Baryon self-energy due to the pion-quark interaction

K. G. Horacsek, Y. Iwamura,* and Y. Nogami
Department of Physics, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 25 March 1985)

We examine the baryon self-energy due to the pion-quark interaction, more specifically, how the self-energy varies among the ground states of the baryon octet and decuplet, and also from the ground to excited states. A nonrelativistic constituent quark model is used in which quarks are bound in a harmonic-oscillator potential. For the excited states we examine the lowest even- and odd-parity states in the nucleon sector. We find that the variation of the self-energy within the same strangeness sector is quite small, i.e., of the order of 30 MeV or smaller. For example, the pion contribution to the Δ-N splitting is $\lesssim 20$ MeV. In arriving at this small variation it is crucial to take account of the spin dependence of the quark energy that enters in the energy denominator of the self-energy calculation. The individual energy shift is sensitive to the (*ad hoc*) form factor associated with the pion-quark interaction, and also to the quark excitation in the intermediate state. However, this sensitivity is strongly reduced in the difference between energy shifts for different baryon states.

I. INTRODUCTION

In the last several years a number of papers have appeared examining the pion-quark interaction. In those papers, which we will cite in due course, the pion is treated (phenomenologically) as an elementary particle, and is incorporated either into the MIT bag model[1] or into the nonrelativistic constituent quark model[2] (NRQM). Despite the considerable effort expended on the pion interaction, however, there still seems much to be clarified. Among other problems, in the present paper we will concentrate on the effect of the pion-quark interaction on the baryon masses, in particular, on the mass splitting.

A good example that illustrates the problems we have in mind is the pion effect on the Δ-N mass splitting. It is generally accepted that this and other mass splittings among the octet and decuplet baryons are mainly due to the spin-spin interaction between quarks which arises from the gluon exchange.[3] Is the pion effect also important in this connection? For this question one finds conflicting results in the literature.

Consider the one-pion-exchange (OPE) potential between quarks, which is proportional to $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$, where $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$ are the isospin and spin operators of the quark, and the subscripts i and j refer to the quarks. The expectation value of $\sum_{i>j}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$ is 15 for N, and 3 for Δ. This is essentially how an appreciable pion contribution to the Δ-N splitting was obtained.[4] This is in contrast with several other calculations, which all indicate that the pion effect in the Δ-N splitting is much smaller.[2,5] The origin of the difference can be traced to the following. In all the calculations of Refs. 2 and 5, in addition to the OPE process, the self-energy diagram of Fig. 1 is taken into account. The crucial point is, when a pion is emitted by a quark, the quark spin can flip. If there is already a spin-spin correlation among the quarks in the system (which presumably is due to the gluon interaction), it affects the quark energy in the intermediate state. The self-energy effect of Fig. 1 therefore becomes state dependent. This is a pionic analog of the atomic Lamb shift. The OPE effect is also affected by the spin-spin correlation among the quarks, and it becomes quite different from the expectation value of the OPE *potential.* What is remarkable is that, when the contributions of the self-energy and OPE are combined, the state dependence of the pion effect on the baryon mass shift becomes much smaller than that of the expectation value of the OPE potential. Although there are differences in detail among the models and calculations of Refs. 2 and 5, the feature described above is common. In contrast, this feature is not considered in Ref. 4.

The problem discussed above naturally leads to the following. In evaluating the self-energy effect of Fig. 1 for the ground-state baryons such as N and Δ, it was assumed in all of the works cited in Refs. 2 and 5 that, apart from the spin and isospin flipping the quarks remain in the ground-state configuration. This (often implicit) assumption has no logical basis. In the last few years it has been realized that effects of the quark excitation in the intermediate state can be very large—so large that the self-energy may diverge.[6,7] When the MIT bag model with a sharp, fixed bag radius is used, the self-energy diverges even if the finite (spatial) size of the pion is introduced.[7] In order to suppress the divergence, two modifications of the bag model have been considered. One is to make the

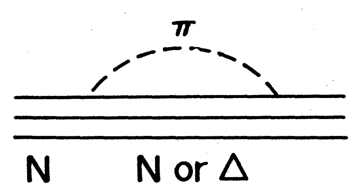

FIG. 1. Nucleon self-energy due to the pion-quark interaction. In addition there are one-pion-exchange diagrams which are not shown here.

bag surface smeared,[7–9] and the other is to assume that the pion-quark interaction is smeared both in space and time.[10] An advantage of the potential model is that it has no such divergence, provided that the finite size of the pion is taken into account. Still it is interesting to know how important the effect of the quark excitation is; we will see that it is not very important as far as the mass splitting is concerned.

Yet another problem that is closely related to those discussed above is how the baryon self-energy varies from the ground state to the excited state. For example, consider the self-energy diagram of Fig. 2 for an excited state N^*. The lowest-energy intermediate state is such that N^* is de-excited into N. In that case, depending on the pion energy, the signature of the energy denominator can change. Hence the self-energy of N^* could be quite different from that of the ground state N. This variation of the self-energy has to be taken account of in fitting the nucleon excitation spectrum.

The purpose of this paper is to examine the problems we have pointed out above: (i) the pion effect on the baryon mass with and without taking account of the spin dependence of the quark energy, and (ii) the effect of the quark excitation in the intermediate state. Throughout, we are interested in the *state dependence,* rather than the absolute value, of the baryon self-energy due to the pion interaction.

For the model of the pion-quark interaction, we choose the NRQM. Our model baryon consists of three nonrelativistic (hence massive) quarks bound in a common harmonic-oscillator potential. We assume that the gluon field has been eliminated already and replaced by the harmonic-oscillator confining potential and an effective spin-spin interaction between quarks. The pion is treated as an elementary particle (of a finite size), and is assumed to interact with quarks through the pseudovector coupling. For the ground-state baryons we examine all of the octet and decuplet, except Ω which does not interact with the pion. For excited states we consider only the lowest even- and odd-parity states in the nucleon sector.

There are other possible models for the pion-quark interaction, for example, those of Ref. 5. However, as far as the properties of the ground-state baryons are concerned, all of the models of Ref. 5 lead to very similar results. Yet our model is the simplest; all the matrix elements that we need for the pion effect can be obtained explicitly in a closed form. We believe that the features that we examine are common to those models. At this point one may raise a natural question: why do we not choose the bag model with relativistic quarks? It would be sufficient to point out that the bag model, in its usual static-cavity approximation, is beset with difficulty in calculating transition matrix elements. Since we discussed this earlier[7,8,11] let us only note here that the bag radius varies from one state to another and the quark wave functions of different states are not orthogonal to each other. The pion-quark interaction depends on the bag radius but what bag radius should one take if two states of different radii are involved? There is a way out of this difficulty by means of a "dynamical bag model,"[11] which we shall pursue elsewhere.

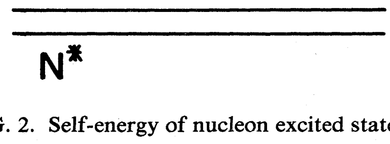

FIG. 2. Self-energy of nucleon excited state N^*.

II. MODEL

The Hamiltonian for our model baryon is given by

$$H = H_q + H_\pi + H_{\text{int}} \ , \tag{2.1}$$

$$H_q = \sum_{i=1}^{3} \left[\frac{p_i^2}{2m_i} + V(r_i) \right] + \sum_{i>j} V_{s,ij} \ , \tag{2.2}$$

$$H_{\text{int}} = (4\pi)^{1/2} \frac{f_q}{m_\pi} \sum_{i\alpha} \tau_{i\alpha} \int d\mathbf{r}\, \rho(\,|\mathbf{r}-\mathbf{r}_i|\,) \boldsymbol{\sigma}_i \cdot \nabla \phi_\alpha(\mathbf{r}) \ . \tag{2.3}$$

Here H_π is the standard Hamiltonian for the free pion field. The notation is essentially the same as that of Ref. 2. In H_q, $V(r)$ is the "shell-model potential" which we assume to be

$$V(r) = \tfrac{1}{2} m \eta^2 r^2 \ , \tag{2.4}$$

where m is the mass of the u (and d) quark. With this shell-model potential we use single-particle wave functions for the quarks. A disadvantage of this approach is that the wave functions contain spurious center-of-mass (c.m.) motion. However, all of the wave functions that we will use contain the c.m. motion in the same form, and hence we believe that the c.m. effect varies little from one baryon to another, and from the ground to excited states.

The other potential V_s in H_q is the "hyperfine interaction" which represents the spin-dependent part of the gluon-exchange potential. As a simple illustration we consider

$$V_{s,ij} = \lambda \frac{m^2}{m_i m_j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ . \tag{2.5}$$

The expectation values of V_s for the octet and decuplet ground states are listed in Table I; from this all of the mass formulas of De Rújula, Georgi, and Glashow[3] can be derived. Since V_s does not depend on the oscillator coordinates, the three oscillators $(i=1,2,3)$ remain uncoupled.

In H_{int}, $\tau_{i\alpha}$, $\boldsymbol{\sigma}_i$, and $\mathbf{r}_i$ are those of quark i $(=1,2,3)$, ϕ is the pion field, and ρ describes the nonlocality of the interaction. Of course it is understood that only the u and d quarks interact with the pion. The πq coupling constant f_q is related to the πN coupling constant f_N by

$$f_q = \tfrac{3}{5} f_N, \quad f_N{}^2 = 0.08 \ . \tag{2.6}$$

We will use the usual plane-wave expansion of $\phi(\mathbf{r})$. Then it is convenient to introduce the interaction form factor $v_\pi(k) = \int d\mathbf{r}\, \rho(r) e^{i\mathbf{k}\cdot\mathbf{r}}$. For $v_\pi(k)$ we assume the form

TABLE I. Baryon mass splitting due to V_s of Eq. (2.5); $x \equiv m_u/m_s$. The numerical values (in MeV) are those used in the calculation presented in Sec. VI. The parameters used are $\lambda=48.8$, $m_u=m_d=336$ MeV, and $m_s=538$ MeV. The last row shows the empirical mass splittings in MeV.

	$\Delta-N$	$\Sigma-\Lambda$	$\Sigma^*-\Sigma$	$\Xi^*-\Xi$
From V_s	6λ	$4(1-x)\lambda$	$6x\lambda$	$6x\lambda$
	293	73	183	183
Experiment	293	77	192	212

$$v_\pi^2(k)=\exp(-k^2/\Lambda_\pi^2) \ . \tag{2.7}$$

It would be reasonable to assume that the nonlocality represented by ρ is about the size of the pion. The empirical rms radius of the pion is[12] $\langle r^2\rangle_\pi^{1/2}=0.663$ fm. We choose the value of Λ_π which corresponds to this rms radius through $\langle r^2\rangle_\pi=\int d\mathbf{r}\, r^2\rho=3/\Lambda_\pi^2$. Hence $\Lambda_\pi=516$ MeV$=3.74m_\pi$.

III. SINGLE-QUARK CONTRIBUTION WHEN $V_s=0$

Let us begin with a fictitious model which contains only one quark, and evaluate the self-energy due to the diagram of Fig. 3. The single-quark state is determined by

$$H_q\psi_\nu=E_\nu\psi_\nu \ , \tag{3.1}$$

where $\nu=(n,l,m)$ and we set the ground-state energy $E_0=0$. Throughout this section we assume that $V_s=0$. With potential (2.4), $E_\nu=n\eta$, $n=0,1,2,\ldots$. For the initial state of the quark we consider 1s, 1p, and 2s. We denote the ground state by $|0\rangle$ or interchangeably by $|1s\rangle$. Similarly for the excited states,

$$|1\rangle=|1p\rangle, \quad |2\rangle=|2s\rangle \ . \tag{3.2}$$

For the ground state, the self-energy is given by

$$\Delta E_0=-\frac{\gamma}{4\pi}\sum_\nu\int d\mathbf{k}\frac{k^2v_\pi^2(k)}{\omega(\omega+E_\nu)} \times\langle 0|e^{-i\mathbf{k}\cdot\mathbf{r}}|\nu\rangle\langle\nu|e^{i\mathbf{k}\cdot\mathbf{r}}|0\rangle \ , \tag{3.3}$$

where

$$\gamma=\frac{3}{\pi}\left[\frac{f_q}{m_\pi}\right]^2 . \tag{3.4}$$

In evaluating the matrix elements in Eq. (3.3) we write ψ_ν as

$$\psi_\nu(\mathbf{r})=f_{nl}(r)Y_{lm}(\hat{\mathbf{r}}), \quad \hat{\mathbf{r}}=(\theta,\phi) \tag{3.5}$$

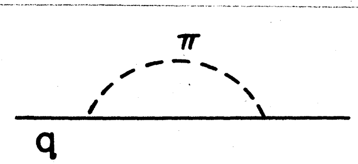

FIG. 3. Self-energy of a single quark.

and obtain

$$\langle\nu|e^{i\mathbf{k}\cdot\mathbf{r}}|0\rangle=\int d\mathbf{r}\,\psi_\nu^*(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}\psi_0(r) =\sqrt{4\pi}\,i^lG_{\nu0}(k)Y_{lm}^*(\hat{\mathbf{k}}) \tag{3.6}$$

with

$$G_{\nu0}(k)=\int_0^\infty dr\, r^2j_l(kr)f_\nu^*(r)f_0(r) \ . \tag{3.7}$$

The ΔE_0 becomes

$$\Delta E_0=-J\left[\sum_{nl}(2l+1)\frac{|G_{\nu0}(k)|^2}{\omega+E_\nu}\right] , \tag{3.8}$$

where

$$J[f(\omega)]=\gamma\int_0^\infty dk\frac{k^4v_\pi^2(k)}{\omega}f(\omega) \ . \tag{3.9}$$

Equations (3.3)–(3.8) are valid for any central potential $V(r)$. With the harmonic-oscillator potential (2.6), Eq. (3.8) can be further reduced to[13]

$$\Delta E_0=-J\left[e^{-t}\sum_{n=0}^\infty\frac{t^n}{n!}\frac{1}{\omega+n\eta}\right] , \tag{3.10}$$

where

$$t=K/\eta, \quad K=k^2/(2m) \ . \tag{3.11}$$

Note that the l summation has been done.

In the tight-binding limit $\eta\to\infty$, only the $n=0$ term contributes, and $\Delta E_{0,\eta\to\infty}=-J[\omega^{-1}]$. In order to examine the loose-binding limit $\eta\to 0$, it is convenient to rewrite Eq. (3.10) as[13]

$$\Delta E_0=-J\left[\int_0^\infty d\lambda\, e^{-\lambda\omega}\exp[t(e^{-\lambda\eta}-1)]\right] . \tag{3.12}$$

Expanding this ΔE_0 into η and carrying out the λ integration for each term, we obtain

$$\Delta E_0=-J\left[\frac{1}{\omega+K}+\frac{K\eta}{(\omega+K)^3}+\cdots\right] . \tag{3.13}$$

The first term in the expansion is $\Delta E_{0,\eta\to 0}$, which is exactly of the form expected. The quark kinetic energy K in the energy denominator is due to the recoil. It is clear that $\Delta E_{0,\eta\to 0}$ is smaller in magnitude than $\Delta E_{0,\eta\to\infty}$. As the binding becomes stronger, the quark becomes less mobile and the effective quark energy in the intermediate state diminishes.

Next let us turn to ΔE_ν for excited states. The wave functions of those states are related to that of the ground state by

$$|1\rangle=\frac{\sqrt{2}}{b}z\,|0\rangle \ , \tag{3.14}$$

$$|2\rangle=\sqrt{2/3}\left[\frac{3}{2}-\frac{z^2}{b^2}\right]|0\rangle \ , \tag{3.15}$$

where $b^2=1/(m\eta)$. For $|1\rangle$ we have taken that of $(l,m)=(1,0)$. Define $S_n(\mathbf{k},\mathbf{k}')$ by

$$S_n(\mathbf{k},\mathbf{k}')=\sum_{\nu'}A_{n'}\langle n|e^{-i\mathbf{k}\cdot\mathbf{r}}|\nu'\rangle\langle\nu'|e^{i\mathbf{k}'\cdot\mathbf{r}}|n\rangle \ , \tag{3.16}$$

where A_n can be any arbitrary (lm-independent) coefficient. The integrand of ΔE_n is of the form of $S_n(\mathbf{k},\mathbf{k})$. For the harmonic oscillator, $S_0(\mathbf{k},\mathbf{k}')$ is given by[12]

$$S_0(\mathbf{k},\mathbf{k}')=\exp\left[-\frac{b^2}{4}(k^2+k'^2)\right]\sum_{n=0}^{\infty}\frac{A_n}{n!}\left[\frac{b^2\mathbf{k}\cdot\mathbf{k}'}{2}\right]^n . \tag{3.17}$$

Using Eqs. (3.14) and (3.15), other S_n's are obtained by

$$S_1(\mathbf{k},\mathbf{k}')=\frac{2}{b^2}\frac{\partial^2}{\partial k_z \partial k_z'}S_0(\mathbf{k},\mathbf{k}') , \tag{3.18}$$

$$S_2(\mathbf{k},\mathbf{k}')=\frac{2}{3}\left[\frac{3}{2}+\frac{\Delta_k}{b^2}\right]\left[\frac{3}{2}+\frac{\Delta_{k'}}{b^2}\right]S_0(\mathbf{k},\mathbf{k}') . \tag{3.19}$$

In the actual calculation that follows, A_n of Eq. (3.17) depends on k, but in using Eqs. (3.18) and (3.19) the differentiations with respect to k and k' do not operate on A_n. It is now straightforward to derive ΔE for those excited states. They are[14]

$$\Delta E=-J\left[e^{-t}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{t^n}{\omega+n'\eta}F_n(t)\right] , \tag{3.20}$$

where

$$F_n(t)=\begin{cases}\dfrac{1}{3t}[(t-n)^2+2n],\\ \dfrac{1}{6t^2}\{[(t-n)^2-n]^2+4n(n-1)\}\end{cases} \tag{3.21}$$

and

$$n'=\begin{cases}n-1 & \text{for } 1p,\\ n-2 & \text{for } 2s .\end{cases} \tag{3.22}$$

So far we have considered only one quark. When there are three quarks, the total ΔE due to diagrams of the type of Fig. 3 is simply a sum of three single-quark (SQ) contributions. For example, for $0^21\equiv(1s)^2(1p)$ we obtain

$$\Delta E(0^21)=2\Delta E_0+\Delta E_1 . \tag{3.23}$$

IV. SINGLE-QUARK CONTRIBUTION WHEN $V_s\neq 0$

We continue to examine the SQ contribution but with $V_s\neq 0$ in this section. Consider the process of Fig. 1 for N. In the initial state the total spin of the three quarks is $S=\frac{1}{2}$. When the pion is emitted the quark spin can flip, and hence the total spin of the three quarks in the intermediate state is either $S=\frac{1}{2}$ or $\frac{3}{2}$. Therefore, the factor $(\omega+E_v)^{-1}$ in ΔE_0 of Eq. (3.3) has to be replaced with an appropriate combination of $(\omega+E_v)^{-1}$ and $(\omega+E_v+\delta)^{-1}$ where δ is the energy splitting between $S=\frac{1}{2}$ and $\frac{3}{2}$ due to V_s. The weighting factors for these two terms can be determined as follows.

Introduce the projection operators P_S for $S=\frac{1}{2}$ and $\frac{3}{2}$ by

$$P_{1/2}=\frac{1}{6}\left[3-\sum_{i>j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right] , \tag{4.1}$$

$$P_{3/2}=\frac{1}{6}\left[3+\sum_{i>j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right] . \tag{4.2}$$

Then $(H_\pi+H_q-E_0)^{-1}$ in the calculation of the SQ contribution can be written in the form of

$$aP_{1/2}+bP_{3/2} , \tag{4.3}$$

where

$$a=\begin{cases}(\omega+E_v)^{-1} ,\\ (\omega+E_v-\delta)^{-1} ,\end{cases} \quad b=\begin{cases}(\omega+E_v+\delta)^{-1}, & \text{for } N ,\\ (\omega+E_v)^{-1}, & \text{for } \Delta ,\end{cases} \tag{4.4}$$

and δ is the energy difference between $S=\frac{1}{2}$ and $\frac{3}{2}$ states; V_s of Eq. (2.5) gives $\delta=6\lambda$. Expression (4.3) has to be placed between two vertex factors: $(\boldsymbol{\sigma}_1\cdot\mathbf{k})\cdots(\boldsymbol{\sigma}_1\cdot\mathbf{k})$. Anticipating the angular integration with respect to $\mathbf{k}$ we find

$$(\boldsymbol{\sigma}_1\cdot\mathbf{k})\left[\sum_{i>j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right](\boldsymbol{\sigma}_1\cdot\mathbf{k})\to\frac{k^2}{3}\{-\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\}$$
$$=\frac{k^2}{3}\times\begin{cases}-1 & \text{for } N ,\\ 1 & \text{for } \Delta .\end{cases} \tag{4.5}$$

Hence

$$\sum_\alpha \tau_{1\alpha}(\boldsymbol{\sigma}_1\cdot\mathbf{k})(aP_{1/2}+bP_{3/2})\tau_{1\alpha}(\boldsymbol{\sigma}_1\cdot\mathbf{k})$$
$$=\frac{k^2}{3}\times\begin{cases}(5a+4b) & \text{for } N ,\\ (4a+5b) & \text{for } \Delta .\end{cases} \tag{4.6}$$

When $V_s=0$, i.e., $\delta=0$, then $a=b=(\omega+E_v)^{-1}$. The prescription is now clear; $(\omega+E_v)^{-1}=(\omega+n\eta)^{-1}$ of Sec. III is to be replaced by $(5a+4b)/9$ for N, and by $(4a+5b)/9$ for Δ, where a and b are those of Eq. (4.4). Exactly the same prescription applies to Ξ and Ξ^* except that $\delta=6x\lambda$ with $x=m_u/m_s$.

For Λ, we take $\delta=4(1-x)\lambda$ for the $S=\frac{1}{2}$ intermediate state and $\delta=2(2+x)\lambda$ for $S=\frac{3}{2}$. For Σ and Σ^*, the intermediate state with $S=\frac{1}{2}$ can have isospin $I=0$ or 1. Therefore the intermediate state has to be projected into $I=0$ and 1, and appropriate δ's have to be incorporated accordingly. This prescription is correct for the $n=0$ intermediate states, but only approximate for $n\geq 1$ states. The origin of this complication is that the spin-projection operator P_J does not commute with V_s when the masses of the quarks are not all equal. For Ξ and Ξ^* this does not cause any problem, but it leads to a complication for Λ, Σ, and Σ^*. However, we believe that the above prescription which is summarized in Table II is a very good approximation.

Before ending this section we note that the term "single-quark" contribution is a misnomer. It obviously depends on the spin correlation among the quarks through δ that enters into the energy denominator. The SQ contribution is therefore state dependent. It cannot simply be renormalized away.

TABLE II. Projection of the intermediate state; generalization of Eqs. (4.6) and (5.3) to other octet and decuplet baryons. For Σ, the subscripts 0 and 1 refer to the isospin of the intermediate state. SQ and ex refer to the single-quark and exchange contributions, respectively. For ex only the $n=0$ intermediate state contributes. The coefficients in SQ + ex correspond to those of Eqs. (2.6)—(2.12) of Ref. 2.

	SQ	ex	SQ + ex
N	$\frac{1}{3}(5a+4b)$	$\frac{10}{9}(a+2b)$	$\frac{1}{9}(25a+32b)$
Δ	$\frac{1}{3}(4a+5b)$	$\frac{2}{9}(-2a+5b)$	$\frac{1}{9}(8a+25b)$
Λ	$\frac{2}{3}(a+2b)$	$\frac{2}{3}(a+2b)$	$\frac{4}{3}(a+2b)$
Σ	$\frac{14}{27}(a_0+2a_1)+\frac{4}{9}b$	$\frac{2}{27}(-a_0+2a_1+2b)$	$\frac{4}{27}(3a_0+8a_1+4b)$
Σ^*	$\frac{8}{27}(a_0+2a_1)+\frac{10}{9}b$	$\frac{2}{27}(2a_0-4a_1+5b)$	$\frac{4}{27}(3a_0+2a_1+10b)$
Ξ	$\frac{1}{9}(a+8b)$		
Ξ^*	$\frac{1}{9}(4a+5b)$		

V. PION-EXCHANGE CONTRIBUTION

In addition to the SQ contribution ΔE_{SQ} examined in Sec. IV, we have the contribution from the OPE diagram, which we denote by ΔE_{ex}. Let us begin with ΔE_{ex} for the nucleon ground state, i.e., $0^3 \equiv (1s)^3$ with $I=S=\frac{1}{2}$. If we assume that $V_s=0$, we find

$$E_{ex}(0^3) = -\tfrac{10}{3} J[e^{-t}/\omega] \ . \tag{5.1}$$

The factor $\frac{10}{3}$ arises from the spin-isospin algebra. The same result is obtained by taking the expectation value of the OPE potential

$$V_{OPE} = \frac{f_q^2}{3} \sum_{i>j} \tau_i \cdot \tau_j \sigma_i \cdot \sigma_j \left[\frac{e^{-m_\pi r_{ij}}}{r_{ij}} - \frac{4\pi}{m_\pi^2} \delta(\mathbf{r}_{ij}) \right] . \tag{5.2}$$

If it is for the Δ ground state, with $I=S=\frac{3}{2}$, the factor $\frac{10}{3}$ of Eq. (5.1) will be replaced by $\frac{2}{3}$. This difference between ΔE_{ex} for N and Δ seems to have caused a widespread impression that the pion effect contributes significantly to the $N\Delta$ mass splitting. This is a fallacy as we will show in Sec. VI.

So far we have assumed $V_s=0$. In order to take account of V_s, again we have to project out the quark intermediate state into $S=\frac{1}{2}$ and $\frac{3}{2}$. For N and Δ this goes as follows:

$$\sum_\alpha \tau_{1\alpha}(\sigma_1 \cdot \mathbf{k})(aP_{1/2}+bP_{3/2})\tau_{2\alpha}(\sigma_2\cdot\mathbf{k}) + (1 \rightleftarrows 2)$$
$$\rightarrow \frac{2k^2}{9} \times \begin{cases} (5a+10b) & \text{for } N , \\ (-2a+5b) & \text{for } \Delta , \end{cases} \tag{5.3}$$

where a and b are those of Eq. (4.4) with $\nu=0$. When $V_s=0$, then $a=b=\omega^{-1}$, and ΔE_{ex} of Eq. (5.1) follows for N. Hence the prescription for including V_s is clear. *Note that ΔE_{ex} is no longer equal to the expectation value of the OPE potential of Eq. (5.2).*

It would be in order at this point to examine the relation between the present calculation and that of Ref. 2. The ΔE_{SQ} for N and Δ are given by

$$\Delta E_{SQ}(N) = -J \left[e^{-t} \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{1}{3} \left[\frac{5}{\omega+n\eta} + \frac{4}{\omega+n\eta+\delta} \right] \right] , \tag{5.4}$$

$$\Delta E_{SQ}(\Delta) = -J \left[e^{-t} \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{1}{3} \left[\frac{4}{\omega+n\eta-\delta} + \frac{5}{\omega+n\eta} \right] \right] . \tag{5.5}$$

The ΔE_{ex} for N and Δ are

$$\Delta E_{ex}(N) = -J \left[e^{-t} \frac{10}{9} \left[\frac{1}{\omega} + \frac{2}{\omega+\delta} \right] \right] , \tag{5.6}$$

$$\Delta E_{ex}(\Delta) = -J \left[e^{-t} \frac{2}{9} \left[\frac{-2}{\omega-\delta} + \frac{5}{\omega} \right] \right] . \tag{5.7}$$

If we take only the $n=0$ part of ΔE_{SQ}, we find $\Delta E_{tot} = \Delta E_{SQ} + \Delta E_{ex}$ to be

$$\Delta E_{tot}(N) = -\tfrac{1}{9} J \left[e^{-t} \left[\frac{25}{\omega} + \frac{32}{\omega+\delta} \right] \right] , \tag{5.8}$$

$$\Delta E_{tot}(\Delta) = -\tfrac{1}{9} J \left[e^{-t} \left[\frac{8}{\omega-\delta} + \frac{25}{\omega} \right] \right] . \tag{5.9}$$

Remembering that $J/9 = f_q^2 I$ of Ref. 2, and Eq. (2.4), one can see that Eqs. (5.8) and (5.9) agree exactly with Eqs. (2.6) and (2.7) of Ref. 2, with the understanding that $v_\pi^2(k)e^{-t}$ in the present calculation corresponds to $v^2(k)$ of Ref. 2.

Although we are going to give numerical results in the next section, let us have some preliminary note regarding the $N\Delta$ mass splitting. In Ref. 2 ΔE_{tot} was calculated by setting δ equal to the observed $N\Delta$ mass difference; it was found to be practically degenerate (within a few MeV) between N and Δ. This means that the pion contribution to the $N\Delta$ mass splitting is very small. Uehara and Kondo[5] used the same formulas with different form factors, and found that the pion contribution is somewhat larger but still only about 10% of the observed $N\Delta$ splitting.

For the excited states we consider the lowest even-parity and odd-parity states of the nucleon. For the even-parity excited state the spatial part of the wave function is given by

$$\psi(L=0^+)=\sqrt{2/3}(1s)^2(2s)+\sqrt{1/3}(1s)(1p)^2 \tag{5.10}$$

which is totally symmetric among the three quarks.[15] This is combined with the totally symmetric spin-isospin function, i.e., the same spin-isospin function as that of the ground state. The SQ contribution is given by

$$\Delta E_{SQ}(L=0^+)=\tfrac{1}{3}[5\Delta E(1s)+2\Delta E(1p)+2\Delta E(2s)]\ . \tag{5.11}$$

The OPE contribution is somewhat tedious. Let us illustrate the calculation by taking only $0^2 2=(1s)^2(2s)$. First, note that e^{-t} in Eq. (5.1) is $G_{00}{}^2$ where G_{00} is that of Eq. (3.7); i.e., $G_{00}=\langle 0|e^{i\mathbf{k}\cdot\mathbf{r}}|0\rangle$. If we assume $V_s=0$, the $\Delta E_{ex}(0^2 2)$ is obtained from $\Delta E_{ex}(0^3)$ by the following substitution:

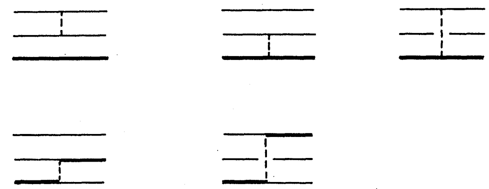

FIG. 4. One-pion-exchange contributions to the self-energy. The thick line represents a quark in an excited state.

$$\frac{3G_{00}{}^2}{\omega}\rightarrow\frac{1}{\omega}(G_{00}{}^2+2G_{00}G_{22})+G_{02}{}^2\left[\frac{1}{\omega-2\eta}+\frac{1}{\omega+2\eta}\right], \tag{5.12}$$

where

$$G_{02}=\langle 0|e^{i\mathbf{k}\cdot\mathbf{r}}|2\rangle=\sqrt{2/3}\frac{t}{2}e^{-t/2}\ , \tag{5.13}$$

$$G_{22}=\langle 2|e^{i\mathbf{k}\cdot\mathbf{r}}|2\rangle=\left[1-\frac{2t}{3}+\frac{t^2}{6}\right]e^{-t/2}\ . \tag{5.14}$$

The correspondence between the terms in Eq. (5.3) and the diagrams of Fig. 4 should be obvious. The effect of V_s can be included by using Eq. (5.3), and we arrive at

$$\Delta E_{ex}(L=0^+)=-\frac{10}{9}J\left[e^{-t}\left\{\left[1-\frac{4t}{9}+\frac{t^2}{9}\right]\left[\frac{1}{\omega}+\frac{2}{\omega+\delta}\right]+\frac{t^2}{27}\left[\frac{1}{\omega+2\eta}+\frac{1}{\omega-2\eta}+\frac{2}{\omega+\delta+2\eta}+\frac{2}{\omega+\delta-2\eta}\right]\right.\right.$$
$$\left.\left.-\frac{t}{9}\left[1-\frac{2t}{3}\right]\left[\frac{1}{\omega+\eta}+\frac{1}{\omega-\eta}+\frac{2}{\omega+\delta+\eta}+\frac{2}{\omega-\delta-\eta}\right]\right\}\right], \tag{5.15}$$

where $\delta=6\lambda$. The total contribution is given by this ΔE_{ex} plus ΔE_{SQ} of Eq. (5.11).

For the odd-parity state which we denote by $\psi(L=1^-)$, we combine the orbital angular momentum $L=1^-$ and the spin $S=\frac{1}{2}$ into $J^P=\frac{1}{2}^-$. The orbital, spin and isospin parts are all of mixed symmetry. We obtain

$$\Delta E_{SQ}(L=1^-)=2\Delta E(1s)+\Delta E(1p)\ , \tag{5.16}$$

$$\Delta E_{ex}(L=1^-)=-\frac{2}{3}J\left[e^{t}\left\{\left[1+\frac{2t}{3}\right]\left[\frac{1}{\omega}+\frac{2}{\omega+\delta}\right]+\frac{35}{162}t\left[\frac{1}{\omega+\eta}+\frac{1}{\omega-\eta}+\frac{2}{\omega+\eta+\delta}+\frac{2}{\omega-\eta+\delta}\right]\right\}\right]\ . \tag{5.17}$$

The $\Delta E_{tot}(L=1^-)$ is the sum of ΔE_{SQ} and ΔE_{ex}.

VI. RESULTS AND DISCUSSIONS

In the preceding sections we have derived the pion contribution to the baryon mass, with and without V_s. Let us now present numerical results. For the parameters in the model, the pion-quark coupling constant f_q and the cutoff momentum Λ have been specified in Sec. II. We assume the u-quark mass to be $m_u=336$ MeV from the magnetic-moment consideration. For m_s we take $m_s=538$ MeV, which, together with λ, gives a reasonable fit for the baryon mass splitting (Table I). The value of λ could be chosen such that V_s and the pion effect com-

TABLE III. The (negative of the) single-quark self-energy in MeV derived in Secs. III and IV. For $\Delta E(1s)$ the results with $\delta=0$ and those for N and Δ when $\delta\neq0$ are shown. For $1p$ and $2s$, they are for N with $\delta\neq0$. Breakdown for the contributions from intermediate states ($n=0,1,2,3$) is shown. The last row for each value of η is the total $\sum_{n=0}^{\infty}$.

η (GeV)	n	$\delta=0$	1s N	Δ	1p N	2s N
0.4	0	26	22	43	15	2
	1	12	11	15	17	7
	2	6	6	7	9	17
	3	3	3	3	5	9
	Total	51	45	72	53	45
0.5	0	31	26	51	15	−5
	1	12	11	14	20	4
	2	5	5	6	9	20
	3	2	2	2	5	9
	Total	53	46	75	54	36

bined yield a best fit for the baryon mass splitting. We have not done so because, as we will see, the pion effect on the mass of splitting is quite small. For the oscillator constant η, we take 0.4 and 0.5 GeV, which we believe to cover a reasonable range of η.[16]

Table III shows $-\Delta E$ for a single quark, given by Eqs. (3.10) without V_s, and also with V_s for N and Δ. Contributions from intermediate states are listed. It is evident that the assumption of the frozen quark configuration is grossly misleading. The contribution from the frozen configuration (the intermediate state with the same spatial configuration as that of the initial state) could be only a fraction of the total sum. This is very similar to what was found in Ref. 8. Note also that $|\Delta E_0|$ decreases as η decreases. The weak-binding limit of Eq. (3.12) turns out to be 44.2 MeV.

Table IV gives ΔE_{tot} with the breakdown for ΔE_{SQ} and ΔE_{ex}. The table also compares the results with V_s and those without V_s. *Note that the mass splitting Δ-N and Σ-Λ become much smaller when V_s is taken into account.*[17] When V_s is included the self-energy tends to be larger in magnitude for the decuplet than for the octet. This effect is in the direction opposite to that of the V_s. Note that the pion effect lowers Δ relative to N. For the Σ-Λ, although the pion effect is small, it is in the same direction as that of V_s. Table V shows the pion contribution to the nucleon excited states. Again the state dependence of the self-energy is substantially reduced when V_s is taken into account. Overall, with V_s included, the pion effect on the mass splitting within the same strangeness sector is $\lesssim 30$ MeV.

In our calculation we took the shell-model approach with H_q of Eq. (2.2) and used single-particle wave functions. However, all of the three quark wave functions that we have used, when rewritten in terms of the c.m. coordinate $\mathbf{R}$ and the Jacobi coordinates $\boldsymbol{\rho}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$ and $\boldsymbol{\lambda}=(\mathbf{r}_1+\mathbf{r}_2+2\mathbf{r}_3)/\sqrt{6}$, contain exactly the same $\mathbf{R}$-dependent part.[15] Therefore the effects of the spurious c.m. motion cancel, in a very good approximation, in the difference between self-energies of different baryons or between the ground and excited states.

The purpose of this paper was to examine two problems stated in Sec. I. In summary we have found that (i) it is important to include V_s, which strongly reduces the pion

TABLE IV. The (negative of the) self-energies in MeV of the ground-state baryons with and without V_s. For the SQ contribution, that from $n=0$ intermediate state is also shown. When $V_s=0$, the self-energies are degenerate between Σ and Σ^*, and between Ξ and Ξ^*.

η (GeV)		$V_s=0$ N	Δ	Λ	Σ	Ξ	$V_s\neq0$ N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*
0.4	SQ $n=0$	79	79	53	53	26	65	129	37	52	81	19	39
	SQ all n	154	154	103	103	51	134	215	80	102	136	41	66
	ex	88	18	53	6		64	1	37	4	3		
	Total	242	172	155	108	51	198	216	117	106	139	41	66
0.5	SQ $n=0$	94	94	63	63	31	77	153	44	62	94	23	45
	SQ all n	158	158	106	106	53	137	225	82	104	141	42	68
	ex	104	21	63	7		77	1	44	5	4		
	Total	263	179	168	112	53	214	226	126	109	144	42	68

TABLE V. The (negative of the) total self-energy of the nucleon in MeV, consisting of the single-quark (SQ) and exchange (ex) contributions. For the "total," the results with $V_s = 0$ are shown in parentheses.

η (GeV)		Ground state	Excited states $L=1^-$	0^+
0.4	SQ	134	142	140
	ex	64	81	52
	Total	198	224	192
		(242)	(279)	(215)
0.5	SQ	137	145	136
	ex	77	92	55
	Total	214	237	191
		(263)	(286)	(223)

effect on the mass splitting, and (ii) the effect of the virtual quark excitation on the energy itself is substantial. However, this effect is not very state dependent. *The small variations that we found, after incorporating V_s of course, do not sensitively depend on whether or not the $n \geq 1$ contributions are included.* On the other hand, for other quantities like the magnetic moment, one is interested in the quantity itself for each baryon rather than the difference of them among different baryons. Then the effect of the intermediate quark excitation will have more serious implications.[18]

Before ending perhaps we should discuss an inevitable question and/or criticism about the nonrelativistic nature of our model. The self-energy process involves large virtual excitation of the quark, which may require relativistic corrections. As we emphasized, however, high quark excitation on the mass splitting turned out to be unimportant. There is another source of ambiguity, i.e., the pion-quark interaction form factor $v_\pi(k)$. We assumed $v_\pi(k)$ of Eq. (2.7) and related Λ_π to the rms radius of the pion. We believe that this is a reasonable choice for a phenomenological model as ours. We are of course aware that the baryon self-energy depends on Λ_π; it diverges when $\Lambda_\pi \rightarrow \infty$ (point pion). However we confirmed that, as long as Λ_π is not very different from what we have chosen, the mass splitting is quite insensitive to Λ_π. Therefore, we believe that the features we found, for example, how the Δ-N splitting depends on the spin dependence of the quark energy, are quite model independent.

In Sec. I we said that the state dependence of the self-energy (Fig. 1) is a pionic analog of the atomic Lamb shift. This analogy should not be taken too literally. In the atomic case the state dependence of the electron self-energy arises mainly through the electron wave function, whereas in the present pionic case it is due to the spin-dependent quark energy. Our self-energy calculation is similar to the famous nonrelativistic Lamb shift calculation of Bethe.[19] Bethe introduced an *ad hoc* cutoff ($\sim mc$) for the virtual-photon momentum; the result depends on this cutoff. It was shown later that this dependence on the cutoff can be suppressed by taking account of relativistic effects.[20] In our case, the momentum cutoff is provided by $v_\pi(k)$. Can we suppress the dependence on the cutoff in a way similar to the atomic case? We tend to think not. In the atomic case the high-momentum contribution can be related to the electron-scattering S matrix.[19] For the quark we do not have such a scattering matrix. Another more important difference is that in our case we have to know more details of the pion-quark interaction at high-momentum transfers. This is beyond the scope of, not only our model, but all the models considered in Refs. 2, 4, and 5.

ACKNOWLEDGMENTS

One of the authors (Y.I.) wishes to thank the Theory Group of McMaster University for the hospitality he received during his stay. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

*Present address: Department of Physics, Science University of Tokyo, Tokyo 162, Japan.

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[16]If we identify the first odd-parity state with $(1s)^2(1p)$ then $\eta \sim 0.5$ GeV. On the other hand, if the first even-parity (Roper) state is interpreted as $(1s)^2(2s)$, then $2\eta \sim 0.5$ GeV. It is well known that these even- and odd-parity states cannot be simultaneously fitted in this way (Ref. 11). We are inclined to believe that $\eta \approx 0.5$ GeV is about the right choice.
[17]This effect of V_s, which reduces the state dependence of the pion effect, was discussed by F. Myhrer *et al.* (Ref. 5). We are under the impression that this effect of V_s is not widely known. None of the papers quoted in Ref. 4 mentions this.
[18]With the $m_u = 336$ MeV that we have chosen the proton magnetic moment is fitted. But this is so without considering the pion effect on the magnetic moment. In Ref. 2 the baryon masses and magnetic moments are fitted including the pion contributions. That required varying the quark masses, but by less than 5% (see Table III of Ref. 2). Note, however, that only the $n = 0$ quark intermediate state was considered in Ref. 2. Note also that the difference between the self-energies of different strangeness also affects the choice of m_s. With the contribution from $n \geq 1$, the variation of m's has to be greater, but probably of the order of 10%. In the ΔE's m_u appears only through t of Eq. (3.11), and ΔE's are not very sensitive to the variation of m_u of that order. We believe that our conclusion on the mass splitting is not affected.
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