Interpreting p-p polarization at high energies

Gary R. Goldstein
Department of Physics, Tufts University, Medford, Massachusetts 02155

Michael J. Moravcsik
Department of Physics and Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

(Received 19 June 1984)

It is shown that the recent measurements of the analyzing power A in high-energy p-p elastic scattering can be most conveniently interpreted in the optimal transversity frame. The deviations from $A=0$ predicted by QCD are parametrized and a step-by-step experimental program is outlined for the further pinpointing of the causes of the deviations.

In a recent paper,[1] experimental information was presented on analyzing power (or simple polarization) for p-p elastic scattering at 28 GeV/c in a substantial range of t values, and further experiments are in progress to extend that t range. One of the motivations for such experiments has been the QCD prediction that such polarization should vanish, something that does not appear to be borne out by the experimental results.

In this Brief Report, we want to discuss the most suitable way of interpreting this situation and to point at further experiments that can shed light on the extent of the discrepancy and on the causes of it. The main point we want to stress is that, as is the case for all parity-conserving reactions,[2] the most suitable formalism for such a discussion is the optimal transversity frame. Indeed, we will show that using that frame we can interpret the existing results very simply, that is, in terms of a small number of parameters, and can also easily suggest a step-by-step experimental program which successively narrows down the determination of the structure of the amplitudes.

The optimal transversity frame, in which the commonly known transversity amplitudes are defined, has been discussed extensively in previous papers,[2] both in general and in the context of p-p elastic scattering. We will not repeat the derivation of that formalism but simply borrow the results from the previous papers. In particular, we need the relationship between helicity amplitudes and transversity amplitudes, and the experimental observables in terms of the transversity amplitudes. The five helicity amplitudes are denoted by

$$\begin{aligned} &a\equiv(+,+,+,+),\quad b\equiv(+,-,+,+)\ ,\\ &c\equiv(+,-,+,-),\quad d\equiv(-,-,+,+)\ ,\\ &e\equiv(-,+,+,-)\ , \end{aligned} \tag{1}$$

where the four arguments denote the helicities of the first and second incoming particles, and the first and second outgoing particles, respectively. The transversity amplitudes are correspondingly denoted by

$$\begin{aligned} &\alpha\equiv(\uparrow,\uparrow,\uparrow,\uparrow),\quad \beta\equiv(\downarrow,\downarrow,\downarrow,\downarrow),\quad \gamma\equiv(\uparrow,\downarrow,\uparrow,\downarrow)\ ,\\ &\delta\equiv(\downarrow,\downarrow,\uparrow,\uparrow),\quad \epsilon\equiv(\downarrow,\uparrow,\uparrow,\downarrow)\ , \end{aligned} \tag{2}$$

where the arguments denote, in the same order as in the helicity amplitudes, the spin projections of the particles with respect to a quantization axis that is normal to the scattering plane. Equation (7.10) of Ref. 2 gives the relationship between the two sets of amplitudes:

$$\begin{aligned} a&=\tfrac{1}{4}[\alpha+\beta+2(\gamma-\delta-\epsilon)]\ ,\\ b&=\tfrac{1}{4}(\alpha-\beta)\ ,\\ c&=\tfrac{1}{4}[\alpha+\beta+2(\gamma+\delta+\epsilon)]\ ,\\ d&=\tfrac{1}{4}[\alpha+\beta+2(-\gamma-\delta+\epsilon)]\ ,\\ e&=\tfrac{1}{4}[-\alpha-\beta+2(\gamma-\delta+\epsilon)]\ . \end{aligned} \tag{3}$$

As to the relationship between the observables and the transversity amplitudes, they can be found in Table VI of Ref. 2. We will need a particular set of five of these relations:

$$\begin{aligned} \sigma&=|\alpha|^2+|\beta|^2+2(|\gamma|^2+|\delta|^2+|\epsilon|^2)\ ,\\ A&=|\alpha|^2-|\beta|^2\ ,\\ A_{NN}&=|\alpha|^2+|\beta|^2+2(-|\gamma|^2+|\delta|^2-|\epsilon|^2)\ \ (=C_{NN})\ ,\\ D_{NN}&=|\alpha|^2+|\beta|^2+2(|\gamma|^2-|\delta|^2-|\epsilon|^2)\ ,\\ K_{NN}&=|\alpha|^2+|\beta|^2+2(-|\gamma|^2-|\delta|^2+|\epsilon|^2)\ . \end{aligned} \tag{4}$$

We see already the crucial feature of the transversity amplitudes which brings about the simplicity we desire, namely, that in the transversity frame both the unpolarized differential cross section and A, the analyzing power or polarization, can be expressed in terms of the absolute value squared of the amplitudes, with no reference at all to the phases among these complex amplitudes. This is not the case for other formalisms, such as, for example, the helicity formalism, in which the simple polarization does involve phases also. This difference has important practical implications, since the analysis of the situation using transversity amplitudes can be carried out in terms of five parameters only (namely, the five magnitudes), while the corresponding analysis in the helicity frame would involve all nine amplitude parameters (namely the five magnitudes and the four relative phases among them). In particular, the five observables listed in Eq. (4) can then lead to an unambiguous determination of the five magnitudes of the transversity amplitudes. Another way of saying this is that in the transversity frame the determination of the magnitudes of the five amplitudes is simply and completely decoupled from the more involved task of the determination of the four phases, and hence the former can be carried out with high precision and on the basis of an only partial set of polarization experiments.

Let us assume now that the analyzing power A is found to be zero (which is not actually the case). We see from Eq. (4) that the sufficient and necessary condition for this to happen, in terms of the amplitudes, is $|\alpha|=|\beta|$. Note that this is the only possible way the analyzing power can vanish. In contrast, in the helicity frame, the vanishing of the analyzing power ($A=P$ in Table VI of Ref. 2) would mean

$$\mathrm{Im}(a+c+d-e)b^*=0 ,$$

which, however, can come about in three different ways (namely, either of the two magnitudes or the angle between the two could vanish).

If A does not vanish, but is equal to a value ζ, then we have $|\alpha|^2=\zeta+|\beta|^2$ and therefore we can rewrite the remaining three observables of Eq. (4) as

$$\begin{aligned} C_{NN}&=\zeta+2|\beta|^2+(-|\gamma|^2+|\delta|^2-|\epsilon|^2)=A_{NN} ,\\ D_{NN}&=\zeta+2|\beta|^2+(|\gamma|^2-|\delta|^2-|\epsilon|^2) ,\\ K_{NN}&=\zeta+2|\beta|^2+(-|\gamma|^2-|\delta|^2+|\epsilon|^2) . \end{aligned} \tag{5}$$

Let us now turn to the feature of QCD that predicts that the analyzing power vanishes. In the approximation of negligible quark masses, QCD requires helicity conservation for the quarks which, in a given approximation (see Appendix), translates into $b=d=e=0$ for the p-p scattering helicity amplitudes. Thus only the two amplitudes a and c are nonzero, with no relationship between them.

The above three constraints, transposed to the transversity amplitudes, are as follows:

$$b=0 \rightarrow \alpha=\beta , \tag{6}$$

$$d=0 \rightarrow \alpha+\beta+2(-\gamma-\delta+\epsilon)=0 , \tag{7}$$

$$e=0 \rightarrow -\alpha-\beta+2(\gamma-\delta+\epsilon)=0 . \tag{8}$$

Equations (7) and (8) together give

$$\delta=\epsilon , \tag{9}$$

$$\alpha+\beta=2\gamma , \tag{10}$$

and these equations, together with Eq. (6), then give

$$\alpha=\beta=\gamma,\quad \delta=\epsilon . \tag{11}$$

From Eq. (4) we see that the $b=0$ requirement has a simple consequence by itself only on the analyzing power. The other two constraints, namely, $d=0$ and $e=0$, together require that

$$A_{NN}=C_{NN}=K_{NN} . \tag{12}$$

Finally, the three constraints together require

$$A_{NN}=C_{NN}=K_{NN}=0 . \tag{13}$$

We see, therefore, that the three constraints on the helicity amplitudes can be tested in two distinct steps, which are completely independent of each other, one testing only $b=0$ and the other testing the other two constraints together. The first step involves only the measurement of the analyzing power or polarization while the other step involves the measurement of C_{NN} ($=A_{NN}$) and K_{NN}. The observable D_{NN} is not utilized at this stage. It and the unpolarized differential cross section together can be used to determine the values of the two remaining nonzero magnitudes.

Since we already know that, in reality, the above three constraints are not satisfied or at least not exactly satisfied, it is also useful to describe how the five observables of Eq. (4) can be used to actually determine the five magnitudes, quite independently of whether or not the above three constraints hold. This can be done by simply inverting Eq. (4):

$$\begin{aligned} |\alpha|^2&=\tfrac{1}{8}(\sigma+C_{NN}+D_{NN}+K_{NN})+\tfrac{1}{2}P ,\\ |\beta|^2&=\tfrac{1}{8}(\sigma+C_{NN}+D_{NN}+K_{NN})-\tfrac{1}{2}P ,\\ |\gamma|^2&=\tfrac{1}{4}(\sigma-C_{NN}+D_{NN}-K_{NN}) ,\\ |\delta|^2&=\tfrac{1}{4}(\sigma+C_{NN}-D_{NN}-K_{NN}) ,\\ |\epsilon|^2&=\tfrac{1}{4}(\sigma-C_{NN}-D_{NN}+K_{NN}) , \end{aligned} \tag{14}$$

where C_{NN} can be replaced by A_{NN} and P could be replaced by A.

In summary, we can say that using the transversity amplitudes to interpret high-energy p-p elastic-scattering experiments we can easily describe the deviations from the constraints predicted by QCD for the reaction amplitudes. In particular, the experimental procedure involves first measuring the analyzing power or simple polarization, which gives information on one of the three constraints. The next step is to measure C_{NN} (or A_{NN}) and K_{NN} which give information on the other two constraints. If it is found that these last two constraints are not satisfied, a measurement of D_{NN} should be carried out. The altogether five measurements thus obtained together allow us to determine the magnitudes of the five transversity amplitudes, independently of the relative phases among these amplitudes, and with complete uniqueness and high precision. If desired, an additional experimental program can be undertaken afterwards, involving at least four more measurements, to determine also the relative phases. The knowledge of these phases would provide additional information on the validity of the three constraints. It should be noted that the experimental program of the first part, involving five observables, involves exclusively polarizations normal to the reaction plane, something that is likely to be good news to those in charge of the experiments.

Analyses like the present one should also be helpful in checking specific dynamical models like the recent one by Anselmino.[3] Such models need to be tested not through isolated observables but through a comparison amplitude by amplitude. As we have seen, this is facilitated by choosing the most appropriate formalism for the description of the reaction.

ACKNOWLEDGMENTS

We gratefully acknowledge the comments and suggestions of Professor Alan Krisch concerning a preliminary draft of this paper. This research was in part supported by the U.S. Department of Energy.

APPENDIX: SOME COMMENTS ON DYNAMICAL MODELS

The direct (uncrossed) q-q one-gluon-exchange scattering diagram, in the limit of zero quark mass, predicts helicity conservation. The actual p-p scattering cross section may, however, deviate from this prediction for at least three reasons: (a) the finite quark mass disturbs the perfect helicity conservation, (b) the crossed diagram for q-q scattering violates helicity conservation for a given proton in certain ways, and (c) the processes of dissociation into and recombination from quark states may be such as to upset helicity conservation.

The second of these mechanisms is perhaps the easiest to deal with, because it is likely to be significant only at angles not very far from 90°, and because it would still preserve the overall helicity of the two protons together. In other words, it would still demand that helicity amplitudes with an odd number of minus signs in their arguments vanish. In particular, it would require $b=0$, so that the simple polarization (or analyzing power) would still have to vanish, in contradiction to the experimental findings. For that reason the Anselmino model in Ref. 3, even if it gives qualitative agreement with some measurements of C_{LL} (or A_{LL}), as Ref. 4 claims, is not likely to be correct, at least in its present form.

Concerning the effects of nonzero quark masses, reliable calculations are not possible, at least on the basis of our present knowledge, but one can attempt to make an estimate on the basis of a particular model of the q-q interaction, such as one-gluon exchange. In the one-gluon exchange with zero quark mass, only the two amplitudes a and c are nonzero. When we take into account the quark masses and take these to be about 300 MeV, the values of a and c, for p-p elastic scattering above 10 GeV or so, will change only by at most 2%. The values of the amplitudes d and e will, similarly, be only a percent or two of the values of the amplitudes a or c. In all the cases so far the correction is proportional to the square of the ratio of the quark mass to the quark center-of-mass momentum. In contrast, the magnitude of the amplitude b (single helicity flip) is proportional to the first power of the above ratio, and hence this amplitude could be as large as 10% of the magnitude of a. Thus we would expect a larger deviation from the zero-mass prediction for the simple polarization or analyzing power than for the two spin-correlation parameters.

Finally, the effect of dissociation and recombination is even more difficult to ascertain. Reference 3 makes the assumption that these two processes do not disturb the spin structure as it can be gleened from the q-q interaction, but others, for example Ref. 5, make a quite different assumption.

Similar uncertainties arise in connection with possible multiple-scattering corrections,[6] which have also been considered[7] in this context. For predictions of simple polarizations in the "massive-quark model," see Ref. 8.

Quite independently of the validity of any particular dynamics, however, one can say something about the amplitudes at 90° on the basis of measurements of the unpolarized differential cross section, C_{NN} (or A_{NN}) and C_{LL} (or A_{LL}). In terms of helicity amplitudes at 90° these three observables can be written as

$$\sigma=2|a|^2+4|c|^2+2|d|^2 \ , \tag{A1}$$

$$C_{NN}=4\,\mathrm{Re}(ad^*)+4|c|^2=A_{NN} \ , \tag{A2}$$

$$C_{LL}=-2|a|^2+4|c|^2-2|d|^2=A_{LL} \ . \tag{A3}$$

Since these relationships involve three magnitudes and one relative phase, one cannot make a complete determination of the values of these four amplitude parameters. Nevertheless one can derive a lower limit for the magnitude of d. If one uses Ref. 4 for A_{LL} and Ref. 9 for A_{NN}, one obtains, in a straightforward way, that the magnitude of d must at least be 12% of the magnitude of a, the latter being the largest of the three magnitudes.

This result is thus incompatible with models predicting a zero value for d. Reference 3 is not among them. It predicts a sizable d at 90°, but using a mechanism (the crossed diagram) which gives $P=A=0$ at all angles, as mentioned earlier. In view of the foregoing, it is appropriate to conclude that, with the already existing measurements of the various polarization quantities the attempts to construct dynamical models for p-p elastic scattering at these energies need to be reoriented.

[1]D. C. Peaslee *et al.*, Phys. Rev. Lett. **51**, 2359 (1983).

[2]Gary R. Goldstein and M. J. Moravcsik, Ann. Phys. (N.Y.) **142**, 219 (1982), and references therein.

[3]M. Anselmino, Z. Phys. C **13**, 63 (1982).

[4]J. P. Auer *et al.*, Phys. Rev. Lett. **52**, 800 (1984).

[5]G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[6]P. V. Landshoff and D. J. Pritchard, Z. Phys. C **6**, 69 (1980); A. H. Mueller, Phys. Rep. **73C**, 237 (1981); B. Pire and J. P. Ralston, Phys. Lett. **117B**, 233 (1982).

[7]J. Szwed, Nucl. Phys. B **229**, 53 (1983); J. P. Ralston and B. Pire, in *High Energy Spin Physics—1982*, proceedings of the 5th Symposium, Brookhaven National Laboratory and Westhampton Beach, New York, edited by G. M. Bunce (AIP, New York, 1983), p. 329.

[8]P. Chiapetta and J. Soffer, Phys. Rev. D **28**, 2162 (1983).

[9]E. A. Crosbie *et al.*, Phys. Rev. D **23**, 600 (1981).