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Detection rates for "invisible"-axion searches

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Experiments are described to search for axions floating about in the halo of our galaxy and for axions emitted by the sun. Expressions are given for the signal strengths in these experiments.

I. INTRODUCTION

Some time ago, it was shown that the strong *CP* problem[1] can be solved[2] by the introduction of a light pseudoscalar particle,[3] called the axion. The axion mass and its couplings to ordinary particles are all inversely proportional to the magnitude v of the vacuum expectation value that spontaneously breaks the $U_{PQ}(1)$ quasisymmetry which was postulated by Peccei and Quinn[2] and of which the axion is the pseudo-Nambu-Goldstone boson. As far as the solution to the strong *CP* problem is concerned, the value of v is arbitrary.[14] However, constraints from past experiments[5] attempting to produce and detect axions in the laboratory, from stellar evolution,[6] and from cosmology[7] suggest that

$$10^8 \lesssim \frac{v}{N} v \lesssim 10^{12}\ \text{GeV}\ , \tag{1.1}$$

where N is a nonzero integer, defined below, which measures the strength of the coupling of axions to gluons and photons.

When $(v/N)v$ is near 10^{12} GeV, axions can provide the critical energy density for closing the Universe. In that case, they are an excellent candidate for constituting the dark matter of galactic halos.[8] When $(v/N)v$ is near 10^8 GeV, approximately $\frac{1}{500}$ the luminosity of our sun is in axions. Two of the experiments[9] described below would attempt to detect galactic-halo axions. A third experiment[9] would attempt to detect axions emitted by our sun.

In Sec. II we derive expressions for the axion mass and for the electromagnetic coupling of the axion. These are essential ingredients for the calculation of the axion detection rates. Sections III—V describe the three axion-search experiments and give expressions for their signal strengths.

II. MASS AND ELECTROMAGNETIC COUPLING OF THE AXION

Several years ago, Bardeen and Tye[10] gave explicit rules for the application of current algebra techniques to the properties of the axion. Using these techniques, Kaplan[11] and Srednicki[12] recently derived expressions for the mass and electromagnetic coupling of the axion. We give here a brief account of their results.

The current j_μ^a which enters the PCAC (partial conservation of axial-vector current) relation satisfied by the axion,

$$\partial^\mu j_\mu^a = 2v\partial_\mu\partial^\mu a\ , \tag{2.1}$$

is the Peccei-Quinn current j_μ^{PQ} minus a linear combination of the chiral currents $\bar{u}\gamma_\mu\gamma_5 u$ and $\bar{d}\gamma_\mu\gamma_5 d$ which is such that j_μ^a has no color anomaly. Hence

$$j_\mu^a = j_\mu^{PQ} - \frac{N}{1+z}(\bar{u}\gamma_\mu\gamma_5 u + z\bar{d}\gamma_\mu\gamma_5 d)\ , \tag{2.2}$$

where z is a number which is left arbitrary thus far, and N is a nonzero integer which expresses the color anomaly of the Peccei-Quinn charge Q^{PQ}:

$$N = \mathrm{Tr}[Q^{PQ}(Q^\alpha_{\text{color}})^2]\ . \tag{2.3}$$

Here Q^α_{color} is any one of the eight generators of $SU^c(3)$ and the trace is over all Weyl fermions. One has

$$\begin{aligned}\partial^\mu j_\mu^a = {}& \frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\left[\mathrm{Tr}[Q^{PQ}(Q_{EM})^2] - \frac{N}{1+z}\frac{2}{3}(4+z)\right] \\ & - \frac{N}{1+z}(2im_u\bar{u}\gamma_5 u + 2izm_d\bar{d}\gamma_5 d)\ ,\end{aligned} \tag{2.4}$$

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor. In a grand unified theory, the electromagnetic anomaly of the Peccei-Quinn charge is related to its color anomaly:

$$\mathrm{Tr}[Q^{PQ}(Q_{EM})^2]\frac{1}{\sin^2\theta_W^0}\mathrm{Tr}[Q^{PQ}(Q^\alpha_{\text{color}})^2]\ , \tag{2.5}$$

where θ_W^0 is the value of the electroweak angle at the grand unification scale. In grand unified theories in which the successful Georgi-Quinn-Weinberg[13] prediction of $\sin^2\theta_W$ holds, one has $\sin^2\theta_W^0 = \frac{3}{8}$. We will assume this value from now on. Substitution of Eqs. (2.5) and (2.3) into (2.4) yields

$$\begin{aligned}\partial^\mu j_\mu^a = {}& +\frac{e^2}{16\pi^2}N\left[\frac{8}{3} - \frac{2}{3}\frac{4+z}{1+z}\right]F_{\mu\nu}\tilde{F}^{\mu\nu} \\ & - \frac{N}{1+z}2i(m_u\bar{u}\gamma_5 u + zm_d\bar{d}\gamma_5 d)\ .\end{aligned} \tag{2.6}$$

The analogous equation for the neutral pion is

$$\begin{aligned}\partial^\mu j_\mu^{53} &= f_\pi\partial_\mu\partial^\mu\pi^0 \\ &= -\frac{e^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} + 2i[m_u\bar{u}\gamma_5 u - m_d\bar{d}\gamma_5 d]\ .\end{aligned} \tag{2.7}$$

One can derive expressions for the pion and axion masses using Eqs. (2.6) and (2.7) and Dashen's formula.[14] The value of z is fixed by the requirement that j_μ^a is the current associated with the physical axion, i.e., that the axion-pion mixing terms in Dashen's formula vanish. One finds

$$z=\frac{m_u}{m_d} \tag{2.8}$$

and

$$m_a=\frac{f_\pi m_\pi}{v}N\frac{\sqrt{z}}{1+z}=\frac{f_\pi m_\pi}{v}N\frac{(m_u m_d)^{1/2}}{m_u+m_d} . \tag{2.9}$$

Equations (2.1) and (2.6) imply that the low-energy effective theory of axions and photons contains the interaction term

$$\begin{aligned}\mathscr{L}_{a\gamma\gamma}&=+\frac{e^2}{32\pi^2}N\frac{a}{v}\left[\frac{8}{3}-\frac{2}{3}\frac{4+z}{1+z}\right]F_{\mu\nu}\tilde{F}^{\mu\nu}\\&=-g_\gamma\frac{\alpha}{\pi}N\frac{a}{v}\mathbf{E}\cdot\mathbf{B} ,\end{aligned} \tag{2.10}$$

where

$$g_\gamma=\frac{4}{3}-\frac{1}{3}\frac{4m_d+m_u}{m_u+m_d}\simeq 0.36 . \tag{2.11}$$

Note that the same combination N/v of unknown axion-model parameters enters the expression for the axion mass [Eq. (2.9)] and the expression for the $a\gamma\gamma$ coupling strength [Eq. (2.10)], so that for a given axion mass the $a\gamma\gamma$ coupling strength is known.

III. CAVITY HALOSCOPE

Consider a cylindrical electromagnetic cavity of arbitrary cross-sectional shape, permeated by a static homogeneous longitudinal magnetic field $\mathbf{B}_0=\hat{\mathbf{z}}B_0$. Galactic halo axions have velocities of the order $\beta\simeq 10^{-3}$, and hence their energies

$$\epsilon_a=m_a+\tfrac{1}{2}m_a\beta^2=m_a[1+O(10^{-6})] \tag{3.1}$$

have a spread of order $\frac{1}{2}10^{-6}$ above the axion mass. When the frequency ω of an appropriate cavity mode equals $m_a[1+O(10^{-6})]$ galactic halo axions can convert to quanta of excitation (photons) of that cavity mode. Provided that the cavity is much shorter that the de Broglie wavelength $\lambda_a=2\pi(\beta m_a)^{-1}\simeq 2\pi 10^3 m_a{}^{-1}$ of the galactic halo axions, the coupling of a given cavity mode to the galactic-halo axions is

$$\frac{\alpha}{\pi}g_\gamma\frac{N}{v}aB_0\int_V d^3x\,\mathbf{E}_\omega(\mathbf{x},t)\cdot\hat{\mathbf{z}} , \tag{3.2}$$

where $\mathbf{E}_\omega$ is the electric-field amplitude for that mode and V is the volume of the cavity. Hence the modes into which galactic-halo axions can convert their energy are the TM_{nl0} modes

$$\begin{aligned}&\mathbf{E}_\omega=e^{i\omega t}\psi(x,y)\hat{\mathbf{z}} ,\\&(\nabla^2+\mu\epsilon\omega^2)\psi=0 ,\\&\psi\,|_c=0 ,\end{aligned} \tag{3.3}$$

where c is the boundary of the cross-sectional area, and μ and ϵ are the magnetic susceptibility and dielectric constant of whatever medium is present inside the cavity. The equation of motion for ψ if there is a galactic halo axion field a is

$$\left[\mu\epsilon\frac{\partial^2}{\partial t^2}-\nabla^2\right]\psi=+\frac{\alpha}{\pi}g_\gamma\frac{N}{v}B_0\mu\frac{\partial^2 a}{\partial t^2} . \tag{3.4}$$

The calculation of the power on resonance ($\omega_{nl}=m_a$) into a given TM_{nl0} cavity mode from axion$\to$photon conversion proceeds in a straightforward manner. The result is

$$P_{nl}=\left[\frac{\alpha}{\pi}g_\gamma\right]^2\frac{N^2}{v^2}V(B_0)^2\rho_a\frac{1}{\epsilon}C_{nl}\min(Q_{nl},Q_a)\frac{1}{m_a} , \tag{3.5}$$

where $Q_a\simeq\frac{1}{2}10^{-6}$, V is the volume of the cavity, ρ_a is the energy density of galactic-halo axions, Q_{nl} is the cavity's quality factor for the mode in question, and the coefficient C_{nl} is given by

$$C_{nl}=\frac{1}{S}\left|\int_S d^2x\,\psi_{nl}\right|^2\Big/\int_S d^2x\,|\psi_{nl}|^2 , \tag{3.6}$$

where S is the cross-sectional area of the cavity. For a rectangular cross section, one has

$$C_{nl}=\frac{64}{\pi^4}\frac{1}{n^2l^2} . \tag{3.7}$$

For a circular cross section

$$C_{nm}=\frac{4}{(\chi_{0n})^2}\delta_{m0} , \tag{3.8}$$

where χ_{0n} is the nth zero of the Bessel function $J_0(x)$. Expressing Eq. (3.5) in laboratory units one obtains

$$P_{nl}=0.8\times10^{-19}\ \mathrm{W}\left[\frac{V}{500\ \text{liter}}\right]\left[\frac{B_0}{8\ \mathrm{T}}\right]^2\frac{1}{\epsilon}\frac{\rho_a}{\rho_{\text{halo}}}C_{nl}\left[\frac{m_a}{2\pi(3GH_Z)}\right]\min(Q_a{}^{-1}Q_{nl},1) , \tag{3.9}$$

where[16] $\rho_{\text{halo}}\equiv 10^{-24}\ \mathrm{g/cm}^3$.

I would like to finish this section with the following remarks.

(1) In Eqs. (3.5) and (3.9), the power P increases linearly with the quality factor Q provided $Q\lesssim Q_a$. This is of course due to the fact that the galactic-halo axion-energy spectrum has width of order $\frac{1}{2}10^{-6}$ [see Eq. (3.1)]. To detect the power, a hole must be made in the cavity walls through which the electromagnetic radiation can be brought to "shine" upon a microwave detector. The qual-

ity factor therefore is given by

$$\frac{1}{Q}=\frac{1}{Q_h}+\frac{1}{Q_w}\ , \tag{3.10}$$

where $1/Q_w$ is the contribution due to absorption into the cavity walls and $1/Q_h$ the contribution due to the hole. The maximum power that can be brought to shine upon the microwave detector is

$$P_{\text{detector}}=\frac{Q}{Q_h}P \tag{3.11}$$

with P given by Eq. (3.9).

(2) The power into a given cavity mode decreases sharply with increasing mode number [see Eqs. (3.7) and (3.8)]. It is therefore preferable to use the fundamental TM mode as much as possible. On the other hand, for cavities of rectangular or circular transverse cross section, the frequency ω of the fundamental TM mode is $\sim\pi/d$ where d is the shortest linear dimension of the transverse cross section. To explore high values of the axion mass using the fundamental TM mode of such cavities, their volume will have to be correspondingly reduced. One may want to use instead many smaller cavities whose fundamental TM frequencies (before coupling) are close to each other and which are then strongly coupled together. This design[16] allows one to have simultaneously large volume, high frequency, and low mode number. An alternate means of achieving the same result is through the use of dielectric plates.[17] Further ideas on how to carry out a cavity haloscope experiment may be found in Refs. 16—18.

IV. HALOSCOPE II

The cavity haloscope may become impractical for the exploration of the largest values of the allowed range of axion masses. One may instead attempt to detect galactic-halo axions through their conversion to microwave photons in a static inhomogeneous magnetic field $\mathbf{B}_0(\mathbf{x})$. The inhomogeneity is necessary because three-momentum must be provided for the transition to occur. Using the coupling (2.10), one finds the following result for the partial differential cross section for axion$\rightarrow$photon conversion in a region of space of dielectric constant ϵ and magnetic permeability μ which is permeated by a static magnetic field $\mathbf{B}_0(\mathbf{x})$ and a static electric field $\mathbf{E}_0(\mathbf{x})$:

$$\frac{d\sigma}{d\Omega}=\frac{\mu\sqrt{\mu\epsilon}(\epsilon_a)^2}{16\pi^2|\boldsymbol{\beta}_a|}\left[\frac{\alpha N}{\pi v}g_\gamma\right]^2\left|\int_V d^3x\, e^{i(\mathbf{k}_\gamma-\mathbf{k}_a)\cdot\mathbf{x}}\mathbf{n}\times[\mathbf{B}_0(\mathbf{x})-\boldsymbol{\beta}_a\times\mathbf{E}_0(\mathbf{x})]\right|^2 . \tag{4.1}$$

Equation (4.1) is valid for arbitrary incoming-axion momentum $(\epsilon_a,\mathbf{k}_a)=\epsilon_a(1,\boldsymbol{\beta}_a)$. The outgoing photon momentum is $(\omega,\mathbf{k}_\gamma)=\omega(1,\sqrt{\mu\epsilon}\,\mathbf{n})$ with $\omega=\epsilon_a$. Integrating Eq. (4.1) over solid angles and multiplying by the axion-energy flux, one finds for the power from axion$\rightarrow$photon conversion

$$\begin{aligned}P&=\frac{\rho_a}{16\pi^2}\left[\frac{\alpha N}{\pi v}g_\gamma\right]^2\frac{1}{\epsilon}\int d^3k_\gamma\delta(\omega-\epsilon_a)\left|\int_V d^3x\, e^{i(\mathbf{k}_a-\mathbf{k}_\gamma)}\mathbf{n}\times[\mathbf{B}_0(\mathbf{x})-\boldsymbol{\beta}_a\times\mathbf{E}_0(\mathbf{x})]\right|^2\\&=\frac{\pi}{2}\left[\frac{\alpha N}{\pi v}g_\gamma\right]^2\frac{1}{\epsilon}VB_0{}^2\rho_aR(\mathbf{k}_a)\frac{1}{\epsilon_a}\ ,\end{aligned} \tag{4.2}$$

where $R(\mathbf{k}_a)$ is a dimensionless function that describes the detector's response

$$R(\mathbf{k}_a)=\frac{\epsilon_a}{VB_0{}^2}\int\frac{d^3k_\gamma}{(2\pi)^3}\delta(\omega-\epsilon_a)\left|\int_V d^3x\, e^{i(\mathbf{k}_a-\mathbf{k}_\gamma)\cdot\mathbf{x}}\mathbf{n}\times[\mathbf{B}_0(\mathbf{x})-\boldsymbol{\beta}_a\times\mathbf{E}_0(x)]\right|^2 . \tag{4.3}$$

One can choose to have large values of R over a small range of axion momenta by making the spatial dependence of $\mathbf{B}_0(\mathbf{x})$ and/or $\mathbf{E}_0(\mathbf{x})$ periodic over the appropriate length scale. Expressing Eq. (4.2) in laboratory units, one obtains

$$P=(1.3\times10^{-24}\ \mathrm{W})\left[\frac{V}{100\ \text{liter}}\right]\frac{1}{\epsilon}\left[\frac{B_0}{8\ \mathrm{T}}\right]^2\frac{\rho_a}{\rho_{\text{halo}}}\left[\frac{m_a}{2\pi(300GH_Z)}\right]R(m_a)\ . \tag{4.4}$$

A possible design[16] for such a detector is to embed parallel superconducting wires ($\sim100\ \mu$m apart) in a block of material transparent to microwave radiation. By running currents through the wires, inhomogeneous magnetic fields of maximum strength ~8 T can be obtained. In order to scan through a range of possible axion masses, the currents in the individual wires would have to be changed in a slow continuous fashion. The photons produced are well collimated and hence easy to focus on a microwave detector.

V. AXION HELIOSCOPE

Inhomogeneous static magnetic or electric fields can also be used to convert solar axions to photons. The photon energy will equal the axion energy which is about 1 keV $=10^7$ K, the temperature in the solar interior. Hence the change in three-momentum is

$$q_z=k_\gamma-k_a=\epsilon_a-(\epsilon_a{}^2-m_a{}^2)^{1/2}=\frac{1}{2}\frac{m_a{}^2}{\epsilon_a}=\frac{2\pi}{70\text{ cm}}N^2\left[\frac{10^8\text{ GeV}}{v}\right]^2\frac{\text{keV}}{\epsilon_a}\ . \tag{5.1}$$

This sets the length scale over which one should make the magnetic and/or electric field inhomogeneous to obtain resonant conversion. Consider then a detector of length L in the direction $\mathbf{n}$ of the sun, inside of which there is a transverse magnetic field

$$\mathbf{B}_0=\hat{\mathbf{t}}B_0\cos\left[\frac{2\pi}{d}\mathbf{n}\cdot\mathbf{x}\right]\ .$$

The response function (4.3) is

$$R=\frac{E_aL}{8\pi}\left[\frac{\sin(2\pi/d-q_Z)L/2}{(2\pi/d-q_Z)L/2}+\frac{\sin(2\pi/d+q_Z)L/2}{(2\pi/d+q_Z)L/2}\right]^2\ . \tag{5.2}$$

Note that R is huge ($\sim 2\times 10^8$ for $L=1$ m) when q_z is near $2\pi/d$. The spectrum of solar axion energies is broad since it is the convolution of the blackbody spectrum at temperature T with the distribution of temperatures T inside the sun. The flux of solar axions on Earth is approximately

$$f_{0a}=\frac{3\times 10^{13}\text{ axions}}{\text{sec cm}^2}\left[\frac{N}{6}\right]^2\left[\frac{10^8\text{ GeV}}{v}\right]^2\ . \tag{5.3}$$

Integrating (4.1) over solid angles and multiplying by (5.3) one finds the event rate on resonance ($d\simeq 2\pi/q_Z$)

$$\frac{\text{No. of x rays}}{\text{time}}\simeq 3\frac{10^{-3}}{\text{sec}}\frac{V}{(\text{meter})^3}\frac{1}{\epsilon}\left[\frac{B_0}{8\text{ T}}\right]^2\left[\frac{N}{6}\right]^4\left[\frac{10^8\text{ GeV}}{v}\right]^4\frac{d}{\text{meter}}\ , \tag{5.4}$$

where V is the detector volume.

Note added. I understand that several authors, including F. Wilczek and L. Krauss, are carrying out an independent calculation of the detection rates for the experiments described herein.

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