Brief Reports

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Another look at $Z \rightarrow l \bar{l} \gamma$ decay

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We first make some comments on the angular distribution of the photon observed in the anomalous $Z \rightarrow l \bar{l} \gamma$ decay, and then calculate explicitly the lowest-order one-loop contribution within the standard electroweak model. It is finally pointed out that, possibly in some composite models with hyperstrong coupling, large electromagnetic moments for Z may exist to accommodate the anomalous decay rate without violating the present accuracy of the $g-2$ factors of the electron and muon.

The observation of the anomalous $Z \rightarrow l \bar{l} \gamma$ decay events at CERN[1] has drawn considerable attention from particle physicists. It is relevant to the fundamental understanding of the physics at the energy scale around 100 GeV. Various possible explanations have been suggested since the events were reported. Most of them are related to the composite scenario for obvious reasons. Within this context, there still exist several possibilities, in general, that are potentially able to give an account for these events observed. Among those, there are two specific suggestions which are in some sense more attractive. One relates to the existence of a spin-0 boson X (which may be a parity doublet); the other assumes the existence of excited or heavy leptons.[2–4] Though they explain some aspects of the experimental results, there are some points which remain unclear or not very satisfactory. For example, in the first case, the mass of the hypothetical X boson should be[2] around 40–50 GeV; this leaves unexplained the fact that similar events are not observed at the e^+e^- colliders. Also, the lack of comparable $W \rightarrow l \nu \gamma$ decay rates implies that the X boson is an isoscalar which is not very natural from the composite point of view. In the second case, there exists an excited electron (muon) with mass around 75 (60) GeV which lifts some of the difficulties associated with the X bosons.[3] But it is not understood why the invariant mass for the photon and one of the leptons is found to be less than the order of 10 GeV. On the other hand, the statistics of the experiment are after all not very good; we cannot exclude the possibility that the standard model may turn out to be correct after more data are collected. In view of the fact that no further events are reported so far with the second run of the experiment, it is not unlikely that the standard model may eventually survive the test.

Because of the above reasons, we felt it necessary to look more carefully at the various aspects of the prediction from the standard model.[5] Here we shall first study the detailed kinematics of the events observed. The one thing that deserves some attention is the angle distribution of the photon with respect to the leptons. The angles recorded for the three events observed so far are 8°, 15°, and 30°, respectively, which, as usually thought, do not seem small enough to characterize an inner-bremsstrahlung process. Nonetheless, they are not very large either. Actually, as is well known, the pole associated with inner bremsstrahlung is sensitive directly to the cosine of the particular opening angle; we should take more seriously the cosines of the above angles. They turn out to be 0.99, 0.97, and 0.87, which are essentially close to 1, as some signature of the inner bremsstrahlung. To make this more precise, we have actually integrated the photon spectrum over different ranges of the emerging angles for the photon. As an example, with 5 GeV as the cutoff energy, integrating over the angle between the photon and one of the charged particles from 5° to 20°, we obtain a branching ratio (with respect to $Z \rightarrow l \bar{l}$) of about 2.1%. If the angle integration is performed from 20° to 40°, the branching ratio obtained would be around 1%. Regardless of their exact values, the relative magnitude of the two is about 2–1. This relative ratio basically remains the same even when different values of cutoff energy are used. This seems to fit quite well with the angular distribution of the three events observed in spite of its poor statistics.

To see more completely the result based on the standard model, we also carried out the calculation for the one-loop diagrams that contribute to this particular process. These results could be employed to estimate similar types of contributions based upon composite models. The one-loop contribution within the standard model is given by the set of diagrams as shown in Fig. 1, where we have neglected those with a running W loop and its corresponding ghost loop, which do not contribute in this particular process.

The detailed calculations of these diagrams will not be presented in this Brief Report. We shall only give the explicit results for the two channels involving a virtual photon and a virtual Z, respectively. For the amplitude with a virtual photon, we have

$$\mathscr{M}_{\gamma^*}=-\frac{eD_{\gamma^*}}{m_Z^2}P_2{}^2\epsilon_{\mu\nu\lambda\rho}\epsilon^{\mu}j^{\nu}P_1{}^{\lambda}\mathscr{E}^{\rho}\ , \quad (1)$$

where ϵ^{μ} and $\mathscr{E}^{\rho}$ are the polarization four-vectors of the photon and Z, respectively, j^{ν} is the electromagnetic current associated with the virtual photon, and

$$D_{\gamma^*}=\sum_f \frac{e_f{}^2 b_f m_Z{}^2}{16\pi^2 e}\int_0^1 dx\, \frac{P^2x(1-x)-m_f{}^2}{(P_1\cdot P_2)^2x}\times\ln\left[\frac{P^2x(1-x)-m_f{}^2}{P_2{}^2x(1-x)-m_f{}^2}\right] . \quad (2)$$

Here, we denote the coupling constants between the Z and the fermions in the theory by a_f and b_f as shown in Fig. 1. The mass-independent parts have already canceled like anomalies when summing over the fermion contents. This amplitude is proportional to $P_2{}^2$; it vanishes in the on-shell limit as could be expected from Yang's theorem.[6] Similarly, for the case involving a virtual Z, we have

$$\mathscr{M}_{Z^*}=-\frac{eD_{Z^*}}{m_Z{}^2}P_2{}^2\epsilon_{\mu\nu\lambda\rho}\epsilon^{\mu}\mathscr{J}^{\nu}P_1{}^{\lambda}\mathscr{E}^{\rho}\ , \quad (3)$$

where

$$D_{Z^*}=\sum_f \frac{2e_f a_f b_f m_Z{}^2}{16\pi^2 e}\int_0^1 dx\left[\frac{P^2x(1-x)-m_f{}^2}{(P_1\cdot P_2)^2x}-\frac{m_f{}^2}{P_2{}^2(P_1\cdot P_2)x}\right]\ln\left[\frac{P^2x(1-x)-m_f{}^2}{P_2{}^2x(1-x)-m_f{}^2}\right] , \quad (4)$$

and $\mathscr{J}^{\nu}$ is the neutral current associated with Z^*. The mass-independent parts also cancel.

As is well known, unlike the W boson, Z does not have any electromagnetic moments at the tree level within the standard model. The above two amplitudes in Eqs. (1) and (3) show, however, nonvanishing one-loop contributions, though small they may be. We shall first examine from the general ground what kind of electromagnetic moments are allowed, and then we will show that the two amplitudes obtained above indeed correspond to these allowed transitions only before we look at their numerical implications.

In this decay process, only the spin-1 part of the neutral current contributes because of the small masses of the leptons involved. As already known, there are four possible electromagnetic-moment interactions that correspond to $E1$, $M2$, $M1$, and $E2$ transitions, respectively.[7] Assuming also CP invariance, $M1$ and $E2$ contributions are then absent; only $E1$ and $M2$ transitions are allowed. Now we see that the amplitudes given in Eqs. (1) and (3) correspond to the effective-Lagrangian density $F^*_{\mu\nu}Z^{\mu}\Box J^{\nu}$, where J^{ν} may be the electromagnetic current j^{ν} or the neutral current $\mathscr{J}^{\nu}$. When it is realized in the rest frame of Z, we have the following two terms:

(i) $2\epsilon_{0ijk}F^{0k}Z^iJ^j$,

(ii) $-2\epsilon_{0ijk}(\partial_j A_k)Z^iJ^0$.

The first term (i) can be written as

$$2\epsilon_{ijk}E_kZ_iJ_j=2\mathbf{E}\cdot(\mathbf{Z}\times\mathbf{J})\ .$$

Similarly, the second term (ii) becomes

$$-2\epsilon_{ijk}(\partial_jA_k)Z^iJ^0=-2(\mathbf{B}\cdot\mathbf{Z})J^0=2(\mathbf{B}\cdot\mathbf{Z})(\mathbf{P}_1\cdot\mathbf{J})/\omega_2 =\frac{1}{\omega_2}[\mathbf{Z}\cdot(\overleftrightarrow{\mathbf{Q}}-\overleftrightarrow{\mathbf{M}})\cdot\mathbf{J}] =\frac{1}{\omega_2}\mathbf{Z}\cdot\overleftrightarrow{\mathbf{Q}}\cdot\mathbf{J}-\frac{\omega_1}{\omega_2}\mathbf{E}\cdot(\mathbf{Z}\times\mathbf{J})\ ,$$

where we have used the relations

$$\omega_2J^0-\mathbf{P}_2\cdot\mathbf{J}=P_{2\mu}J^{\mu}=0,\quad \mathbf{P}_1+\mathbf{P}_2=0\ ,$$

and

$$\overleftrightarrow{\mathbf{Q}}\equiv\mathbf{P}_1\mathbf{B}+\mathbf{B}\mathbf{P}_1,\quad \overleftrightarrow{\mathbf{M}}\equiv\mathbf{P}_1\mathbf{B}-\mathbf{B}\mathbf{P}_1\ ,$$

$$P_1\equiv(\omega_1,\mathbf{P}_1),\quad P_2\equiv(\omega_2,\mathbf{P}_2)\ ,$$

$$\mathbf{Z}\cdot\overleftrightarrow{\mathbf{M}}\cdot\mathbf{J}=Z_i(P_{1i}B_j-P_{1j}B_i)J_j=Z_i[\epsilon_{ijk}(\mathbf{P}_1\times\mathbf{B})_k]J_j =\omega_1\epsilon_{ijk}Z_iJ_jE_k=\omega_1\mathbf{E}\cdot(\mathbf{Z}\times\mathbf{J})\ .$$

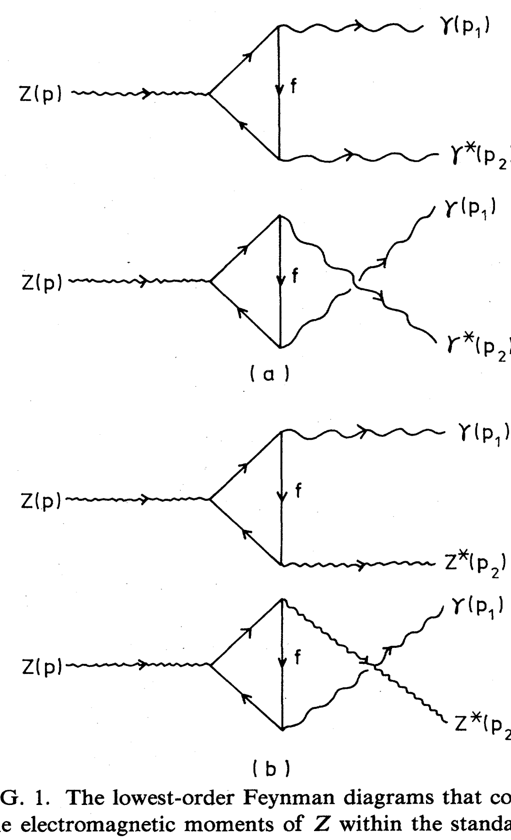

FIG. 1. The lowest-order Feynman diagrams that contribute to the electromagnetic moments of Z within the standard electroweak model. The coupling at the Zff vertex is given by $-i\gamma_\rho(a_f+b_f\gamma_5)$.

Combining the two results, we obtain the following two terms:

$$\left[2-\frac{\omega_1}{\omega_2}\right]\mathbf{E}\cdot(\mathbf{Z}\times\mathbf{J})+\frac{1}{\omega_2}\mathbf{Z}\cdot\overleftrightarrow{\mathbf{Q}}\cdot\mathbf{J}$$

that correspond to the electric dipole and magnetic quadrupole transitions as mentioned to be the only two allowed by the requirement of CP invariance.

We now turn to the numerical aspects of the amplitudes in Eqs. (1) and (3). From Eqs. (2) and (4), we see both D_{γ^*} and D_{Z^*} are of the order $1/16\pi^2$. While in order to give the desired decay rate observed, separately, D_{γ^*} should be of the order 30, and D_{Z^*} has to be as large as 80. Thus, indeed, the one-loop contribution from the standard model is too small to accommodate the anomalous decay rate observed. This conclusion is, of course, expected. Based on the results in Eqs. (2) and (4), we can, nonetheless, make some naive speculations in going beyond the standard model and not be concerned too much at this moment with the detailed consistency within the full scope of any complete theory possible. As an example, first we notice that there are terms directly proportional to m_f^2 in these two expressions. Hence, one might try to assume the existence of some fermionic constituents with masses $m_f \gg m_Z$ in certain possible composite models to enhance the values of D_{γ^*} and D_{Z^*}. However, as we look more carefully at the results obtained, we find that when $m_f \gg m_Z$, Eq. (2) turns out as

$$D_{\gamma^*}=\sum_f \frac{e_f{}^2 b_f m_Z{}^2}{16\pi^2 e}\int_0^1 dx\,\frac{-m_f{}^2}{(P_1\cdot P_2)^2 x}\ln\left[1-\frac{(P^2-P_2{}^2)x(1-x)}{m_f{}^2}\right]\simeq\sum_f \frac{e_f{}^2 b_f m_Z{}^2}{16\pi^2 e}\cdot\frac{1}{P_1\cdot P_2}\simeq\frac{1}{16\pi^2}\ .$$

Similarly, Eq. (4) becomes

$$D_{Z^*}=\sum_f \frac{2e_f a_f b_f m_Z{}^2}{16\pi^2 e}\left[\frac{1}{P_1\cdot P_2}+\frac{1}{P_2{}^2}\right]\simeq\frac{1}{16\pi^2}\ .$$

The situation is evidently not improved. However, in order to be consistent at this level, the effective coupling between Z and its fermionic constituents with this particular mass scale may be extremely strong. D_{γ^*} and D_{Z^*} may therefore be highly enhanced though we still have no idea about the relevant nonperturbative effects which, of course, should be essential. Hence, it may not be so unlikely that some composite model with superheavy fermionic constituents could provide a solution for this problem.

Putting aside, at this moment, any possible theoretical uncertainties, we shall go ahead and see now if the existence of anomalous electromagnetic moments for Z would cause any contradiction with other well-established experimental facts, especially, the $g-2$ factors of the electron and muon.[8] To see this, we have to evaluate the diagrams shown in Fig. 2. Under the assumption of this particular hyperstrong coupling, it could be understood that the contribution from diagram (a) with $ZZ\gamma$ coupling dominates. From the results given in Refs. 9 and 10, we can make a similar estimate for this particular diagram. If the anomalous decay rate of the process $Z\to l\bar{l}\gamma$ is to be counted for by the large electromagnetic moments of Z assumed, we then have the following results for the $g-2$ factors of the electron and the muon. For the muon, the contribution would be of the order of 10^{-8}, which is just around the current errors allowed.[11] Because the contribution is proportional to $m_l{}^2$, for the case of the electron, we see that the corrections due to this diagram is of the order of 10^{-12}, which is well within the current errors of order 10^{-10} for the electron $g-2$ factor.[11]

Therefore, we conclude that the possible existence of some large electromagnetic moments for Z could, in principle, give an account for the anomalous decay rate of $Z\to l\bar{l}\gamma$ observed without violating the present accuracy achieved for the electron and muon $g-2$ factors. Though the angular distribution of the photon predicted in this model is not in good agreement with that observed as was already pointed out by the authors in Ref. 12, it seems too early, however, to draw a final conclusion at this stage concerning any specific candidate of explanation like this. As one can see from above, if the accuracy of the muon $g-2$ factor could be pushed down one order further, this hypothesis can then be checked. On the contrary, this effect is still quite insensitive with respect to the measurement of this factor for the electron. Hence, further experiments performed for the muon would give

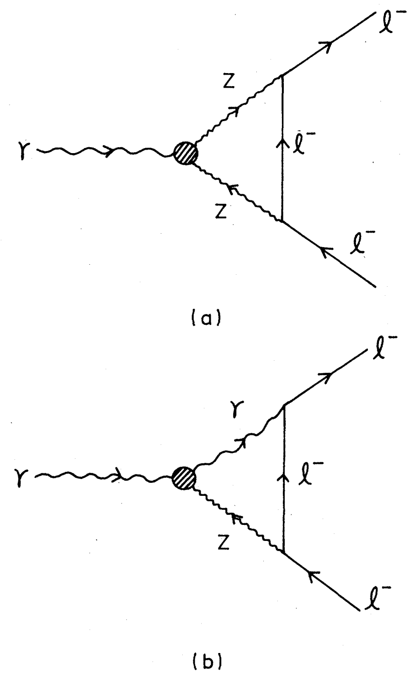

FIG. 2. The contributions to the $g-2$ factor of the electron (muon) due to $ZZ\gamma$ and $Z\gamma\gamma$ couplings.

an important consistent check concerning this particular hypothesis, if the desired accuracy can be achieved. Of course, the most direct check of this should come from a well-established photon spectrum that will be observed if the anomalous decay rate for $Z \to l\bar{l}\gamma$ persists to stand.

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