Peebles, Astrophys. J. **263**, L1 (1982).

[27]M. S. Turner, 1983 (unpublished); also, see J. M. Bardeen, 1984 (unpublished); J. M. Bardeen, J. Bond, N. Kaiser, and A. Szalay, Astrophys. J. (to be published).

[28]Angular scale ϕ and linear scale l on the last scattering surface are related by $\phi \simeq 0.55'(l/\text{Mpc})h$. For a detailed discussion of the predicted microwave anisotropies see J. R. Bond and G. Efstathiou, Astrophys. J. **285**, L44 (1984); N. Vittorio and J. Silk, *ibid.* **285**, L39 (1984).

[29]The predicted microwave anisotropies for the isothermal case are discussed in more detail in J. R. Bond and G. Efstathiou, Astrophys. J. (to be published).

[30]E. Witten, Phys. Lett. **149B**, 351 (1985); K. Choi and J. E. Kim, *ibid.* **154B**, 393 (1985); **156B**, 452(E) (1985).

[31]We note that if $\Omega_a < 1$, our formula for Δ_{iso} should be reduced proportionally.