Gravitational counterterms in an axial gauge

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All the counterterms of quantum Einstein gravity are calculated up to bilinear terms and one-loop order in an axial gauge as a sum of gauge-invariant and the Becchi-Rouet-Stora- (BRS) invariant terms. Contrary to the de Donder gauge condition, 10 out of 33 coefficients for counterterms remain undetermined in this gauge. Some relations among the counterterms, which satisfy the BRS invariance, are implicitly obtained in the course of the calculations.

I. INTRODUCTION

The first successful attempt to obtain a gravitational theory renormalizable up to one-loop order was done by utilizing the background-field method.[1] It is well known that the background-field method applied to gauge theories in an appropriate gauge gives us only gauge-invariant counterterms.[2] Furthermore, no Faddeev-Popov ghost fields appear in the one-particle-irreducible vertex functional. Hence it is very easy to show whether or not a theory is renormalizable because only gauge-invariant counterterms appear in the functional even for non-Abelian theories.

However, for the sake of unitarity one should also work in the conventional approach, which includes the Faddeev-Popov ghost fields, since the background-field method does not give any straightforward proof for unitarity of the theory due to our ignorance of how to treat the Faddeev-Popov determinant. Although a pure quantum Einstein gravity is not renormalizable in a normal sense, the unitarity should be kept intact since it describes only a massless spin-2 physical particle. A standard method can be applied to prove unitarity if the Faddeev-Popov fields are included.[3]

The counterterms, which cancel divergences arising from the original Lagrangian, are endowed with a remarkable feature if a certain condition is met. That is, once all the counterterms are given as a form of the gauge-invariant term plus the Becchi-Rouet-Stora- (BRS) invariant terms, all the Slavnov-Taylor (or BRS) identities among divergent vertex functions are automatically derived. That is, all miraculous identities among vertex functions stem from the same origin, the BRS-invariant counterterms. Only some of them have been checked by explicit calculations.[4,5]

There are two interesting gauges of quantum gravity, i.e., the de Donder gauge or any other Lorentz-invariant gauge condition and an axial-gauge condition. The latter was proposed some time ago by the analogy with the one in non-Abelian theories and has an advantage that the Faddeev-Popov ghost fields decouple from other fields in the Lagrangian,[6] although the Feynman integrals become involved.[5,7]

In a previous paper,[8] we have succeeded in uniquely obtaining all coefficients for counterterms of quantum gravity in the de Donder gauge condition up to bilinear terms and to one-loop order. In this paper, we study quantum gravity in the axial gauge. Although the Faddeev-Popov ghost fields are absent in the S matrix in this gauge condition, it will be shown that they are necessary to formulate the BRS identities as in the case of Yang-Mills theories.[9] Our discussions are confined in four space-time dimensions but the form of counterterms in an axial gauge is general enough to be applied to the one in the light-cone gauge. Things remaining to do are to recalculate the Feynman integrals in the light-cone gauge in certain space-time dimensions by adopting an appropriate calculation method.

What we will actually do in this paper is to obtain BRS-invariant counterterms which include gauge-invariant as well as -noninvariant terms. Namely, we calculate appropriate one-loop Feynman diagrams and/or borrow the results from other references, compare them with the most general BRS-invariant counterterms, and determine those coefficients. In order to perform all of this, we need to formulate a method which gives us the most general form of the BRS-invariant counterterms. This will be given in Sec. II. All necessary one-loop Feynman diagrams will be calculated in Sec. III. Section IV will be devoted to calculating all coefficients of counterterms. Finally, remarks and comments on the results derived in Sec. IV will be given in Sec. V. Feynman rules and some calculations which are necessary for Secs. III and IV will be given in Appendixes.

II. STRUCTURE OF COUNTERTERMS

We consider a pure Einstein gravity in the axial-gauge condition in this paper, whose Lagrangian is given by $\mathscr{L}_0+\mathscr{L}_{\rm GF}+\mathscr{L}_{\rm FP}$:

$$\mathscr{L}_0=\frac{1}{\kappa^2}\sqrt{-g}R\ , \tag{1}$$

$$\mathscr{L}_{\rm GF}=-\frac{1}{2\alpha\kappa^2 n^2}[n^\mu(g_{\mu\nu}-\eta_{\mu\nu})]^2\ , \tag{2}$$

$$\mathscr{L}_{\rm FP}=-\frac{i}{2\kappa(n^2)^{1/2}}(n^\mu\bar{c}^\nu+n^\nu\bar{c}^\mu)\times[(\partial_\mu c^\lambda)g_{\lambda\nu}+(\partial_\nu c^\lambda)g_{\mu\lambda}-c^\lambda\partial_\lambda g_{\mu\nu}]\ , \tag{3}$$

where $\kappa^2=16\pi G$ with G a gravitational Newton constant, $\mathscr{L}_{\rm GF}$ is a gauge-fixing term with α a gauge parameter and with n^μ an arbitrary four-vector but $n^2\neq 0$, and $\mathscr{L}_{\rm FP}$ is the Faddeev-Popov ghost term. We define the graviton by lower indices of the metric, i.e., $g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$. This Lagrangian with a gauge-fixing term and the Faddeev-Popov ghost term is invariant under the so-called Becchi-Rouet-Stora transformation given as follows:[10]

$$\begin{aligned}
&\delta(g^{\mu\nu})=-(\partial_\lambda c^\mu)g^{\lambda\nu}-(\partial_\lambda c^\nu)g^{\mu\lambda}+c^\lambda\partial_\lambda g^{\mu\nu} ,\\
&\delta(g_{\mu\nu})=(\partial_\mu c^\lambda)g_{\lambda\nu}+(\partial_\nu c^\lambda)g_{\mu\lambda}-c^\lambda\partial_\lambda g_{\mu\nu} ,\\
&\delta(c^\rho)=c^\lambda\partial_\lambda c^\rho ,\\
&\delta(\bar c^\rho)=\frac{i}{2\alpha\kappa(n^2)^{1/2}}(\eta^{\rho\mu}n^\nu+\eta^{\rho\nu}n^\mu)(g_{\mu\nu}-\eta_{\mu\nu}) .
\end{aligned}\tag{4}$$

We need to introduce external source terms for fields and the BRS-invariant source terms for constructing a simple functional formalism:

$$\mathscr{L}_{\rm S}=J^{\mu\nu}h_{\mu\nu}+\bar\xi_\rho c^\rho+\bar c^\rho\xi_\rho ,\tag{5}$$

$$\mathscr{L}_{\rm IS}=\frac{1}{\kappa}u^{\mu\nu}\delta(g_{\mu\nu})+v_\rho\delta(c^\rho) .\tag{6}$$

Now we can define a generating functional $Z(J,\bar\xi,\xi,u,v)$ for general Green's functions by

$$Z(J,\bar\xi,\xi,u,v)=\int d\mu\exp[i(S_0+S_{\rm GF}+S_{\rm FP}+S_{\rm S}+S_{\rm IS})] ,\tag{7}$$

where a functional measure $d\mu$ is given by

$$d\mu=dg_{\mu\nu}dc^\rho d\bar c^\sigma ,\tag{8}$$

and an action S is obtained by integrating the corresponding Lagrangian, e.g., $S_0=\int d^4x\,\mathscr{L}$. One can derive the functional equations for Z given by (7), which are derived from a BRS invariance and a ghost equation. A generating functional for the connected Green's functions is defined through

$$W(J,\bar\xi,\xi,u,v)=-i\ln Z(J,\bar\xi,\xi,u,v) ,\tag{9}$$

with the same arguments as Z. There are corresponding functional equations for W, too. Finally, we can define a generating functional for the one-particle-irreducible vertices through the Legendre transformation of W given by

$$\Gamma(h,c,\bar c,u,v)=W(J,\bar\xi,\xi,u,v)-(J^{\mu\nu}h_{\mu\nu}+\bar\xi_\rho c^\rho+\bar c^\rho\xi_\rho) ,\tag{10}$$

where integrations in space-time of the last three terms are tacitly assumed. By using functional relations between W and Γ given by

$$\begin{aligned}
&\frac{\delta W}{\delta J^{\mu\nu}}=h_{\mu\nu},\quad \frac{\delta W}{\delta\bar\xi_\rho}=c^\rho,\quad \frac{\delta W}{\delta\xi_\rho}=-\bar c^\rho ,\\
&\frac{\delta\Gamma}{\delta h_{\mu\nu}}=-J^{\mu\nu},\quad \frac{\delta\Gamma}{\delta c^\rho}=\bar\xi_\rho,\quad \frac{\delta\Gamma}{\delta\bar c^\rho}=-\xi_\rho ,
\end{aligned}\tag{11}$$

functional equations in terms of Γ are given by

$$\frac{\delta\tilde\Gamma}{\delta h_{\mu\nu}}\frac{\delta\tilde\Gamma}{\delta u^{\mu\nu}}+\frac{\delta\tilde\Gamma}{\delta c^\rho}\frac{\delta\tilde\Gamma}{\delta v_\rho}=0 ,\tag{12}$$

$$\frac{\delta\tilde\Gamma}{\delta\bar c^\rho}=-\frac{i}{\kappa}n^\sigma\frac{\delta\tilde\Gamma}{\delta u^{\rho\sigma}} ,\tag{13}$$

where integration in space-time of the whole equation is tacitly assumed as before and we have simplified equations by utilizing a new functional defined by

$$\tilde\Gamma=\Gamma+\frac{1}{2\alpha\kappa^2n^2}\int d^4x[n^\mu(g_{\mu\nu}-\eta_{\mu\nu})]^2 .\tag{14}$$

By using the ghost equation, the external source $u^{\mu\nu}$ can always be replaced with

$$\tilde u^{\mu\nu}=u^{\mu\nu}-\frac{i}{2(n^2)^{1/2}}(n^\mu\bar c^\nu+n^\nu\bar c^\mu) .\tag{15}$$

These functional equations [(12) and (13)] should hold for finite vertices as well as divergent counterterms and hence they can be used to determine counterterms. In order to obtain one-loop counterterms in bilinear forms, one must solve the following equation for the one-loop divergent vertex $\Gamma^{(1)}_{\rm div}$:

$$\Delta\Gamma^{(1)}_{\rm div}=0 ,\tag{16}$$

where Δ is the so-called BRS operator which gives the BRS transformation when it operates on a field and is given by

$$\Delta=\frac{\delta\tilde S}{\delta h_{\mu\nu}}\frac{\delta}{\delta\tilde u^{\mu\nu}}+\frac{\delta\tilde S}{\delta\tilde u^{\mu\nu}}\frac{\delta}{\delta h_{\mu\nu}}+\frac{\delta\tilde S}{\delta c^\rho}\frac{\delta}{\delta v_\rho}+\frac{\delta\tilde S}{\delta v_\rho}\frac{\delta}{\delta c^\rho}\tag{17}$$

with $\tilde S=S_0+S_{\rm FP}+S_{\rm IS}$. The divergent counterterm $S_{\rm counter}$ is well known to be written in a general form as[11]

$$\begin{aligned}
-S_{\rm counter}&=\Gamma^{(1)}_{\rm div}\\
&=\int d^4x[\sqrt{-g}\,(c_1R^2+c_2R_{\mu\nu}R^{\mu\nu})]+\Delta G ,
\end{aligned}\tag{18}$$

where $R_{\mu\nu}$ is the Ricci tensor. The G is an arbitrary functional of fields and has the mass dimension 3 and the ghost number -1 which are determined by looking at Table I. Since we are interested in only the lowest- and one-loop-order calculations, G can be written in bilinear forms and the term κ^2 must be factored out. The reason why (18) gives a general solution to (16) is because gauge-invariant terms are automatically BRS invariant and because the operator Δ satisfies the nilpotency condition, i.e., $\Delta^2=0$. The most general form of G is given in Appendix A with its functional differentiations in terms of $h_{\mu\nu}$, $\tilde u^{\mu\nu}$, c^ρ, and v_ρ. We also give the functional differentiations of $\tilde S$ in terms of the same fields in Appendix A, which will be needed in Sec. IV.

III. ONE-LOOP-ORDER CALCULATIONS

At first we must give Feynman rules for the Lagrangian given by $\mathscr{L}_0+\mathscr{L}_{\rm GF}+\mathscr{L}_{\rm FP}+\mathscr{L}_{\rm IS}$ and they are given in Appendix B together with corresponding expressions of the one-particle-irreducible vertices. In the following, we will calculate three one-loop vertex functions which are enough to determine counterterms up to bilinear terms.

TABLE I. Mass dimensions and ghost numbers assigned to each field, operators and parameters.

	Mass dimension	Ghost number
$h_{\mu\nu}$	1	0
c_μ	0	1
$\bar{c}_\mu$	2	-1
$u_{\mu\nu}$	2	-1
v_μ	3	-2
κ	-1	0
∂_μ	1	0
n_μ	0	0
α	-2	0
G	3	-1

(1) Luckily enough, the most involved one-loop diagram in the axial gauge, i.e., the graviton self-energy shown in Fig. 1, was already calculated for the gauge parameter α chosen to be zero,

$$\alpha=0 ,$$

which will be kept intact in the following calculations. The result can be written as

$$\Pi_{\mu\nu,\rho\sigma}(p)=i\frac{\delta^2\tilde{\Gamma}^{(1)}_{\rm div}}{\delta h^{\rho\sigma}(p)\delta h^{\mu\nu}(-p)}=\kappa^2p^4I\sum_j d_j(T_j)_{\mu\nu,\rho\sigma} , \tag{19}$$

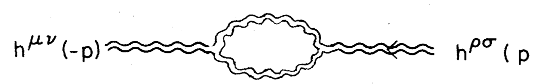

FIG. 1. A one-loop diagram for a graviton self-energy. Notice that there is no ghost contribution in this gauge. A double-wavy line denotes a graviton $h_{\mu\nu}$.

where the divergent integral I is given by

$$I=\frac{2}{(4\pi)^2(4-D)} \tag{20}$$

with D a number of space-time dimensions, the T_j's are 14 independent tensors which are given in Table II, and their coefficients are given by Table III. In the middle of (19), it is understood that this is written in the momentum space even though the same notations as in the configuration space are used and one should set all fields equal to zero after functional differentiation, whose rule is also kept in the following similar expressions.

(2) Next we need to calculate a divergent part of the one-loop correction to the v-c-c vertex as depicted in Fig. 2, which is given by

$$V^{\lambda}{}_{\mu\nu}(p,q,r)=i\frac{\delta^3\tilde{\Gamma}^{(1)}_{\rm div}}{\delta v_\lambda(r)\delta c^{\nu}(q)\delta c^{\mu}(p)}$$

$$\underset{r=0}{\longrightarrow} -i\kappa^2(p\cdot n)^2\int d^4k\frac{1}{[(p+k)\cdot n]^2}G_{\nu\sigma,\mu\rho}(k)[(p+k)_\rho\delta^\lambda_\sigma+(p+k)_\sigma\delta^\lambda_\rho] , \tag{21}$$

where $iG_{\mu\nu,\rho\sigma}$ is the tree graviton propagator. Without explicitly evaluating this integral, the property of $V^{\lambda}{}_{\mu\nu}(p,-p,0)$ is easily derived by using the famous property of the graviton propagator in the axial gauge, i.e.,

$$n^\mu G_{\mu\nu,\rho\sigma}=n^\rho G_{\mu\nu,\rho\sigma}=0 . \tag{22}$$

With the help of this equation, one can easily derive the following which is enough to determine suitable coefficients:

TABLE II. Fourteen independent symmetric tensors which consist of k_μ, n_μ, and $\eta_{\mu\nu}$.

T_1	$2^{-1}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})$
T_2	$\eta_{\mu\nu}\eta_{\rho\sigma}$
T_3	$(k^2)^{-1}(\eta_{\mu\nu}k_\rho k_\sigma+\eta_{\rho\sigma}k_\mu k_\nu)$
T_4	$(2k\cdot n)^{-1}(\eta_{\mu\nu}k_\rho n_\sigma+\eta_{\mu\nu}k_\sigma n_\rho+\eta_{\rho\sigma}k_\mu n_\nu+\eta_{\rho\sigma}k_\nu n_\mu)$
T_5	$(\eta^2)^{-1}(\eta_{\mu\nu}n_\rho n_\sigma+n_{\rho\sigma}n_\mu n_\nu)$
T_6	$(k^2)^{-1}(\eta_{\mu\rho}k_\nu k_\sigma+\eta_{\mu\sigma}k_\nu k_\rho+\eta_{\nu\rho}k_\mu k_\sigma+\eta_{\nu\sigma}k_\mu k_\rho)$
T_7	$(2k\cdot n)^{-1}[(\eta_{\mu\rho}k_\nu+\eta_{\nu\rho}k_\mu)n_\sigma+(\eta_{\mu\sigma}k_\nu+\eta_{\nu\sigma}k_\mu)n_\rho+(\eta_{\mu\rho}n_\nu+\eta_{\nu\rho}n_\mu)k_\sigma+(\eta_{\mu\sigma}n_\nu+\eta_{\nu\sigma}n_\mu)k_\rho]$
T_8	$(n^2)^{-1}(\eta_{\mu\rho}n_\nu n_\sigma+\eta_{\mu\sigma}n_\nu n_\rho+\eta_{\nu\rho}n_\mu n_\sigma+\eta_{\nu\sigma}n_\mu n_\rho)$
T_9	$(k^2)^{-1}k_\mu k_\nu k_\rho k_\sigma$
T_{10}	$(4k^2k\cdot n)^{-1}(n_\mu k_\nu k_\rho k_\sigma+n_\nu k_\mu k_\rho k_\sigma+n_\rho k_\mu k_\nu k_\sigma+n_\sigma k_\mu k_\nu k_\rho)$
T_{11}	$(2k^2n^2)^{-1}(k_\mu k_\nu n_\rho n_\sigma+k_\rho k_\sigma n_\mu n_\nu)$
T_{12}	$[4(k\cdot n)^2]^{-1}(k_\mu n_\nu+k_\nu n_\mu)(k_\rho n_\sigma+k_\sigma n_\rho)$
T_{13}	$(4k\cdot nn^2)^{-1}(k_\mu n_\nu n_\rho n_\sigma+k_\nu n_\mu n_\rho n_\sigma+k_\rho n_\mu n_\nu n_\sigma+k_\sigma n_\mu n_\nu n_\rho)$
T_{14}	$(n^2)^{-1}n_\mu n_\nu n_\rho n_\sigma$

$$n^\mu V^\lambda{}_{\mu\nu}(p,-p,0)=0\ . \tag{23}$$

(3) One more one-loop diagram we can readily estimate is a divergent part of the vertex $\tilde{u}$-c shown in Fig. 3, which is given by

$$U^\lambda{}_{\mu\nu}(p)=i\frac{\delta^2\tilde{\Gamma}^{(1)}_{\text{div}}}{\delta c_\lambda(p)\delta\tilde{u}^{\mu\nu}(-p)}$$

$$=-\frac{i}{2}\kappa^2(n\cdot p)\int d^4k\frac{1}{(p-k)\cdot n}[(p-k)_\mu G_{\rho\nu}{}^{,\rho\lambda}(k)+(p-k)_\nu G_{\rho\mu}{}^{,\rho\lambda}(k)-k_\rho G_{\mu\nu}{}^{,\rho\lambda}(k)]\ . \tag{24}$$

In order to study this integral, one must know the following property of the dimensional regularization:

$$\int d^4k\frac{1}{k^{2l}(kn)^m}=0\ , \tag{25}$$

where l and m are integers. This describes one peculiar feature of the dimensional regularization, namely, the integral without dimensional parameters should vanish. Using this together with (22), one obtains

$$n_\lambda U^\lambda{}_{\mu\nu}(p)=0\ , \tag{26}$$

$$n^\mu U^\lambda{}_{\mu\nu}(p)=0\ . \tag{27}$$

Using Table III, Eqs. (23), (26), and (27), we will determine coefficients for the counterterms in the next section.

IV. DETERMINATION OF THE COEFFICIENTS

In this section, we derive the forms for three vertex functions up to κ^2, which are calculated from a general form of the counterterms (18).

(1) First we derive the one corresponding to the vertex v-c-c from (A4) and (A9) in Appendix A, which is given as follows:

$$\frac{1}{\kappa^2}V^\lambda{}_{\mu\nu}(p,-p,0)=\frac{i}{\kappa^2}\frac{\delta^3}{\delta v_\lambda(0)\delta c^\nu(-p)\delta c^\mu(p)}\times\left[\frac{\delta\tilde{S}}{\delta v_\rho}\frac{\delta G}{\delta c^\rho}\right]\ . \tag{28}$$

TABLE III. Coefficients of tensors T_i for a one-loop calculation of a graviton self-energy, where $a=k^2n^2/(k\cdot n)^2$.

d_1	$-(19+128a^{-1})/120$
d_2	$(63+352a^{-1}-336a^{-2})/120$
d_3	$-(63+440a^{-1})/120$
d_4	$2a^{-1}(11+62a^{-1})/15$
d_5	$-(11+62a^{-1}+20a^{-2})/15$
d_6	$(19+328a^{-1})/240$
d_7	$a^{-1}(5+4a^{-1})/3$
d_8	$(5+8a^{-1})/6$
d_9	$\frac{11}{30}$
d_{10}	$\frac{56}{15}$
d_{11}	$2(11+20a^{-1})/15$
d_{12}	$2a^{-1}(25+124a^{-1})/15$
d_{13}	$8a^{-1}(21+10a^{-1})/15$
d_{14}	$-2(21+20a^{-1})/15$

An explicit expression for the above equation in terms of undetermined coefficients is given in Appendix C. Substituting (C1) into (23), we find that all coefficients included in this counterterm should vanish:

$$a_6=a_7=b_{20}=b_{21}=b_{22}=b_{23}=b_{24}=0\ . \tag{29}$$

(2) Next we calculate counterterms for the graviton self-energy from (A3) and (A6), which is calculated as

$$\Pi_{\mu\nu,\rho\sigma}(p)=i\frac{\delta^3}{\delta h^{\rho\sigma}(p)\delta h^{\mu\nu}(-p)}\left[\frac{\delta\tilde{S}}{\delta h_{\kappa\lambda}}\frac{\delta G}{\delta u^{\kappa\lambda}}\right]$$

$$=\tfrac{1}{2}\kappa^2p^4I\sum_j e_j(T_j)_{\mu\nu,\rho\sigma}\ , \tag{30}$$

where the coefficients e_j are given in Table IV. Coefficients for the gauge-invariant counterterms in (18) were given by the background-field calculations in a particular background gauge as[1]

$$c_1=\tfrac{1}{120}I,\quad c_2=\tfrac{7}{20}I\ , \tag{31}$$

with I given by (20). However, we cannot adopt these values as c_1 and c_2 since these coefficients are known to be gauge dependent,[12] unlike the Yang-Mills case. Hence we leave these coefficients unknown at this stage.

(3) In order to determine coefficients as far as possible, we need the help of a vertex function for $\tilde{u}$-c, which is given by

$$U^\lambda{}_{\mu\nu}(p)=i\frac{\delta^2}{\delta c(p)\delta\tilde{u}^{\mu\nu}(-p)}\left[\frac{\delta\tilde{S}}{\delta\tilde{u}^{\mu\nu}}\frac{\delta G}{\delta h_{\mu\nu}}+\frac{\delta\tilde{S}}{\delta c^\rho}\frac{\delta G}{\delta v_\rho}\right]\ , \tag{32}$$

where the second term in the last line vanishes because of (29). Constraints on $U^\lambda{}_{\mu\nu}$, (26) and (27), are reduced to 12 independent relations among coefficients given by (C2)

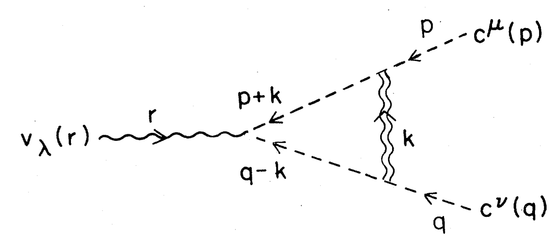

FIG. 2. A one-loop diagram for a v-c-c vertex. A wavy line denotes an external field v_μ and the dotted line the Faddeev-Popov ghost field.

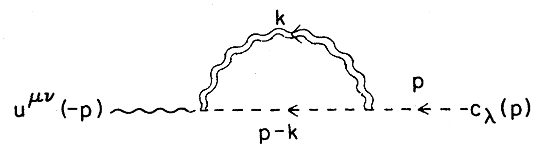

FIG. 3. A one-loop diagram for a $\tilde{u}$-c vertex. A wavy line denotes an external field $\tilde{u}_{\mu\nu}$.

and lead us to the form given by (C3) in Appendix C, which does not give us any new information about coefficients. That is, the numerical value for this vertex correction is completely determined by the graviton self-energy calculations, whose result is given in Table III. With the help of relations (C2), leaving ten coefficients undetermined, we have the following solutions for coefficients,

$$b_1=-b_6=-b_8=b_{13}=-\tfrac{20}{3}I,\ \ b_2=\tfrac{16}{5}I\ ,$$
$$b_7=b_{17}=-\tfrac{28}{5}I,\ \ b_{10}=b_{16}=0,\ \ b_{11}=\tfrac{56}{5}I\ , \tag{33}$$
$$b_{12}=-\tfrac{32}{5}I,\ \ b_{14}=-\tfrac{32}{15}I,\ \ b_{18}=\tfrac{32}{3}I,\ \ b_{19}=-\tfrac{16}{3}I\ ,$$

together with eight relations among ten undetermined coefficients,

$$a_1=\tfrac{19}{30}I+c_2,\ \ a_2=a_4=-2^{-1}a_1\ ,$$
$$a_5=-a_3,\ \ 2a_3+b_3=\tfrac{11}{15}I-(2c_1+c_2)\ ,$$
$$b_5=-b_3,\ \ b_9=-2b_4,\ \ b_{15}=b_4\ , \tag{34}$$
$$b_3+2b_4=\tfrac{28}{15}I\ .$$

V. COMMENTS ON UNDETERMINED COEFFICIENTS

In this paper, we have obtained the counterterm (18) up to one-loop order and to bilinear terms, coefficients of which are given by Eqs. (29), (33), and (34). This result contains implicit relations among vertex functions, i.e., BRS identities. One remarkable example is given by the $\tilde{u}$-c vertex (C4) whose coefficients are completely determined by the graviton self-energy. However, 10 of 33 coefficients remain undetermined.

Let us discuss what is a necessary condition for solving undetermined coefficients. We need at least one more Feynman diagram in order to get new information about coefficients. There remains only one triple-vertex $\tilde{u}$-c-h diagram. Let us consider a counterterm for this vertex. Given an arbitrary functional G, one can write a counterterm as

$$\frac{\delta\tilde{S}}{\delta h_{\mu\nu}}\frac{\delta G}{\delta\tilde{u}^{\mu\nu}}+\frac{\delta\tilde{S}}{\delta\tilde{u}^{\mu\nu}}\frac{\delta G}{\delta h_{\mu\nu}}+\frac{\delta G}{\delta c^{\rho}}\frac{\delta G}{\delta v_{\rho}}\ . \tag{35}$$

When one carefully looks at this form, one immediately notices the presence of other contributions as well as those coming from bilinear terms in G. That is, trilinear terms such as $\kappa^3 c^{\rho} v_{\rho}\partial^{\mu}\partial^{\nu}h_{\mu\nu}$ and $\kappa^3 h_{\rho\sigma}\tilde{u}^{\rho\sigma}\partial^{\mu}\partial^{\nu}h_{\mu\nu}$ may contribute to the counterterm for this vertex. These terms and other similar terms should be added to G if one wants to determine the counterterms for this vertex completely. In that case, their coefficients should also be determined as well as a_i and b_i. This is beyond our scope of research since we have confined ourselves up to bilinear terms in G and up to one-loop order.

Final comments are on the coefficients of c_i in (18). These can be calculated by the background-field method. However, this may not be a good way to obtain them since c_i's are generally gauge dependent[12] and since there is no simple relation between the axial gauge and the background-field gauge.

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TABLE IV. Coefficients of tensors T_i for a gauge-noninvariant counterterm of a graviton self-energy.

e_1	$a_4+a^{-1}b_{14}+2^{-1}c_2$
e_2	$-a_4-2a_5+(a^{-1}-1)b_5-a^{-1}b_{14}-2a^{-1}b_{15}+a^{-1}(a^{-1}-1)b_{17}+2c_1+2^{-1}c_2$
e_3	$-a_3+a_4+a_5+2^{-1}(a^{-1}-1)b_3+2^{-1}b_5+(2a)^{-1}b_{13}+a^{-1}b_{14}+a^{-1}b_{15}+(2a)^{-1}b_{17}-2c_1-2^{-1}c_2$
e_4	$-a^{-1}b_5+(2a)^{-1}b_6-a^{-1}b_9+(2a)^{-1}(a^{-1}-1)b_{11}-(2a)^{-1}b_{13}-a^{-2}b_{17}+2^{-1}a^{-2}b_{18}$
e_5	$-b_4+2^{-1}b_5-2^{-1}b_6+2^{-1}(a^{-1}-1)b_7-a^{-1}b_{16}+(2a)^{-1}b_{17}-(2a)^{-1}b_{18}+(2a)^{-1}(a^{-1}-1)b_{19}$
e_6	$-2^{-1}a_4-(4a)^{-1}b_{13}-(2a)^{-1}b_{14}-4^{-1}c_2$
e_7	$-(4a)^{-1}b_6+(4a)^{-1}b_{13}-4^{-1}a^{-2}b_{18}$
e_8	$4^{-1}b_6+(4a)^{-1}b_{18}$
e_9	$2a_3+b_3+2c_1+c_2$
e_{10}	$-2a^{-1}b_3+2a^{-1}b_9+a^{-1}b_{11}$
e_{11}	$b_3+2b_4+b_6+b_7+2a^{-1}b_{16}+a^{-1}b_{18}+a^{-1}b_{19}$
e_{12}	$-a^{-1}b_6-2a^{-2}b_{11}-a^{-2}b_{18}$
e_{13}	$-2a^{-1}b_7+a^{-1}b_{11}-2a^{-2}b_{19}$
e_{14}	$b_7+a^{-1}b_{19}$

APPENDIX A

In this appendix, we give the most general form of the functional G in (18) up to bilinear forms and order of κ^2 as follows:

$$\kappa^{-2}G=\int d^4x\,[(a_1\partial^\mu h_{\mu\nu}\partial_\rho\tilde{u}^{\rho\nu}+a_2\partial_\rho h\partial_\sigma\tilde{u}^{\rho\sigma}+a_3\partial^\mu h_{\mu\nu}\partial^\nu\tilde{u}+a_4\partial^\rho h_{\mu\nu}\partial_\rho\tilde{u}^{\mu\nu}+a_5\partial^\rho h\partial_\rho\tilde{u}+b_1n^\mu n_\rho\partial^\nu h_{\mu\nu}\partial_\sigma\tilde{u}^{\rho\sigma}$$
$$+b_2n^\mu n^\nu\partial_\rho h_{\mu\nu}\partial_\sigma\tilde{u}^{\rho\sigma}+b_3n_\rho n_\sigma\partial^\mu h_{\mu\nu}\partial^\nu\tilde{u}^{\rho\sigma}+b_4n^\mu n^\nu\partial^\rho h_{\mu\nu}\partial_\rho\tilde{u}+b_5n_\mu n_\nu\partial^\rho h\partial_\rho\tilde{u}^{\mu\nu}$$
$$+b_6n^\mu n_\sigma\partial^\rho h_{\mu\nu}\partial_\rho\tilde{u}^{\sigma\nu}+b_7n^\mu n^\nu n_\kappa n_\lambda\partial^\rho h_{\mu\nu}\partial_\rho\tilde{u}^{\kappa\lambda}+b_8n_\mu n\cdot\partial h\partial_\nu\tilde{u}^{\mu\nu}+b_9n^\mu\partial^\nu h_{\mu\nu}n\cdot\partial\tilde{u}$$
$$+b_{10}n^\mu n^\nu n_\rho n\cdot\partial h_{\mu\nu}\partial_\sigma\tilde{u}^{\rho\sigma}+b_{11}n^\mu n_\rho n_\sigma\partial^\nu h_{\mu\nu}n\cdot\partial\tilde{u}^{\rho\sigma}+b_{12}n^\mu n\cdot\partial h_{\mu\nu}\partial_\rho\tilde{u}^{\rho\nu}+b_{13}n_\rho n\cdot\partial h_{\mu\nu}\partial^\mu\tilde{u}^{\rho\nu}$$
$$+b_{14}n\cdot\partial h_{\mu\nu}n\cdot\partial\tilde{u}^{\mu\nu}+b_{15}n\cdot\partial hn\cdot\partial\tilde{u}+b_{16}n^\mu n^\nu n\cdot\partial h_{\mu\nu}n\cdot\partial\tilde{u}+b_{17}n_\mu n_\nu n\cdot\partial hn\cdot\partial\tilde{u}^{\mu\nu}$$
$$+b_{18}n^\mu n_\rho n\cdot\partial h_{\mu\nu}n\cdot\partial\tilde{u}^{\rho\nu}+b_{19}n^\mu n^\nu n_\rho n_\sigma n\cdot\partial h^{\mu\nu}n\cdot\partial\tilde{u}^{\rho\sigma})$$
$$+(a_6\partial_\mu c^\mu\partial^\nu v_\nu+a_7\partial^\rho c^\mu\partial_\rho v_\mu+b_{20}n_\mu n^\nu\partial^\rho c^\mu\partial_\rho v_\nu+b_{21}n_\mu n\cdot\partial c^\mu\partial^\nu v_\nu+b_{22}n^\nu\partial_\mu c^\mu n\cdot\partial v_\nu$$
$$+b_{23}n\cdot\partial c^\mu n\cdot\partial v_\mu+b_{24}n_\mu n^\nu n\cdot\partial c^\mu n\cdot\partial v_\nu)]\;, \tag{A1}$$

where $h=h^\rho{}_\rho$ and $\tilde{u}=\tilde{u}^\rho{}_\rho$. In the following, we give the functional differentiations of G in terms of $h_{\mu\nu}$, $\tilde{u}^{\mu\nu}$, c^ρ, and v_ρ, which are necessary for calculating the counterterms of (28), (30), and (32).

$$-\kappa^{-2}\frac{\delta G}{\delta h_{\mu\nu}}=a_1\partial^\mu\partial_\rho\tilde{u}^{\rho\nu}+a_2\eta^{\mu\nu}\partial_\rho\partial_\sigma\tilde{u}^{\rho\sigma}+a_3\partial^\mu\partial^\nu\tilde{u}+a_4\partial^2\tilde{u}^{\mu\nu}+a_5\eta^{\mu\nu}\partial^2\tilde{u}+b_1n^\mu n_\rho\partial^\nu\partial_\sigma\tilde{u}^{\rho\sigma}+b_2n^\mu n^\nu\partial_\rho\partial_\sigma\tilde{u}^{\rho\sigma}$$
$$+b_3n_\rho n_\sigma\partial^\mu\partial^\nu\tilde{u}^{\rho\sigma}+b_4n^\mu n^\nu\partial^2\tilde{u}+b_5n_\rho n_\sigma\eta^{\mu\nu}\partial^2\tilde{u}^{\rho\sigma}+b_6n^\mu n_\rho\partial^2\tilde{u}^{\rho\nu}+b_7n^\mu n^\nu n_\rho n_\sigma\partial^2\tilde{u}^{\rho\sigma}$$
$$+b_8n_\rho\partial_\sigma\eta^{\mu\nu}n\cdot\partial\tilde{u}^{\rho\sigma}+b_9n^\mu\partial^\nu n\cdot\partial\tilde{u}+b_{10}n^\mu n^\nu n_\rho\partial_\sigma n\cdot\partial\tilde{u}^{\rho\sigma}+b_{11}n^\mu n_\rho n_\sigma\partial^\nu n\cdot\partial\tilde{u}^{\rho\sigma}+b_{12}n^\mu\partial_\rho n\cdot\partial\tilde{u}^{\rho\nu}$$
$$+b_{13}n_\rho\partial^\mu n\cdot\partial\tilde{u}^{\rho\nu}+b_{14}(n\cdot\partial)^2\tilde{u}^{\mu\nu}+b_{15}\eta^{\mu\nu}(n\cdot\partial)^2\tilde{u}+b_{16}n^\mu n^\nu(n\cdot\partial)^2\tilde{u}+b_{17}n_\rho n_\sigma\eta^{\mu\nu}(n\cdot\partial)^2\tilde{u}^{\rho\sigma}$$
$$+b_{18}n^\mu n_\rho(n\cdot\partial)^2\tilde{u}^{\rho\nu}+b_{19}n^\mu n^\nu n_\rho n_\sigma(n\cdot\partial)^2\tilde{u}^{\rho\sigma}\;, \tag{A2}$$

$$-\kappa^{-2}\frac{\delta G}{\delta\tilde{u}^{\mu\nu}}=a_1\partial_\mu\partial^\rho h_{\rho\nu}+a_2\partial_\mu\partial_\nu h+a_3\eta_{\mu\nu}\partial^\rho\partial^\sigma h_{\rho\sigma}+a_4\partial^2h_{\mu\nu}+a_5\eta_{\mu\nu}\partial^2h+b_1n_\mu n^\rho\partial_\nu\partial^\sigma h_{\rho\sigma}+b_2n^\rho n^\sigma\partial_\mu\partial_\nu h_{\rho\sigma}$$
$$+b_3n_\mu n_\nu\partial^\rho\partial^\sigma h_{\rho\sigma}+b_4n^\rho n^\sigma\eta_{\mu\nu}\partial^2h_{\rho\sigma}+b_5n_\mu n_\nu\partial^2h+b_6n_\mu n^\rho\partial^2h_{\rho\nu}+b_7n_\mu n_\nu n^\rho n^\sigma\partial^2h_{\rho\sigma}$$
$$+b_8n_\mu\partial_\nu n\cdot\partial h+b_9n^\rho\eta_{\mu\nu}\partial^\sigma n\cdot\partial h_{\rho\sigma}+b_{10}n_\mu n^\rho n^\sigma\partial_\nu n\cdot\partial h_{\rho\sigma}+b_{11}n_\mu n_\nu n^\rho\partial^\sigma n\cdot\partial h_{\rho\sigma}$$
$$+b_{12}n^\rho\partial_\mu n\cdot\partial h_{\rho\nu}+b_{13}n_\mu\partial^\rho n\cdot\partial h_{\rho\nu}+b_{14}(n\cdot\partial)^2h_{\mu\nu}+b_{15}\eta_{\mu\nu}(n\cdot\partial)^2h+b_{16}n^\rho n^\sigma\eta_{\mu\nu}(n\cdot\partial)^2h_{\rho\sigma}$$
$$+b_{17}n_\mu n_\nu(n\cdot\partial)^2h+b_{18}n_\mu n^\rho(n\cdot\partial)^2h_{\rho\nu}+b_{19}n_\mu n_\nu n^\rho n^\sigma(n\cdot\partial)^2h_{\rho\sigma}\;, \tag{A3}$$

$$-\kappa^{-2}\frac{\delta G}{\delta c^\rho}=a_6\partial_\rho\partial^\mu v_\mu+a_7\partial^2v_\rho+b_{20}n_\rho n^\mu\partial^2v_\mu+b_{21}n_\rho\partial^\mu n\cdot\partial v_\mu+b_{22}n^\mu\partial_\rho n\cdot\partial v_\mu+b_{23}(n\cdot\partial)^2v_\rho+b_{24}n_\rho n^\mu(n\cdot\partial)^2v_\mu\;, \tag{A4}$$

and

$$-\kappa^{-2}\frac{\delta G}{\delta v_\rho}=a_6\partial^\rho\partial_\mu c^\mu+a_7\partial^2c^\mu+b_{20}n^\rho n_\mu\partial^2c^\mu$$
$$+b_{21}n_\mu\partial^\rho n\cdot\partial c^\mu+b_{22}n^\rho\partial_\mu n\cdot\partial c^\mu$$
$$+b_{23}(n\cdot\partial)^2c^\rho+b_{24}n^\rho n_\mu(n\cdot\partial)^2c^\mu\;. \tag{A5}$$

The following functional differentiations of the action $\tilde{S}$ in terms of the same fields as above are also necessary for calculating the counterterms (28), (30), and (32):

$$\frac{-\delta\tilde{S}}{\delta h_{\mu\nu}}=\tfrac{1}{2}\partial^2h^{\mu\nu}-\partial^\mu\partial_\rho h^{\rho\nu}+\tfrac{1}{2}\partial^\mu\partial^\nu h$$
$$+\tfrac{1}{2}\eta^{\mu\nu}\partial^\rho\partial^\sigma h_{\rho\sigma}-\tfrac{1}{2}\eta^{\mu\nu}\partial^2h$$
$$+(\partial_\rho c^\mu+\partial^\mu c_\rho)\tilde{u}^{\rho\nu}+\partial_\rho(c^\rho\tilde{u}^{\mu\nu})\;, \tag{A6}$$

FIG. 4. A graviton propagator.

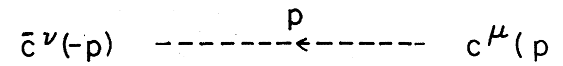

FIG. 5. The Faddeev-Popov ghost propagator. Notice the unbalanced appearance of two indices μ and ν in the propagator.

$$\frac{\delta\widetilde{S}}{\delta\widetilde{u}^{\mu\nu}}=\kappa^{-1}\delta(g_{\mu\nu})$$

$$=\kappa^{-1}(\partial_\mu c^\rho\cdot g_{\rho\nu}+\partial_\nu c^\rho\cdot g_{\mu\rho}-c^\rho\partial_\rho g_{\mu\nu})\ , \tag{A7}$$

$$\frac{\delta\widetilde{S}}{\delta c^\rho}=\partial_\mu(g_{\rho\nu}\widetilde{u}^{\mu\nu})+\partial_\nu(g_{\mu\rho}\widetilde{u}^{\mu\nu})+\widetilde{u}^{\mu\nu}\partial_\rho g_{\mu\nu}$$

$$+\partial_\mu(v_\rho c^\mu)+v_\mu\partial_\rho c^\mu\ , \tag{A8}$$

$$\frac{\delta\widetilde{S}}{\delta v_\rho}=\delta(c^\rho)=c^\mu\partial_\mu c^\rho\ . \tag{A9}$$

APPENDIX B

The graviton self-energy shown in Fig. 4 is

$$iG_{\mu\nu,\rho\sigma}(k)=i\kappa^{-2}(2I_1-I_2)_{\mu\nu,\rho\sigma}\ ,$$

where

$$n^\mu(I_1)_{\mu\nu,\rho\sigma}=n^\rho(I_1)_{\mu\nu,\rho\sigma}$$

$$=n^\mu(I_2)_{\mu\nu,\rho\sigma}=n^\rho(I_2)_{\mu\nu,\rho\sigma}=0\ ,$$

and the explicit forms of I_1 and I_2 are given in terms of tensors in Table II as

$$I_1=T_1+2^{-1}aT_6-T_7+a^2T_9-4aT_{10}+2aT_{11}+2T_{12}\ ,$$

$$I_2=T_2+aT_3-2T_4+a^2T_9-4aT_{10}+4T_{12}\ .$$

The Faddeev-Popov ghost propagator shown in Fig. 5 as the dotted line is given by

$$G_{\mu\nu}=\frac{-i\kappa}{(k\cdot n)}\left[\eta_{\mu\nu}-\frac{k_\mu n_\nu}{2(k\cdot n)}\right].$$

The h-c-c vertex shown Fig. 6 is

$$i\frac{\delta^3\Gamma^{(0)}}{\delta h^{\rho\sigma}(q)\delta c^\mu(p)\delta c^\nu(p')}=-\frac{i}{2}[(p\cdot n)(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})$$

$$+p_\nu(\eta_{\mu\rho}n_\sigma+\eta_{\mu\sigma}n_\rho)$$

$$+q_\mu(\eta_{\nu\sigma}n_\rho+\eta_{\nu\rho}n_\sigma)]\ .$$

The two-point vertex $\widetilde{u}$-c shown in Fig. 7 is

$$i\frac{\delta^2\Gamma^{(0)}}{\delta c_\lambda(p)\delta\widetilde{u}^{\mu\nu}(-p)}=p_\mu\delta_\nu^\lambda+p_\nu\delta_\mu^\lambda\ .$$

The three-point vertex h-$\widetilde{u}$-c shown in Fig. 8 is

$$i\frac{\delta^2\Gamma^{(0)}}{\delta h^{\rho\sigma}(p)\delta c_\lambda(q)\delta\widetilde{u}^{\mu\nu}(r)}=\tfrac{1}{2}\kappa[q_\mu(\delta_\rho^\lambda\eta_{\nu\sigma}+\delta_\sigma^\lambda\eta_{\nu\rho})$$

$$+q_\nu(\delta_\rho^\lambda\eta_{\mu\sigma}+\delta_\sigma^\lambda\eta_{\mu\rho})$$

$$-p^\lambda(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})]\ .$$

The three-point vertex v-c-c shown in Fig. 9 is

$$i\frac{\delta^2\Gamma^{(0)}}{\delta v_\lambda(r)\delta c^\nu(q)\delta c^\mu(p)}=q_\mu\delta_\nu^\lambda-p_\nu\delta_\mu^\lambda\ .$$

APPENDIX C

The counterterm (28) is calculated using (A4) and (A9) and is given by

$$\kappa^{-2}V^\lambda{}_{\mu\nu}(p,-p,0)=2a_6p^\lambda p_\mu p_\nu+2b_{22}(n\cdot p)n^\lambda p_\mu p_\nu+(p_\mu\delta_\nu^\lambda+p_\nu\delta_\mu^\lambda)[(a_6+2a_7)p^2+(b_{22}+2b_{23})(n\cdot p)^2]$$

$$+(p_\mu n_\nu+p_\nu n_\mu)[b_{20}p^2n^\lambda+b_{21}(n\cdot p)p^\lambda+b_{24}(n\cdot p)^2n^\lambda]$$

$$+(n_\mu\delta_\nu^\lambda+n_\nu\delta_\mu^\lambda)[(b_{20}+b_{21})p^2(n\cdot p)+b_{24}(n\cdot p)^3]\ . \tag{C1}$$

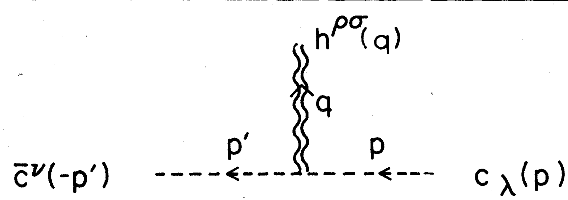

FIG. 6. A tree triple vertex h-c-c.

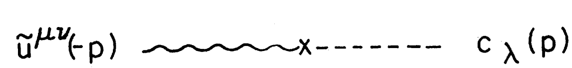

FIG. 7. A tree vertex $\widetilde{u}$-c.

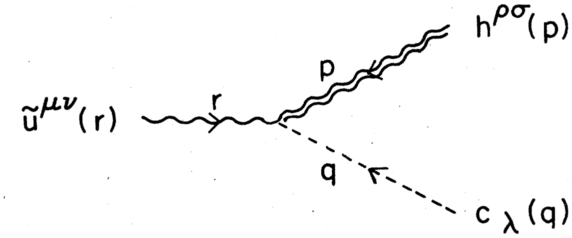

FIG. 8. A tree triple vertex h-$\tilde{u}$-c.

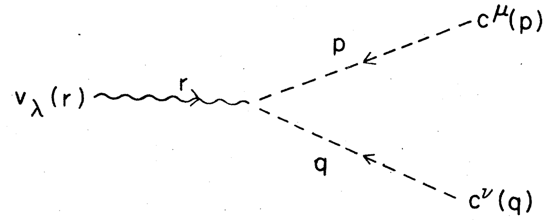

FIG. 9. A tree triple vertex v-c-c.

The constraints on the coefficients a_i and b_i derived from the constraints (26) and (27) on $U^\lambda{}_{\mu\nu}$ (32) are given as follows:

$$a_1+2a_2+b_1+b_8=0\ , \tag{C2a}$$

$$a_1+2a_4+b_1+b_6=0\ , \tag{C2b}$$

$$a_3+a_5+b_3+b_5=0\ , \tag{C2c}$$

$$b_9=2(b_3-b_4+b_5)\ , \tag{C2d}$$

$$b_{10}=-(b_1+b_8)\ , \tag{C2e}$$

$$b_{11}=-(b_1+2b_3+2b_5+b_6+2b_7)\ , \tag{C2f}$$

$$b_{12}=b_1-b_2+b_8\ , \tag{C2g}$$

$$b_{13}=b_1\ , \tag{C2h}$$

$$b_{16}=-(b_3-b_4+b_5+b_{15})\ , \tag{C2i}$$

$$b_{17}=b_4+2^{-1}b_6+b_7-2^{-1}b_8-b_{15}\ , \tag{C2j}$$

$$b_{18}=-b_1+2b_2-b_8-2b_{14}\ , \tag{C2k}$$

$$b_{19}=b_1-b_2+b_3-b_4+b_5+b_8+b_{14}+b_{15}\ . \tag{C2l}$$

The form of $U^\lambda{}_{\mu\nu}$ which satisfies (26) and (27), using all relations (C2), is given as follows:

$$\begin{aligned}-\kappa^{-2}U^\lambda{}_{\mu\nu}(p)=&\,2^{-1}[(b_1+b_6)p^2-(b_1-2b_2+b_8+2b_{14})(n\cdot p)^2]\\ &\times\{[(n\cdot p)n_\mu-p_\mu](\delta^\lambda_\nu-n^\lambda n_\nu)+[(n\cdot p)n_\nu-p_\nu](\delta^\lambda_\mu-n^\lambda n_\mu)\}\\ &+2[-(b_3+b_5)p^2+(b_3-b_4+b_5+b_{15})(n\cdot p)^2][p^\lambda-(n\cdot p)n^\lambda](\eta_{\mu\nu}-n_\mu n_\nu)\\ &-(b_1+b_8)[p^\lambda-(n\cdot p)n^\lambda][p_\mu-(n\cdot p)n_\mu][p_\nu-(n\cdot p)n_\nu]\end{aligned} \tag{C3}$$

$$=\tfrac{16}{3}I\{[(n\cdot p)n_\mu-p_\mu](\delta^\lambda_\nu-n^\lambda n_\nu)+[(n\cdot p)n_\nu-p_\nu](\delta^\lambda_\mu-n^\lambda n_\mu)\}\ . \tag{C4}$$

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