Bianchi type-I model with cosmological constant in a generalized scalar-tensor theory of gravitation

A. Banerjee, S. B. Duttachoudhury, and N. Banerjee
Department of Physics, Jadavpur University, Calcutta 700 032, India
(Received 6 June 1985)

Bianchi type-I cosmological models are discussed in the Bergmann-Wagoner-Nordtvedt scalar-tensor theory where both the so-called cosmological constant Λ and the coupling parameter ω are taken to be functions of the scalar field ϕ. Exact solutions are obtained in Dicke's revised units assuming a very simple relationship between Λ and ϕ. The properties of the models are discussed in special cases.

I. INTRODUCTION

The concepts of contemporary field theory have recently led to the belief that in the early stage of the Universe the properties of the vacuum were different from what they are now. The Λ term, the so-called cosmological parameter, was big and strongly affected the beginning of the expansion of the Universe. The parameter Λ is nowadays believed to correspond to the vacuum energy (Zel'dovich[1]) which gives rise to the corresponding mass density of the vacuum. Dreitlein[2] suggested that the mass of the Higgs boson is connected with Λ as well as the so-called gravitational constant G. Some workers have even suggested the possibility of Λ being a variable quantity depending on the scalar field (see Bergmann[3] and Wagoner[4]). Linde[5] proposed that Λ is a function of temperature and related it to the spontaneous-symmetry-breaking process. In cosmology, however, the cosmological term Λ may be understood by incorporating Mach's principle which simulates the interest of the work in Brans-Dicke[6] theory. The application of the Brans-Dicke homogeneous cosmology with $\Lambda\neq 0$ to the inflationary-universe scenario with radiation was given by Dominici, Holman, and Kim.[7] Very recently, Lorenz-Petzold[8] investigated the Brans-Dicke field equations for Bianchi type-I space-time and obtained vacuum as well as dust solutions in the presence of the cosmological parameter Λ. Previously Endo and Fukui[9] and later Banerjee and Santos[10] added the Λ term in the Brans-Dicke Lagrangian to obtain modified field equations involving Λ, which is now a function of the scalar field. The parameter Λ, in Lorenz-Petzold's work, however, is assumed to be a constant quantity.

In the present paper we investigate the Bianchi type-I cosmological models which include Λ as a variable quantity in Nordtvedt's scalar-tensor theory[11] considering the parameter ω a function of the scalar field, too. The interests in such an investigation are twofold. First, Nordtvedt's theory generalizes Brans-Dicke theory in considering ω as a variable parameter, so that it keeps open the possibility of ω having smaller magnitudes at different epochs giving results appreciably different from those in Einstein's theory. It should be remarked in this context that the recent experimental data of Will[12] indicate very large values for ω ($\omega\sim 500$) which makes the results practically indistinguishable from Einstein's theory. Second, the variable Λ involves wider prospects for its specific role in cosmology currently discussed by particle physicists.

We consider the modified Lagrangian and the corresponding field equations in Dicke's revised units,[13] in which the particle masses vary whereas the gravitational constant G remains fixed. In these units, however, the test particles do not follow the geodesic trajectories. We work in such units to get the set of equations in a much simpler form. In Sec. II we write the complete set of field equations in empty space and in Sec. III the solutions are obtained for a very simple relationship between Λ and ϕ, namely, $\Lambda/\phi=\text{const}$. The dynamical properties of the models are analyzed in different special cases. The solutions for the scalar field for different choices of the functional relationships between the parameter ω and the scalar field ϕ are also explicitly given. Brans-Dicke theory is only a special case, where ω is assumed to be a constant. It is not difficult to obtain explicitly the solution in the original atomic units,[6] for which the masses of test particles remain fixed and the so-called gravitational constant varies with time, by a simple transformation like $g_{\mu\nu}=\phi\bar{g}_{\mu\nu}$, where an overbar indicates atomic units and a variable without an overbar is given in revised units. An example is worked out for the special case of Brans-Dicke theory.

Finally in Sec. IV the isotropic Friedmann-Robertson-Walker model with zero spatial curvature ($k=0$) is considered as a special case of the Bianchi type-I model. One set of solutions is seen to reduce to a vacuum de Sitter[14] cosmological solution.

II. FIELD EQUATIONS

Assuming the cosmological term to be an explicit function of the scalar field $\bar{\phi}$, Nordtvedt's variational principle with modified $\Lambda(\bar{\phi})$ will be

$$\delta\int\left[\bar{\phi}[\bar{R}-2\Lambda(\bar{\phi})]+\frac{\omega(\bar{\phi})}{\bar{\phi}}\bar{\phi}_{,\mu}\bar{\phi}^{,\mu}+\frac{16\pi}{c^4}\bar{L}_m\right]\sqrt{-g}\,d^4x=0\,, \quad (2.1)$$

where $\bar{R}$ is the Ricci scalar and $\bar{L}_m$ is the Lagrangian density due to matter. In Dicke's revised units, that is, with the transformation $\bar{g}_{\mu\nu}=(1/\phi)g_{\mu\nu}$ and $\phi=\bar{\phi}/\bar{\phi}_0$, where $\bar{\phi}_0$ is a constant, the variational principle looks like

$$\delta\int\left[R-\frac{2\Lambda(\phi)}{\phi}+\frac{(2\omega+3)}{2}\frac{\phi_{,\mu}\phi^{,\mu}}{\phi^2}+\frac{16\pi G_0}{c^4}L_m\right]\sqrt{-g}\,d^4x=0\ . \quad (2.2)$$

Variables with an overbar are in the original atomic units of Brans and Dicke and those without an overbar are in Dicke's revised units. With $\Lambda(\phi)=0$, Eq. (2.2) reduces to the usual Nordtvedt's variational principle in Dicke's revised units. The usual variation of Eq. (2.2) with respect to $g_{\mu\nu}$ and ϕ will yield, respectively, the field equations

$$\begin{aligned}G_{\alpha\beta}&\equiv R_{\alpha\beta}-\tfrac{1}{2}Rg_{\alpha\beta}\\&=-T_{\alpha\beta}-\frac{(2\omega+3)}{2\phi^2}(\phi_{,\alpha}\phi_{,\beta}-\tfrac{1}{2}g_{\alpha\beta}\phi_{,\mu}\phi^{,\mu})\\&\quad-\left[\frac{\Lambda}{\phi}\right]g_{\alpha\beta}\ ,\end{aligned} \quad (2.3)$$

and the wave equation

$$\begin{aligned}\Box(\ln\phi)&\equiv(\ln\phi)^{;\mu}{}_{;\mu}\\&=\frac{1}{(2\omega+3)}\left[T+\frac{2\Lambda}{\phi}-\frac{2\,d\Lambda}{d\phi}-\frac{1}{\phi}\phi_{,\mu}\phi^{,\mu}\frac{d\omega}{d\phi}\right]\ .\end{aligned} \quad (2.4)$$

Here $8\pi G_0/c^4$ is taken to be unity and $T_{\alpha\beta}$ represents the energy-momentum tensor of matter.

The diagonal Bianchi type-I line element for an anisotropic but homogeneous space-time is given by

$$ds^2=dt^2-e^{2\alpha}dx^2-e^{2\beta}dy^2-e^{2\gamma}dz^2\ , \quad (2.5)$$

where α, β, and γ are functions of time t alone. With this metric, the nontrivial field equations in the case of empty space according to (2.3) and (2.4) are

$$\tfrac{9}{2}(\dot{R}/R)^2-\tfrac{1}{2}(\dot{\alpha}^2+\dot{\beta}^2+\dot{\gamma}^2)=\frac{(2\omega+3)}{4}(\dot{\phi}/\phi)^2+\Lambda/\phi\ , \quad (2.6)$$

$$\ddot{\beta}+\ddot{\gamma}+\tfrac{3}{2}(\dot{R}/R)(\dot{\beta}+\dot{\gamma}-\dot{\alpha})+\tfrac{1}{2}(\dot{\alpha}^2+\dot{\beta}^2+\dot{\gamma}^2)=-\frac{(2\omega+3)}{4}(\dot{\phi}/\phi)^2+\frac{\Lambda}{\phi}\ , \quad (2.7)$$

$$\ddot{\gamma}+\ddot{\alpha}+\tfrac{3}{2}(\dot{R}/R)(\dot{\gamma}+\dot{\alpha}-\dot{\beta})+\tfrac{1}{2}(\dot{\alpha}^2+\dot{\beta}^2+\dot{\gamma}^2)=-\frac{(2\omega+3)}{4}(\dot{\phi}/\phi)^2+\frac{\Lambda}{\phi}\ , \quad (2.8)$$

$$\ddot{\alpha}+\ddot{\beta}+\tfrac{3}{2}(\dot{R}/R)(\dot{\alpha}+\dot{\beta}-\dot{\gamma})+\tfrac{1}{2}(\dot{\alpha}^2+\dot{\beta}^2+\dot{\gamma}^2)=-\frac{(2\omega+3)}{4}(\dot{\phi}/\phi)^2+\frac{\Lambda}{\phi}\ , \quad (2.9)$$

$$(\ln\phi)^{\cdot\cdot}+\frac{3\dot{R}}{R}(\ln\phi)^{\cdot}=\frac{1}{(2\omega+3)}\left[\frac{2\,d\Lambda}{d\phi}-\frac{2\Lambda}{\phi}-\frac{1}{\phi}\dot{\phi}^2\frac{d\omega}{d\phi}\right]\ , \quad (2.10)$$

where $R^3=\sqrt{-g}=\exp(\alpha+\beta+\gamma)$ and an overdot represents differentiation with respect to t.

With these field equations, the Raychaudhuri[15] equation looks like

$$\dot{\theta}+\tfrac{1}{3}\theta^2-\dot{u}^{\alpha}{}_{;\alpha}+2(\sigma^2-\omega^2)+\tfrac{1}{2}(\rho_\phi+3p_\phi)-\frac{\Lambda}{\phi}=0\ , \quad (2.11)$$

where $\theta=3\dot{R}/R$ is the expansion scalar, σ and ω are shear and rotation scalars, respectively, and $\dot{u}_\alpha$ is the acceleration vector. From the field equations, one observes that the scalar field plays the role of a matter source with effective density ρ_ϕ and pressure p_ϕ given by the relation

$$\rho_\phi=p_\phi=\frac{(2\omega+3)}{4}(\dot{\phi}/\phi)^2\ . \quad (2.12)$$

For a rotation-free homogeneous model in general, $\dot{u}^{\alpha}{}_{;\alpha}=0$ and $\omega^2=0$. With $(2\omega+3)>0$, ρ_ϕ and p_ϕ are positive and one observes from (2.10) that for $\Lambda/\phi<0$, θ is always negative and the singularity of zero proper volume is unavoidable. For $\Lambda/\phi>0$, however, the sign of $\dot{\theta}$ cannot be determined by (2.11).

To solve the system of equations in this model, we see that there are six variables, namely, α, β, γ, ω, Λ, ϕ but only four independent equations, (2.5)—(2.8). For empty space, the wave equation follows from the field equations in view of the Bianchi identity. In what follows, we would assume two relations. A very simple linear relationship between Λ and ϕ will be assumed,

$$\Lambda=b\phi\ , \quad (2.13)$$

where b is a constant. It will be seen that the solution for the metric components is independent of the functional relationship between $\omega(\phi)$ and ϕ. But the exact functional form of ω will be necessary when one goes to solve for the scalar field. For this purpose, we would assume a few functional forms already in the literature as examples.

III. SOLUTION OF THE FIELD EQUATIONS AND BEHAVIOR OF THE MODEL

Equation (2.13) yields $(d/d\phi)(\Lambda/\phi)=(d/d\phi)(b)=0$ and it considerably simplifies the wave equation (2.10) which becomes

$$\ddot{\psi}+\frac{3\dot{R}}{R}\dot{\psi}=-\frac{1}{(2\omega+3)}\dot{\psi}\dot{\omega}\ , \quad (3.1)$$

where $\psi=\ln\phi$. Equation (3.1) can be integrated to the form

$$(2\omega+3)^{1/2}\dot{\psi}=\frac{C}{R^3}\ , \quad (3.2)$$

where C is a constant of integration.

Adding (2.6), (2.8), and (2.9) together and subtracting (2.7) from the result, one obtains

$$2\ddot{\alpha}+\tfrac{9}{2}(\dot{R}/R)^2+\tfrac{3}{2}(\dot{R}/R)(3\dot{\alpha}-\dot{\beta}-\dot{\gamma})=2b$$

which, with the help of the relation $R^3=\exp(\alpha+\beta+\gamma)$ becomes

$$\ddot{\alpha}+\frac{3\dot{R}}{R}\dot{\alpha}=b \; . \tag{3.3a}$$

Addition of (2.6) with (2.7) yields

$$6(\dot{R}/R)^2+\frac{3\ddot{R}}{R}-\frac{3\dot{R}}{R}\dot{\alpha}-\ddot{\alpha}=2b \; . \tag{3.3b}$$

From (3.3a) and (3.3b) we obtain

$$\frac{3\ddot{R}}{R}+\frac{6\dot{R}^2}{R^2}=3b \; ,$$

that is,

$$(R^3)^{\cdot\cdot}=3bR^3=n^2R^3 \; , \tag{3.4}$$

where $n^2=3b$.

For $b<0$, that is, $n^2<0$, the solution for R^3 is harmonic and we have a model between a minimum of zero proper volume R^3 and the maximum of a finite volume. The other case for $n^2>0$ can be given by a general solution

$$R^3=Ae^{nt}+Be^{-nt} \; , \tag{3.5}$$

where A and B are arbitrary constants.

From (3.3a) one can have

$$(\dot{\alpha}R^3)^{\cdot}=bR^3 \; ,$$

which, in view of (3.4), yields

$$(\dot{\alpha}R^3)^{\cdot}=\tfrac{1}{3}(R^3)^{\cdot\cdot} \; .$$

This readily integrates to yield

$$\dot{\alpha}=\frac{\dot{R}}{R}+\frac{a_1}{R^3} \; , \tag{3.6a}$$

where a_1 is a constant of integration. Proceeding similarly for $\dot{\beta}$ and $\dot{\gamma}$ we obtain

$$\dot{\beta}=\frac{\dot{R}}{R}+\frac{a_2}{R^3} \; , \tag{3.6b}$$

$$\dot{\gamma}=\frac{\dot{R}}{R}+\frac{a_3}{R^3} \; , \tag{3.6c}$$

where a_2 and a_3 are constants of integration with $(a_1+a_2+a_3)=0$.

With the help of (3.5), Eqs. (3.6) can be integrated to yield the solutions for the metric. We shall state the solutions corresponding to $n^2>0$ and discuss the dynamics of the model in different cases without giving the details of calculations. We use the notation R_i^2 for the metric components, where $R_1^2=e^{2\alpha}$, $R_2^2=e^{2\beta}$, $R_3^2=e^{2\gamma}$. The signs of A and B should be chosen in such a manner that the proper volume R^3 is positive. The constants of integrations are absorbed in metric coefficients wherever possible without loss of generality.

Case I: A,B both positive

$$R_i^2=(k^2e^{nt}+e^{-nt})^{2/3}\exp[p_i\tan^{-1}(ke^{nt})] \; , \tag{3.7a}$$

where $k=\sqrt{A/B}$ and p_i's are constants with

$$\sum_{i=1}^{3}p_i=0 \; .$$

Here the expansion scalar $\theta=3\dot{R}/R$ is given by

$$\theta=\frac{n(Ae^{nt}-Be^{-nt})}{R^3} \tag{3.7b}$$

and $\dot{\theta}=4n^2AB/R^6$, which is positive. So there is a lower bound when $\theta=0$. From (3.7b) one finds that $\theta=0$ at $t=(1/2n)\ln(B/A)$. Hence, the singularity of zero proper volume is avoidable in this case. This is because here $\Lambda>0$, which indicates that it causes a repulsion and thus can give rise to a model with a bounce at a minimum volume.

If we choose the time scale such that the lower bound occurs at $t=0$, that is $A=B$, the solution can be expressed as

$$R^3=2A\cosh nt \tag{3.7c}$$

and

$$\theta=n\tanh nt \; . \tag{3.7d}$$

With this choice of the time scale, the metric is given by

$$R_i^2=(2A\cosh nt)^{2/3}\exp[p_i\tan^{-1}(e^{nt})] \; . \tag{3.7e}$$

Case II: $A>0$, $B\le 0$

$$R^3=|A|e^{nt}-|B|e^{-nt} \; , \tag{3.8a}$$

$$R_i^2=(|A|e^{nt}-|B|e^{-nt})^{2/3}\times\left[\frac{\sqrt{|A|}e^{nt}-\sqrt{|B|}}{\sqrt{|A|}e^{nt}+\sqrt{|B|}}\right]^{p_i} \; , \tag{3.8b}$$

where the p_i's are constants and

$$\sum_{i=1}^{3}p_i=0 \; .$$

Here,

$$\theta=\frac{n(|A|e^{nt}+|B|e^{-nt})}{R^3} \; , \tag{3.8c}$$

which is positive and not zero for any finite t and so the model has no turning point. The Universe starts from the singularity of zero proper volume and is ever expanding. At $t\to\infty$, R^3 becomes infinitely large. If $B\neq 0$, the singularity occurs at a finite past given by $t=\frac{1}{2}\ln|B/A|$. One can fix the origin of the time scale $(t=0)$ at $R^3=0$ by the choice $|A|=|B|$. If however $B=0$, the singularity occurs at infinite past $(t\to-\infty)$, and the rate of expansion is steady $(\theta=n)$.

Case III: $B>0$, $A\le 0$

$$R^3=|B|e^{-nt}-|A|e^{nt} \; , \tag{3.9a}$$

$$R_i^2=(|B|e^{-nt}-|A|e^{nt})^{2/3}\times\left[\frac{|B|e^{-nt}+|A|e^{nt}}{|B|e^{-nt}-|A|e^{nt}}\right]^{q_i} \; , \tag{3.9b}$$

where the q_i's are constants with

$$\sum_{i=1}^{3} q_i = 0 ,$$

$$\theta = -\frac{n(|B|e^{-nt}+|A|e^{nt})}{R^3} , \tag{3.9c}$$

which is negative and never zero for any finite t. The Universe is ever contracting, R^3 is infinitely large at $t \to -\infty$ and it reduces to the singularity of zero volume at a finite time $t = \frac{1}{2} n \ln|B/A|$. If $|B| = |A|$, $R^3=0$ occurs at $t=0$. If however, $A=0$, the singularity of $R^3=0$ occurs at infinite future $t \to \infty$ with a steady rate of contraction ($\theta = -n$).

From the above we note that the only case where there is a bounce from a minimum finite volume is for $A>0$ and $B>0$. In all other cases the singularity cannot be avoided even if $\Lambda > 0$ producing a repulsive effect.

The geometric shear scalar σ is given by (Raychaudhuri[15])

$$\sigma^2 = \frac{1}{12}\left[\left[\frac{\dot{g}_{11}}{g_{11}} - \frac{\dot{g}_{22}}{g_{22}}\right]^2 + \left[\frac{\dot{g}_{22}}{g_{22}} - \frac{\dot{g}_{33}}{g_{33}}\right]^2 + \left[\frac{\dot{g}_{33}}{g_{33}} - \frac{\dot{g}_{11}}{g_{11}}\right]^2\right] ,$$

and in all the cases mentioned above, it is found that $\sigma^2 \sim R^{-6}$, that is, the shear, which is a measure of the anisotropy becomes infinitely large when $R^3 \to 0$. From (3.2), we note that ρ_ϕ, the energy density due to the scalar field, also attains an infinitely large value ($\sim R^{-6}$) at $R^3 \to 0$. For a Bianchi type-I cosmological model, the curvature of the three-space is always zero. The curvature scalar of the four-space, $g^{\mu\nu}R_{\mu\nu}$, will be given by

$$g^{\mu\nu}R_{\mu\nu} = -2n^2 + 6\dot{R}^2/R^2 + D/R^6 ,$$

in this theory. Here D is a constant. From the above relation we note that as $R^3 \to 0$, R^{-6} will be the dominant term and $g^{\mu\nu}R_{\mu\nu}$ explodes to infinity as R^{-6} like σ^2 and ρ_ϕ.

Solution for the scalar field

Once the solution for R^3 is known, Eq. (3.2) can be integrated to yield the solution for ϕ provided the exact functional form of $\omega(\phi)$ is known. We shall use a few functional forms of ω already in the literature to solve for ϕ. The solutions corresponding to two special cases $A>0, B>0$ and $A>0, B=0$ will be given in what follows omitting the details of calculations.

(A) Brans-Dicke theory: $\omega = \omega_0$, a constant.

(i) $A>0$, $B>0$

$$e^{\psi} = \phi = A_1 \exp\left[\frac{C}{nk(2\omega_0+3)^{1/2}} \tan^{-1}(ke^{nt})\right] . \tag{3.10a}$$

(ii) $A>0$, $B=0$

$$e^{\psi} = \phi = A_2 \exp\left[-\frac{C}{nA(2\omega_0+3)^{1/2}} e^{-nt}\right] . \tag{3.10b}$$

(B) Barker's theory:[16] $2\omega+3 = 1/(\phi-1)$.

(i) $A>0$, $B>0$

$$e^{\psi} = \phi = 1 + \tan^2\left[\frac{C}{2nk}\tan^{-1}(ke^{nt}) + A_3\right] . \tag{3.11a}$$

(ii) $A>0$, $B=0$

$$e^{\psi} = \phi = 1 + \tan^2\left[A_4 - \frac{C}{nA} e^{-nt}\right] . \tag{3.11b}$$

(C) Schwinger's theory:[17,18] $2\omega+3 = 1/\alpha\phi$, α being a constant.

(i) $A>0$, $B>0$

$$e^{\psi} = \phi = \frac{4k^2n^2}{C^2\alpha}[\tan^{-1}(ke^{nt}) + A_5]^{-2} . \tag{3.12a}$$

(ii) $A>0$, $B=0$

$$e^{\psi} = \phi = \frac{4n^2A^2}{C^2\alpha}(e^{nt} + A_6)^{-2} . \tag{3.12b}$$

(D) Curvature coupling:[18] $2\omega+3 = 3/(1-\phi)$.

(i) $A>0$, $B>0$

$$e^{\psi} = \phi = \frac{4A_7 \exp\left[\frac{C}{2\sqrt{3}kn}\tan-1(ke^{nt})\right]}{\left[1 + A_7\exp\left[\frac{C}{2\sqrt{3}kn}\tan^{-1}(ke^{nt})\right]\right]^2} . \tag{3.13a}$$

(ii) $A>0$, $B=0$

$$e^{\psi} = \phi = \frac{4A_8\left[\exp\left[-\frac{C}{2\sqrt{3}nA}e^{-nt}\right]\right]}{\left[1 + A_8\exp\left[-\frac{C}{2\sqrt{3}nA}e^{-nt}\right]\right]^2} . \tag{3.13b}$$

In the above $A_1, A_2, \ldots, A_8$ are constants of integration.

The cosmological parameter can be immediately obtained from (2.13) once the scalar field ϕ is known. Λ, being linearly proportional to ϕ, has the same time behavior as ϕ.

Transformation of the solutions into atomic units

With the solutions for ϕ at hand, one can transform the metric components into the original atomic units of Brans and Dicke. The transformation relation is

$$\bar{g}_{\mu\nu} = \frac{1}{\phi} g_{\mu\nu} . \tag{3.14}$$

We shall cite only one example here. For $A,B>0$, Eqs. (3.7a) and (3.10a) give the solutions for the metric and the scalar field in Brans-Dicke theory, respectively. The transformed metric, with the help of Eq. (3.14) is

$$R_i^2 = \frac{1}{A_1}(k^2e^{nt} + e^{-nt})^{2/3} \times \exp\left[\left[p_i - \frac{C}{nk(2\omega_0+3)^{1/2}}\right]\tan^{-1}(ke^{nt})\right] . \tag{3.15}$$

We have noticed that the R_i^2's are independent of the choice of $\omega(\phi)$ in the gravitational units of Dicke (where G remains fixed). But when the solutions are transformed into the atomic units, they become dependent on the solution of ϕ through the transformation relation (3.14). The scalar field ϕ, in turn, depends on the choice of $\omega(\phi)$ via the wave equation. So the metric becomes dependent on the choice of the functional form of $\omega(\phi)$.

IV. ISOTROPIC MODELS

When $\alpha=\beta=\gamma$, the metric (2.5) reduces to the Friedmann-Robertson-Walker line element with zero spatial curvature. In this case, for obvious reasons, the shear scalar vanishes. The general solution for the isotropic case also is given by Eq. (3.5) and the dynamical behavior will be similar to that discussed in Sec. III for anisotropic models.

One interesting case is for $n^2>0$ and $B=0$. The solution in this case is

$$R^3=Ae^{nt}$$

or (4.1)

$$R^2=A^{2/3}e^{2/3nt} .$$

This reduces to the de Sitter solution in empty space,

$$R^2\sim e^{2Ht}$$

with $H=\sqrt{\Lambda/3}$. In (4.1), however, the constant n is given by

$$n^2=3b=\frac{3\Lambda}{\phi} ;$$

that is,

$$R^2=A^{2/3}e^{2[(1/3)(\Lambda/\phi)]^{1/2}t} .$$

We note that Λ/ϕ in the present case plays the role as Λ does in the empty-space de Sitter model.

V. DISCUSSION

In this paper exact solutions are given for Bianchi type-I empty space-time in Bergmann-Wagoner-Nordtvedt generalized scalar tensor theory where both the coupling parameter ω and the cosmological parameter Λ are assumed to be functions of the scalar field. To the best of our knowledge the solutions are new. Singh and Singh[19] and Singh and Rai[20] in previous papers obtained such solutions in the presence of dust and fluid, but in their work the parameter ω was constant. They made another assumption in the wave equation for the scalar field introducing a constant μ which measured the deviation of the theory from Brans-Dicke theory. The explicit form of the equation was

$$\Box\phi=\frac{\mu kT}{(2\omega+3)} .$$

However in empty space $T=0$ and this particular form of the equation has no influence on the general nature of the solutions. In a recent publication of Lorenz-Petzold,[21] apart from the one mentioned at the beginning of this paper, some exact vacuum solutions were obtained in Brans-Dicke theory with nonzero cosmological constant Λ. So our attempt here is to extend such solutions in a more general theory. We have been successful in getting them with, however, a restriction imposed on the functional form of Λ, that is, in the form $\Lambda\propto\phi$. At the end we refer here to some other recent investigations of interest in this connection (Refs. 22—30).

ACKNOWLEDGMENT

One of the authors (N.B.) acknowledges financial support from the Council of Scientific and Industrial Research (India).

[1] Ya. B. Zel'dovich, Usp. Fiz. Nauk **95**, 209 (1968) [Sov. Phys. Usp. **11**, 381 (1968)].
[2] J. Dreitlein, Phys. Rev. Lett. **33**, 1243 (1974).
[3] P. G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968).
[4] R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970).
[5] A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 320 (1974) [JETP Lett. **19**, 183 (1974)].
[6] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
[7] D. Dominici, R. Holman, and C. W. Kim, Phys. Rev. D **28**, 2983 (1983).
[8] D. Lorenz-Petzold, Phys. Rev. D **29**, 2399 (1984).
[9] M. Endo and T. Fukui, Gen. Relativ. Gravit. **8**, 833 (1977).
[10] A. Banerjee and N. O. Santos, Gen. Relativ. Gravit. **14**, 559 (1982).
[11] K. Nordtvedt, Jr., Astrophys. J. **161**, 1059 (1970).
[12] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University, Cambridge, 1981).
[13] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).
[14] W. de Sitter, Proc. K. Ned. Acad. Wet. **19**, 1217 (1917); **20**, 229 (1917).
[15] A. K. Raychaudhuri, Phys. Rev. **98**, 1123 (1955).
[16] B. M. Barker, Astrophys. J. **219**, 5 (1978).
[17] J. Schwinger, *Particles, Sources and Fields* (Addison-Wesley, Reading, Mass., 1970).
[18] N. Van den Bergh, Gen. Relativ. Gravit. **14**, 17 (1982).
[19] T. Singh and Tarkeshwar Singh, J. Math. Phys. **25**, 2800 (1984).
[20] T. Singh and L. N. Rai, Acta Phys. Pol. B **13**, 3 (1982).
[21] D. Lorenz-Petzold, Astrophys. Space Sci. **100**, 475 (1984).
[22] A. Banerjee and N. O. Santos, Phys. Rev. D **23**, 2111 (1981).
[23] A. Banerjee and N. O. Santos, J. Phys. A **14**, 2829 (1981).
[24] D. Lorenz-Petzold, Astrophys. Space Sci. **100**, 461 (1984).
[25] D. Lorenz-Petzold, Astrophys. Space Sci. **106**, 211 (1984).
[26] L. O. Pimentel, Astrophys. Space Sci. **112**, 175 (1985).
[27] B. L. Altshuler, Int. J. Theor. Phys. **24**, 109 (1985).
[28] B. L. Altshuler, Dok. Akad. Nauk SSSR **274**, 1076 (1984) [Sov. Phys. Dokl. **29**, 119 (1984)].
[29] K. Maslanka, Acta Cosmol. **12**, 57 (1983).
[30] D. Lorenz-Petzold, Astrophys. Space Sci. **106**, 419 (1985).