Derivation of mass relations for composite W^* and Z^* from effective-Lagrangian approach

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In an effective-Lagrangian model with gauge bosons (W,Z,γ) and their neighboring spin $J=1$ composites (W^*,Z^*), we find relations among their masses, m_W, m_Z, m_{W^*}, and m_{Z^*}: $m_W m_{W^*}=\cos\theta m_Z m_{Z^*}$ (as a generalization of $m_W=\cos\theta m_Z$) and $m_W{}^2+m_{W^*}{}^2+\tan^2\theta m_{W0}{}^2=m_Z{}^2+m_{Z^*}{}^2$ with m_{W0} being the mass of W in the standard model provided that the system respects the $SU(2)_L\times U(1)_Y$ symmetry. W^* and Z^* are taken as the lowest-lying excited states belonging to an $SU(2)_L$ triplet in the symmetric limit. The existence of W^* coupling to the $V-A$ current modifies the relation between G_F and m_W and that of Z^* generates a new interaction of the $(J^{EM})^2$ type as well as the deviation of $\sin\theta_W$ observed at low energies from the mixing angle $\sin\theta$ in neutral-current interactions.

PROLOGUE

In composite models of quarks and leptons as well as gauge bosons,[1] there will naturally emerge various resonances:[2-4] spin $J=0$ resonances as partners of $J=1$ gauge bosons, $J=\frac{1}{2}$ resonances as excited states of quarks and leptons, $J=1$ resonances as excited states of gauge bosons, and so on. Their properties and possible observable signatures have been extensively discussed lately.[4] In a previous paper,[5] we studied the masses of $J=1$ excited W and Z bosons, W^* and Z^*, using the charge commutation relations (CR's) of the underlying $SU(2)_L\times U(1)_Y$ symmetry.[6] The breaking of $SU(2)_L\times U(1)_Y$ is characterized by the presence of the CR $[\dot T_+,T_+]=0$, where $\dot T_+=i[H,T_+]$, with T_+ being the $SU(2)_L$-raising generator $(T_+=T_1+iT_2)$ and H being the Hamiltonian of the system. Since the realization of $[\dot T_+,T_+]=0$ depends crucially on the spectrum of physical $J=1$ bosons present in the models, it yields, after simple calculations (carried out in the infinite-momentum frame), $m_W=\cos\theta m_Z$ for the system[6] (W,Z,γ) and $m_W m_{W^*}=\cos\theta m_Z m_{Z^*}$ for the system[5] (W,Z,γ,W^*,Z^*), where m_W, m_Z, m_{W^*}, and m_{Z^*} are the masses of W, Z, W^*, and Z^*, respectively; W^* and Z^* belong to an $SU(2)_L$ triplet in the symmetric limit.

In the present Brief Report, we construct an explicit model containing W^* and Z^* as well as W, Z, γ by the use of an effective Lagrangian based on the $SU(2)_L\times U(1)_Y$ symmetry. The CR constraint $[\dot T_+,T_+]=0$ previously used will be automatically realized if the $SU(2)_L$-doublet Higgs scalar ϕ is produced at the level of composites.[7] From the effective Lagrangian involving kinetic mixing among non-Abelian fields (W,Z,γ,W^*,Z^*), the relation $m_W m_{W^*}=\cos\theta m_Z m_{Z^*}$ is found to be satisfied (as one may expect from the work in Ref. 5). We then study in detail the influence of W^* and Z^* on low-energy weak interactions.

EFFECTIVE LAGRANGIAN

We introduce the following set of fields: (1) Gauge bosons[8] $V_\mu\equiv T_i V_\mu^{(i)}$ and B_μ associated with $SU(2)_L$ and $U(1)_Y$, respectively; (2) composite vector bosons $V_\mu^*\equiv T_i V_\mu^{*(i)}$; and (3) Higgs scalar ϕ. Their transformation under $SU(2)_L$ is given by $V_\mu\to UV_\mu U^{-1}+iU\partial_\mu U^{-1}$, $V_\mu^*\to UV_\mu^*U^{-1}$, and $\phi\to U\phi$, where U represents the gauge transformation. For simplicity, the possible presence of B_μ^* or other $SU(2)_L$-singlet $J=1$ resonances is neglected as a first approximation.

The $SU(2)_L\times U(1)_Y$-invariant coupling among V_μ and V_μ^* involving their kinetic mixing[9] can be expressed in terms of $V_{\mu\nu}$, $V_{\mu\nu}^*$, and $[V_\mu^*,V_\nu^*]$, where

$$V_{\mu\nu}\equiv T_i V_{\mu\nu}^{(i)}=\partial_\mu V_\nu-\partial_\nu V_\mu+ig[V_\mu,V_\nu]$$

and

$$V_{\mu\nu}^*\equiv T_i V_{\mu\nu}^{*(i)}=\partial_\mu V_\nu^*-\partial_\nu V_\mu^*+ig[V_\mu,V_\nu^*]-ig[V_\nu,V_\mu^*]$$

The $SU(2)_L\times U(1)_Y$-invariant effective Lagrangian is given by $L_{\rm eff}=L_0+L_1+L_{\rm mix}$, with

$$L_0=-\tfrac{1}{4}V_{\mu\nu}^{(i)}V^{(i)\mu\nu}-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}+|D_\mu\phi|^2-V(\phi)\ , \tag{1a}$$

$$L_1=-\tfrac{1}{4}V_{\mu\nu}^{*(i)}V^{*(i)\mu\nu}+\frac{m_V{}^2}{2}V_\mu^{*(i)}V^{*(i)\mu}+\frac{\delta m^2}{v^2}(\phi^\dagger V_\mu^* V^{*\mu}\phi)+ih[\phi^\dagger V_\mu^*(D^\mu\phi)-(D^\mu\phi)^\dagger V_\mu^*\phi]\ , \tag{1b}$$

$$L_{\rm mix}=-\tfrac{1}{2}\lambda_{VV^*}V_{\mu\nu}^{(i)}V^{*(i)\mu\nu}-\tfrac{1}{2}\lambda_V V_{\mu\nu}^{(i)}(i[V^{*\mu},V^{*\nu}])^{(i)}-\tfrac{1}{2}\lambda_{V^*}V_{\mu\nu}^{*(i)}(i[V^{*\mu},V^{*\nu}])^{(i)}-\tfrac{1}{2}\lambda_{V^*V^*}(i[V_\mu^*,V_\nu^*])^{(i)}(i[V^{*\mu},V^{*\nu}])^{(i)}\ , \tag{1c}$$

where

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu,\quad D_\mu=\partial_\mu-igV_\mu-ig'(Y/2)B_\mu\ ,$$

with Y being the $U(1)_Y$ charge,

$$[V_\mu^*,V_\nu^*]^{(i)}=i\epsilon_{ijk}V_\mu^{*(j)}V_\nu^{*(k)}\ .$$

The scale v is to be identified in $\langle\phi\rangle=(0,\ v/\sqrt{2})^T$.

After some calculations,[10] $L_{\rm eff}$ is transformed into $L'_{\rm eff}$ without the mixed term $V^{(i)}_{\mu\nu}V^{*(i)\mu\nu}$: $L'_{\rm eff}=L'_0+L'_1+L'_{\rm mix}$ with

$$L'_0=-\tfrac{1}{4}Y^{(i)}_{\mu\nu}Y^{(i)\mu\nu}-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}+|D_\mu\phi|^2-V(\phi)\ , \tag{2a}$$

$$L'_1=-\tfrac{1}{4}V^{*(i)}_{\mu\nu}V^{*(i)\mu\nu}+\frac{m_V{}^2}{2}V^{*(i)}_{\mu}V^{*(i)\mu}+\frac{\delta\tilde m^2}{v^2}(\phi^\dagger V^*_\mu V^{*\mu}\phi)+i\tilde h[\phi^\dagger V^*_\mu(D^\mu\phi)-(D^\mu\phi)^\dagger V^*_\mu\phi]\ , \tag{2b}$$

$$L'_{\rm mix}=-\tfrac{1}{2}\tilde\lambda_V Y^{(i)}_{\mu\nu}(i[V^{*\mu},V^{*\nu}])^{(i)}-\tfrac{1}{2}\tilde\lambda_{V^*}V^{*(i)}_{\mu\nu}(i[V^{*\mu},V^{*\nu}])^{(i)}-\tfrac{1}{2}\tilde\lambda_{V^*V^*}(i[V^*_\mu,V^*_\nu])^{(i)}(i[V^{*\mu},V^{*\nu}])^{(i)}\ . \tag{2c}$$

The new gauge fields denoted by $Y^{(i)}_\mu$ are defined as $Y_\mu=T_iY^{(i)}_\mu=V_\mu+[\lambda_{VV^*}/(1-\lambda_{VV^*}{}^2)^{1/2}]V^*_\mu$ and $Y_{\mu\nu}=\partial_\mu Y_\nu-\partial_\nu Y_\mu+ig[Y_\mu,Y_\nu]$. V_μ in $V^*_{\mu\nu}$ has been replaced by Y_μ in $L'_{\rm eff}$. The factor $1-\lambda_{VV^*}{}^2$ appears, owing to the rescaling $V^*_\mu\to V^*_\mu/(1-\lambda_{VV^*}{}^2)^{1/2}$. Other parameters are defined by

$$\tilde m_V{}^2=m_V{}^2/(1-\lambda_{VV^*}{}^2)\ ,\quad \tilde m^2=\delta m^2/(1-\lambda_{VV^*}{}^2)\ ,$$

$$\tilde h=h/(1-\lambda_{VV^*}{}^2)^{1/2}\ ,$$

$$\tilde\lambda_{V^*}=[\lambda_{V^*}-\lambda_V\lambda_{VV^*}+2g\lambda_{VV^*}(\lambda_{VV^*}-1)]/(1-\lambda_{VV^*}{}^2)^{3/2}\ ,$$

$$\tilde\lambda_V=(\lambda_V-g\lambda_{VV^*}{}^2)/(1-\lambda_{VV^*}{}^2)\ ,$$

$$\tilde\lambda_{V^*V^*}=\frac{\lambda_{V^*V^*}-2g\lambda_V\lambda_{VV^*}{}^2+g^2\lambda_{VV^*}{}^2(5\lambda_{VV^*}{}^2-4)}{(1-\lambda_{VV^*}{}^2)^2}\ .$$

The effect of the inclusion of V^*_μ in the "physical" gauge field Y_μ arises in their interactions with quarks and leptons. In particular, low-energy weak interactions will be modified. We will discuss this point later.

MASS RELATIONS

From the Lagrangian $L'_{\rm eff}$ we can find mass matrices $M_{\rm ch}$ on the $(Y^{(\pm)}_\mu,V^{*(\pm)}_\mu)$ basis and M_n on the $(\tilde Z_\mu,V^{*(3)}_\mu)$ basis where $\tilde Z_\mu=\cos\theta Y^{(3)}_\mu-\sin\theta B_\mu$,

$$M_{\rm ch}=\begin{pmatrix} m_{W0}{}^2 & (\tilde h/g-\tilde\lambda_{VV^*})m_{W0}{}^2\\ (\tilde h/g-\tilde\lambda_{VV^*})m_{W0}{}^2 & \tilde m_V{}^2+\delta\tilde m^2+\tilde\lambda_{VV^*}(\tilde\lambda_{VV^*}-2\tilde h/g)m_{W0}{}^2\end{pmatrix}\ , \tag{3a}$$

$$M_n=\begin{pmatrix} m_{Z0}{}^2 & (\tilde h/g-\tilde\lambda_{VV^*})c_\theta m_{Z0}{}^2\\ (\tilde h/g-\tilde\lambda_{VV^*})c_\theta m_{Z0}{}^2 & \tilde m_V{}^2+\delta\tilde m^2+\tilde\lambda_{VV^*}(\tilde\lambda_{VV^*}-2\tilde h/g)m_{W0}{}^2\end{pmatrix}\ , \tag{3b}$$

where $m_{W0}=gv/2$ and $m_{Z0}=(g^2+g'^2)^{1/2}v/2$ are the masses for W and Z in the standard $SU(2)_L\times U(1)_Y$ model: $m_{W0}=\cos\theta m_{Z0}$ $(\equiv c_\theta m_{Z0})$; v is defined as $\langle\phi\rangle=(0,\ v/\sqrt{2})^T$; $\tilde\lambda_{VV^*}=\lambda_{VV^*}/(1-\lambda_{VV^*}{}^2)^{1/2}$. Since the local $U(1)_{\rm EM}$ symmetry remains, the photon stays massless. The physical photon A_μ in this model is given as an orthogonal state of $\tilde Z_\mu$ $(c_\theta=\cos\theta, s_\theta=\sin\theta)$:

$$A_\mu=c_\theta B_\mu+s_\theta Y^{(3)}_\mu=A^0_\mu+s_\theta\tilde\lambda_{VV^*}V^{*(3)}_\mu\ , \tag{4}$$

where $A^0_\mu=c_\theta B_\mu+s_\theta V^{(3)}_\mu$ is the photon in the standard model.

One can find the following mass relations from $M_{\rm ch}$ and M_n:

$$m_W{}^2m_{W^*}{}^2=\frac{m_{W0}{}^2}{1-\lambda_{VV^*}{}^2}[\hat m_V{}^2-(h/g)^2m_{W0}{}^2]\ , \tag{5a}$$

$$m_W{}^2+m_{W^*}{}^2=m_{W0}{}^2+\frac{1}{1-\lambda_{VV^*}{}^2}\times[\hat m_V{}^2-(h/g)^2m_{W0}{}^2+(\lambda_{VV^*}-h/g)^2m_{W0}{}^2]\ , \tag{5b}$$

and

$$m_Z{}^2m_{Z^*}{}^2=\frac{m_{Z0}{}^2}{1-\lambda_{VV^*}{}^2}[\hat m_V{}^2-(h/g)^2m_{W0}{}^2]\ , \tag{6a}$$

$$m_Z{}^2+m_{Z^*}{}^2=m_{Z0}{}^2+\frac{1}{1-\lambda_{VV^*}{}^2}\times[\hat m_V{}^2-(h/g)^2m_{W0}{}^2+(\lambda_{VV^*}-h/g)^2m_{W0}{}^2]\ , \tag{6b}$$

where $\hat m_V{}^2\equiv m_V{}^2+\delta m^2$. It is thus clear that m_W, m_Z, m_{W^*}, and m_{Z^*} satisfy the relation

$$m_Wm_{W^*}=\cos\theta m_Zm_{Z^*}\ , \tag{7a}$$

since $m_{W0}=\cos\theta m_{Z0}$. In addition,

$$m_Z{}^2+m_{Z^*}{}^2=m_W{}^2+m_{W^*}{}^2+\tan^2\theta m_{W0}{}^2 \tag{7b}$$

is also derived.

LOW-ENERGY WEAK INTERACTIONS

Since the $SU(2)_L$ triplet V^*_μ (thus, W^*_μ and Z^*_μ) may couple to quark or lepton pairs via the $V-A$ currents, they mediate low-energy weak interactions and give additional contributions to $n\to pe\bar\nu_e$, $\mu\to e\nu_\mu\bar\nu_e$, and so on. Let g^* be a coupling constant of $V^{*(i)}_\mu$ to quark and lepton currents, $J^{*(i)}_\mu$. For simplicity, g^* has been taken to be universal over all quark and lepton doublets.

The interaction Lagrangian can be expressed as

$$L_{\rm int}=gJ^{(i)}_\mu V^{(i)\mu}+g'J^B_\mu B^\mu+g^*J^{*(i)}_\mu V^{*(i)\mu}\ , \tag{8}$$

which is subsequently transformed into

$$L_{\rm int}=\frac{g}{\sqrt2}(J^{(+)}_\mu Y^{(-)\mu}+{\rm H.c.})+\frac{1}{\sqrt2}[(\tilde g^*J^{*(+)}_\mu-\tilde\lambda_{VV^*}gJ^{(+)}_\mu)V^{*(-)\mu}+{\rm H.c.}]+g_ZJ^Z_\mu\tilde Z^\mu+eJ^{\rm EM}_\mu A^\mu+(\tilde g^*J^{*(3)}_\mu-\tilde\lambda_{VV^*}gJ^{(3)}_\mu)V^{*(3)\mu}\ , \tag{9}$$

with $\tilde g^*=g^*/(1-\lambda_{VV^*}{}^2)^{1/2}$, $g_Z=(g^2+g'^2)^{1/2}$ and $e=gg'/g_Z$;

$\tilde{Z}_\mu = \cos\theta Y_\mu^{(3)} - \sin\theta B_\mu$, and $A_\mu = \sin\theta Y_\mu^{(3)} + \cos\theta B_\mu$ as have been already defined; $J_\mu^Z (= J_\mu^{(3)} - \sin^2\theta J_\mu^{\rm EM})$ and $J_\mu^{\rm EM}$ are the neutral current of the standard $SU(2)_L \times U(1)_Y$ model and electromagnetic current, respectively. Since $(Y_\mu^{(\pm)}, V_\mu^{*(\pm)})$ and $(\tilde{Z}_\mu, V_\mu^{*(3)})$ are the bases of $M_{\rm ch}$ in Eq. (3a) and M_n in Eq. (3b), respectively, physical states denoted by $(W_\mu^{(\pm)}, W_\mu^{*(\pm)})$, and (Z_μ, Z_μ^*) can be defined to be ($c_\delta = \cos\delta, s_\delta = \sin\delta$, etc.)

$$\begin{pmatrix} W_\mu^{(\pm)} \\ W_\mu^{*(\pm)} \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} Y_\mu^{(\pm)} \\ V_\mu^{*(\pm)} \end{pmatrix} , \tag{10a}$$

$$\begin{pmatrix} Z_\mu \\ Z_\mu^* \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \tilde{Z}_\mu \\ V_\mu^{*(3)} \end{pmatrix} . \tag{10b}$$

For quarks and leptons, $J_\mu^{(i)} = J_\mu^{*(i)}$. The low-energy effective Lagrangian, $L_{\rm eff}^{\rm ch}$ for weak charged-current interactions and $L_{\rm eff}^n$ for weak neutral-current ones, can be constructed by the method described in Refs. 6 and 11 (suppressing the Lorentz indices):

$$L_{\rm eff}^{\rm ch} = 4\sqrt{2} G_F^0 \rho_1 J^{(+)} J^{(-)} , \tag{11a}$$

$$L_{\rm eff}^n = 4\sqrt{2} G_F^0 \rho_2 [(J^{(3)} - \sin^2\theta_w J^{\rm EM})^2 + C_{\rm EM} (J^{\rm EM})^2] , \tag{11b}$$

with

$$4\sqrt{2} G_F^0 = g^2/m_W^2, \quad \rho_i = C_i + 2\epsilon\Delta_i + \epsilon^2 S_i, \quad i = 1, 2 ,$$

$$C_{\rm EM} = \sin^2\theta \epsilon^2 (C_2 S_2 - \Delta_2^2)/\rho_2 ,$$

and

$$\sin^2\theta_W = \sin^2\theta[1 - \epsilon(\Delta_2 + \epsilon S_2)/\rho_2] , \tag{12}$$

where

$$C_i = c_i^2 + s_i^2 (m_i/m_{i^*})^2, \quad S_i = (c_i \leftrightarrow s_i \text{ in } C_i) ,$$

$$\Delta_i = c_i s_i [1 - (m_i/m_{i^*})^2] ,$$

and $\epsilon = [(g^*/g) - \lambda_{VV^*}]/\sqrt{1 - \lambda_{VV^*}^2}$. The mixing angles and masses are given as $c_i = c_\delta$ (c_α), $s_i = s_\delta$ (s_α), $m_i = m_W$ (m_Z), and $m_{i^*} = m_{W^*}$ (m_{Z^*}) for $i = 1$ $(i = 2)$. It can be found that C_i, S_i, and Δ_i are independent of i and calculated to be

$$C (= C_i) = 1 + (m_W^2 - m_{W0}^2)/m_{W^*}^2 , \tag{13a}$$

$$S (= S_i) = (m_{W0}/m_{W^*})^2 , \tag{13b}$$

and

$$\Delta (= \Delta_i) = (m_{W0}/m_{W^*})^2 [(h/g) - \lambda_{VV^*}]/(1 - \lambda_{VV^*}^2)^{1/2} , \tag{13c}$$

by using M_n ($M_{\rm ch}$) with Eqs. (10a) and (10b) together with Eqs. (7a) and (7b). We then observe that

$$\rho_1(\text{in } L_{\rm eff}^{\rm ch}) = \rho_2(\text{in } L_{\rm eff}^n) \equiv \rho . \tag{14}$$

The mixing angles α and δ are expressed as

$$s_\alpha^2 = (m_{Z^*}^2 - m_W^2)(m_{Z^*}^2 - m_{W^*}^2)/s_\theta^2 m_{Z^*}^2 (m_{Z^*}^2 - m_Z^2)$$

and

$$s_\delta^2 = (c_\theta^2 m_{Z^*}^2 - m_W^2)(m_{Z^*}^2 - m_{W^*}^2)/s_\theta^2 m_{Z^*}^2 (m_{W^*}^2 - m_W^2)$$

with $m_{Z^*} > m_{W^*}$.

The influence of W^* and Z^* in the quark and lepton interactions is characterized by the parameter ϵ, which involves g^*/g. In $L_{\rm eff}^n$, it can be seen from the appearance of $(J^{\rm EM})^2$, which is also expected by the general argument made by Bjorken.[12] The mixing angle observed at low energies is $\sin^2\theta_W$ defined in Eq. (12) but not $\sin^2\theta$ itself.[13]

The present phenomenology suggests[14] that $G_F^0 \rho \equiv G_F = (1.16638 \pm 0.00002) \times 10^{-5}$ GeV^{-2} and $\sin^2\theta_W(0) = 0.207 \pm 0.012$ (average value of $\nu_\mu N$ and eD).[15] Shown in Tables I and II are typical values of various parameters in $L_{\rm eff}^{\rm ch}$ and $L_{\rm eff}^n$. The experimentally determined values of ρ for given s_θ^2 are expressed in Table II as $\rho_{\rm expt}$: $\rho_{\rm expt} = \sqrt{2} G_F m_W^2 \sin^2\theta/\pi\alpha_{\rm EM}$ with $\alpha_{\rm EM}^{-1} = 137.036$. One can then find from Table II that appropriate values of

$$\epsilon (= [(g^*/g) - \lambda_{VV^*}]/(1 - \lambda_{VV^*}^2)^{1/2}) ,$$

i.e., those of g^* and λ_{VV^*}, reproduce consistent results with the low-energy phenomenology. The vacuum expectation value v can also be calculated to be $v = 2m_{W0}/g = m_{W^*} \sin\theta \sqrt{S/\pi\alpha_{\rm EM}}$, whose values are listed in Table I.

SUMMARY

For the system with W, Z, W^*, Z^*, and γ, we have derived two mass relations

$$m_W m_{W^*} = \cos\theta m_Z m_{Z^*} ,$$

$$m_Z^2 + m_{Z^*}^2 = m_W^2 + m_{W^*}^2 + \tan^2\theta m_{W0}^2 ,$$

with $m_{Z^*} > m_{W^*}$, where $m_{W0} = v\sqrt{\pi\alpha_{\rm EM}}/\sin\theta$. These sum rules will not be disturbed as long as possible $SU(2)_L$-singlet spin-1 resonances such as B^* are much heavier than W^* and Z^*.

The existence of W^* and Z^* with masses in the range of 150–200 GeV is not incompatible with the present low-energy weak-interaction phenomenology if $m_{W^*} \sim m_{Z^*}$ and $\epsilon (= [(g^*/g) - \lambda_{VV^*}]/(1 - \lambda_{VV^*}^2)^{1/2})$ is arranged to the ap-

TABLE I. Typical values of various parameters in effective low-energy Lagrangian for $m_W = 82$ GeV and $m_Z = 93$ GeV. The unit for m_{W^*} and v is GeV.

s_θ^2	m_{Z^*}/m_{W^*}	m_{W^*}	s_δ^2	s_α^2	C	S	Δ	v
0.21	1.004	150	0.028	0.009	0.981	0.325	0.114	259
		200	0.030	0.009	0.976	0.197	0.140	268
0.22	1.010	150	0.067	0.022	0.954	0.352	0.173	276
		200	0.071	0.030	0.941	0.231	0.213	298
0.23	1.017	150	0.102	0.037	0.929	0.377	0.210	292
		200	0.110	0.035	0.910	0.263	0.258	325
0.24	1.024	150	0.135	0.051	0.906	0.400	0.237	307
		200	0.144	0.048	0.881	0.292	0.291	349

TABLE II. Calculated values of ϵ, $\sin^2\theta_W$, and $C_{\rm EM}$ for the given values of $\rho_{\rm expt}$.

s_θ^2	$\rho_{\rm expt}$	m_{W^*}	ϵ	$\sin^2\theta_W$	$C_{\rm EM}$
0.21	1.041	150	0.205(−0.294)	0.203(0.211)	$5.44\times10^{-4}(1.12\times10^{-3})$
		200	0.204(−0.320)	0.203(0.215)	$3.05\times10^{-4}(7.45\times10^{-4})$
0.22	1.090	150	0.301(−0.452)	0.203(0.221)	$1.23\times10^{-3}(2.78\times10^{-3})$
		200	0.301(−0.495)	0.203(0.230)	$6.91\times10^{-4}(1.87\times10^{-3})$
0.23	1.140	150	0.375(−0.562)	0.203(0.230)	$2.00\times10^{-3}(4.49\times10^{-3})$
		200	0.375(−0.615)	0.203(0.242)	$1.12\times10^{-3}(3.02\times10^{-3})$
0.24	1.190	150	0.437(−0.649)	0.204(0.237)	$2.83\times10^{-3}(6.24\times10^{-3})$
		200	0.436(−0.710)	0.203(0.252)	$1.61\times10^{-3}(4.28\times10^{-3})$

propriate value of order $\frac{1}{10}$. In this case, the $V-A$ charged-current interactions are caused by the admixture of $W^{(\pm)}$ and $W^{*(\pm)}$; therefore, the observed G_F in $\mu\to e\nu\bar\nu$ must include both contributions while the inclusion of Z^* induces the term $(J^{\rm EM})^2$ in low-energy neutral-current interactions. Its relative strength to $(J^{(3)}-\sin^2\theta_W J^{\rm EM})^2$ is of order 10^{-3}. The mixing angle $\sin\theta$ for the neutral gauge bosons ($V_\mu^{(3)}$ and B_μ) is expected to differ from the one observed in the low-energy neutral-current interactions,[13] $\sin^2\theta_W$, as shown in Eq. (12).

Owing to the presence of the interactions of $\phi^\dagger T_i \overline{\rm D}^\mu\phi V_\mu^{*(i)}$, decays of W^* and Z^* can proceed as

$$W^{*(\pm)}\to W^{(\pm)}+H^0,\quad Z^*\to(Z\text{ or }\gamma)+H^0\ ,$$

if kinematically allowed, where H^0 is the physical Higgs boson. A radiative decay of W^*,

$$W^{*(\pm)}\to W^{(\pm)}+\gamma\ ,$$

can be induced by $L'_{\rm mix}$ with $V^{*(3)}_{\mu\nu}V^{*(3)\mu\nu}$ and $Y^{(3)}_{\mu\nu}(i[V^{*\mu},V^{*\nu}])^{(3)}$ giving

$$s_\theta s_8 c_8(\partial_\mu A_\nu-\partial_\nu A_\mu)(W^{(+)\mu}W^{*(-)\nu}-W^{(+)\nu}W^{*(-)\mu}+\text{H.c.})\ ,$$

where $s_\theta^2 s_8^2 c_8^2$ lies in the ranges of 10^{-2}–10^{-3} from Table I. Therefore, it is likely that *the radiative decay of W^* into $W\gamma$ is more suppressed than usually expected.* On the other hand, the present $L_{\rm eff}$ does not contain the interaction term, which causes $Z^*\to Z\gamma$.[16] Finally, it should be noted that $\Gamma(W^*\to f\bar f)/\Gamma(W\to f\bar f)\simeq\epsilon^2(m_{W^*}/m_W)$ ($f=$ a quark or lepton) with $\epsilon^2\simeq10^{-1}$ from Table II for $m_{W^*}=150$–200 GeV.

We hope that the properties of W^* and Z^* presented in this paper, especially the relation $m_W m_{W^*}=\cos\theta m_Z m_{Z^*}$, can be examined in future high-energy collider experiments.

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