Symmetry-breaking patterns in left-right-symmetric models: How to ensure natural flavor conservation and a soft *CP* violation

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A detailed analysis of the symmetry-breaking patterns in left-right-symmetric models is performed, looking for a soft violation of *CP* and requiring the absence of Higgs-boson-induced flavor-changing neutral currents in the light-Higgs-boson sector. The difficulties typical of pseudomanifest left-right-symmetric theories in ensuring both properties are discussed. An alternative approach is proposed in which the Weinberg-Salam projection of the left-right model is naturally flavor conserving and *CP* is spontaneously violated in the manner of Weinberg without effects of Kobayashi-Maskawa phases.

I. GENERALITIES

Left-right-symmetric models have been introduced about ten years ago[1] as a natural extension of the standard model,[2] mainly in order to justify on physical grounds the typical *P*-violating structure of weak interactions.

Starting from the gauge group $G_{LR}^W \equiv SU(2)_L \times SU(2)_R \times U(1)$, one is led to assume an initial left-right symmetry of the Lagrangian, by ascribing to the spontaneous-symmetry-breaking mechanism of the local gauge symmetry the natural basis of the maximal parity violation at low energies. As a consequence, parity restoration is to be expected beyond the mass scale at which the spontaneous breakdown is supposed to happen.

According to the L-R-symmetry requirements, quarks (and similarly leptons) are symmetrically placed in left and right doublets:

$$Q_{\alpha L} = \begin{bmatrix} u_\alpha \\ d_\alpha \end{bmatrix}_L \equiv (2,1,\tfrac{1}{3}), \quad Q_{\alpha R} = \begin{bmatrix} u_\alpha \\ d_\alpha \end{bmatrix}_R \equiv (1,2,\tfrac{1}{3}) , \qquad (1)$$

where $\alpha = 1, \ldots, N$ is the generation index, and the representation content with respect to the gauge group is explicitly indicated.

Similarly, gauge vector bosons consist of two triplets $\mathbf{W}_L^\mu \equiv (3,1,0)$, $\mathbf{W}_R^\mu \equiv (1,3,0)$, and a singlet $B^\mu \equiv (1,1,1)$, with covariant derivative

$$D^\mu = \partial^\mu - i g_L \mathbf{T}_L \cdot \mathbf{W}_L^\mu - i g_R \mathbf{T}_R \cdot \mathbf{W}_R^\mu - i g_1 \frac{Y}{2} B^\mu , \qquad (2)$$

where $g_L = g_R \equiv g_{LR}$ is required. It is an easy matter to verify[3] the well-known correspondence with the standard Weinberg-Salam (WS) model at low energies (i.e., in the limit of neglecting all right-handed effects) by identifying

$$\left.\begin{array}{l} g \leftrightarrow g_{LR} \\ g' \leftrightarrow g'' = \dfrac{g_{LR} g_1}{(g_{LR}{}^2 + g_1{}^2)^{1/2}} \end{array}\right\} \Longrightarrow \tan\theta_W = \frac{g'}{g} \leftrightarrow \tan\theta_{LR} = \frac{g''}{g} , \qquad (3)$$

where g and g' are the two couplings appearing in the usual analysis of the gauge group $G_{WS} \equiv SU(2)_L \times U(1)_Y$.

Two kinds of problems are raised by an approach based on an initial left-right symmetry. First, the agreement of a similar model with the present status of the low-energy phenomenology. This on general grounds involves the specific structure of the neutral currents, to be parametrized in a consistent way in terms of five parameters, and more specifically concerns the "contamination" coming from the right-handed counterpart: the present limit of the W_L-W_R mixing, derived using semileptonic decay data in a very general framework,[4] constrains a realistic model based on G_{LR}^W and leads to a lower bound on the $M_R \equiv M_{W_R}$ mass scale.

The second order of problems is more strictly related to the theoretical framework in which L-R models can be developed. They involve (i) the possibility of ascribing not only *P* violation but also *CP* violation to the same spontaneous-symmetry-breaking mechanism, by requiring both violations be regulated in a natural way by the typical ratio M_L/M_R which fixes the hierarchy of the mass scale[5] and (ii) the possibility of ensuring in a natural way the suppression of flavor-changing neutral currents in these models in which neutral-Higgs-boson interaction terms proliferate because of the rich Higgs structure required by the larger group.

This latter problem has been examined in the past.[6] Here it will be analyzed more deeply, by paying particular attention to the requirements that natural flavor conservation (NFC) imposes on the general structure of the symmetry-breaking patterns of a L-R-symmetric theory.

The paper is organized as follows. In Sec. II the difficulties arising in a minimal model of both manifest and pseudomanifest L-R-symmetric theories are considered. The different solutions to the symmetry-breaking-pattern problem are analyzed in light of the effects of the introduction of a further discrete symmetry in Sec. III. A realistic model is developed in Sec. IV, the conclusions being drawn in Sec. V. Three appendixes contain the technical details.

II. A MINIMAL MODEL

The minimal Higgs content assuring a symmetry-breaking pattern of the group G_{LR} which gives mass to the fermions, preserves only electromagnetic (EM) gauge invariance, and breaks parity spontaneously, consists of two kinds of Higgs multiplets, $\chi_{L,R}$ and Φ. In particular

$$\chi_L \equiv \begin{bmatrix} \chi_L^{(+)} \\ \chi_L^{(0)} \end{bmatrix} \equiv (\tfrac{1}{2},0,1)$$

and (4)

$$\chi_R \equiv \begin{bmatrix} \chi_R^{(+)} \\ \chi_R^{(0)} \end{bmatrix} \equiv (0,\tfrac{1}{2},1)$$

doublets under $SU(2)_L$ and $SU(2)_R$, respectively, uncoupled to quarks and leptons, are responsible for the first stage of the symmetry breaking of G_{LR}, whereas the

$$\Phi(\tfrac{1}{2},\tfrac{1}{2},0) \equiv \begin{bmatrix} \phi_1^{(0)} & \phi_1^{(+)} \\ \phi_2^{(-)} & \phi_2^{(0)} \end{bmatrix} \tag{5}$$

doublet under both $SU(2)_L$ and $SU(2)_R$, has the role of inducing the final breaking [down to $U(1)_{EM}$] and, at the same time, it gives mass to the fermions through the Yukawa Lagrangian (only quarks are considered here)

$$L_Y = Q_{\alpha L}\Gamma_1^{\alpha\beta}\Phi Q_{\beta R} + Q_{\alpha L}\Gamma_2^{\alpha\beta}\tilde{\Phi} Q_{\beta R} + \text{H.c.} , \tag{6}$$

where α, β run on the different generations and $\tilde{\Phi} = \tau_2\Phi^*\tau_2$. [Alternatively, the breaking can be realized by two L-R-conjugate triplets $\Delta_L(1,0,\frac{2}{3})$ and $\Delta_R(0,1,\frac{2}{3})$, which can couple to the leptons. However, no substantial differences can be found as far as the general properties of the symmetry-breaking patterns are concerned.]

As it will be seen in Sec. III, the first stage of the symmetry breaking is properly realized by taking

$$\langle \chi_L \rangle = 0 , \quad \langle \chi_R \rangle = \begin{bmatrix} 0 \\ v \end{bmatrix} . \tag{7}$$

By allowing $\langle \Phi \rangle$ to have the form

$$\langle \Phi \rangle = \begin{bmatrix} k & 0 \\ 0 & k' \end{bmatrix} \tag{8}$$

the second stage of the symmetry-breaking pattern is, along usual lines, dependent on the specific structure of the most general quartic (renormalizable) Higgs potential built up in terms of the fields χ_L, χ_R, Φ. In Appendix B a detailed study is reported, whose results will be used here.

From the Yukawa Lagrangian (6) the mass matrices of the quarks of given charges q_u and q_d are easily derived in the form

$$M_u = k\Gamma_1 + k'^*\Gamma_2 , \quad M_d = k'\Gamma_1 + k^*\Gamma_2 , \tag{9}$$

where Γ_1 and Γ_2 are symmetrical matrices, real if the Lagrangian is supposed CP invariant.

The NFC requirements[7] in the sector of neutral currents (NC's) induced by the neutral bosons are satisfied by L-R models. But problems arise in the sector of Higgs-boson-induced NC. Let us discuss, by looking for a spontaneous CP violation, the possibility of realizing NFC in the present minimal model, by considering the Higgs-boson-induced flavor-changing NC originated by direct neutral Higgs exchange. Let $U_{u,L(R)}, U_{d,L(R)}$ be the biunitary transformations called to transform quarks of given charge defined in the "weak basis" into mass eigenstates ("strong basis")

$$q_{u,L(R)}^{(s)} = U_{u,L(R)} q_{u,L(R)}^{(w)} , \quad q_{d,L(R)}^{(s)} = U_{d,L(R)} q_{d,L(R)}^{(w)} , \tag{10}$$

$U_{u,L(R)}, U_{d,L(R)}$ are required to satisfy

$$U_{uL}^\dagger M_u U_{uR} = D_u , \quad U_{dL}^\dagger M_d D_{dR} = D_d . \tag{11}$$

D_u and D_d being the (not necessarily real) diagonal mass matrices of u- and d-like quarks, respectively. [In order to simplify the notation, we do not absorb extra phases into the quark fields, so that D_u, D_d do not necessarily have positive eigenvalues and a further biunitary transformation involving diagonal unitary matrices will be, in general, considered in order to get positive eigenvalues (but the problem is irrelevant here).]

The general structure of the piece of L_Y which describes the coupling of the neutral Higgs bosons is easily found

$$L_Y^{(0)} = \frac{1}{(|k|^2 - |k'|^2)} \{ \bar{q}_{uL}^{(s)} [D_u(k^*\phi_1^{(0)} - k'\phi_2^{(0)*}) + U_{cL} D_d U_{cR}^\dagger(-k'^*\phi_1^{(0)} + k\phi_2^{(0)*})] q_{uR}^{(s)} + q_{dL}^{(s)} [D_d(k\phi_1^{(0)*} - k'^*\phi_2^{(0)}) + U_{cL}^\dagger D_u U_{cR}(-k'\phi_1^{(0)*} + k^*\phi_2^{(0)})] q_{dR}^{(s)} \} + \text{H.c.} , \tag{12}$$

where

$$U_{cL(R)} = U_{uL(R)}^\dagger U_{dL(R)} \tag{13}$$

corresponds to the generalized Cabibbo mixing in the $L(R)$ sector.

Now, when the explicit form (8) of $\langle \phi \rangle$ is considered, if δ is the relative phase of k and k' (in the following, without loss of generality $k = h$, $k' = h'e^{i\delta}$ with h, h', δ real parameters, since the features of the symmetry-breaking pattern depend only on the relative phase δ), we are faced with two possibilities.

(1) The relative phase δ satisfies $\sin\delta = 0$ (k and k' relatively real). This is the case known in the literature as manifest left-right symmetry (MLRS) characterized by $U_{cL} = U_{cR}$ because of the assumed L-R symmetry of the Lagrangian. An inspection of the mass-squared matrix of the neutral-Higgs-boson sector [partially diagonalized in Eq. (B10)] shows that in the usual scheme of mass hierarchy required by the $V-A$ structure of the low-energy phenomenology, i.e., $v^2 \gg h^2, h'^2$, the Higgs-boson com-

bination which induces diagonal couplings in $L_Y^{(0)}(H^2=h^2+h'^2,\ c_\delta\equiv \cos\delta,\ s_\delta\equiv\sin\delta)$

$$H_d=[h\phi_{1r}^{(0)}-h'(\phi_{2r}^{(0)}c_\delta+\phi_{2i}^{(0)}s_\delta)]/H \tag{14}$$

is characterized by light mass $[(\text{mass})^2\sim h^2,h'^2]$, whereas nondiagonal couplings are induced by

$$H_{nd}=[h\phi_{2r}^{(0)}-h'(\phi_{1r}^{(0)}c_\delta-\phi_{1i}^{(0)}s_\delta)]/H \tag{15}$$

which is a combination of heavy-mass Higgs bosons $(M^2\sim v^2)$. Light and heavy Higgs bosons are not disconnected: the mixing angle, however, is very small $[\tan 2\theta_{\rm mix}\simeq(h^2-h'^2)/v^2]$. It follows that, to a good level of approximation but not in an exact way, neutral light Higgs bosons induce a diagonal NC and reproduce at the $SU(2)_L\times U(1)$ level the main features of NFC.

But this scheme does not allow for a spontaneous *CP* violation. If *CP* is not a symmetry of the Lagrangian, then in general the Yukawa coupling matrices Γ_1,Γ_2 are Hermitian, but not real. *CP* violation is "hard"[8] and takes place via the usual mechanism of Kobayashi-Maskawa[9] (KM). If conversely *CP* is a good symmetry at the Lagrangian level, U_{cL} and then U_{cR} can be made real. The vacuum expectation values of the scalar fields are also real and there is no viable mechanism inducing *CP* violation spontaneously.

(2) The relative phase δ of k and k' is different from zero. We disregard the case in which Γ_1 and Γ_2 are not real: as above, it leads to a "hard" *CP* violation, which can be parametrized in the manner of Kobayashi-Maskawa within a scheme of nonmanifest *L*-*R* symmetry with a large number of unspecified parameters.

Let us consider then Γ_1,Γ_2 real and symmetric. It is realized that the so-called pseudomanifest *L*-*R* symmetry, where the two *L* and *R* Cabibbo matrices are not independent, but related through

$$U_{cL}=U_{cR}^* \tag{16}$$

and M_u,M_d of Eq. (9) are symmetrical but not, in general, Hermitian. A large amount of attention has been paid recently to this kind of approach, the interest due to the possibility of connecting *CP* violation to the quantity $(h/h')\sin\delta$, and then to the amount of W_L-W_R mixing.[10,11] *CP* violation here can be described in a simple way only in a four-quark approach, since in the (realistic) case of three generations, two sources of *CP* violation can be found, the first due to the interference between *L* and *R* currents, the second one purely left-handed in character and due to the KM structure of the mixing matrix, without the possibility of obtaining definite predictions concerning their relative importance (even though under the rather arbitrary assumption that the KM mechanism is not the main contribution to the well-known parameter ϵ of the K-$\bar K$ mixing an upper bound to M_R can be derived.[11])

The above description of *CP* violation is rather unsatisfying, at least from an aesthetical point of view, because of the appearance of the KM phase. But this is not the main problem that pseudomanifest *L*-*R* symmetric theories have to face. The analysis of the most general Higgs-boson potential consistent with gauge and *L*-*R* symmetries shows that, if the two vacuum expectation values (VEV) are relatively complex (i.e., $\sin\delta\neq 0$), then the minimum condition applied to the potential leads to well-specified constraints involving VEV's and Higgs-boson couplings. As shown in Appendix B, this enlarges the light Higgs-boson sector and introduces flavor-changing NC induced by light Higgs bosons $[(\text{mass})^2\sim h^2,h'^2]$: therefore, the approach appears incompatible with the requisite of NFC in the WS projection of a pseudomanifest *L*-*R*-symmetric gauge theory.

Let us discuss the possibility of removing the above difficulty. It is obvious to discard the solution $\sin\delta=0$ which clashes with the requirement of a spontaneous *CP* violation induced by the relative phase of k and k'. The possibility that also the coupling which induces the flavor-changing contribution is diagonal must be equally ruled out: from the structure of Eq. (12) it follows that the condition being exactly what is required by a simultaneous diagonalizability of both M_u and M_d, it would imply a meaningless Cabibbo mixing. The only way out is then an approximate vanishing of the Higgs-boson coupling: this has been analyzed by Gilman and Reno[12] for both manifest and pseudomanifest theories. For both it is shown that the $s\leftrightarrow d$ flavor-changing neutral-Higgs-boson coupling cannot be made to vanish and, therefore, the relevant neutral-Higgs-boson mass must be raised in the multi-TeV region in order to avoid too large contributions to ΔM_K and/or ϵ. Even though the situation is a bit less drastic in the pseudomanifest case because of the presence of further (unknown) phases, whose value can be "adjusted" in order to allow a smaller Higgs-boson coupling to $\bar s d$ and $\bar d s$ (Ref. 11), it seems very difficult (and unattractive too) to force the scheme by demanding these phases to preserve from the appearance of flavor-changing effects induced by Higgs-bosons belonging to the mass scale of the left-handed vector bosons.

We can conclude this section by noting that, even though a minimal model may account for the main features expected in a $SU(2)_L\times SU(2)_R\times U(1)$ gauge theory of the electroweak interactions (in particular it gives mass to the fermions and breaks parity spontaneously), several problems are raised by the further requirements of spontaneous *CP* violation and natural flavor conservation in the Higgs-boson sector. There are two possibilities.

(i) NFC is accomplished to a good level of approximation in the light-Higgs-boson sector (small mixing among light and heavy Higgs bosons, the light component being flavor diagonal), but *CP* violation is "hard" in character since it can be related only to a KM phase in the Lagrangian. This is the typical situation of a manifest *L*-*R*-symmetric approach.

(ii) Alternatively, *CP* violation can be connected to the relative phases of k and k' in the expectation value (8). Even if the three-generation case is rather involved, it is reasonable, as shown by Chang,[10] that the appearance of a KM phase does not change the general feature of the two-generation case, where *CP* violation is directly related to the relative phases of k and k'. However $\sin\delta\neq 0$, required by a spontaneous *CP* violation, enlarges the light Higgs-boson sector with uncontrollably large flavor-changing contributions which cannot agree with the requirement of NFC in the light sector.

In both its versions, the minimal model is rather unpleasant and characterized by either unspecified parameters or specific constraints, with a general inadequacy to satisfy NFC and spontaneous CP violation. In Sec. III the possibility of circumventing these difficulties will be analyzed.

III. A DISCRETE SYMMETRY

It is evident from the arguments of Sec. II that to enlarge the Higgs-boson structure by no means modifies the conclusions, which are related to the difficulty of obtaining at the same time NFC in the light sector and a clear source of spontaneous CP violation. The difficulty, in fact, resides in the symmetry-breaking pattern itself. According to the approach proposed in Ref. 6, we consider here the effect of a further discrete symmetry imposed on the Lagrangian, whose effect is essentially to force each Φ Higgs boson to contribute to the mass matrix of specific kinds of quark (u-like and d-like quarks). Let us assume then the following invariance requirements (D-symmetry requirements):

$$\begin{aligned} &Q_{\alpha L}\to Q_{\alpha L}\ ,\quad \Phi\to e^{i\pi/2}\Phi\ ,\\ &Q_{\alpha R}\to e^{-i\pi/2}Q_{\alpha R}\ ,\quad \widetilde{\Phi}\to e^{-i\pi/2}\widetilde{\Phi}\ . \end{aligned} \tag{17}$$

D symmetry cannot be justified in the framework of the gauge group G_{LR}. But it could be the remnant of the symmetry breaking of a larger unifying group.

The most general renormalizable Higgs-boson potential compatible with gauge invariance and the discrete L-R and D symmetries takes on the form

$$V_D=V(\chi_{L,R})+V_D(\Phi)+V_D(\chi_{L,R},\Phi)\ , \tag{18}$$

where

$$\begin{aligned} &V(\chi_{L,R})=-\mu^2(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+\tfrac14\rho_1(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)^2-\tfrac14\rho_2(\chi_L^\dagger\chi_L-\chi_R^\dagger\chi_R)^2\ ,\\ &V_D(\Phi)=-\mu_1{}^2\mathrm{Tr}(\Phi^\dagger\Phi)+\tfrac12\lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2+\tfrac1{16}\lambda_2[\mathrm{Tr}(\Phi^\dagger\Phi)+\mathrm{Tr}(\widetilde{\Phi}^\dagger\Phi)]^2+\tfrac1{16}\lambda_3[\mathrm{Tr}(\Phi^\dagger\widetilde{\Phi})-\mathrm{Tr}(\widetilde{\Phi}^\dagger\Phi)]^2\ ,\\ &V_D(\chi_{L,R},\Phi)=\alpha_1[\mathrm{Tr}(\Phi^\dagger\Phi)](\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+\alpha_2(\chi_L^\dagger\Phi\Phi^\dagger\chi_L+\chi_R^\dagger\Phi^\dagger\Phi\chi_R)\ . \end{aligned} \tag{19}$$

(Properties related to the structure of $\widetilde{\Phi}$ have been taken into account in order to simplify the general structure of the potential.)

First step of the symmetry-breaking pattern

This is related to the vacuum expectation values of χ_L and χ_R. By assuming

$$\langle\chi_L\rangle=\begin{bmatrix}0\\ v_L\end{bmatrix}\ ,\quad \langle\chi_R\rangle=\begin{bmatrix}0\\ v_R\end{bmatrix}\ , \tag{20}$$

and disregarding the trivial solution $v_L=v_R=0$, which would correspond to an unbroken gauge symmetry without any Higgs-boson mechanism at work, we have three possible solutions to the symmetry-breaking-pattern problem:

$$\begin{aligned} &(\mathrm{I\,a})\ v_L=v_R\ ,\ \text{symmetric solution with residual } L\text{-}R \text{ symmetry.}\\ &(\mathrm{I\,b})\ \left.\begin{matrix} v_L \text{ and } v_R\neq 0\\ v_L=v_R\end{matrix}\right\}\ \text{partially asymmetric solution; } L\text{-}R \text{ symmetry is broken.}\\ &(\mathrm{I\,c})\ \left.\begin{matrix} v_L=0\\ v_R\neq 0\end{matrix}\right\}\ \text{totally asymmetric solution, with total breakdown of } L\text{-}R \text{ symmetry.} \end{aligned} \tag{21}$$

The above solutions, analyzed in terms of the gauge hierarchy problem, correspond to (I a) no hierarchy, (I b) finite hierarchy, and (I c) infinite hierarchy, respectively, a possible parametrization being the ratio of the masses of the gauge bosons associated with the L and R parts of the gauge group.[13] It has been stressed[13] that no freedom is left to arrange for a finite hierarchy [solution (I b)] at the tree level, the transition region separating the two remaining solutions not being smoothed by inserting radiative corrections in the manner of Coleman and Weinberg[14] that leave intact the form of the phase transition.

More generally it can be shown that both solutions (I a) and (I b) have to be ruled out on the basis of the appearance of a number of massless Higgs-bosons larger than that required in order to give mass to R and L gauge bosons. According to the detailed analysis of the extremum conditions and Higgs-boson masses performed in Appendix A,

$$\text{if } v_L\neq 0\ ,\ v_R\neq 0, \text{ it follows that } \rho_2(v_L{}^2-v_R{}^2)=0\ , \tag{22}$$

so that the partially asymmetric solution (I b) either goes into (I a) or implies $\rho_2=0$. The latter represents a very restrictive condition on the Higgs-boson potential whose symmetry is in fact enlarged to SU(4), with the appearance of two more massless Higgs bosons induced by the breaking of the larger symmetry, not reabsorbed by the gauge vector bosons through the Higgs-boson mechanism.

We are led then to discard solution (I b), corresponding to the partially asymmetric solution. But a similar situation happens also in the case of the symmetric solution (I a): as shown in Appendix A, independent of the specific choice concerning the symmetry breaking induced by $\langle\Phi\rangle$, a further neutral massless Higgs boson appears, not "eaten" by the massive neutral vector bosons. Also in this case the approach is characterized by a symmetry larger than that of the gauge group G_{LR}: it corresponds to

$$SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$$

[further enlarged if $\rho_2=0$ is assumed, as required by the partially asymmetric solution (I b) discussed above] whose generators, with usual notations, are given by $I^{(\pm)}_{L(R)}$, $I^{(3)}_{L(R)}$, and $Y_{L(R)}$. A symmetry-breaking pattern corresponding to both v_L and v_R different from zero breaks both $U(1)_L$ and $U(1)_R$ separately [and not only $U(1)_{L+R}$ which describes the gauged U(1)]. It follows that an extra massless Higgs boson, which cannot be "eaten" by massive gauge bosons, will be present at the end of the symmetry-breaking mechanism, this independent of the specific form of the second stage of the symmetry breaking itself.

Even though physical Goldstone particles have been often called upon to play a physical role in the literature, we are led to discard both solutions (I a) and (I b), which do not seem to satisfy phenomenological requirements, and lead to instability of the residual symmetry.[15]

Let us, therefore, select solution (I c), corresponding to the total asymmetric solution $v_L=0$, $v_R=v\neq 0$, as that evading the unwanted massless Higgs boson in the theory and satisfying the physical requirements of a maximal breakdown of the L-R symmetry. The problem raised by the infinite hierarchy[13] is solved in a realistic approach which involves Φ Higgs bosons, responsible to the second stage of the symmetry breaking.

It is worth noting that D symmetry is not relevant in the analysis of the first stage of the symmetry-breaking pattern: it merely simplifies the form of the potential (and then the calculations). What is relevant is the assumed L-R symmetry, maximally broken by the assumed solution.

The second stage of the symmetry-breaking pattern

According to the arguments developed above, let us assume the first stage of the symmetry-breaking pattern satisfies the stability conditions leading to the select solution (I c), corresponding to the totally asymmetric solution $v_L=0$, $v_R=v\neq 0$. Our attention is now turned to the second stage of the symmetry breaking, from $SU(2)_L \times U(1)$ down to $U(1)_{EM}$, under the effect of the D symmetry displayed at the beginning of this section. The details of the analysis are reported in Appendix A, where extremum conditions and Higgs-boson masses are explicitly calculated.

In a way rather similar to that adopted in the above analysis of the first stage of the symmetry breaking, we may distinguish three different symmetry-breaking patterns, corresponding to

$$\begin{aligned}
&(\text{II a})\ k=k'\neq 0\ , \ \text{symmetric solution.}\\
&(\text{II b})\ k,k'\neq 0,\ k\neq k'\ , \ \text{partially asymmetric solution.}\\
&(\text{II c})\ k\neq 0\ ,\ k'=0\ \ (\text{or vice versa}),\ \text{totally asymmetric solution}\ .
\end{aligned} \tag{23}$$

It can be shown that (II a) is a particular case of (II b), obtained in the limit $\alpha_2=0$ in the potential $V_D(\chi_{L,R},\Phi)$ of Eq. (19): no phase transition is present going from (II a) to (II b) and the two cases can be analyzed together.

In the analysis of the solution (II b), it is easily seen that the effect of the D symmetry is to force δ, the relative phase of k and k', to satisfy either of the following:

$$\begin{aligned}
&(\text{i})\ \sin\delta=0\ ,\\
&(\text{ii})\ \cos\delta=0\ .
\end{aligned} \tag{24}$$

Alternative (i) corresponds to Hermitian mass matrices M_u and M_d, characteristic of a manifest L-R symmetry. If CP is supposed a good symmetry at the Lagrangian level, violated only because of the spontaneous symmetry breaking characterizing the theory (spontaneous CP violation), then M_u and M_d become real and symmetric without any viable mechanism of CP violation.

Alternative (ii), conversely, describes a pseudomanifest L-R symmetry, with k and k' relatively complex: even if the Lagrangian is CP invariant, the two mass matrices M_u and M_d are complex (and symmetric), but not, in general, Hermitian, and the relative phase δ can be called to produce the desired CP-violating effect in the theory.

The two solutions above do not correspond to the only possible value of the phase δ. A third possibility is open, δ arbitrary but

$$(\text{iii})\ \lambda_2+\lambda_3=0 \tag{25}$$

in the potential $V_D(\Phi)$ of Eq. (19). Condition (iii) introduces in the potential a rotational invariance which eliminates the δ dependence: this however enlarges the symmetry of the Lagrangian and leads to a further massless Higgs boson (besides the would-be-Goldstone bosons due to the spontaneous breakdown of the gauge symmetry). According to the general attitude adopted in the analysis of the first stage of symmetry breaking, alternative (iii) will be disregarded.

The two alternatives (i) and (ii) of Eq. (24) can be treated together. In Appendix A the mass-squared matrices of charged and neutral Higgs bosons are derived and diagonalized, in order to obtain the mass-squared eigenstates (only a partial diagonalization is performed in the case of the neutral Higgs bosons). From the analysis of the mass-squared matrices (A27), (A28), (A29), and (A30) of Appendix A the following statements can be made.

(a) Of the two pairs of charge-conjugate massive Higgs bosons, one, $\chi_L^{(\pm)}$, uncoupled to the quarks, is heavy. The other one, however, given by the combination $[T^2=H^2v^2+(h^2-h'^2)^2,\ H^2=h^2+h'^2]$

$$\frac{1}{T}[(h^2-h'^2)\chi_R^{(\pm)}+v(h\phi_1^{(\pm)}+h'e^{\pm i\delta}\phi_2^{(\pm)})]\ , \tag{26}$$

i.e., dominated, in the limit $v \gg h,h'$, by $\phi_1^{(\pm)}$, $\phi_2^{(\pm)}$, which are coupled to the quarks, has a light mass squared [given by $M^2=\alpha_2(h^2-h'^2)-H^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)$, where either $c_\delta=0$ or $s_\delta=0$ must be taken] so that effects of transitions operated by a light charged Higgs boson are to be expected in the theory.

(b) Neutral Higgs bosons consist (apart from the two would-be-Goldstone bosons of the theory) of six physical particles: two of them, $\chi_{Li}^{(0)}$ and $\chi_{Lr}^{(0)}$, uncoupled to the quarks, are heavy, the combination $(H^2=h^2+h'^2)$

$$H_3 \equiv [h'\phi_{1i}^{(0)}-h(c_\delta\phi_{2i}^{(0)}-s_\delta\phi_{2r}^{(0)})]/H \tag{27}$$

has light mass squared [see Eq. (A31)]

$$M_{H_3}{}^2=-(\lambda_2+\lambda_3)(h^2+h'^2)(c_\delta{}^2-s_\delta{}^2)\ . \tag{28}$$

Of the remaining three Higgs, two are light and weakly mixed to the heavy "quasieigenstate" $\chi_{Rr}^{(0)}$. More precisely, H_d and H_{nd} given by Eqs. (14) and (15), respectively, correspond to the combinations

$$H_d=(hH_1-h'H_2)/H\ , \tag{29}$$

$$H_{nd}=(hH_2-h'H_1)c_\delta/H-s_\delta H_3\ , \tag{30}$$

where H_1 and H_2 are combinations of light eigenstates, weakly mixed with $\chi_{Rr}^{(0)}$ [the mass-squared matrix $M_{H_1}^{(0)2}$ of $\chi_{Lr}^{(0)}$, $\chi_{Rr}^{(0)}$, H_1, and H_2 is given in (A29)]. It follows that, in both cases (i) and (ii) of Eq. (24), flavor-changing NC's induced by light-Higgs-boson combinations are present in the theory and cannot be avoided. Moreover, also in the case of a further, rather arbitrary, mass hierarchy $h \gg h'$ (or vice versa), flavor-changing NC's cannot be suppressed, the only effect being, in the $s_\delta=0$ case, of varying the relative importance of H_1 and H_2 in inducing nondiagonal transitions. In conclusion, NFC cannot be ensured in the light-Higgs-boson sector, unless specific (and quite arbitrary) fine-tuning of the parameters appearing in the Higgs-boson potential is performed.

Let us consider now the case (II c), in which only one of the complex neutral Higgs bosons in Φ, say $\phi_1^{(0)}$, develops a nonzero vacuum expectation value, i.e.,

$$\langle\Phi\rangle=\mathrm{diag}(k,0)\ . \tag{31}$$

According to the mass-squared matrix (A36) all massive charged Higgs bosons are now heavy in the scheme $v \gg h=|k|$. As far as neutral Higgs bosons are concerned, $\chi_{Ri}^{(0)}$ and $\phi_{1i}^{(0)}$ play the role of would-be-Goldstone bosons ("eaten" by the neutral gauge vector bosons) and $\chi_{Li}^{(0)}$, $\chi_{Lr}^{(0)}$, $\phi_{2i}^{(0)}$, $\phi_{2r}^{(0)}$ are mass-squared eigenstates of large mass [see Eq. (A38)]. The two remaining Higgs bosons, $\chi_{Rr}^{(0)}$ and $\phi_{1r}^{(0)}$, whose mass-squared matrix is given by (A39), are "quasieigenstates" weakly mixed, the former heavy and the latter light. Being in this case

$$H_d \equiv \phi_{1r}^{(0)}\ ,\quad H_{nd} \equiv \phi_{2r}^{(0)}\ , \tag{32}$$

the only light neutral Higgs boson is strictly flavor diagonal and weakly mixed with a heavy Higgs boson uncoupled to the quarks. Conversely, flavor-changing NC's are induced by a heavy Higgs boson $[M^2(\phi_{2r}^{(0)})=\alpha_2v^2+\lambda_2h^2]$ and are suppressed to the same extent as right-handed currents are suppressed too. It is worthwhile to note that the above symmetry-breaking pattern realizes exactly the features of the minimal WS model in the light-Higgs-boson sector: only a neutral Higgs-boson doublet participates to the play, one component "eaten" by the neutral (light) gauge vector boson, the other component, the only physical Higgs boson, being flavor diagonal. No charged light Higgs bosons are present, the remaining rich Higgs-boson structure involved by the enlarged gauge symmetry being confined to the larger mass typical of the right-handed vector bosons.

A first-order phase transition then takes place, with a discontinuous variation of the state of the physical system: the symmetry-breaking pattern characterized by $\langle\Phi\rangle\equiv\mathrm{diag}(k,0)$ cannot be obtained as the limit for $k'\to 0$ of the symmetry-breaking pattern in which $\langle\Phi\rangle \equiv\mathrm{diag}(k,k')$, and the two symmetry-breaking patterns give rise to a different physics. Only the former realizes NFC in the light sector and does not admit light charged Higgs bosons, i.e., in reproducing the main features of the WS model, it agrees with the present status of the "low-energy" phenomenology, in which flavor-changing NC's are highly suppressed and evidence of light charged Higgs bosons is still lacking.

A further argument in favor of the totally asymmetric solution (II c) comes from the analysis of the different solutions in terms of the parameters entering the Higgs-boson potential. As is well known, they represent free parameters, subjected only to the requirement of renormalizability of the theory. However, they are, in general, constrained by the minimization conditions. It seems reasonable to argue that the most general solution corresponds to that solution which leads to less severe restrictions on the parameters. The comparison of the symmetry-breaking patterns analyzed before (without loss of generality let us assume $s_\delta=0$: the case $c_\delta=0$ is obtained by interchanging the role of λ_2 and $-\lambda_3$), can be performed by considering the regions in the parameter space allowed by the different solutions. In principle, we have to do with a large number of parameters, as it can be seen from the general expression of the Higgs-boson potential [Eqs. (18) and (19)]. However, only two of them are relevant to our purposes, α_2 and λ_2, since the restrictions imposed by the minimization conditions do not differ in a sensitive way for all the other parameters involved. In the case of solution (II c), corresponding to $k'=0$, the allowed region in the plane (α_2,λ_2) is represented in Fig. 1 by the shaded area, delimited by the λ_2 positive semiaxis and the straight line $\alpha_2v^2+\lambda_2h^2=0$. Conversely, solution (II b) is allowed only on the points of the straight line $\alpha_2v^2+\lambda_2(h^2-h'^2)=0$ belonging to the half-plane $\lambda_2<0$, the two solutions being disconnected.

It is worthwhile to observe that in the limit $v^2 \gg h^2$ the dashed area is maximal and tends to the half-plane $\alpha_2>0$, whereas the straight line corresponding to the region allowed in case (II b) goes into the negative λ_2 semiaxis which, on the other hand, corresponds to the allowed region in case (II a), i.e., $k=k'$.

We can conclude that the totally asymmetric solution of the symmetry-breaking-pattern problem not only ensures the absence of flavor-changing NC induced by light Higgs bosons, but also represents the solution compatible with the largest generality of the parameters of the potential, as a result of the minimization problem.

IV. A REALISTIC MODEL

On the basis of the arguments developed in the above sections, it follows that the totally asymmetric solutions are to be preferred on physical grounds, as far as both the two stages of the symmetry-breaking pattern are concerned.

However, the totally asymmetric solution is not able to give mass to both u-like and d-like quarks in a minimal version of the model, i.e., when only a Φ Higgs boson is present. Moreover, if the Higgs-boson structure is not enriched, a spontaneous CP violation cannot take place. A minimal version in which all quarks take mass requires two Φ Higgs bosons

$$\Phi_1 \equiv \begin{bmatrix} \phi_1^{(0)} & \phi_1^{(+)} \\ \phi_2^{(-)} & \phi_2^{(0)} \end{bmatrix}, \quad \Phi_2 \equiv \begin{bmatrix} \psi_1^{(0)} & \psi_1^{(+)} \\ \psi_2^{(-)} & \psi_2^{(0)} \end{bmatrix}, \tag{33}$$

transforming in an opposite way under D symmetry

$$\Phi_1 \to e^{i\pi/2}\Phi_1\;, \quad \Phi_2 \to e^{-i\pi/2}\Phi_2 : \tag{34}$$

the most general Yukawa coupling takes on the form

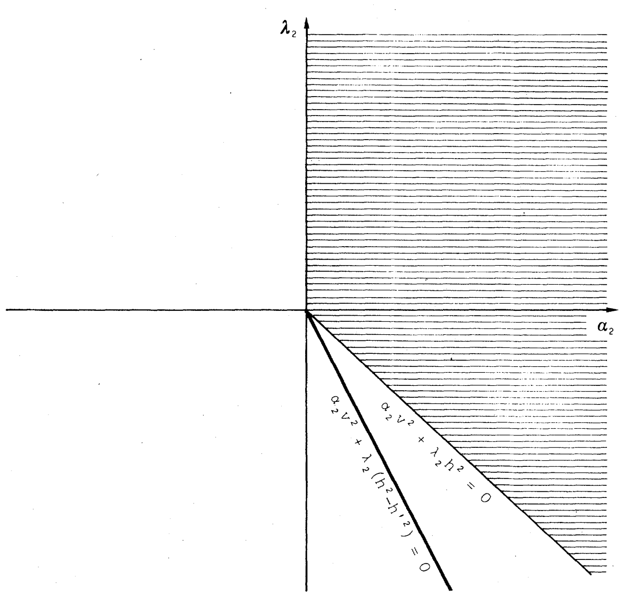

FIG. 1. Comparison of the allowed regions in the parameter subspace corresponding to the plane (α_2, λ_2) of the two solutions to the symmetry-breaking-pattern problem. The dashed area is the allowed region of the totally asymmetric solution (II c), whereas only the points belonging to the straight line $\alpha_2 v^2 + \lambda_2(h^2 - h'^2) = 0$ are allowed in the case of the partially asymmetric solution (II b).

$$L_Y = \bar{Q}_{\alpha L}\Gamma_1^{\alpha\beta}\Phi_1 Q_{\beta R} + \bar{Q}_{\alpha L}\Gamma_2^{\alpha\beta}\tilde{\Phi}_2 Q_{\beta R} + \text{H.c.} \tag{35}$$

so that the mass matrices (9) become now

$$M_u = k_1\Gamma_1\;, \quad M_d = k_2^*\Gamma_2\;, \tag{36}$$

in terms of the totally asymmetric solution to the symmetry-breaking-pattern problem

$$\langle \Phi_1 \rangle = \text{diag}(k_1, 0)\;, \quad \langle \Phi_2 \rangle = \text{diag}(k_2, 0)\;. \tag{37}$$

If the Lagrangian is supposed CP invariant, Γ_1 and Γ_2 are real and symmetric matrices and the transition from the "weak" to the "strong" quark basis can be expressed in terms of biorthogonal transformations O_u, O_d (with $O_{uL} = O_{uR}$, $O_{dL} = O_{dR}$ because of the L-R symmetry), in terms of which the mass matrices can be diagonalized. The phases coming from the, in general, complex values of k_1 and k_2 can be reabsorbed through a redefinition of the physical fields.

Generalized Cabibbo mixing is then realized through an orthogonal matrix $O_c = O_u^T O_d$, and there is no KM CP-violating phases, either in the left or in the right sector. CP cannot be violated in the manner of Kobayashi-Maskawa, and a mechanism like that of Weinberg,[16] requiring at least three Φ Higgs bosons, appears as the only possible source of CP violation.[5] A detailed analysis of the mechanism of CP violation in this approach, and of its physical implications, will be given elsewhere.[17]

Despite the fact that adding a second Φ Higgs boson makes more cumbersome the analysis of the corresponding Higgs-boson potential, the main aspects of the totally asymmetric solution analyzed in the above section are maintained. Moreover, now all quarks take mass through the symmetry-breaking mechanism. In Appendix C some detail of the features of the model are reported. From the analysis of the most general renormalizable Higgs-boson potential [it consists of 22 terms and is given in Eqs. (C1) and (C2)] these statements follow.

(1) The extremum conditions require all phases to be zero, and, in a way similar to that considered at the end of Sec. III, the most general solution compatible with the parameters entering the potential corresponds to the totally asymmetric solution (37). A more accurate analysis will be performed elsewhere:[17] it will be seen that a third Higgs boson Φ_3, uncoupled to the quarks, is required in order to have solutions leading to a spontaneous CP violation.)

(2) The mass-squared matrices of the neutral Higgs bosons are given by Eqs. (C5), (C6), and (C7). All physical Higgs bosons belonging to the light mass scale ($M^2 \sim k^2$) are expressed in terms of $\phi_1^{(0)}$ and $\psi_1^{(0)}$ (neglecting the small mixing with components uncoupled to the quarks), whereas all other eigenstates have large mass-squared $\sim v^2$, in particular those expressed as combinations of $\phi_2^{(0)}$ and $\psi_2^{(0)}$.

(3) The above masses realize NFC in the light Higgs-boson sector, in this way reproducing the typical situation of the WS model, even though not in its minimal version, since two light doublets are present. By extracting from L_Y of Eq. (35) the single terms corresponding to Higgs-boson-induced NC's, it is easily seen that NC's induced by $\phi_1^{(0)}, \psi_1^{(0)}$ are diagonal:

$$\bar{q}\,^{(w)}_{uL}\Gamma_1 q^{(w)}_{uR}\phi_1^{(0)}=\bar{q}\,^{(s)}_{uL}D_u q^{(s)}_{uR}\phi_1^{(0)}/k_1\ ,$$
$$\bar{q}\,^{(w)}_{dL}\Gamma_2 q^{(w)}_{dR}\psi_1^{(0)*}=\bar{q}\,^{(s)}_{dL}D_d q^{(s)}_{dR}\psi_1^{(0)*}/k_2^*\ , \tag{38}$$

and those induced by $\phi_2^{(0)}$ and $\psi_2^{(0)}$ are nondiagonal:

$$\bar{q}\,^{(w)}_{dL}\Gamma_1 q^{(w)}_{dR}\phi_2^{(0)}=\bar{q}\,^{(s)}_{dL}O_c^T D_u O_c q^{(s)}_{dR}\phi_2^{(0)}/k_1\ ,$$
$$\bar{q}\,^{(w)}_{uL}\Gamma_2 q^{(w)}_{uR}\psi_2^{(0)*}=\bar{q}\,^{(s)}_{uL}O_c D_d O_c^T q_{uR}\psi_2^{(0)*}/k_2^*\ , \tag{39}$$

where $O_c=O_u^T O_d$ is the generalized Cabibbo mixing. It follows that, as required, flavor-changing NC's are suppressed to the same extent as right-handed currents are suppressed too.

It is worthwhile to note that, by deriving along usual methods the mass matrices of charged and neutral vector bosons, the above approach leads to disconnecting *CP* violation from the W_L-W_R mixing, which is absent when the totally asymmetric solution (37) is adopted: W_L and W_R are mass eigenstates, in agreement with the limits derived from the present estimates of the mixing effects in the weak-interaction data.[4]

V. CONCLUSION

If L-R symmetry is a good approach to weak-interaction physics, in the appealing picture of justifying through the spontaneous breakdown of the gauge symmetry the appearance of a maximal effect of parity violation in weak interactions at "low" energies, then a considerable relevance is played by the specific pattern of the symmetry breaking, because of the different physical effects which follow.

In particular, it is of interest to enlarge the effect of the spontaneous breakdown of the gauge symmetry in such a way of including also *CP* violation within the physical consequences of this peculiar mechanism: it becomes the natural source of all the violation effects, starting from a highly symmetric Lagrangian. Furthermore, if realized, this allows to avoid the well-known difficulties related to a hard *CP* violation.

At the same time, in order to reproduce the "low"-energy physics well described within the $SU(2)_L\times U(1)_Y$ features of the WS model, a peculiar character has to be required, known as "natural flavor conservation," i.e., the absence of flavor-changing neutral currents, which are experimentally suppressed at least to the order αG_F. This requirement, because of the structure itself of the theory, cannot be satisfied in a rigorous way: Higgs-boson-induced flavor-changing NC cannot be avoided without spoiling the meaning of the generalized Cabibbo mixing.[18] But, from a less restrictive point of view, it can be reasonably required that NFC is realized in the "light" Higgs boson sector: the WS projection of the L-R-symmetric theory is then flavor conserving, and flavor-changing NC's are naturally suppressed, the typical parameter regulating this suppression being the ratio $(M_L/M_R)^2$, which operates the suppression of right-handed currents.

In light of the above arguments, NFC in the light sector and a spontaneous violation of *CP* are assumed as guiding principles in the analysis of the symmetry-breaking patterns of an L-R symmetric gauge theory.

A possible source of spontaneous *CP* violation can be found in a pseudomanifest L-R-symmetric theory, by ascribing it to the relative phase δ of k and k' in the expectation value of Φ (Refs. 10 and 11). But it was shown in Sec. II that this leads to flavor-changing contributions in the "light" Higgs-boson sector, in disagreement with the NFC requirement stated above.

The phase δ can be constrained to assume specific values by imposing to the Lagrangian a further discrete symmetry, given as D symmetry in Sec. III, which forces the relative phase between k and k' to assume only trivial values. Once D symmetry is applied, then, as has been shown in Sec. III, the only solution of the symmetry-breaking pattern problem which ensures NFC corresponds to the totally asymmetric solution. A first-order phase transition takes place [so that solution (31) cannot be derived in the limit $k'\to 0$ from solution (8)] and an analysis in terms of the parameters appearing in the Higgs-boson potential shows that the solution ensuring NFC is, at the same time, the most general solution when represented in the parameter space.

A realistic model based on the above approach requires however that all quarks take their mass from the spontaneous symmetry-breaking mechanism, the same mechanism being responsible of the *CP* violation. Both arguments lead to an enlargement of the Higgs-boson content: at least two Φ Higgs bosons are to be introduced in order to give mass to u-like and d-like quarks, and a third Higgs boson, uncoupled to the quarks, allows us to select[5,17] *CP*-violating solutions in the manner of Weinberg.

The approach then ensures in a natural way, without recursion to fine-tuning of parameters, (i) the absence of Higgs-boson-induced flavor-changing NC, suppressed to the same degree of suppression which characterizes right-handed currents; (ii) the absence of W_L-W_R mixing, in agreement with the present limits deduced from the experimental data; (iii) a source of spontaneous *CP* violation, mainly related to $\Delta S=2$ transitions with a strong suppression of $\Delta S=1$ contributions, which satisfies the Sanda-Deshpande argument,[19] is superweak in character (being $\epsilon'/\epsilon \lesssim 10^{-5}$), and appears as the first "palpable" indirect evidence of right-handed-current effects.[5,17]

APPENDIX A: EXTREMUM CONDITIONS AND HIGGS-BOSON MASSES (D SYMMETRY ASSUMED)

Here the technical details related to the discussion of Sec. III are reported. Extremum conditions and corresponding mass-squared matrices of charged and neutral Higgs bosons are derived under the assumption that a discrete symmetry D, given by Eq. (17), characterizes the problem.

We start then from the Higgs-boson potential (18) and (19) and take for the VEV's $\langle\chi_L\rangle,\langle\chi_R\rangle,\langle\Phi\rangle$ the general forms (20) and (8), respectively. Without any loss of generality, we can assume $k=h$, $k'=h'e^{i\delta}$ with h, h', δ real parameters, since only the relative phase between k and k' is relevant. Since no dependence on the phases of v_L and v_R is present, they can both be taken as real parameters.

The extremum conditions read ($c_\delta=\cos\delta$, $s_\delta=\sin\delta$)

$$\frac{\partial V_D}{\partial v_L}=v_L[-\mu^2+\tfrac{1}{2}\rho_1(v_L{}^2+v_R{}^2)-\tfrac{1}{2}\rho_2(v_L{}^2+v_R{}^2)+\alpha_1(h^2+h'^2)+\alpha_2 h'^2]=0\ , \tag{A1}$$

$$\frac{\partial V_D}{\partial v_R}=v_R[-\mu^2+\tfrac{1}{2}\rho_1(v_L{}^2+v_R{}^2)+\tfrac{1}{2}\rho_2(v_L{}^2+v_R{}^2)+\alpha_1(h^2+h'^2)+\alpha_2 h'^2]=0\ , \tag{A2}$$

$$\frac{\partial V_D}{\partial h}=h[-\mu_1{}^2+\alpha_1(v_L{}^2+v_R{}^2)+\lambda_1(h^2+h'^2)+h'^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)]=0\ , \tag{A3}$$

$$\frac{\partial V_D}{\partial h'}=h'[-\mu_1{}^2+\alpha_1(v_L{}^2+v_R{}^2)+\alpha_2(v_L{}^2+v_R{}^2)+\lambda_1(h^2+h'^2)+h^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)]=0\ , \tag{A4}$$

$$\frac{\partial V_D}{\partial \delta}=-h^2h'^2(\lambda_2+\lambda_3)c_\delta{}^2 s_\delta{}^2=0\ . \tag{A5}$$

The above conditions induce well-defined constraints on the parameters, depending on the specific choice of the symmetry-breaking pattern. Let us consider the case $v_L\neq0$, $v_R\neq0$: from (A1) and (A2) it follows that

$$\left.\begin{matrix} v_L\neq0 \\ v_R\neq0\end{matrix}\right\}\Longrightarrow \rho_2(v_L{}^2-v_R{}^2)=0 \tag{A6}$$

so that we have either (compare with Sec. III)

$$(\text{I a})\ v_L=v_R\neq0\ , \tag{A7}$$

or

$$(\text{I b})\ v_L\neq0\ ,\quad v_R\neq0\ ,\quad v_L\neq v_R,\quad \text{but}\ \rho_2=0\ . \tag{A8}$$

Further restrictions come from the relations (A3), (A4), and (A5). Let us assume $h\neq0$, $h'\neq0$. It is easily derived

$$\left.\begin{matrix} h\neq0 \\ h'\neq0\end{matrix}\right\}\Longrightarrow \left\{\begin{matrix}\alpha_2(v_L{}^2+v_R{}^2)+(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)(h^2-h'^2)=0\ , \\ (\lambda_2+\lambda_3)c_\delta s_\delta=0\ ,\end{matrix}\right. \tag{A9}$$

the two conditions on the right to be simultaneously satisfied. The former imposes a specific constraint on the parameters, which will be discussed later. The latter relation leads us to select one of the following possible cases:

$$(\text{i})\ c_\delta=0\ , \tag{A10}$$

$$(\text{ii})\ s_\delta=0\ , \tag{A11}$$

$$(\text{iii})\ \lambda_2+\lambda_3=0\ . \tag{A12}$$

The two solutions (I a) and (I b) given in (A7) and (A8), respectively, can be discussed together, by applying the restriction (A6) to the elements of the Higgs-boson mass-squared matrices. As far as charged Higgs bosons are concerned, we find the (Hermitian) mass-squared matrix

$$\begin{matrix} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} & \\ \end{matrix}$$
$$\begin{bmatrix} \alpha_2(h^2-h'^2) & 0 & \alpha_2 v_L h' e^{i\delta} & \alpha_2 v_L h \\ & \alpha_2(h^2-h'^2) & \alpha_2 v_R h & \alpha_2 v_R h' e^{i\delta} \\ & & -\alpha_2 v_L{}^2-h^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) & -hh'(\lambda_2 c_\delta - i\lambda_3 s_\delta) \\ & & & \alpha_2 v_L{}^2-h'^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \end{bmatrix} \begin{matrix}\chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)}\end{matrix}\ , \tag{A13}$$

where, in case (I a), $v_L=v_R$ must be taken.

Whichever of the conditions (A10)—(A12) is applied, i.e., independently of the second step of the symmetry-breaking pattern, we find two pairs of charge-conjugate massless charged Higgs bosons (exactly those required in order to give mass to the charged gauge bosons $W_{L,R}^{(\pm)}$). They can be evidenced by applying successively to the above mass-squared matrix $M_H^{(\pm)2}$ the two unitary transformations

$$U=\begin{bmatrix} \frac{H}{\Delta} & 0 & -\frac{v_L}{H}\frac{h'}{\Delta}e^{-i\delta} & \frac{v_L}{H}\frac{h}{\Delta} \\ 0 & 1 & 0 & 0 \\ \frac{v_L}{\Delta} & 0 & \frac{h'}{\Delta}e^{-i\delta} & -\frac{h}{\Delta} \\ 0 & 0 & \frac{h}{\Delta} & \frac{h'}{H}e^{i\delta} \end{bmatrix}\ , \tag{A14}$$

$$U' = \begin{bmatrix} 2hh'\dfrac{v_L}{\Delta T}e^{i\delta} & v_R\dfrac{H}{T} & 0 & -\dfrac{h^2-h'^2}{T} \\ -v_R\dfrac{H}{S} & 2hh'\dfrac{v_L}{\Delta S}e^{-i\delta} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2(h^2-h'^2)\dfrac{hh'}{\Delta ST}v_L e^{i\delta} & (h^2-h'^2)\dfrac{Hv_R}{ST} & 0 & \dfrac{S}{T} \end{bmatrix}, \tag{A15}$$

where

$$H^2=h^2+h'^2\ ,\quad \Delta^2=H^2+v_L^2\ ,\quad S^2\Delta^2=H^2v_R^2\Delta^2+4h^2h'^2v_L^2\ ,\quad T^2=S^2+(h^2-h'^2)^2\ .$$

It is easy to find

$$U'UM_H^{(\pm)2}U^+U'^+ = \begin{bmatrix} 0 & 0 & 0 & 0 \\ & \alpha_2(h^2-h'^2)\left[1+\dfrac{v_L^2v_R^2}{S^2}\right] & 0 & -2\alpha_2 hh'v_L v_R\dfrac{v_L^2T}{H\Delta S^2}e^{-i\delta} \\ & & 0 & 0 \\ & & & -\left[\alpha_2(h^2-h'^2)\dfrac{v_L^2}{H^2}+H^2(\lambda_2c_\delta^2-\lambda_3s_\delta^2)\right]\dfrac{T^2}{S^2} \end{bmatrix}. \tag{A16}$$

The dependence on v_L, v_R, h, h' is rather involved. In the physically most interesting case $v_R \gg h,h',v_L$ (and eventually $h,h' \gg v_L$) the submatrix corresponding to the two massive Higgs bosons takes on the form

$$\begin{bmatrix} \alpha_2(h^2-h'^2)\left[1+\dfrac{v_L^2}{H^2}\right] & -2\alpha_2 hh'\dfrac{v_L^3}{H^3}e^{-i\delta} \\ & -\alpha_2(h^2-h'^2)\dfrac{v_L^2}{H^2}-H^2(\lambda_2c_\delta^2-\lambda_3s_\delta^2) \end{bmatrix} \tag{A17}$$

which describes two light $[(\text{mass})^2 \sim h^2, h'^2]$ charged Higgs bosons.

Let us now consider the mass terms of the neutral Higgs bosons: after some standard algebra one finds

$$M^2(\chi_{Li}^{(0)})=0\ ,\quad M^2(\chi_{Ri}^{(0)})=0\ , \tag{A18}$$

the (symmetric) mass-squared matrix of the remaining Higgs boson being given by

$$\begin{array}{cccccc|c} \chi_{Lr}^{(0)} & \chi_{Rr}^{(0)} & \phi_{1r}^{(0)} & \phi_{2r}^{(0)} & \phi_{1i}^{(0)} & \phi_{2i}^{(0)} & \\ (\rho_1-\rho_2)v_L^2 & (\rho_1+\rho_2)v_Lv_R & 2\alpha_1v_Lh & 2(\alpha_1+\alpha_2)v_Lh'c_\delta & 0 & (\alpha_1+\alpha_2)v_Lh's_\delta & \chi_{Lr}^{(0)} \\ & (\rho_1-\rho_2)v_R^2 & 2\alpha_1v_Rh & 2(\alpha_1+\alpha_2)v_Rh'c_\delta & 0 & 2(\alpha_1+\alpha_2)v_Rh's_\delta & \chi_{Rr}^{(0)} \\ & & 2\lambda_1h^2 & 2(\lambda_1+\lambda_2)hh'c_\delta & 0 & 2(\lambda_1-\lambda_3)hh's_\delta & \phi_{1r}^{(0)} \\ & & & 2\lambda_1h'^2c_\delta^2+(\lambda_2+\lambda_3)h^2c_\delta & -(\lambda_2+\lambda_3)hh's_\delta & 2\lambda_1h'^2s_\delta c_\delta & \phi_{2r}^{(0)} \\ & & & & (\lambda_2+\lambda_3)h'^2(s_\delta^2-c_\delta^2) & -(\lambda_2+\lambda_3)hh'c_\delta & \phi_{1i}^{(0)} \\ & & & & & 2\lambda_1h'^2s_\delta^2-(\lambda_2+\lambda_3)h^2c_\delta^2 & \phi_{2i}^{(0)} \end{array} . \tag{A19}$$

A separation into two disjoint sets can be obtained through a rotation O_δ acting on the components of $\phi_2^{(0)}$:

$$M_{H_1}^{(0)2} \equiv \begin{bmatrix} (\rho_1-\rho_2)v_L^2 & (\rho_1+\rho_2)v_Lv_R & 2\alpha_1v_Lh & 2(\alpha_1+\alpha_2)v_Lh' \\ & (\rho_1-\rho_2)v_R^2 & 2\alpha_1v_Rh & 2(\alpha_1+\alpha_2)v_Rh' \\ & & 2\lambda_1h^2 & 2hh'(\lambda_1+\lambda_2c_\delta^2-\lambda_3s_\delta^2) \\ & & & 2\lambda_1h'^2 \end{bmatrix}, \tag{A20}$$

$$M_{H_2}^{(0)2} \equiv \begin{bmatrix} -h'^2(\lambda_2+\lambda_3)(c_\delta^2-s_\delta^2) & -hh'(\lambda_2+\lambda_3)(c_\delta^2-s_\delta^2) \\ & -h'^2(\lambda_2+\lambda_3)(c_\delta^2-s_\delta^2) \end{bmatrix}. \tag{A21}$$

$M_{H_2}^{(0)2}$ exhibits a further massless Higgs boson in both cases (i) and (ii) of Eqs. (A10) and (A11), whereas case (iii) of Eq. (A12) implies two massless Higgs bosons. We are led to disregard this last case, $-\lambda_2=\lambda_3$, which introduces a further symmetry in the Higgs-boson potential and an excess of massless Higgs bosons. The relative phase of k and k' is then constrained, this being the effect of imposing D symmetry, to take on the trivial values corresponding to either $c_\delta=0$ or $s_\delta=0$.

As far as $M_{H_1}^{(0)2}$ is concerned, it can be rotated through the matrix ($V^2=v_L{}^2+v_R{}^2$)

$$O_V \equiv \begin{bmatrix} v_R/V & -v_L/V & 0 & 0 \\ v_L/V & v_R/V & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{A22}$$

By applying condition (A6), it follows that

$$O_V M_{H_1}^{(0)2} O_V^T \equiv \begin{bmatrix} -4\rho_2 v_L{}^2 v_R{}^2/V^2 & 0 & 0 & 0 \\ & \rho_1 V^2 & 2\alpha_1 hV & 2(\alpha_1+\alpha_2)h'V \\ & & 2\lambda_1 h^2 & 2hh'(\lambda_1+\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \\ & & & 2\lambda_1 h'^2 \end{bmatrix} \tag{A23}$$

with evidence of a further massless Higgs boson in the partially asymmetric case (I b) of Eq. (A8).

In conclusion, as discussed in Sec. III, in cases (I a) (symmetric solution: $v_L=v_R\neq 0$) and (I b) (partially asymmetric solution : $v_L\neq 0$, $v_R\neq 0$, $v_L\neq v_R$, but $\rho_2=0$) we find three and four massless Higgs bosons, respectively, i.e., in both cases more massless Higgs bosons than those required by the gauge symmetry breaking. Moreover, because of the D symmetry, the relative phase δ between k and k' is restricted to the trivial values required by either $c_\delta=0$ or $s_\delta=0$, if the possibility $\lambda_2+\lambda_3=0$, which introduces a further massless Higgs boson into play, is excluded.

Let us discuss now the totally asymmetric solution:

$$\text{(I c)}\ v_L=0\ ,\quad v_R=v\neq 0\ , \tag{A24}$$

where the alternative choice $v_L\neq 0$, $v_R=0$, leading to similar conclusions, is evidently discarded on physical grounds. Let us assume $h\neq 0$, $h'\neq 0$, corresponding to Eq. (A9) and characterized by the three possible cases listed as (i) , (ii), and (iii) in Eqs. (A10)–(A12). The (Hermitian) mass-squared matrix of the charged Higgs-boson sector is now given by

$$M_H^{(\pm)2} = \begin{matrix} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} & \\ \left[\begin{matrix} \rho_2 v^2+\alpha_2(h^2-h'^2) \\ \\ \\ \\ \end{matrix}\right. & \begin{matrix} 0 \\ \alpha_2(h^2-h'^2) \\ \\ \\ \end{matrix} & \begin{matrix} 0 \\ \alpha_2 vh \\ -h^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \\ \\ \end{matrix} & \left.\begin{matrix} 0 \\ \alpha_2 vh' e^{i\delta} \\ -hh'(\lambda_2 c_\delta - i\lambda_3 s_\delta) \\ -h'^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \end{matrix}\right] & \begin{matrix} \chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)} \end{matrix} \end{matrix} . \tag{A25}$$

It can be diagonalized through the unitary matrix [now $H^2=h^2+h'^2$ and $T^2=H^2v^2+(h^2-h'^2)^2$]

$$U'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & v\dfrac{H}{T} & -\dfrac{h^2-h'^2}{T}\dfrac{h}{H} & -\dfrac{h^2-h'^2}{T}\dfrac{h'}{H}e^{i\delta} \\ 0 & 0 & \dfrac{h'}{H}e^{-i\delta} & -\dfrac{h}{H} \\ 0 & \dfrac{h^2-h'^2}{T} & v\dfrac{h}{T} & v\dfrac{h'}{T}e^{i\delta} \end{bmatrix} , \tag{A26}$$

with the resulting diagonal mass-squared matrix

$$U'' M_H^{(\pm)2} U''^{+} = \begin{bmatrix} \rho_2 v^2+\alpha_2(h^2-h'^2) & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & \alpha_2(h^2-h'^2)-H^2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \end{bmatrix} . \tag{A27}$$

In the usual scheme $v \gg h,h'$, one of the massive Higgs bosons is heavy, the other one light, whereas both the two massive Higgs bosons were light in the previous cases (I a) and (I b) [compare with matrix (A17)].

Going to the neutral Higgs bosons, we have

$$M^2(\chi_{Li}^{(0)})=\rho_2 v^2\ ,\quad M^2(\chi_{Ri}^{(0)})=0\ . \tag{A28}$$

The (symmetric) mass-squared matrix of the remaining Higgs bosons can be made block diagonal, as in the case of matrix (A19), through the same rotation O_δ: the two disjoint sets of Higgs bosons correspond to

$$M_{H_1}^{(0)2}=\begin{bmatrix} \rho_2 v^2 & 0 & 0 & 0 \\ & \rho_1 v^2 & 2\alpha_1 vh & 2(\alpha_1+\alpha_2)vh' \\ & & 2\lambda_1 h^2 & 2hh'(\lambda_1+\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2) \\ & & & 2\lambda_1 h'^2 \end{bmatrix}, \tag{A29}$$

$$M_{H_2}^{(0)2}=\begin{bmatrix} -h'^2(\lambda_2+\lambda_3)(c_\delta{}^2-s_\delta{}^2) & -hh'(\lambda_2+\lambda_3)(c_\delta{}^2-s_\delta{}^2) \\ & -h^2(\lambda_2+\lambda_3)(c_\delta{}^2-s_\delta{}^2) \end{bmatrix}. \tag{A30}$$

$M_{H_2}^{(0)2}$ is identical to the matrix (A21) obtained above [cases (I a) and (I b)] and leads to the same conclusions: if solution (iii) is excluded (two massless Higgs bosons from $M_{H_2}^{(0)2}$), then we have one massless Higgs boson if either $c_\delta=0$ or $s_\delta=0$. The other mass eigenstate has light mass

$$M^2\left[\frac{h'\phi_{1i}^{(0)}+h(c_\delta\phi_{2i}^{(0)}-s_\delta\phi_{2r}^{(0)})}{(h^2+h'^2)^{1/2}}\right] = -(h^2+h'^2)(\lambda_2+\lambda_3)(c_\delta{}^2-s_\delta{}^2)\ . \tag{A31}$$

$M_{H_1}^{(0)2}$ given in (A29) must be compared with the corresponding matrix (A23), deduced in cases (I a) and (I b). The only difference is

$$M^2(\chi_{Lr}^{(0)})=\rho_2 v^2 \tag{A32}$$

without any constraint on ρ_2. In conclusion, we find in the (I c) case, i.e., when the totally asymmetric solution is adopted, only two massless neutral Higgs bosons, as required by the gauge-symmetry breaking, and of the six massive neutral Higgs bosons, three have heavy mass ($M^2\sim v^2$) [see Eqs. (A28), (A30), and (A32)], the remaining three are light. With respect to the previous cases (I a) and (I b), a massless neutral Higgs boson becomes of heavy mass. Moreover, the constraint (A6) does not exist: ρ_2 has only to be positive.

Let us assume the totally asymmetric solution (I c) as the most significant one in the description of the first step of the symmetry-breaking pattern. It is interesting to compare, as far as the second step is concerned, the solution considered above, $k,k'\neq 0$, corresponding to the condition (A9) rewritten now in the form

$$\left.\begin{matrix} h\neq 0 \\ h'\neq 0 \end{matrix}\right\} \Longrightarrow \begin{cases} \alpha_2 v^2+(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)(h^2-h'^2)=0\ , \\ (\lambda_2+\lambda_3)c_\delta s_\delta=0\ , \end{cases} \tag{A33}$$

with the totally asymmetric case, in which either k or k' is zero. Let us assume

$$k\neq 0\ ,\quad k'=0\ . \tag{A34}$$

The totally asymmetric solution (A34) leads to a charged-Higgs-boson mass-squared matrix given by

$$M_H^{(\pm)2}=\begin{array}{c} \begin{matrix} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} \end{matrix} \\ \begin{bmatrix} \rho_2 v^2+\alpha_2 h^2 & 0 & 0 & 0 \\ & \alpha_2 h^2 & \alpha_2 vh & 0 \\ & & \alpha_2 v^2 & 0 \\ & & & 0 \end{bmatrix} \end{array} \begin{matrix} \chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)} \end{matrix} \tag{A35}$$

which can be diagonalized in the form

$$OM_H^{(\pm)2}O^T=\begin{bmatrix} \rho_2 v^2+\alpha_2 h^2 & 0 & 0 & 0 \\ & \alpha_2(v^2+h^2) & 0 & 0 \\ & & 0 & 0 \\ & & & 0 \end{bmatrix} \tag{A36}$$

under the action of the orthogonal matrix

$$O=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & h/\Delta & v/\Delta & 0 \\ 0 & -v/\Delta & h/\Delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A37}$$

All massive charged Higgs bosons are now heavy [(mass)$^2\sim v^2$], whereas in the $k,k'\neq 0$ case one of them was light, as can be seen from the mass matrix (A27).

Going to the neutral Higgs bosons, it is easy to derive

$$M^2(\chi_{Li}^{(0)})=\rho_2 v^2\ ,\quad M^2(\chi_{Ri}^{(0)})=0\ ,\quad M^2(\chi_{Lr}^{(0)})=\rho_2 v^2\ ,$$
$$M^2(\phi_{1i}^{(0)})=0\ ,\quad M^2(\phi_{2i}^{(0)})=\alpha_2 v^2-\lambda_3 h^2\ , \tag{A38}$$
$$M^2(\phi_{2r}^{(0)})=\alpha_2 v^2+\lambda_2 h^2\ .$$

The only mixed states correspond to the mass-squared matrix

$$M_H^{(0)2}=\begin{array}{c} \begin{matrix} \chi_{Rr}^{(0)} & \phi_{1r}^{(0)} \end{matrix} \\ \begin{bmatrix} \rho_1 v^2 & 2\alpha_1 vh \\ & 2\rho_1 h^2 \end{bmatrix} \end{array} \begin{matrix} \chi_{Rr}^{(0)} \\ \phi_{1r}^{(0)} \end{matrix} \tag{A39}$$

which indicates a heavy Higgs boson and a light Higgs boson slightly mixed. In particular $\phi_{1r}^{(0)}$ (see Sec. III) is flavor diagonal. Its small mixing with $\chi_{Rr}^{(0)}$, uncoupled to quarks, ensures naturally flavor conservation in the light Higgs-boson sector.

APPENDIX B: EXTREMUM CONDITIONS AND HIGGS-BOSON MASSES (D SYMMETRY RELAXED)

Here the most general case is considered, in which D symmetry [see Eq. (17)] is relaxed. According to the arguments of Sec. III, the first stage of the symmetry-breaking pattern is assumed to obey the requirements of the totally asymmetric solution (I c) of Eq. (A24). To the D-symmetric potential V_D of Eqs. (18) and (19) a D-violating piece must be added. The most general potential can be written as $V=V_D+V_{DV}$, where

$$V_{DV}=[-\mu_{12}{}^2+\alpha_3(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+\tfrac{1}{2}\lambda_4\mathrm{Tr}(\Phi\Phi^\dagger)]\tfrac{1}{2}\,\mathrm{Tr}(\Phi\tilde{\Phi}^\dagger+\Phi^\dagger\tilde{\Phi})\ . \tag{B1}$$

(The trilinear terms are neglected, by assuming, as usual in these cases, a reflection symmetry acting on the Φ field. It is evident that, in the case of D symmetry, trilinear terms are zeroed directly by the D symmetry itself.) The extremum conditions take on the form

$$\frac{\partial V}{\partial v}=v[-\mu^2+\tfrac{1}{2}(\rho_1-\rho_2)v^2+\alpha_1(h^2+h'^2)+\alpha_2 h'^2+2\alpha_3 hh'c_\delta]=0\ , \tag{B2}$$

$$\frac{\partial V}{\partial h}=h[-\mu_1{}^2+\alpha_1 v^2+\lambda_1(h^2+h'^2)+(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)h'^2]+h'c_\delta[-\mu_{12}{}^2+\alpha_3 v^2+\tfrac{1}{2}\lambda_4(h^2+h'^2)+\lambda_4 h^2]=0\ , \tag{B3}$$

$$\frac{\partial V}{\partial h'}=h'[-\mu_1{}^2+(\alpha_1+\alpha_2)v^2+\lambda_1(h^2+h'^2)+(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)h^2]+hc_\delta[-\mu_{12}{}^2+\alpha_3 v^2+\tfrac{1}{2}\lambda_4(h^2+h'^2)+\lambda_4 h'^2]=0\ , \tag{B4}$$

$$\frac{\partial V}{\partial \delta}=hh's_\delta[-\mu_{12}{}^2+\alpha_3 v^2+\tfrac{1}{2}\lambda_4(h^2+h'^2)+(\lambda_2+\lambda_3)hh'c_\delta]=0\ . \tag{B5}$$

Let us consider the symmetry-breaking pattern characterized by

(II a) $s_\delta\neq0\ ,\ \ h,h'\neq0\ ,$

which corresponds to the pseudomanifest L-R symmetry discussed in Sec. II. The (Hermitian) mass-squared matrix of the charged Higgs boson is easily deduced after some algebra:

$$M_H^{(\pm)2}=\begin{array}{c} \\ \end{array}\begin{pmatrix} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} \\ \rho_2 v^2+\alpha_2(h^2-h'^2) & 0 & 0 & 0 \\ & \alpha_2(h^2-h'^2) & \alpha_2 vh & \alpha_2 vh'e^{i\delta} \\ & & \lambda_3 h^2 & \lambda_3 hh'e^{i\delta} \\ & & & \lambda_3 h'^2 \end{pmatrix}\begin{matrix} \\ \chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)} \end{matrix}\ . \tag{B6}$$

The corresponding eigenstates are given by $[H^2=h^2+h'^2,\ T^2=H^2v^2+(h^2-h'^2)^2]$

$$\begin{aligned}
&\chi_L^{(\pm)}\ \text{Higgs boson with } M^2=\rho_2 v^2+\alpha_2(h^2-h'^2)\ ,\\
&\frac{1}{T}\left[vH\chi_R^{(\pm)}-\frac{h^2-h'^2}{H}(h\phi_1^{(\pm)}+h'e^{\pm i\delta}\phi_2^{(\pm)})\right]\ \text{would-be-Goldstone boson}\ ,\\
&\frac{1}{H}(h'e^{\mp i\delta}\phi_1^{(\pm)}-h\phi_2^{(\pm)})\ \text{would-be-Goldstone boson}\ ,\\
&\frac{1}{T}[(h^2-h'^2)\chi_R^{(\pm)}+v(h\phi_1^{(\pm)}+h'e^{\pm i\delta}\phi_2^{(\pm)})]\ \text{Higgs boson with } M^2=\lambda_3 H^2+\alpha_2(h^2-h'^2)\ ,
\end{aligned} \tag{B7}$$

so that, in the usual scheme $v\gg h,h'$, one pair of charge-conjugate Higgs bosons is heavy, the other one of light mass.

Going to the neutral-Higgs-boson sector, we find [as in (A38)]

$$M^2(\chi_{Li}^{(0)})=\rho_2 v^2\ ,\ \ M^2(\chi_{Ri}^{(0)})=0\ ,\ \ M^2(\chi_{Lr}^{(0)})=\rho_2 v^2\ , \tag{B8}$$

the remaining states being characterized by the (symmetric) mass-squared matrix

$$\begin{array}{cccccc} \chi_{Rr}^{(0)} & \phi_{1r}^{(0)} & \phi_{2r}^{(0)} & \phi_{1i}^{(0)} & \phi_{2i}^{(0)} & \\ \rho_1 v^2 & 2\alpha_1 vh+2\alpha_3 vh'c_\delta & 2(\alpha_1+\alpha_2)vh'c_\delta+2\alpha_3 vh & -2\alpha_3 vh's_\delta & 2(\alpha_1+\alpha_2)vh's_\delta & \chi_{Rr}^{(0)} \\ & 2\lambda_1 h^2+(\lambda_2+\lambda_3)h'^2c_\delta{}^2 & (2\lambda_1+\lambda_2-\lambda_3)hh'c_\delta & -(\lambda_2+\lambda_3)h'^2 s_\delta c_\delta & 2(\lambda_1-\lambda_3)hh's_\delta & \phi_{1r}^{(0)} \\ & +2\lambda_4 hh'c_\delta & +\lambda_4(h^2+h'^2c_\delta{}^2) & -\lambda_4 hh's_\delta & +\lambda_4 h'^2 s_\delta c_\delta & \\ & & 2\lambda_1 h'^2 c_\delta{}^2+(\lambda_2+\lambda_3)h^2 & -(\lambda_2+\lambda_3)hh's_\delta & 2\lambda_1 h'^2 s_\delta c_\delta & \phi_{2r}^{(0)} \\ & & +2\lambda_4 hh'c_\delta & -\lambda_4 h'^2 s_\delta c_\delta & +\lambda_4 hh's_\delta & \\ & & & (\lambda_2+\lambda_3)h'^2 s_\delta{}^2 & -\lambda_4 h'^2 s_\delta{}^2 & \phi_{1i}^{(0)} \\ & & & & 2\lambda_1 h'^2 s_\delta{}^2 & \phi_{2i}^{(0)} \end{array} \quad . \tag{B9}$$

With rotations similar to those applied in Appendix A, the above matrix can be partially diagonalized in the form

$$M_{H'}^{(0)2}=O_H O_\delta M_H^{(0)2} O_\delta^T O_H^T$$

$$= \begin{pmatrix} \rho_1 v^2 & 2\alpha_1 vH+2\alpha_2 v\dfrac{h'^2}{H}+4\alpha_3 v\dfrac{hh'}{H}c_\delta & 2\alpha_2 vhh'+2\alpha_3 v\dfrac{h^2-h'^2}{H}c_\delta & -2\alpha_3 vHs_\delta & 0 \\ & 2\lambda_1 H^2+4(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)\dfrac{h^2h'^2}{H^2}+4\lambda_4 hh'c_\delta & 2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)(h^2-h'^2)\dfrac{hh'}{H^2}+\lambda_4(h^2-h'^2)c_\delta & -2(\lambda_2+\lambda_3)hh'c_\delta s_\delta-\lambda_4 H^2 s_\delta & 0 \\ & & (\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)\dfrac{(h^2-h'^2)^2}{H^2}+\lambda_3 H^2 & -(\lambda_2+\lambda_3)(h^2-h'^2)c_\delta s_\delta & 0 \\ & & & (\lambda_2+\lambda_3)H^2 s_\delta{}^2 & 0 \\ & & & & 0 \end{pmatrix} \tag{B10}$$

the corresponding "quasieigenstates" (so called since weakly mixed each other if $v \gg h,h'$) being, respectively,

$$\begin{aligned} &\chi_{Rr}^{(0)} \equiv \chi_{Rr}^{(0)} \,, \\ &H_1 \equiv [h\phi_{1r}^{(0)}+h'(\phi_{2r}^{(0)}c_\delta+\phi_{2i}^{(0)}s_\delta)]/H \,, \\ &H_2 \equiv [-h'\phi_{1r}^{(0)}+h(\phi_{2r}^{(0)}c_\delta+\phi_{2i}^{(0)}s_\delta)]/H \,, \\ &H_3 \equiv [h'\phi_{1i}^{(0)}+h(-\phi_{2r}^{(0)}s_\delta+\phi_{2i}^{(0)}c_\delta)]/H \,, \\ &G \equiv [-h\phi_{1i}^{(0)}+h'(-\phi_{2r}^{(0)}s_\delta+\phi_{2i}^{(0)}c_\delta)]/H \,, \end{aligned} \tag{B11}$$

where G corresponds to the second would-be-Goldstone boson of the theory. H_1, H_2, and H_3, in the usual scheme $v \gg h,h'$, all are of light mass: more precisely, the eigenstates constructed predominantly in terms of H_1, H_2, and H_3 have light mass and are weakly mixed to the heavy "quasieigenstate" $\chi_{Rr}^{(0)}$ (the mixing goes to zero in the limit $v \gg h,h'$). It follows that Higgs-boson terms inducing diagonal transitions between quarks [compare with Eq. (14)]

$$\begin{aligned} H_d &= \frac{1}{H}[h\phi_{1r}^{(0)}-h'(\phi_{2r}^{(0)}c_\delta+\phi_{2i}^{(0)}s_\delta)] \\ &= \frac{1}{H^2}[(h^2-h'^2)H_1-hh'H_2] \end{aligned} \tag{B12}$$

correspond to a combination of light Higgs bosons, as expected, but also those Higgs bosons inducing flavor changing transitions are of light mass, being

$$H_{nd}=\frac{1}{H}[h\phi_{2r}^{(0)}-h'(\phi_{1r}^{(0)}c_\delta+\phi_{1i}^{(0)}s_\delta)]=H_2c_\delta-H_3s_\delta \,. \tag{B13}$$

The situation is made more transparent in the specific case $h=h'$, which, in this case, can be obtained by merely taking the limit $h' \to h$ and $\alpha_2=0$. Massive charged Higgs bosons correspond to

$$\begin{aligned} &\chi_L^{(\pm)} \text{ with } M^2=\rho_1 v^2 \,, \\ &\frac{1}{\sqrt{2}}(\phi_1^{(\pm)}e^{\mp i\delta}-\phi_2^{(\pm)}) \text{ with } M^2=2\lambda_3 h^2 \,, \end{aligned} \tag{B14}$$

and the neutral-Higgs-boson mass matrix (B10) reduces to

$$M_{H'}^{(0)2}(h=h')=\begin{pmatrix} \rho_1 v^2 & 2\sqrt{2}(\alpha_1 vh+\alpha_3 vhc_\delta) & 0 & -2\sqrt{2}\alpha_3 vhs_\delta & 0 \\ & 4\lambda_1 h^2+2(\lambda_2 c_\delta{}^2-\lambda_3 s_\delta{}^2)h^2+4\lambda_4 h^2 c_\delta & 0 & -2(\lambda_2+\lambda_3)h^2 c_\delta s_\delta-2\lambda_4 h^2 s_\delta & 0 \\ & & 2\lambda_3 h^2 & 0 & 0 \\ & & & 2(\lambda_2+\lambda_3)h^2 s_\delta{}^2 & 0 \\ & & & & 0 \end{pmatrix} \tag{B15}$$

which shows that H_2, which induces flavor-changing NC into play, is now diagonal, with $M^2(H_2)=2\lambda_3 h^2$.

Let us now consider the symmetry-breaking pattern corresponding to manifest L-R-symmetric theories:

$$\text{(IIb)}\ s_\delta=0\ ,\quad h,h'\neq 0\ . \tag{B16}$$

The (symmetric) mass-squared matrix of the charged Higgs boson is now given by ($\epsilon=\pm 1$)

$$M_H^{(\pm)2}=\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} \end{array} \left[\begin{array}{cccc} \rho_2 v^2+\alpha_2(h^2-h'^2) & 0 & 0 & 0 \\ & \alpha_2(h^2-h'^2) & \alpha_2 v h & \alpha_2 v h'\epsilon \\ & & \dfrac{\alpha_2 v^2 h^2}{(h^2-h'^2)} & \dfrac{\alpha_2 v^2 h h'\epsilon}{(h^2-h'^2)} \\ & & & \dfrac{\alpha_2 v^2 h'^2}{(h^2-h'^2)} \end{array}\right] \begin{array}{c} \chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)} \end{array} . \tag{B17}$$

The mass-squared eigenstates correspond to those of the case examined above and listed in (B7), in the limit $s_\delta=0$. Now, however, both massive Higgs bosons have large mass:

$$\begin{aligned} &M^2(\chi_L^{(\pm)})=\rho_2 v^2+\alpha_2(h^2-h'^2)\ ,\\ &M^2([(h^2-h'^2)\chi_R^{(\pm)}+v(h\phi_1^{(\pm)}+h'\phi_2^{(\pm)}c_\delta)]/H)=\alpha_2 v^2 H^2/(h^2-h'^2)+\alpha_2(h^2-h'^2)\ . \end{aligned} \tag{B18}$$

Going to the neutral Higgs bosons, three of them satisfy also in this case Eq. (B8) whereas the matrix (B9) takes on a block-diagonal form without mixing between real and imaginary components. After partial diagonalization

$$M_{H'}^{(0)2}=O'_H M_H^{(0)2} O_H'^T=\left[\begin{array}{ccccc} \rho_1 v^2 & 2\alpha_1 v H+2\alpha_2 v\dfrac{h'^2}{H}+4\alpha_3 v h h'\dfrac{}{H}\epsilon & 2\alpha_2 v h h'\dfrac{}{H}\epsilon+2\alpha_3 v\dfrac{h^2-h'^2}{H} & 0 & 0 \\ & 2\lambda_1 H^2+4\lambda_2\dfrac{h^2h'^2}{H}+4\lambda_4 hh'\epsilon & 2\lambda_2(h^2-h'^2)\dfrac{hh'\epsilon}{H^2}+\lambda_4(h^2-h'^2) & 0 & 0 \\ & & \alpha_2\dfrac{v^2H^2}{h^2-h'^2}+\lambda_2\dfrac{(h^2-h'^2)^2}{H^2} & 0 & 0 \\ & & & \alpha_2\dfrac{v^2H^2}{h^2-h'^2}-\lambda_3 H^2 & 0 \\ & & & & \end{array}\right] . \tag{B19}$$

The corresponding change of basis leads to

$$\begin{aligned} &\chi_{Rr}^{(0)}\ ,\\ &H_1\equiv(h\phi_{1r}^{(0)}+h'\phi_{2r}^{(0)}\epsilon)/H\ ,\\ &H_2\equiv(-h'\phi_{1r}^{(0)}\epsilon+h\phi_{2r}^{(0)})/H\ ,\\ &H_3\equiv(h'\phi_{1i}^{(0)}\epsilon+h\phi_{2i}^{(0)})/H\ ,\\ &G\equiv(-h\phi_{1i}^{(0)}+h'\phi_{2i}^{(0)}\epsilon)/H\ , \end{aligned} \tag{B20}$$

where G is the would-be-Goldstone boson, and the "quasieigenstates" $\chi_{Rr}^{(0)}$, H_1, H_2, H_3 are weakly mixed in the limit $v\gg h,h'$. It is easily seen that only H_1 has light mass, all the other quasieigenstates being heavy Higgs bosons. The Higgs-boson combination which induces flavor-diagonal NC is given by

$$\begin{aligned} H_d&=(h\phi_{1r}^{(0)}-h'\phi_{2r}^{(0)}\epsilon)/H\\ &=[(h^2-h'^2)H_1-2hh'H_2\epsilon]/H^2\ , \end{aligned} \tag{B21}$$

whereas flavor-changing NC are induced by the combination

$$H_{nd}=(h\phi_{2r}^{(0)}-h'\phi_{1r}^{(0)}\epsilon)=H_2 \tag{B22}$$

which corresponds to a heavy Higgs boson weakly mixed with the others.

Let us finally consider the case in which the symmetry-breaking pattern is realized by the totally asymmetric solution

$$(\text{II c})\ h\neq 0\ ,\quad h'=0\ . \tag{B23}$$

The charged-Higgs-boson mass matrix corresponds to

$$M_H^{(\pm)2}=\begin{array}{c} \begin{array}{cccc} \chi_L^{(+)} & \chi_R^{(+)} & \phi_1^{(+)} & \phi_2^{(+)} \end{array} \\ \left[\begin{array}{cccc} \rho_2 v^2+\alpha_2 h^2 & 0 & 0 & 0 \\ & \alpha_2 h^2 & \alpha_2 vh & 0 \\ & & \alpha_2 v^2 & 0 \\ & & & 0 \end{array}\right] \end{array} \begin{array}{c} \chi_L^{(-)} \\ \chi_R^{(-)} \\ \phi_1^{(-)} \\ \phi_2^{(-)} \end{array}, \tag{B24}$$

which is exactly the same as the mass-squared matrix (A35) obtained in Appendix A (where D symmetry is imposed). It leads to two heavy Higgs bosons in the limit $v \gg h,h'$. From the analysis of the neutral-Higgs-boson sector

$$M^2(\chi_{Li}^{(0)})=\rho_2 v^2\ ,\quad M^2(\chi_{Ri}^{(0)})=0\ ,$$
$$M^2(\phi_{1i}^{(0)})=0\ ,\quad M^2(\phi_{2i}^{(0)})=\alpha_2 v^2-\lambda_3 h^2\ , \tag{B25}$$

and

$$M_H^{(0)2}=\begin{array}{c} \begin{array}{ccc} \chi_{Rr}^{(0)} & \phi_{1r}^{(0)} & \phi_{2r}^{(0)} \end{array} \\ \left[\begin{array}{ccc} \rho_1 v^2 & 2\alpha_1 vh & 2\alpha_3 vh \\ & 2\lambda_1 h^2 & \lambda_4 h^2 \\ & & \alpha_2 v^2+\lambda_2 h^2 \end{array}\right] \end{array} \begin{array}{c} \chi_{Rr}^{(0)} \\ \phi_{1r}^{(0)} \\ \phi_{2r}^{(0)} \end{array}, \tag{B26}$$

i.e., apart from the negligible mixings induced by the α_3 and λ_4 terms, we have also here the same situation as in the analogous case discussed in Appendix A (where D symmetry is imposed).

APPENDIX C: GENERALIZATION TO THE CASE OF TWO Φ HIGGS BOSONS

Here a brief account is given of the generalization of the content of Appendix A to the case in which two Φ Higgs, Φ_1 and Φ_2 [see Eq. (33)], satisfying the condition (34), are considered.

The most general renormalizable Higgs-boson potential satisfying gauge, L-R and D symmetries can be written in the form

$$V_D=V(\chi_{L,R})+\sum_{i=1,2} V_D(\chi_{L,R},\Phi_i)+V_D(\Phi_1)+V_D(\Phi_2)+V_{12}(\Phi_1,\Phi_2)\ , \tag{C1}$$

where $V(\chi_{L,R})$, $V_D(\Phi_i)$, and $V_D(\chi_{L,R},\Phi_i)$ are easily obtained from (19) (if μ_1^2, λ_1, λ_2, λ_3, α_1, α_2 are the couplings characterizing Φ_1, the corresponding coupling of Φ_2 will be indicated as μ_2^2, ϵ_1, ϵ_2, ϵ_3, β_1, β_2). The term $V_{12}(\Phi_1,\Phi_2)$ describes the coupling between Φ_1 and Φ_2: it is given by

$$\begin{aligned} V_{12}(\Phi_1,\Phi_2)=&\sigma_1\mathrm{Tr}(\Phi_1\Phi_1^\dagger)\mathrm{Tr}(\Phi_2\Phi_2^\dagger)+\tfrac{1}{4}\sigma_2[\mathrm{Tr}(\Phi_1\tilde\Phi_2^\dagger)+\mathrm{Tr}(\Phi_1^\dagger\tilde\Phi_2)]^2+\tfrac{1}{4}\sigma_3[\mathrm{Tr}(\Phi_1\tilde\Phi_2^\dagger)-\mathrm{Tr}(\Phi_1^\dagger\tilde\Phi_2)]^2\\ &+\tfrac{1}{4}\sigma_4[\mathrm{Tr}(\Phi_1\Phi_2^\dagger)+\mathrm{Tr}(\Phi_1^\dagger\Phi_2)]^2+\tfrac{1}{4}\sigma_5[\mathrm{Tr}(\Phi_1\Phi_2^\dagger)-\mathrm{Tr}(\Phi_1^\dagger\Phi_2)]^2\\ &+\tfrac{1}{4}\sigma_6[\mathrm{Tr}(\Phi_1\tilde\Phi_1^\dagger)\mathrm{Tr}(\Phi_2\tilde\Phi_2^\dagger)+\mathrm{Tr}(\Phi_1^\dagger\tilde\Phi_1)\mathrm{Tr}(\Phi_2^\dagger\tilde\Phi_2)]+\tfrac{1}{4}\sigma_7[\mathrm{Tr}(\Phi_1\tilde\Phi_1^\dagger)\mathrm{Tr}(\Phi_2^\dagger\tilde\Phi_2)+\mathrm{Tr}(\Phi_1^\dagger\tilde\Phi_1)\mathrm{Tr}(\Phi_2\tilde\Phi_2^\dagger)]\\ &+\sigma_8\mathrm{Tr}(\Phi_1\Phi_1^\dagger\Phi_2\Phi_2^\dagger)\ . \end{aligned} \tag{C2}$$

By assuming the symmetry-breaking pattern

$$\langle\chi_L\rangle=0\ ,\quad \langle\chi_R\rangle=\begin{bmatrix}0\\ v\end{bmatrix}\ ,\quad \langle\Phi_i\rangle=\mathrm{diag}(k_i,k_i')\ , \tag{C3}$$

the analysis of the extremum conditions allows us to verify that, as far as the relative phases among the k_i,k_i' are concerned, the most general solution, not implying specific constraints on the parameters of the potential, requires all phases equal to zero. By assuming then all k_i, k_i' real, the extremum conditions reduce to

$$\frac{\partial V}{\partial v}=v[-\mu^2+\tfrac{1}{2}v^2(\rho_1-\rho_2)+\alpha_1(k_1^2+k_1'^2)+\alpha_2 k_1'^2+\beta_1(k_2^2+k_2'^2)+\beta_2 k_2'^2]=0\ ,$$

$$\frac{\partial V}{\partial k_1}=k_1[\alpha_1 v^2-\mu_1^2+\lambda_1(k_1^2+k_1'^2)+\lambda_2 k_1'^2+\sigma_1(k_2^2+k_2'^2)+\sigma_2 k_2'^2+(\sigma_4+\sigma_8)k_2^2]+(\sigma_2+\sigma_4+\sigma_6+\sigma_7)k_2k_1'k_2'=0\ ,$$

$$\frac{\partial V}{\partial k_1'}=k_1'[(\alpha_1+\alpha_2)v^2-\mu_1^2+\lambda_1(k_1^2+k_1'^2)+\lambda_2 k_1^2+\sigma_1(k_2^2+k_2'^2)+\sigma_2 k_2^2+(\sigma_4+\sigma_8)k_2'^2]+(\sigma_2+\sigma_4+\sigma_6+\sigma_7)k_1k_2k_2'=0\ , \tag{C4}$$

$$\frac{\partial V}{\partial k_2}=k_2[\beta_1 v^2-\mu_2{}^2+\epsilon_1(k_2{}^2+k_2'{}^2)+\epsilon_2 k_2'{}^2+\sigma_1(k_1{}^2+k_1'{}^2)+\sigma_2 k_1'{}^2+(\sigma_4+\sigma_8)k_1{}^2]$$

$$+(\sigma_2+\sigma_4+\sigma_6+\sigma_7)k_1 k_1' k_2'=0\ ,$$

$$\frac{\partial V}{\partial k_2'}=k_2'[(\beta_1+\beta_2)v^2-\mu_2{}^2+\epsilon_1(k_2{}^2+k_2'{}^2)+\epsilon_2 k_2{}^2+\sigma_1(k_1{}^2+k_1'{}^2)+\sigma_2 k_1{}^2+(\sigma_4+\sigma_8)k_1'{}^2]$$

$$+(\sigma_2+\sigma_4+\sigma_6+\sigma_7)k_1 k_1' k_2=0\ .$$

In a way quite similar to that used at the end of Sec. III, it is possible to show that the solution corresponding to $k_1'=k_2'=0$ is the most general one, since it is compatible with the largest generality of the parameters appearing in the potential (C1). To be more specific, we have to compare the two allowed regions, one defined as a part of a hyperplane, the other limited to the intersection of this hyperplane with another hyperplane, in the hyperspace of the parameters of the potential. As in the simplest case described in Sec. III, the two solutions undergo a first-order phase transition.

By assuming therefore the symmetry-breaking pattern corresponding to the totally asymmetric solution (37), the mass matrices of the Higgs bosons can be derived along standard lines (after a rather long algebraic manipulation). By limiting our attention to the neutral-Higgs-boson sector, it is

$$M_H^{(i)2}=\begin{array}{c} \begin{array}{cccc} \phi_{1i}^{(0)} & \psi_{1i}^{(0)} & \phi_{2i}^{(0)} & \psi_{2i}^{(0)} \end{array} \\ \left[\begin{array}{cccc} -(\sigma_4+\sigma_5)k_2{}^2 & (\sigma_4+\sigma_5)k_1k_2 & 0 & 0 \\ & -(\sigma_4+\sigma_5)k_1{}^2 & 0 & 0 \\ & & \alpha_2 v^2-\lambda_3 k_1{}^2-(\sigma_3+\sigma_4+\sigma_8)k_2{}^2 & (\sigma_4-\sigma_3)k_1k_2 \\ & & & \beta_2 v^2-\epsilon_3 k^2-(\sigma_3+\sigma_4+\sigma_8)k_1{}^2 \end{array}\right] \end{array} \begin{array}{c} \phi_{1i}^{(0)} \\ \psi_{1i}^{(0)} \\ \phi_{2i}^{(0)} \\ \psi_{2i}^{(0)} \end{array},\qquad \text{(C5)}$$

$$M_H^{(r)2}=\begin{array}{c} \begin{array}{ccccc} \chi_{Rr}^{(0)} & \phi_{1r}^{(0)} & \psi_{1r}^{(0)} & \phi_{2r}^{(0)} & \psi_{2r}^{(0)} \end{array} \\ \left[\begin{array}{ccccc} \rho_1 v^2 & 2\alpha_1 v k_1 & 2\beta_1 v k_2 & 0 & 0 \\ & 2\lambda_1 k_1{}^2 & 2(\sigma_1+\sigma_4+\sigma_8)k_1k_2 & 0 & 0 \\ & & 2\epsilon_1 k_2{}^2 & 0 & 0 \\ & & & \alpha_2 v^2+\lambda_2 k_1{}^2+(\sigma_2-\sigma_4-\sigma_8)k_2{}^2 & (\sigma_2+\sigma_4+\sigma_6+\sigma_7)k_1k_2 \\ & & & & \beta_2 v^2+\epsilon_2 k_2{}^2+(\sigma_2-\sigma_4-\sigma_8)k_1{}^2 \end{array}\right] \end{array} \begin{array}{c} \chi_{Rr}^{(0)} \\ \phi_{1r}^{(0)} \\ \psi_{1r}^{(0)} \\ \phi_{2r}^{(0)} \\ \psi_{2r}^{(0)} \end{array},\qquad \text{(C6)}$$

the remaining components being mass eigenstates according to

$$M^2(\chi_{Li}^{(0)})=\rho_2 v^2\ ,\quad M^2(\chi_{Ri}^{(0)})=0\ ,\quad M^2(\chi_{Lr}^{(0)})=\rho_2 v^2\ . \qquad \text{(C7)}$$

By simple inspection of the above formulas the conclusions drawn in Sec. IV follow.

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