Scalar mesons and the decay of the iota: A paradox explained

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We show that the "$\delta(980)$ dominance" of the decay $\iota(1440)\to K\bar{K}\pi$, in apparent contradiction to the failure to observe the decay $\iota\to\eta\pi\pi$, can be explained in the $K\bar{K}$ molecule interpretation of the S^* and δ scalar mesons via $K\bar{K}$ final-state interactions in the potential $V_{K\bar{K}}$ which binds the $K\bar{K}$ system. We are unable to distinguish conclusively between a *primary* phase-space and *primary* $K^*\bar{K}+\bar{K}^*K$ distribution; however, we note that the latter interpretation would be consistent with the expected $K^*\bar{K}+\bar{K}^*K$ branching ratio of the (mainly) ω-like state of the radially excited pseudoscalar-meson nonet. This makes the existence of the reported $\eta(1275)$ meson crucial for deciding whether the ι is an ordinary meson or something new.

I. INTRODUCTION

The $\iota(1440)$ and $\delta(980)$ are two of the most curious of the known mesons. In this paper we show that a recent suggestion[1] regarding the nature of the δ may help to explain one of the most puzzling properties of the ι, thereby possibly shedding some light on the nature of both of these interesting states.

The $\delta(980)$ ($IJ^{PC_n}=10^{++}$) and its partner the $S^*(980)$ ($IJ^{PC_n}=00^{++}$) have been known for a very long time. Their most natural interpretation would seem to be as the ρ- and ω-like states of the $^3P_0$ quark-model nonet expected in this general mass range [compare to the $^3P_2$ states $A_2(1310)$ and $f(1280)$, the $^3P_1$ states $A_1(1270)$ and $D(1285)$, and the $^1P_1$ states $B(1235)$ and $H(1190)$]. There are, however, several problems with this picture. The most serious of these, in our opinion, is that the predicted widths of the δ and S^* in this interpretation are radically different from those observed experimentally: for example, $\Gamma(S^*\to\pi\pi)$ is predicted[2] to be about 400 MeV instead of the experimentally observed 25 MeV. There are, however, many other problems, among which are the following: (1) Although in $^3P_2$, $^3P_1$, and $^1P_1$ the $(I=\frac{1}{2})-(I=1)$ splittings are 107 ± 10, 65 ± 40, and 105 ± 20 MeV, respectively,[3] the strange scalar meson $\kappa(1350)$ is nominally 370 MeV above the δ; (2) both the S^* and δ seem to be more strongly coupled to strange particles than nonstrange ones in contrast with a normal ρ- and ω-like pair: their branching ratios to $K\bar{K}$ are substantial even though this mode has hardly any phase space. The $q\bar{q}$ $^3P_0$ interpretation of the δ and S^* therefore seems dubious.

A simple explanation of the properties of these states does emerge, however, from a recent study of the $qq\bar{q}\bar{q}$ system in the quark model.[1] This calculation shows no theoretical evidence for $qq\bar{q}\bar{q}$ bound states analogous to ordinary mesons. On the contrary, it indicates that the $qq\bar{q}\bar{q}$ system tends to cluster very strongly into two ordinary mesons which exhibit weak short-range residual interactions analogous to molecular (or nuclear?) forces. These residual interactions depend on the $qq\bar{q}\bar{q}$ quantum numbers; in fact, in S waves they are attractive in the channel with flavor-octet 0^{++} quantum numbers. Taking the masses of the interacting mesons into account, it was shown in Ref. 1 to be plausible that a weakly bound state could exist in the four $K\bar{K}$ states, but in no others. These states would be expected to form isotriplet and isosinglet scalar mesons just below $K\bar{K}$ threshold; they would, naturally, appear to have a special relationship to the $K\bar{K}$ channel; and finally they would be narrow by virtue of being below threshold and so weakly bound. Obviously, it is very tempting to identify the δ and S^* with these "$K\bar{K}$ molecules."

The $\iota(1440)$ is the other curious character of this story. It was seen in the process $\psi\to\gamma K\bar{K}\pi$ as a $K\bar{K}\pi$ resonance.[4] Although initially identified with the $E(1420)$ ($IJ^{PC_n}=01^{++}$), subsequent analysis revealed it to be an $IJ^{PC_n}=00^{-+}$ (i.e., pseudoscalar) meson[5] and immediately speculations began that this new state (seen, as it is, in a "gluon-rich" channel[6] since $\psi\to\gamma gg$ is the short-distance intermediate state) is a glueball[7] or a hybrid meson.[8] Others have argued that the ι can be plausibly interpreted as an ordinary $q\bar{q}$ meson: a member of the radial excitation of the ground-state nonet.[9,10] Since our conclusions will have some bearing on this argument, we will recall briefly some facts about the ι and the points made by each side in this controversy regarding these facts:

(i) The ι is seen in a gluon-rich channel. Since this channel was predicted to be a good source of glueballs[6] (and, later, hybrids[8]), it can be argued that any meson seen in this channel should be considered to be guilty of having a strong "glue content" until proven innocent. On the other hand, the $\eta(548)$ and $\eta'(958)$ are both produced in ψ radiative decay with strengths comparable to the ι, so this method does not always identify new hadronic objects. Indeed, this argument is based on the approximate validity of the Okubo-Zweig-Iizuka rule which is known to be strongly violated for pseudoscalar mesons; the fact that ι is a pseudoscalar meson therefore obscures its association with non-$q\bar{q}$ states.

(ii) The ι is produced in $\psi\to\gamma X$ more strongly than the η'. In the nonrelativistic limit, a resonance R is produced in $\psi\to\gamma R$, through its "wave function at the origin"

$\psi^R(0)$, and in naive potential models $\psi^R_{2S}(0)$ will be smaller than $\psi^R_{1S}(0)$. These approximations are reasonably good for a heavy meson like $c\bar{c}$; from leptonic decays one indeed finds $\psi^{\psi}_{2S}(0) \simeq 0.6\psi^{\psi}_{1S}(0)$. If such a suppression factor were in operation in ψ radiative decays to the light-quark $1S$ and $2S$ pseudoscalar mesons, then the radial excitation of the η' (for example) would be expected[5] at only about $\frac{1}{4}$ of the rate of the η': this is to be compared to the observation

$$\Gamma(\psi \to \gamma\iota) = (1.2 \pm 0.5)\Gamma(\psi \to \gamma\eta') .$$

Unfortunately, this argument has some weaknesses. The first is that a quantitative nonrelativistic argument like this one is suspect for light quark systems; indeed, $\psi \to \gamma f$ proceeds at a healthy rate even though nonrelativistically $\psi^f(0)$ is zero. [In addition to relativistic nonlocalities, the mechanism for $gg \to q\bar{q}$ has a range $m_q{}^{-1}$: both effects weaken the role of $\psi^R(0)$ in determining $\psi \to \gamma gg \to \gamma R$ rates.] Perhaps more damaging is that the analogy to the ψ system may be faulty due to the existence of the same strong annihilation diagrams mentioned above as being responsible for the Okubo-Zweig-Iizuka violation in the pseudoscalar mesons.[10] Indeed, from the fact that $m_\eta > m_\pi$, it follows that these annihilation amplitudes have a *positive sign* and they therefore tend to cause strong radial mixing in the pseudoscalars which *reduces the normal potential-model* $\psi_{1S}(0)$ *relative to* $\psi_{2S}(0)$. This second fact, like the first, cannot therefore conclusively decide between the various interpretations of the ι.

(iii) The mass of the ι is 1440 MeV. Two members of the radially excited pseudoscalar nonet seem reasonably well established: the $\pi(1300)$ and the $K(1400)$. There is also evidence[11] for an isoscalar state $\eta(1275)$. If the $\eta(1275)$ is confirmed, then this nonet would be near to ideal mixing and its missing $s\bar{s}$ member would then be difficult to associate with the ι purely on the basis of mass.[12] On the other side of this argument is the fact that in most models,[13] the annihilation amplitude which makes $m_\eta \gg m_\pi$ is still reasonably strong in the radially excited nonet and naturally produces a [mainly $(u\bar{u}+d\bar{d})\sqrt{2}$] state about a hundred MeV above the $\pi(1330)$. This state could be identified with the ι if the $\eta(1275)$ does not exist. Models which allow for a pure glue state in this region[14] can account for both the $\eta(1275)$ and the $\iota(1440)$. This underlines the importance of establishing the existence of the $\eta(1275)$.

(iv) There is evidence for a substantial rate for the decay $\iota \to \rho\gamma$. If this observation[15] is confirmed it would rule out an $s\bar{s}$ interpretation of the ι. However, recent calculations have shown that this decay may be consistent with the glueball, bag hybrid, and ω-like $q\bar{q}$ radial interpretations of the ι although it contradicts the naive expectations of all these models. In the glueball case the decay goes by $q\bar{q}$ mixing[14] [the other side of the coin which hurt the glueball arguments in (i) and (ii) above], in the bag-model hybrid case by absorption of the constituent gluon,[16] and in the $q\bar{q}$ radial case via the fact that strong hyperfine interactions prevent the $2\,^1S_0$ wave function of the ι from being orthogonal to the $1\,^3S_1$ wave function of the ρ (Refs. 13 and 14).

Clearly, their unknown characters make both the scalar mesons S^* and δ and ι interesting states: we have something to learn about the strong interactions by understanding them.

II. AN ι PUZZLE NO ONE LIKES AND ITS POSSIBLE RESOLUTION

The main object of this paper is to suggest a resolution of a puzzle associated with the ι which no one seems to like:[17] the $K\bar{K}\pi$ decay of the ι appears to be dominated by $\delta\pi$, but the $\eta\pi\pi$ mode which is expected from $\delta \to \eta\pi$ *is not seen.* We will show that these apparently contradictory facts have a natural explanation if (as discussed above) the δ is not a $q\bar{q}$ meson but rather a $K\bar{K}$ molecule bound by a short-range attractive potential $V_{K\bar{K}}$: the $K\bar{K}$ seen in ι decay then peak at low $K\bar{K}$ mass (see Fig. 1) as a result of final-state interactions produced by $V_{K\bar{K}}$. There is an analog to this behavior which has been known in nuclear physics for some time:[18] the nn system produced in $\pi^- d \to nn\gamma$ shows[19] an extremely strong peaking at low nn mass (see Fig. 2) as a result of the strong short-range attractive nn potential which nearly produces isotriplet partners to the deuteron. Note that physically the peaking of the $K\bar{K}$ spectrum at low mass results from a distortion of the $K\bar{K}$ plane waves and not from direct scattering through the δ pole: it would persist even if, for example, the δ had a width of only 1 MeV.

A. Primary phase-space decay

Although we will consider another option in a moment, let us first show that the $K\bar{K}$ molecule interpretation of the δ, with the parameters deduced in Ref. 1, produces an acceptable description of the data if the primary $K\bar{K}\pi$ spectrum is a phase-space distribution. That is, we assume that we have a Hamiltonian H_0 describing the ι, K, $\bar{K}$, and π states consisting of only kinetic-energy terms and $V_{K\bar{K}}$ which we perturb with a *primary* decay interaction [$O(x)$ is the field of meson O]

$$H_{D3} = g_3\iota(x)\bar{K}(x)K(x)\pi(x) . \tag{1}$$

The usual prejudice is that such a direct three-body decay is much suppressed in strength relative to a two-body decay mode like $K^*\bar{K}$ + c.c. (which is the only kinematically allowed quasi-two-body mode of the ι to two orbital ground-state mesons). Since we will find, however, that the final-state interaction strongly *enhances* the ι decay rate in the sector of the Dalitz plot corresponding to low $K\bar{K}$ masses, we do not believe that this primary mechanism can be ruled out.

Although this option would usually be treated in the coupled-channel partial-wave formalism,[20] we will proceed by another route to lay the groundwork for our later considerations. The physical basis for our treatment is the observation that the coupling (1) leads to an amplitude for ι decay proportional to the $K\bar{K}\pi$ "wave function at the origin" and if $V_{K\bar{K}}$ is attractive, this factor is enhanced over the free-particle case. To see this explicitly we consider the equation for the full (nonrelativistic) propagator for the ι depicted in Fig. 3. This equation is easily solved to give in the ι rest frame

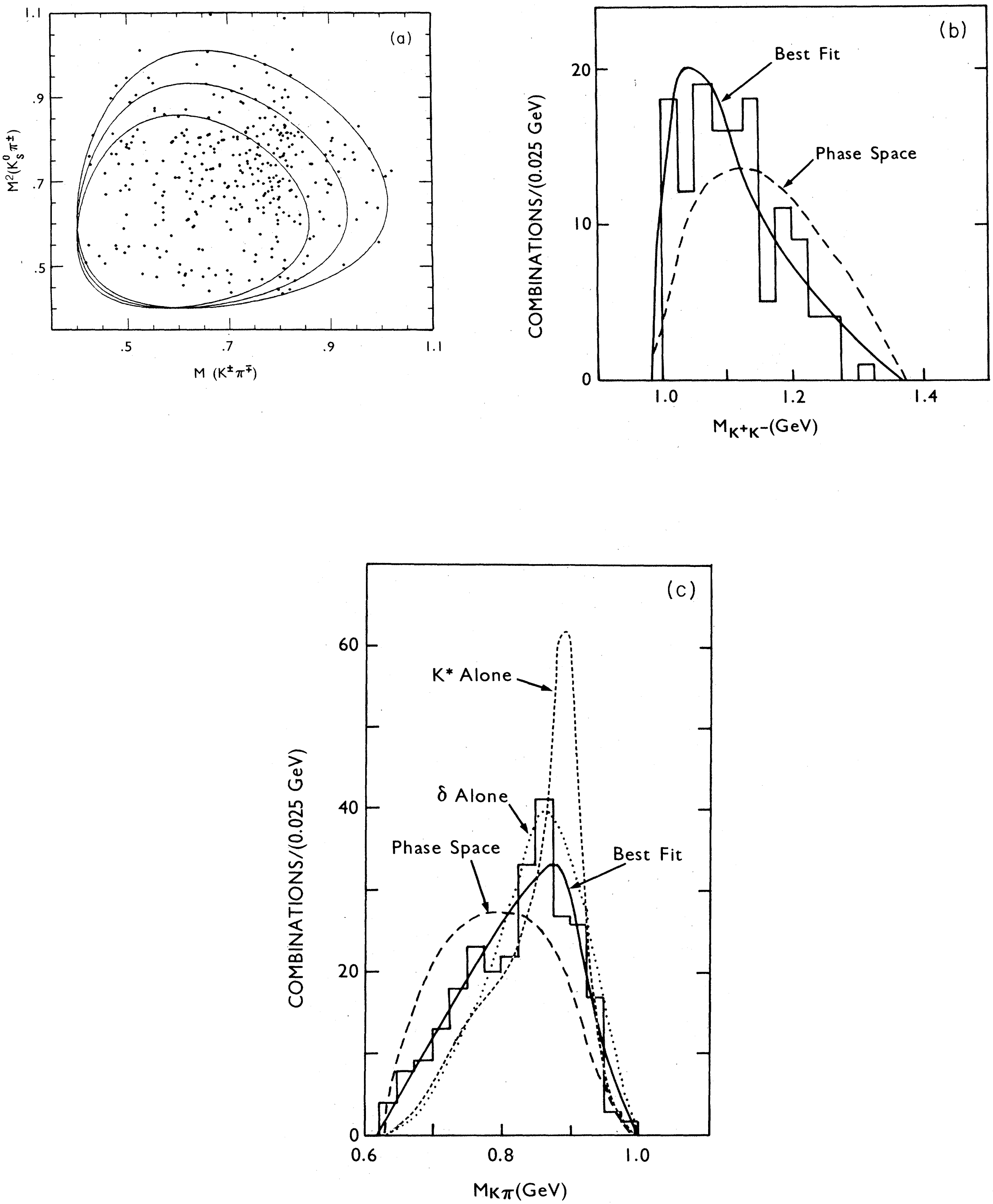

FIG. 1. Experimental results for $\iota \rightarrow K\bar{K}\pi$. (a) The Dalitz plot from Mark III (results from Mark II and Crystal Ball are similar), (b) the $K\bar{K}$ projection of the Dalitz plot from Crystal Ball, and (c) the $K\pi + \bar{K}\pi$ projection, also from Crystal Ball. All data are taken from Refs. 4, where the fits shown are described.

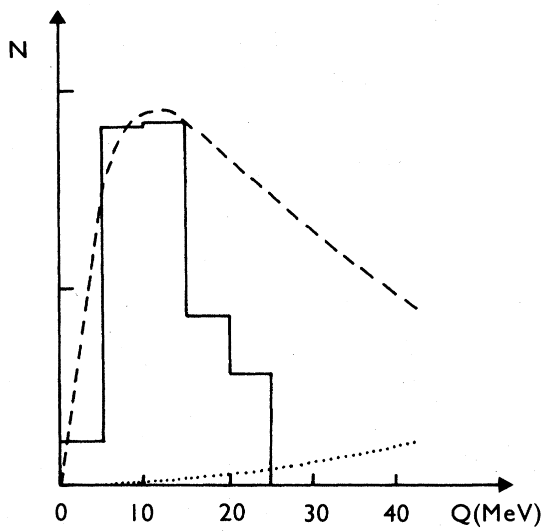

FIG. 2. Experimental results for the spectrum of nn masses in $\pi^- d \to nn\gamma$ taken from Haddock *et al.* (Ref. 19). The absence of a tail at high $Q = |\mathbf{p}_{n1} - \mathbf{p}_{n2}|$ in the data is due to experimental acceptance. The dotted curve is phase space; the dashed curve takes into account nn final-state interactions.

$$G_3(E) = [M_\iota^0 - E - \Sigma_3(E)]^{-1} , \tag{2}$$

where for $E = M_\iota$ (M_ι and M_ι^0 are the mass and unperturbed mass of the ι)

$$\Gamma(\iota \to K\bar{K}\pi) = 2\,\mathrm{Im}\Sigma_3(M_\iota) = \frac{(2\pi)^7}{2M_\iota} \int \frac{d^3p_K}{2E_K} \frac{d^3p_{\bar{K}}}{2E_{\bar{K}}} \frac{d^3p_\pi}{2E_\pi} \delta^4(P_{K\bar{K}\pi} - P_\iota) \, |M|^2 , \tag{3}$$

where

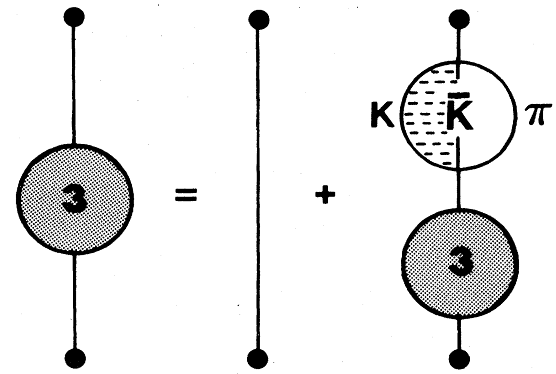

FIG. 3. A graphical representation of the equation $G_3 = G_0 + G_0 \Sigma_3 G_3$ for the full ι propagator with H_{D3}; the dashed lines represent $V_{K\bar{K}}$.

$$M = \langle K\bar{K}\pi \,|\, H_{D3}(0) \,|\, \iota \rangle . \tag{4}$$

Note that here $|K\bar{K}\pi\rangle$ is a *distorted* plane-wave state with four-momentum $P_{K\bar{K}\pi}$. With the choice of three-body coordinates

$$\mathbf{R}_{\mathrm{c.m.}} = \frac{m_K(\mathbf{r}_K + \mathbf{r}_{\bar{K}}) + m_\pi \mathbf{r}_\pi}{2m_K + m_\pi} , \tag{5}$$

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}} (\mathbf{r}_K - \mathbf{r}_{\bar{K}}) , \tag{6}$$

$$\boldsymbol{\lambda} = \frac{1}{\sqrt{6}} (\mathbf{r}_K + \mathbf{r}_{\bar{K}} - 2\mathbf{r}_\pi) , \tag{7}$$

the problem is reduced to three one-body problems: center-of-mass motion, free motion in $\boldsymbol{\lambda}$ with mass

$$m_\lambda = \frac{3m_K m_\pi}{2m_K + m_\pi} ,$$

and motion in ρ with mass $m_\rho = m_K$ in the potential $V_{K\bar{K}}(\sqrt{2}\rho)$. With $\mathbf{P}_{K\bar{K}\pi}$, $\mathbf{p}_\rho$, and $\mathbf{p}_\lambda$ the momenta conjugate to $\mathbf{R}_{\mathrm{c.m.}}$, $\boldsymbol{\rho}$, and $\boldsymbol{\lambda}$,

$$\langle K\bar{K}\pi; \mathbf{P}_{K\bar{K}\pi}, \mathbf{p}_\lambda, p_\rho l_\rho m_\rho \,|\, H_{D3}(0) \,|\, \iota(\mathbf{P}_\iota) \rangle = \int d^3P'_{K\bar{K}\pi} d^3p'_\lambda d^3p'_\rho \langle K\bar{K}\pi; \mathbf{p}_{K\bar{K}\pi}, \mathbf{p}_\lambda, p_\rho l_\rho m_\rho \,|\, (K\bar{K}\pi)_{\mathrm{free}}; \mathbf{P}'_{K\bar{K}\pi}, \mathbf{p}'_\lambda, \mathbf{p}'_\rho \rangle \times \langle (K\bar{K}\pi)_{\mathrm{free}}; \mathbf{P}'_{K\bar{K}\pi}, \mathbf{p}'_\lambda, \mathbf{p}'_\rho \,|\, H_{D3}(0) \,|\, \iota(\mathbf{P}_\iota) \rangle \tag{8}$$

by insertion of the complete set of plane-wave states $|(K\bar{K}\pi)_{\mathrm{free}}\rangle$; one can then show that

$$\langle K\bar{K}\pi; \mathbf{P}_{K\bar{K}\pi}, \mathbf{p}_\lambda, p_\rho l_\rho m_\rho \,|\, H_{D3}(0) \,|\, \iota(\mathbf{P}_\iota) \rangle = \frac{g_3}{(2\pi)^{9/2}} \psi^*_{p_\rho l_\rho m_\rho}(0) , \tag{9}$$

where $\psi_{p_\rho l_\rho m_\rho}(\boldsymbol{\rho})$ is the $\boldsymbol{\rho}$ wave function with energy $E_\rho = p_\rho^2/2m_K = m_{K\bar{K}} - 2m_K$ that is *asymptotically* a spherical wave with momentum p_ρ. The differential decay rate of the ι at any kinematic point within the Dalitz plot will therefore be enhanced by the ratio of $|\psi_{p_\rho 00}(0)|^2$ to its free value. [Only $l_\rho = 0$ contributes since $\psi_{p_\rho l_\rho m_\rho}(0) = 0$ for higher partial waves.] For our purposes it is quite sufficient to consider $V_{K\bar{K}}(r_{K\bar{K}})$ to be a square well of depth V_0 for $r_{K\bar{K}} < a$ (from Ref. 1, $V_0 \simeq 500$ MeV and $a \simeq 1$ fm), in which case

$$\left| \frac{\psi_{p_\rho 00}(0)}{[\psi_{p_\rho 00}(0)]_{\mathrm{free}}} \right|^2 = \frac{E_\rho + V_0}{E_\rho + E_B f(E_\rho)} \equiv d(E_\rho) , \tag{10}$$

where

$$f(E_\rho) = \frac{V_0}{E_B} \cos^2 \left\{ \left[\frac{E_\rho}{V_0} + 1 \right]^{1/2} \left[\frac{\pi}{2} + \left[\frac{E_B}{V_0} \right]^{1/2} \right] \right\} \to 1 \tag{11}$$

for E_ρ small with respect to $(E_B V_0)^{1/2}$. In obtaining this result we have eliminated the range parameter a in favor of the binding energy E_B of the weakly bound $K\bar{K}$ system. We note that as $E_\rho \to 0$ the enhancement factor (10) becomes V_0/E_B, which is a factor of the order of 50. Figure 4 shows the resulting Dalitz plot and its $m_{K\bar{K}}$ and $m_{K\pi}$ projections in comparison to the data. Note that the full width of the ι has been enhanced by $V_{K\bar{K}}$ by about a factor of 4 so one can maintain the prejudice that g_3 should be small even though experimentally $\Gamma_\iota \simeq 100$ MeV.

Thus we conclude that the observed width and Dalitz plot for the decay $\iota \to K\bar{K}\pi$ are consistent with a primary phase-space distribution for $K\bar{K}\pi$ which has been distorted by $K\bar{K}$ final-state interactions expected in the $K\bar{K}$ molecule picture of the δ and S^* scalar mesons. Moreover, the primary coupling (1) may arise naturally via the chain $\iota \to \delta_2 \pi \to K\bar{K}\pi$ where δ_2 is the broad ${}^3P_0$ quark-model state predicted in Ref. 13 (Ref. 21). In this case the decay $\iota \to \eta\pi\pi$ would be expected to proceed with the same primary strength g_3 as $\iota \to K\bar{K}\pi$ (the $\delta_2 K\bar{K}$ and $\delta_2 \eta\pi$ couplings are equal). The absence of final-state interactions means that this decay should eventually be seen in a uniformly populated Dalitz plot with about a factor of 4 smaller rate.

B. Primary $K^*\bar{K}$ + c.c. decay

While the above observations alone may suffice to alleviate "the puzzle no one likes," thereby lifting a dark cloud surrounding the ι (and supporting the $K\bar{K}$ molecule picture of the δ and S^*) we would like to consider a

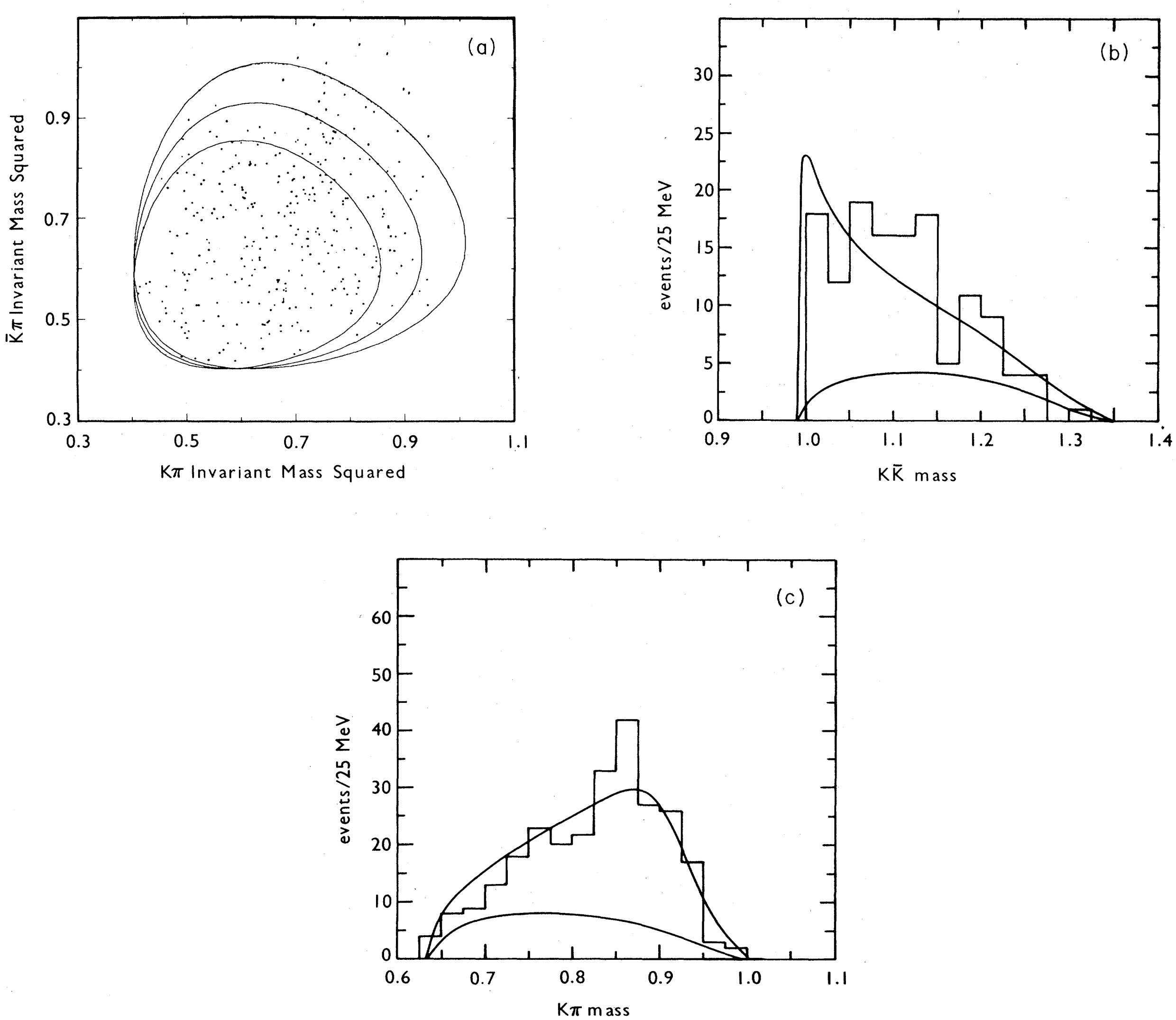

FIG. 4. Dalitz-plot distribution from a Monte Carlo calculation of "primary phase-space" decay and its comparison to the experimental $K\bar{K}$ and $K\pi + \bar{K}\pi$ histogram projections. (a) Dalitz plot, (b) $M_{K\bar{K}}$, and (c) $M_{K\pi}$. The appropriate experimental cuts have been made, but the curves are not corrected for acceptances. A 15% [30%] background has been used in (a) [in (b) and (c)] as indicated by the signal to background ratio seen in Mark III [Crystal Ball] (see Refs. 4).

second possibility. We are less capable of rigorously deducing the consequences of this possibility, but we raise it because it is more in accord with the usual prejudice favoring dominance of quasi-two-body decay modes. This second possibility is that the primary decay of the ι is via the two-body intermediate states $K^*\bar{K}$ + c.c. The general principle in favor of such a cascade decay scheme is enforced in this case by the existence of what should be a reliable prediction that if the ι is a $2\,^1S_0$ $q\bar{q}$ state, it should decay to $K^*\bar{K}$ + c.c. with a width comparable to that observed. [Although the ι is close to $K^*\bar{K}$ threshold, and the decay is in a P wave, the predicted width (see Ref. 2) remains substantial: compare to the SU(3) analog decay $\pi(1300)\to\rho\pi$ which has a measured width of several hundred MeV.] Of course, as long as the nature of the ι remains moot, this particular prediction can only be used as a check on the $q\bar{q}$ content of the ι, but nevertheless the prejudice in favor of a two-body intermediate state can be entertained in a variety of pictures and it is obviously interesting to ask whether it is also consistent with the data.

In the absence of $K\bar{K}$ final-state interactions, the answer is that given originally:[4] the $\iota\to K\bar{K}\pi$ Dalitz plot is not compatible with simple $K^*\bar{K}$ + c.c. dominance. We believe, however, that the distorting effect of $V_{K\bar{K}}$ is so strong that it can substantially modify a primary $K^*\bar{K}$ + c.c. Dalitz plot to look like the data. In this case we expect that the $K\bar{K}$ interaction does not grossly modify the undisturbed width for the simple two-body decay $\iota\to K^*\bar{K}$ + c.c.; however, it can redistribute the events.

Before providing substantiation for this expectation, we will describe the physical picture which we see underlying this effect. If the decay $K^*\to K\pi$ were extremely slow, then for all practical purposes the outgoing particles from the ι decay would be a $K^*\bar{K}$ or $\bar{K}^*K$ (which we are assuming are noninteracting) and the Dalitz plot for $K\bar{K}\pi$ would indeed exhibit extremely narrow K^* bands. However, we believe that the free K^* lifetime is not sufficiently long for this approximation to be valid here: the produced K^*, for example, finds itself in the neighborhood of the $\bar{K}$ with a kinetic energy of 15 MeV and as a result the $K\pi$ plane wave into which the K^* is decaying will *become distorted by the presence of the* $\bar{K}$. This distortion now enhances not the decay rate of the ι (which has already decayed) but rather the decay rate of the K^*. We conclude that $V_{K\bar{K}}$ will distort the K^* bands by *producing an effective* K^* *width which will depend on position within the Dalitz plot* and which will become much broader in the low-$K\bar{K}$-mass sector of the Dalitz plot. Such a phenomenon is not unfamiliar; it occurs in neutron decay in a nucleus where the Dalitz plot can be dramatically affected by distortion of the electron plane wave. An analogous situation occurs for the lifetimes of atomic levels which can be substantially modified if the atoms find themselves in an unusual environment (e.g., inside a crystal).

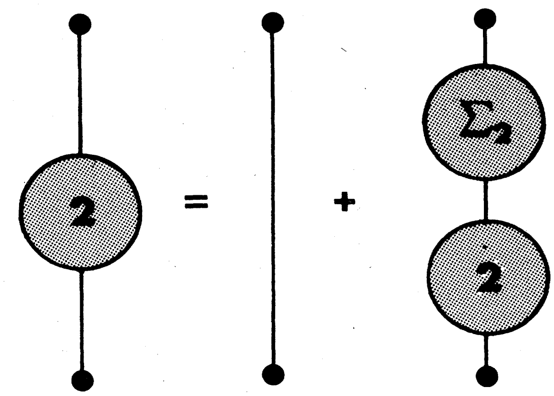

FIG. 5. A graphical representation of the equation $G_2=G_0+G_0\Sigma_2G_2$ for the full ι propagator with H_{D2}; Σ_2 is defined in Fig. 6.

We have already warned that we cannot rigorously substantiate this picture. However, we can make it plausible. For simplicity, consider the analogous case of a state R decaying according to the chain $I\to AR$, $R\to BC$ where all the particles are scalar. (This simplification avoids the complications of the K^* spin and also of the interference between the K^* and $\bar{K}^*$ bands in the ι Dalitz plot, but it will not alter the basic effect at issue here.) In Fig. 5 we depict the equation for the full (nonrelativistic) I propagator which may be solved to give in the I rest frame

$$G_2(E)=[M_I^0-E-\Sigma_2(E)]^{-1}\,, \tag{12}$$

where now at the mass M_I of I

$$\Gamma(I\to ABC)=2\,\mathrm{Im}\Sigma_2(M_I)\,, \tag{13}$$

where

$$\Sigma_2=\Sigma_2^0+\Sigma_2^1+\Sigma_2^2+\cdots \tag{14}$$

in which Σ_2^n represents the effect of the diagram with n virtual R decays inside the I virtual decay (see Fig. 6). In the case where $V_{AB}=0$, $\Sigma_2(E)$ can be calculated exactly (since the momentum of A is unchanged by the virtual R decays) to give in the I rest frame

$$\Gamma(I\to(ABC)_{\rm free})=(2\pi)^7\int\frac{d^3p_A}{2E_A}\frac{d^3p_R}{2E_R}\delta^3(\mathbf{p}_A+\mathbf{p}_R)\,|\langle AR\,|H_{D2}(0)|\,I\rangle|^2W(E_R,M_I-E_A)\,, \tag{15}$$

where H_{D2} is the analog to H_{D3} and

$$W(E_R,E)=\frac{\Gamma_R(E)/2\pi}{(E_R-E)^2+\frac{1}{4}\Gamma_R(E)^2}\,. \tag{16}$$

By considering the expression for Γ_R it can be shown that (15) is just the integral over three-body phase space of the square of the expected full $I\to ABC$ amplitude going through a resonance R.

For $V_{AB}\neq 0$ we have not been able to calculate Σ_2. We have, however, been able to extract information on its structure which suggests that the intuitive picture presented above is roughly valid. Imagine that we are in a sector of the Dalitz plot which is far from the (possibly distorted) resonance band due to the intermediate state R. Based on (15) and (16) we see that the effect of the resonance is then independent of the $\Gamma_R{}^2$ appearing in the denominator of (16); in this region (as can be verified directly), we can approximate the exact result by using

$$\Sigma_2 \simeq \Sigma_2^1 \, . \tag{17}$$

We propose to make this same approximation in the case $V_{AB} \neq 0$ and to identify the factor analogous to $\Gamma_R(E)$ in the numerator of (16) with the distorted width of R due to the presence of A and the potential V_{AB}. Carrying out this calculation indeed gives

$$\Gamma_R^{\text{eff}}(M_{BC}, M_{AB}) \simeq \Gamma_R(M_{BC})[d(E_\rho)]^{1/2} \, , \tag{18}$$

where $d(E_\rho)$ is defined in (10) and where

$$E_\rho \simeq M_{AB} - M_A - M_B \, . \tag{19}$$

Note that the Dalitz plot in this case has a very different structure from a normal plot with an intermediate resonance R. If V_{AB} were zero then every horizontal slice through the Dalitz plot would reveal the same Breit-Wigner resonance shape in M_{BC} with the free resonance width Γ_R, and thus a standard Argand diagram in this variable. If $V_{AB} \neq 0$, then the density of the Dalitz plot will depend on both M_{BC} and M_{AB}. Slices of the Dalitz plot with fixed M_{AB} now show resonant behavior, but with a resonance width Γ_R^{eff} which depends on M_{AB} as indicated by (19): Γ_R^{eff} will approximate the free Γ_R only for large M_{AB} where V_{AB} can be ignored.

Figure 7 shows the Dalitz plot and its $M_{K\bar{K}}$ and $M_{K\pi}$ projections assuming formulas analogous to (18) and (19) for ι decay. Thus, this second (more attractive but more difficult to study) hypothesis also seems consistent with the observed properties of the ι. Note that with more data on $\iota \to K\bar{K}\pi$ at high $K\bar{K}$ mass the two hypotheses we have entertained can be distinguished. Of course the actual situation may correspond to a coherent mixture of the "primary $K^*\bar{K} + \bar{K}^*K$" and "primary phase space" mechanisms. Figure 8 shows, as an example, the Dalitz plot, $K\pi$, and $K\bar{K}$ projections of such a mixture (with a relative probability of "$K^*\bar{K} + \bar{K}^*K$" to "phase space" of two to one) which seems to correspond rather well to the Dalitz plot and $K\pi$ projection of the recent data of Fig. 1(a).

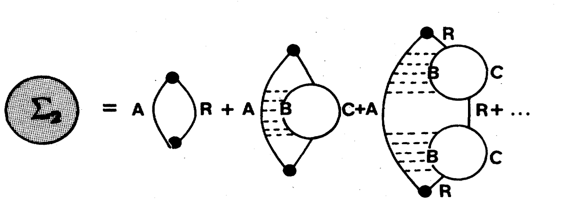

FIG. 6. A graphical representation of the equation $\Sigma_2 = \Sigma_2^0 + \Sigma_2{}^1 + \Sigma_2{}^2 + \cdots$; the dashed lines represent V_{AB}.

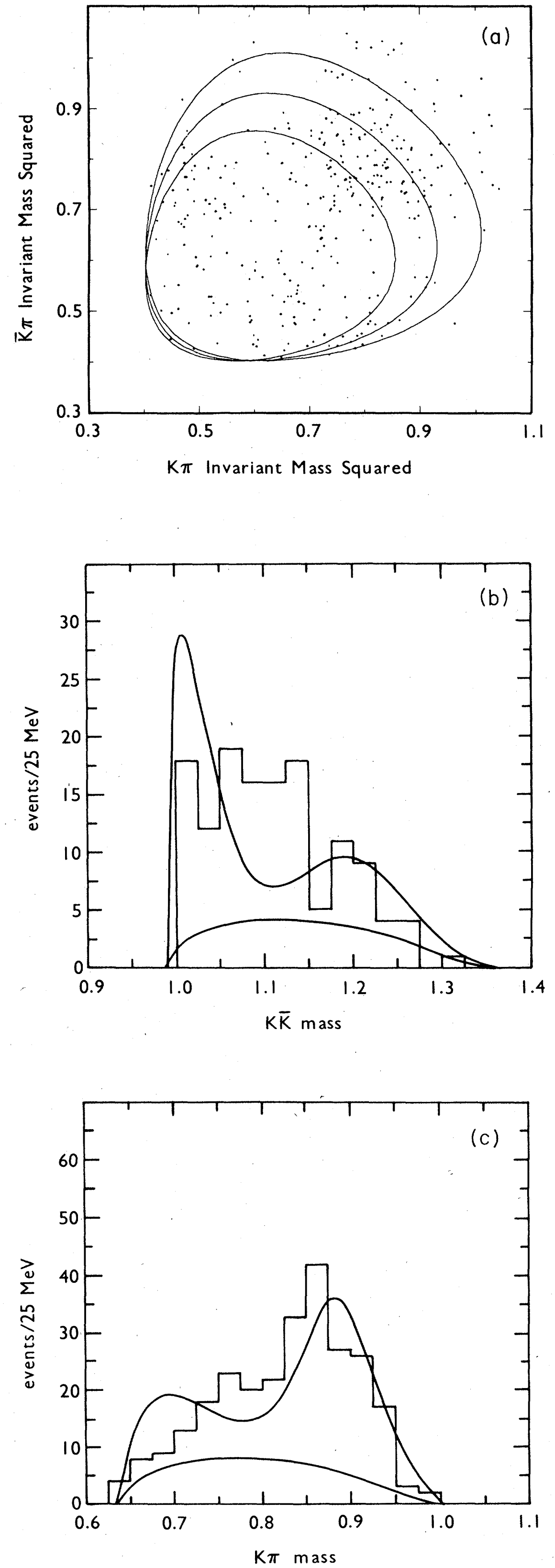

FIG. 7. As in Fig. 4, but for "primary $K^*\bar{K} + \bar{K}^*K$ decay." See also the data in Fig. 8, which are better fit by this mechanism.

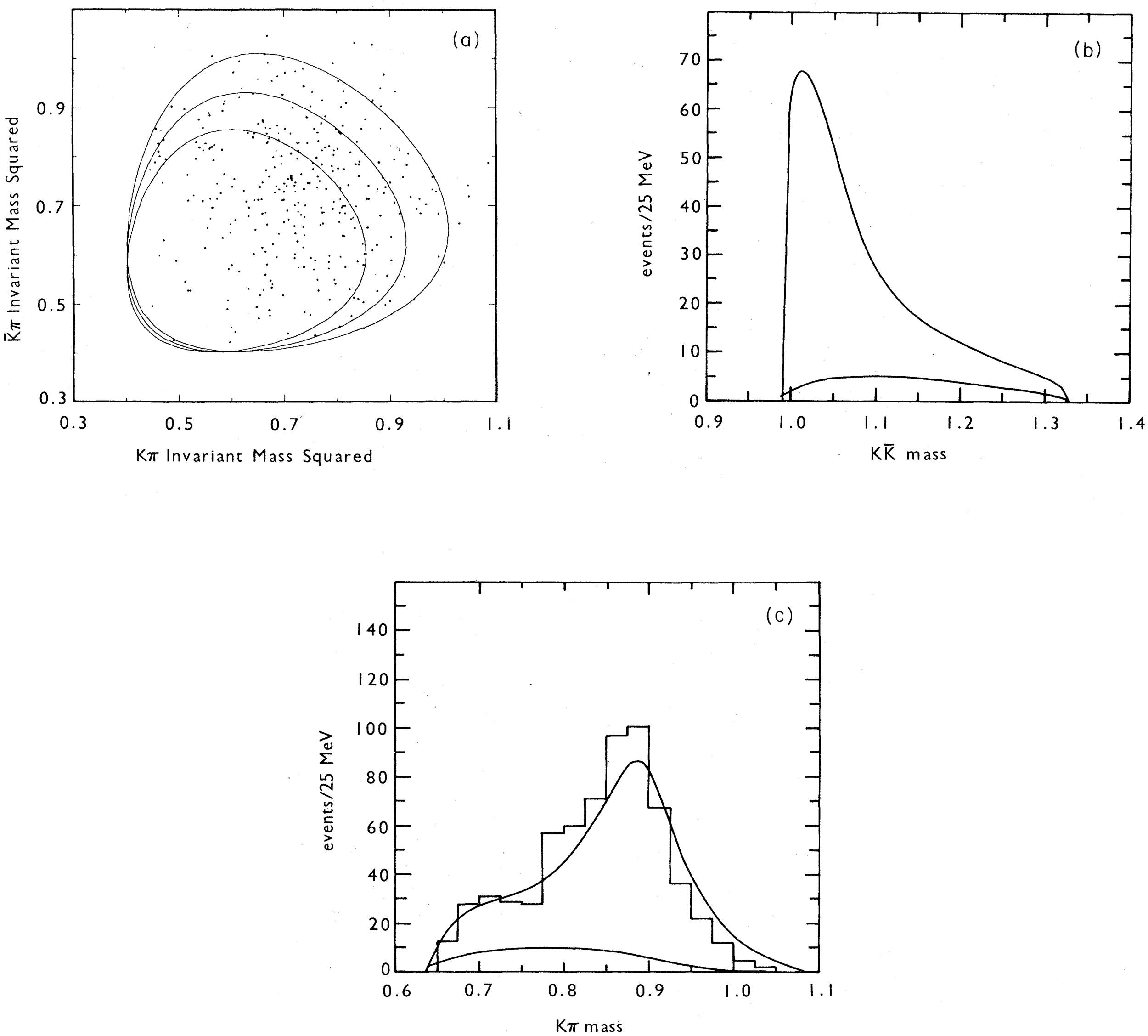

FIG. 8. Dalitz-plot distribution from a Monte Carlo calculation of a mixture of "primary $K^*\bar{K}+\bar{K}^*K$" and "primary phase space" in the ratio of two to one and its comparison to an experimental $K\pi+\bar{K}\pi$ histogram projection. The related $K\bar{K}$ histogram is not published. (See note added in proof.) (a) Dalitz plot [compare to Fig. 1(a)], (b) $M_{K\bar{K}}$, (c) $M_{K\pi}$. The appropriate experimental cuts have been made, but the curves are not corrected for acceptances. A 15% incoherent background has been used as seen in the Mark III ι data (see Ref. 4).

III. CONCLUSIONS

We have shown that the puzzling absence of the $\eta\pi\pi$ decay of the ι relative to its $K\bar{K}\pi$ decay is naturally explained in the $K\bar{K}$ molecule interpretation of the δ and S^*. Our explanation, in the first place, satisfies us that there are no outstanding impediments to accepting the ι as a bona fide resonance. The example of the ι may also serve as a warning against the use of the δ and S^* as simple resonances in isobar analyses.

Finally, of course, we hope that our clarification of the decay of the ι may help in the elucidation of its nature. Indeed, several points seem immediately worth making. The first is that the $K\bar{K}\pi$ dominance of the ι decay modes no longer militates against the ι being an SU(3) singlet (and therefore a glueball candidate). The second is that if the ι decay is really dominated by $K^*\bar{K}$ + c.c., as seems possible, then the properties of the ι would all seem to be consistent with those of the mainly ω-like member of the $2\,^1S_0$ pseudoscalar nonet. This seems to us to make the existence or nonexistence of the reported $\eta(1275)$ and $\iota\to\rho\gamma$ decay mode meson critical for deciding whether the ι is an ordinary meson or something new.

Noted added in proof. After submission of this manuscript we discovered that the $K\bar{K}$ projection of the Mark III data missing from Fig. 8 is available (see J. Richman in Ref. 4). This data is very well fit by the theoretical curve in Fig. 8(b). See J. Weinstein, University of Toronto Report No. UTPT-85-27, 1985 (unpublished).

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[2]This problem is emphasized in S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985) in the context of the elementary pion emission model, and in R. Kokoski and N. Isgur, University of Toronto Report No. UTPT-85-05 in the flux tube breaking and $^3P_0$ models. That a problem exists is apparent from earlier work, but this difficulty was normally either dismissed or not discussed.

[3]We have used the mass $M_{Q_A}=M_{Q_B}=\frac{1}{2}(M_{Q_1}+M_{Q_2})$ following from the Q_A-Q_B mixing angle of 45°.

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[13]See, for example, Godfrey and Isgur in Ref. 2.

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[17]This is not quite true. W. Palmer and S. Pinsky, Phys. Rev. D **27**, 2219 (1983), have pointed out that this situation could arise via an accidental cancellation between $\iota\to\delta\pi\to\eta\pi\pi$ and $\iota\to\eta\epsilon\to\eta\pi\pi$.

[18]We are grateful to R. H. Dalitz for bringing this example to our attention.

[19]For an early experiment, see R. H. Phillips and K. M. Crowe, Phys. Rev. **96**, 484 (1954); a higher resolution spectrum which shows the peaking more clearly is in R. P. Haddock *et al.*, Phys. Rev. Lett. **14**, 318 (1965).

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