Model-independent analysis of hadronic decays of J/ψ and $\eta_c(2980)$

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Implications of broken flavor-SU(3) symmetry are studied for J/ψ and η_c decays to two- and three-meson final states. Nonet-symmetry breaking and pseudoscalar-meson mixing are discussed using recent measurements of $J/\psi \rightarrow$vector+pseudoscalar branching ratios by the Mark III collaboration. Detailed fits are presented to determine phenomenological parameters which describe the various mesonic decays of the J/ψ. We conclude that the η and η' can be mixed with $\iota(1440)$, and predict $B(J/\psi \rightarrow \iota\omega)=(0.75\pm0.12)\times10^{-3}$ and $B(J/\psi\rightarrow\iota\phi)=(0.09\pm0.02)\times10^{-3}$. In $J/\psi \rightarrow$pseudoscalar + pseudoscalar decays, electromagnetic and mass-breaking effects are found to be of similar importance. Different SU(3)-breaking patterns are observed in $J/\psi\rightarrow PV$ and $\eta_c \rightarrow VV$ decays.

I. INTRODUCTION

The numerous decay modes of the J/ψ and η_c allow for a detailed study of meson properties and low-energy hadron dynamics. A particular advantage of these processes is that the initial state, to a very good approximation, is a flavor-SU(3) singlet. (Henceforth, we shall omit the term flavor in characterizing the symmetry group.) To the extent that SU(3) is an approximate symmetry of low-energy hadron interactions, much information on the final-state particles can be obtained by studying the systematics of these decays. The implication of SU(3) symmetry to J/ψ and η_c decays into mesons has been studied previously in the literature.[1] Other theoretical analyses can be found in Ref. 2. The Mark III Collaboration has recently measured with high statistics the branching ratios for all $J/\psi\rightarrow PV$ and $J/\psi\rightarrow PP$ decay modes[3] (we shall denote scalar, pseudoscalar, vector, and tensor mesons by S, P, V, and T, respectively), and for ten decay modes of the η_c into VV and PPP.[4] Given this dramatic improvement in the data, it is now an opportune time to reexamine these issues.

The hadronic decays of the J/ψ and η_c are suppressed by the Okubo-Zweig-Iizuka (OZI) rule[5] and their branching ratios are of the order of 10^{-3}. Furthermore, there exist processes that are doubly OZI suppressed, for example, $J/\psi\rightarrow\phi f$. In applying the group-theoretical analysis to the mesonic final states, the existence of processes of the latter type can be viewed as a breaking nonet symmetry. The $J/\psi\rightarrow VT$ decays ωf, $\omega f'$, ϕf, and $\phi f'$, and the $J/\psi\rightarrow PV$ decays $\omega\pi^0$ and $\phi\pi^0$ will be used to set limits on nonet-symmetry breaking. For decays involving the pseudoscalars, we begin by invoking the standard mixing scheme for η and η' which implies definite relations between the $J/\psi\rightarrow\eta V$ and $J/\psi\rightarrow\eta' V$ decay rates. Inconsistency between the data and the predictions of this simple model of pseudoscalar mixing reveals the presence of additional components in the η' wave function. In the decays of the J/ψ, the effects of electromagnetic and mass breaking of the SU(3) symmetry are found to be of similar importance. This is particularly evident in the decays $J/\psi\rightarrow PP$ which should not occur if SU(3) symmetry were exact. Furthermore, the SU(3)-breaking patterns differ between J/ψ and η_c decays, as seen in $\eta_c\rightarrow VV$ and $J/\psi\rightarrow PV$.

In Sec. II the formalism necessary in order to write down the implications of unbroken SU(3) symmetry is reviewed, as well as the effects of SU(3) symmetry breaking, for hadronic J/ψ and η_c decays. In Sec. III we use the recent Mark III measurements to set upper limits on nonet-symmetry breaking, discuss the mixing of pseudoscalar mesons, and measure the effect of mass and electromagnetic breakings of SU(3). A summary and discussion of our results are given in Sec. IV.

II. DESCRIPTION OF THE FORMALISM

Our notation is as follows.[6] The pseudoscalar and vector-meson octets are denoted by P^a and V^a, respectively ($a=1,\ldots,8$). For example,

$$\pi^{\pm}=\frac{1}{\sqrt{2}}(P^1\mp iP^2),\quad \pi^0=P^3\ ,$$
$$K^{\pm}=\frac{1}{\sqrt{2}}(P^4\mp iP^5),\quad K^0=\frac{1}{\sqrt{2}}(P^6-iP^7)\ ,$$
$$\bar{K}^0=\frac{1}{\sqrt{2}}(P^6+iP^7),\quad \eta_8^0=P^8\ .$$

It is often useful to add the SU(3)-singlet state η_1^0 to the octet P by defining $P^0\equiv\eta_1^0$, thereby creating a nonet, $P=(P^0,P^a)$. If pseudoscalar glueballs and radially excited states are ignored, then the physical states η,η' are related to η_8^0 and the SU(3)-singlet state, η_1^0, via the usual mixing formulas:[7]

$$\begin{aligned}\eta_8^0&=\eta\cos\theta_P+\eta'\sin\theta_P\ ,\\ \eta_1^0&=-\eta\sin\theta_P+\eta'\cos\theta_P\ .\end{aligned}\tag{2.1}$$

For the pseudoscalar octet, diagonalization of the mass-squared matrix yields $\theta_P\approx-10°$, whereas for the vector-meson octet (replacing η,η' by ϕ,ω, respectively) θ_V is close to the ideal mixing value of $\tan\theta_V=(\frac{1}{2})^{1/2}$ which

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corresponds to ϕ being a pure $s\bar{s}$ state. We shall be interested in investigating the possible shortcomings of such a simple mixing scheme [Eq. (2.1)]. To generalize Eq. (2.1), we write[8]

$$P=X_P[(\tfrac{1}{3})^{1/2}\eta_8^0+(\tfrac{2}{3})^{1/2}\eta_1^0] +Y_P[-(\tfrac{2}{3})^{1/2}\eta_8^0+(\tfrac{1}{3})^{1/2}\eta_1^0]+Z_P\eta_0^0 \quad (2.2)$$

for $P=\eta,\eta'$ and any other isoscalar pseudoscalar which may be relevant to the η-η' mixing; η_0^0 is the pseudoscalar state (or states) orthogonal to η_8^0 and η_1^0. That is, $X_P{}^2$ measures the nonstrange quark content of P and $Y_P{}^2$ measures the strange quark content of P. To the extent that $X_P{}^2+Y_P{}^2<1$ for $P=\eta,\eta'$, there is deviation from the simple picture of Eq. (2.1).

The key fact in the subsequent analysis is that, to an extremely good approximation, the J/ψ (or η_c) is an SU(3) singlet. We first consider the decay $J/\psi\to H_1H_2$, where H_1 and H_2 are mesons. To derive the consequences of flavor-SU(3) symmetry, the SU(3) multiplets containing H_1 and H_2 are to be combined in an SU(3)-invariant way. Given two octets O_i and two singlets S_i, the most general interaction term is

$$\mathscr{L}_{\rm int}=\psi(g_8O_1^aO_2^a+g_1S_1S_2)\ , \quad (2.3)$$

where a sum over $a=1,\ldots,8$ is implied. Experimentally, the OZI rule is obeyed to a very good approximation; i.e., in quark language, $q\bar{q}$ annihilation processes are suppressed in meson decays. The OZI rule is easily implemented in Eq. (2.3) by setting $g_8=g_1$; i.e., the full nonet symmetry is exhibited. One further restriction on the couplings can be obtained by applying charge-conjugation (C) invariance. Under C, a meson nonet N^a transforms[9] as $N^a\to C\epsilon^aN^a$ (no sum over a) where C is the C parity of N^3 (the π^0-like state of the nonet), $\epsilon^2=\epsilon^5=\epsilon^7=-1$ and all other $\epsilon^a=+1$. For convenience, we shall regard C as the C parity of the nonet. Since the J/ψ has $C=-1$, C invariance implies that the two octets (or two singlets) which appear in Eq. (2.3) must have opposite C parities. [Note that although $J/\psi\to\pi^0\pi^0$ is forbidden explicitly by ordinary C invariance, $J/\psi\to K^0\bar{K}^0$ is forbidden by C invariance only in the SU(3) limit.] For η_c ($C=+1$) decays, the two octets must have the same C parity.

We now turn to the question of SU(3)-breaking effects. SU(3) breaking in $J/\psi\to H_1H_2$ can be simulated by constructing an SU(3)-invariant amplitude involving three nonets and by choosing one of the nonets (called a "spurion" nonet) to point in a fixed direction of SU(3) space particular to the desired breaking. Two types of SU(3) breaking will be considered. First, SU(3) is broken because $m_s\neq m_u,m_d$ ($m_u=m_d$ is assumed; violations of this inequality as deduced from current-algebra calculations[10] will have little effect on our results). By writing the quark mass term as

$$m_d(\bar{d}d+\bar{u}u)+m_s\bar{s}s=m_0\bar{q}q+(\tfrac{1}{3})^{1/2}(m_d-m_s)\bar{q}\lambda_8q\ , \quad (2.4)$$

where $q=(u,d,s)$ and $m_0=\frac{1}{3}(2m_d+m_s)$ is the average quark mass, we see that this SU(3) breaking corresponds to a spurion M, pointing in the 8 direction; i.e., $M^a=\delta^{a8}$. Second, electromagnetic effects violate SU(3) invariance since the photon coupling to quarks is proportional to the electric charge:

$$\tfrac{2}{3}\bar{u}\gamma_\mu u-\tfrac{1}{3}\bar{d}\gamma_\mu d-\tfrac{1}{3}\bar{s}\gamma_\mu s=\tfrac{1}{2}\bar{q}\gamma_\mu\left[\lambda_3+\frac{\lambda_8}{\sqrt{3}}\right]q\ . \quad (2.5)$$

It follows from Eq. (2.5) that electromagnetic breaking can be simulated by a spurion E, given by $E^a=\delta^{a3}+(\frac{1}{3})^{1/2}\delta^{a8}$. Let us apply these results to $J/\psi\to H_1H_2$ decays. All possible SU(3)-invariant couplings involving three nonets need to be considered. The results are

$$\mathscr{L}_{\rm int}=\psi\left[g_Sd_{abc}O_1^aO_2^bO_3^c+g_Af_{abc}O_1^aO_2^bO_3^c +(\tfrac{2}{3})^{1/2}\sum_{\substack{i,j\neq k\\ i<j}}C_{ijk}O_i^aO_j^aS_k+(\tfrac{2}{3})^{1/2}fS_1S_2S_3\right]. \quad (2.6)$$

In Eq. (2.6), f_{abc} and d_{abc} are the two SU(3)-invariant tensors (respectively, antisymmetric and symmetric). The independent coupling constants are g_S, g_A, C_{ijk}, and f. If nonet symmetry is imposed, tremendous simplification occurs. Only two independent couplings remain: g_S and g_A. In fact, as shown below, C invariance always eliminates one of these two couplings. The nonet-symmetric interaction is obtained by defining $N^a=(S,O^a)$, $a=1,\ldots,8$, $d_{0ab}=(\frac{2}{3})^{1/2}\delta_{ab}$ and $f_{0ab}=0$. Then

$$\mathscr{L}_{\rm int}^{\rm nonet}=\psi(g_Sd_{abc}N_1^aN_2^bN_3^c+g_Af_{abc}N_1^aN_2^bN_3^c)\ .$$

This corresponds to setting $C_{ijk}=f=g_S$ in Eq. (2.6).

C invariance can be imposed by using the following properties:[9]

$$f_{abc}\epsilon^a\epsilon^b\epsilon^c=-f_{abc}\ , \qquad d_{abc}\epsilon^a\epsilon^b\epsilon^c=d_{abc} \quad (2.7)$$

(no summation over the indices), where ϵ^a was defined below Eq. (2.3). This immediately leads to the following rule. For J/ψ or η_c (with charge-conjugation number C) decaying into three nonets (with charge conjugation C_i):

$$\text{If } C_1C_2C_3=\begin{cases}-C & \text{then } g_S=0\ ,\\ +C & \text{then } g_A=0\ .\end{cases} \quad (2.8)$$

[If Eq. (2.6) is used, then in addition to Eq. (2.8), $C_{ijk}=f=0$ in the case $C_1C_2C_3=-C$.] So far, the above results are general; i.e., they can be used to deduce the implications of SU(3) symmetry for $J/\psi\to H_1H_2H_3$ decays. However, we shall apply these results to the study of SU(3)-breaking effects by taking O_3 to be a spurion octet. Note that the implications of C invariance deduced in Eq. (2.8) can still be used if the C parity of the spurion octet is taken to be $C_3=+1$.

As an example, the $J/\psi\to PP$ calculation is considered in detail. First, by C invariance there is no SU(3)-invariant coupling. However, there are contributions when SU(3)-breaking effects are taken into account. The effective Lagrangian, which is C invariant, may be written as follows:

$$\mathscr{L}_{\rm eff}=f_{abc}\psi^\mu P^a \overleftrightarrow{\partial}_\mu P^b(g_M M^c+g_E E^c)\ , \tag{2.9}$$

where M^c and E^c are the spurion fields defined earlier. Two new couplings, g_M and g_E, have been introduced to parametrize the SU(3) breaking. For $J/\psi\to VV$ decays, the group-theoretical structure of the interaction is identical. An interaction Lagrangian of a form similar to that of Eq. (2.9) may be used for $\eta_c\to PV$ decays, with $g_E=0$ since C invariance forbids a first-order electromagnetic decay of the η_c (i.e., a C-even $c\bar{c}$ state annihilating into one virtual photon is prohibited). By expanding Eq. (2.9) in terms of component fields, we obtain the results given in Table I.

TABLE I. Relative coupling strengths for the decays $J/\psi\to PP$. For convenience, we use $K_L^0=(K^0-\bar{K}^0)/\sqrt{2}$, $K_S^0=(K^0+\bar{K}^0)/\sqrt{2}$. The table can also be used for $J/\psi\to VV$ or $\eta_c\to PV$ decays by appropriate change in labeling. The couplings g_M and g_E correspond to SU(3) breaking due to $m_s\neq m_d,m_u$ and electromagnetism, respectively. Note that $g_E=0$ for η_c decays.

Decay mode $J/\psi\to X$	Coupling constant g_M	g_E
$\pi^+\pi^-$	0	1
K^+K^-	$\frac{\sqrt{3}}{2}$	1
$K_S^0K_L^0$	$\frac{\sqrt{3}}{2}$	0

We examine next the decay $J/\psi\to PV$. At first, let us not impose the additional constraint of nonet symmetry (note that this issue did not arise above because $f_{ab0}=0$). Equation (2.3) is used for the SU(3)-symmetric interaction and Eq. (2.6) with spurion fields $O_3^a=M^a$ and $O_3^a=E^a$ (and $S_3=0$) is used for the two SU(3)-breaking terms, as discussed above. In the following we shall denote g_S, C_{132}, and C_{231} by $g_{M,88}$, $g_{M,81}$, and $g_{M,18}$ (mass breaking terms) or $g_{E,88}$, $g_{E,81}$, and $g_{E,18}$ (electromagnetic breaking terms), respectively.

Assuming that nonet symmetry holds to a reasonable approximation, we can take g_8-g_1 to be small and calculate SU(3) breaking to first order in the parameters[11] g_1-g_8, g_M, and g_E (second-order effects are neglected by taking $g_M\equiv g_{M,88}=g_{M,81}=g_{M,18}$, and similarly for g_E). The result is displayed in Table II. The absence of the (isospin-violating) decay $J/\psi\to\phi\pi^0$, which would occur if $g_{E,88}\neq g_{E,18}$, confirms the above approximation.

We now turn to the three-body decays of the J/ψ and η_c. In the limit of SU(3) invariance, the interaction is given by Eq. (2.6). As mentioned earlier, if nonet symmetry is assumed (i.e., the OZI rule is implemented) then only one independent coupling remains—either g_S or g_A, depending on the C parities involved. Thus, in the SU(3)-symmetry limit, all three-body final-state decay amplitudes are related as shown in Tables III and IV. If symmetry breaking is included, the formalism becomes much more complicated and less predictive. Using the spurion technique described above, we must look for SU(3)-invariant combinations of four nonets. It is convenient to use matrix notation. We define

$$\mathbf{P}=\frac{P^a\lambda_a}{\sqrt{2}} \tag{2.10}$$

and, to invoke nonet symmetry, $\lambda_0=(\frac{2}{3})^{1/2}\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix. The interaction given by Eq. (2.6) is modified as follows. Depending on the C parities of the three octets $\mathbf{O}_i$ $(i=1,2,3)$, one of two possible interactions must be used:[12]

$$\begin{aligned}\mathscr{L}_{\rm int}=\psi\{&g_A\,{\rm Tr}[\mathbf{O}_1(\mathbf{O}_2\mathbf{O}_3-\mathbf{O}_3\mathbf{O}_2)]+g_{A_1}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_2\mathbf{O}_3\mathbf{O}_1-\mathbf{O}_1\mathbf{O}_3\mathbf{O}_2)]\\&+g_{A_2}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_3\mathbf{O}_2\mathbf{O}_1-\mathbf{O}_1\mathbf{O}_2\mathbf{O}_3)]+g_{A_3}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_3\mathbf{O}_1\mathbf{O}_2-\mathbf{O}_2\mathbf{O}_1\mathbf{O}_3)]\}\ ,\end{aligned} \tag{2.11}$$

$$\begin{aligned}\mathscr{L}_{\rm int}=\psi\{&g_S\,{\rm Tr}[\mathbf{O}_1(\mathbf{O}_2\mathbf{O}_3+\mathbf{O}_3\mathbf{O}_2)]+g_{S_1}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_2\mathbf{O}_3\mathbf{O}_1+\mathbf{O}_1\mathbf{O}_3\mathbf{O}_2)]\\&+g_{S_2}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_3\mathbf{O}_2\mathbf{O}_1+\mathbf{O}_1\mathbf{O}_2\mathbf{O}_3)]+g_{S_3}{\rm Tr}[\mathbf{O}_4(\mathbf{O}_3\mathbf{O}_1\mathbf{O}_2+\mathbf{O}_2\mathbf{O}_1\mathbf{O}_3)]\\&+g_{S_4}{\rm Tr}(\mathbf{O}_1\mathbf{O}_2){\rm Tr}(\mathbf{O}_3\mathbf{O}_4)+g_{S_5}{\rm Tr}(\mathbf{O}_1\mathbf{O}_4){\rm Tr}(\mathbf{O}_2\mathbf{O}_3)\}\ .\end{aligned} \tag{2.12}$$

In Eq. (2.12), there is no need for a term ${\rm Tr}(\mathbf{O}_1\mathbf{O}_3){\rm Tr}(\mathbf{O}_2\mathbf{O}_4)$ due to the identity[13]

$$\sum{\rm Tr}[\mathbf{O}_4(\mathbf{O}_i\mathbf{O}_j\mathbf{O}_k+\mathbf{O}_k\mathbf{O}_j\mathbf{O}_i)]=\sum{\rm Tr}(\mathbf{O}_4\mathbf{O}_i){\rm Tr}(\mathbf{O}_j\mathbf{O}_k)\ , \tag{2.13}$$

where the sum runs over cyclic permutations of $i,j,k=1,2,3$. Adding interactions involving SU(3) singlets is straightforward. To invoke nonet symmetry, simply replace the octets above with nonets. In the matrix notation, the effect of charge conjugation is to transform $\mathbf{O}\to C\mathbf{O}^T$ where C is the C parity of the π^0-like state of the octet [$\lambda_a^T=\epsilon^a\lambda_a$ (no sum over a) has been used]. Then, Eq. (2.8) determines which of the above interactions to use. The desired SU(3) breaking can be simulated by taking $\mathbf{O}_4$ to be the spurion octet. Clearly, under the most general circumstances, symmetry breaking introduces too many parameters for a general analysis to be useful. However, in special cases, it might be possible to reduce the number of new parameters considerably and make the analysis manageable. For example, if we assume that the dominant interaction term of the decay $J/\psi\to PPP$ (where the three pseudoscalars are members of the same nonet) is

$$\mathscr{L}_{\rm eff}=f_{abc}\epsilon_{\mu\nu\alpha\beta}\psi^\mu\partial^\nu P^a\partial^\alpha P^b\partial^\beta P^c\ , \tag{2.14}$$

TABLE II. Relative coupling strengths for the decays $J/\psi \to PV$. The table can also be used for $\eta_c \to VV$, or $J/\psi \to VT$ decays by appropriate change in labeling. The pseudoscalar mixing angle θ_P has to be replaced by θ_V (θ_T). Note that $g_E=0$ for η_c decays.

Decay mode $J/\psi \to X$	Coupling constant g_8	g_1	g_M	g_E
$\pi^+\rho^-$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
$\pi^-\rho^+$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
$\pi^0\rho^0$	1	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
K^+K^{*-}	1	0	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
K^-K^{*+}	1	0	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$
$K^0\overline{K}^{*0}$	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
$\overline{K}^0K^{*0}$	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{2}{3}$
$\eta\phi$	$(\frac{2}{3})^{1/2}\cos\theta_P$	$(\frac{1}{3})^{1/2}\sin\theta_P$	$-\frac{2}{\sqrt{3}}[(\frac{2}{3})^{1/2}\cos\theta_P+(\frac{1}{3})^{1/2}\sin\theta_P]$	$-\frac{2}{3}[(\frac{2}{3})^{1/2}\cos\theta_P+(\frac{1}{3})^{1/2}\sin\theta_P]$
$\eta\omega$	$(\frac{1}{3})^{1/2}\cos\theta_P$	$-(\frac{2}{3})^{1/2}\sin\theta_P$	$\frac{1}{\sqrt{3}}[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$	$\frac{1}{3}[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$
$\eta\rho^0$	0	0	0	$[(\frac{1}{3})^{1/2}\cos\theta_P-(\frac{2}{3})^{1/2}\sin\theta_P]$
$\eta'\phi$	$(\frac{2}{3})^{1/2}\sin\theta_P$	$-(\frac{1}{3})^{1/2}\cos\theta_P$	$-\frac{2}{\sqrt{3}}[(\frac{2}{3})^{1/2}\sin\theta_P-(\frac{1}{3})^{1/2}\cos\theta_P]$	$-\frac{2}{3}[(\frac{2}{3})^{1/2}\sin\theta_P-(\frac{1}{3})^{1/2}\cos\theta_P]$
$\eta'\omega$	$(\frac{1}{3})^{1/2}\sin\theta_P$	$(\frac{2}{3})^{1/2}\cos\theta_P$	$\frac{1}{\sqrt{3}}[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P)]$	$\frac{1}{3}[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P]$
$\eta'\rho^0$	0	0	0	$[(\frac{1}{3})^{1/2}\sin\theta_P+(\frac{2}{3})^{1/2}\cos\theta_P]$
$\pi^0\phi$	0	0	0	0
$\pi^0\omega$	0	0	0	1

Eq. (2.11) simplifies immediately, i.e., the three SU(3)-breaking terms are identical and only one additional parameter is needed. A similar remark holds for $\eta_c \to PPV$ [here, we would use Eq. (2.14) replacing P^a with η_c and ψ^μ with V_a^μ] and for $\eta_c \to PPP$, assuming that the dominant interaction is

$$\mathscr{L}_{\text{eff}}=d_{abc}\eta P^aP^bP^c \,. \tag{2.15}$$

In the latter case, Eq. (2.12) would be used, and all the g_{s_i} would be equal due to Eq. (2.13). Equations (2.11) and (2.12) may also be used to study the effect of second-order SU(3) breaking by replacing two of four octets with the appropriate spurion fields. Again, one often finds dramatic simplification in the resulting expressions in specific cases of interest. Nevertheless, due to the addition of unknown parameters (albeit small, presumably), the usefulness of such a procedure is marginal. Most decays of the J/ψ into three mesons are dominated by quasi-two-body intermediate states, so that the actual data for nonresonant three-body decays is quite meager. Furthermore, background and small statistics prevent evaluation of the contribution of two-body intermediate states to the $\eta_c \to PPP$ decay rates.[3,4] Hence, the general analysis of the three-body final states will be omitted in this paper.

III. NUMERICAL RESULTS

The width of the decay $J/\psi \to PV$ is given by

$$\Gamma(J/\psi \to PV)=\frac{1}{32\pi^2 m_{J/\psi}}\left[\frac{P_V}{m_{J/\psi}}\right]\sum_{\text{spins}}\int |\mathscr{M}|^2 d\Omega_V \,, \tag{3.1}$$

where $\mathscr{M}=g_{VP}m_{J/\psi}\hat{\mathbf{e}}_{J/\psi}\cdot\hat{\mathbf{e}}_V\times\mathbf{P}_V$, $\hat{\mathbf{e}}_{J/\psi}$ and $\hat{\mathbf{e}}_V$ are the J/ψ and vector-meson polarization vectors, respectively, $\mathbf{P}_V$ is the momentum of the vector meson in the center of mass. On the right-hand side of Eq. (3.1), we factor out the P_V dependence and write the branching ratio as

$$B(J/\psi \to PV)=|A|^2\times P_V{}^3 \,.$$

The coupling strength A has been calculated in Sec. II and presented in Table II. It is useful to define a reduced branching ratio $\widetilde{B}(J/\psi \to PV)\equiv B(J/\psi \to PV)/P_V{}^3$.

A. Tests of nonet symmetry

The decay $J/\psi \to \pi^0\phi$ can occur if the vector mesons V_8^0 and V_1^0 are not ideally mixed or if nonet symmetry is broken. It follows from Eq. (2.6) that the $J/\psi \to \pi^0\phi$ amplitude is equal to

$$g_{E,88}[(\tfrac{1}{3})^{1/2}\cos\theta_V-r(\tfrac{2}{3})^{1/2}\sin\theta_V] \,,$$

TABLE III. Relative coupling strengths for the decays $J/\psi \to PPP$. All amplitudes are proportional to g_A [see Eq. (2.6)]. The table below can be used for $\eta_c \to PPV$ or $J/\psi \to VVP$ by appropriate change in labeling. Because of the fact that the interaction is antisymmetric under interchange of SU(3) indices, all other final-state couplings can be obtained by permuting the final states listed below. For an odd permutation, the corresponding coefficient must be multiplied by -1. All final states which do not appear by permuting the particles below are forbidden by SU(3) symmetry. Note that in the SU(3) limit, none of the final states can be an SU(3) singlet; hence nonet symmetry needs not to be invoked in this limit.

Decay mode	Coupling constant
$\pi^+\pi^-\pi^0$	1
$\pi^- K^+ \bar{K}^0$	$(\frac{1}{2})^{1/2}$
$\pi^+ K^- K^0$	$-(\frac{1}{2})^{1/2}$
$\pi^0 K^+ K^-$	$\frac{1}{2}$
$\pi^0 K^0 \bar{K}^0$	$-\frac{1}{2}$
$K^+K^-\eta$	$\frac{\sqrt{3}}{2}\cos\theta_P$
$K^+K^-\eta'$	$\frac{\sqrt{3}}{2}\sin\theta_P$
$K^0\bar{K}^0\eta$	$\frac{\sqrt{3}}{2}\cos\theta_P$
$K^0\bar{K}^0\eta'$	$\frac{\sqrt{3}}{2}\sin\theta_P$

where θ_V is the mixing angle of the vector mesons and $r = g_{E,81}/g_{E,88}$ [following the notation introduced in the discussion of the $J/\psi \to PV$ decays, below Eq. (2.9)]. Similarly, the $J/\psi \to \pi^0\omega$ amplitude is

$$g_{E,88}[(\tfrac{1}{3})^{1/2}\sin\theta_V + r(\tfrac{2}{3})^{1/2}\cos\theta_V] .$$

Thus

$$\frac{\widetilde{B}(J/\psi \to \pi^0\phi)}{\widetilde{B}(J/\psi \to \pi^0\omega)} = \left[\frac{1-\sqrt{2}r\tan\theta_V}{\tan\theta_V + \sqrt{2}r}\right]^2 . \tag{3.2}$$

If the vector mesons are assumed to be ideally mixed $[\tan\theta_V = (\frac{1}{2})^{1/2}]$, the upper limit on $\widetilde{B}(J/\psi \to \pi^0\phi)$ given in Table V yields $0.74 < r < 1.4$ at the 90% confidence level (C.L.). Note that $r=1$ corresponds to nonet symmetry, so the above observation indicates that it is not unreasonable to neglect violations of nonet symmetry in isospin-violating decays.

The amount of nonet-symmetry breaking can also be determined from the branching ratios $B(J/\psi \to VT)$. The $J/\psi \to VT$ amplitudes are given in Table VI. The parameters g_8, g_M, g_E and the relative phase ϕ_{VT} between the strong and electromagnetic SU(3)-breaking amplitudes $[(g_8, g_M)$ and $g_E]$ are to be determined from the ρA_2, K^*K^{**}, ωf, and $\omega f'$ branching ratios. Unfortunately, the present measurements[7] do not allow for a good determination of these parameters. For this reason, we set $g_M = g_E = 0$ and use the average value of $B(J/\psi \to \rho^0 A_2^0)$ and $B(J/\psi \to K^{*0}\bar{K}^{**0})$ to determine g_8. The V and T mesons are assumed to be in an s-wave final state; thus

TABLE IV. Relative coupling strengths for the decays $\eta_c \to PPP$. We use the notation of Eq. (2.6). In the limit of nonet symmetry, take $g_S = C_{123} = C_{132} = C_{231} = f$. We denote by c_i and s_i the cosine and sine, respectively, of the mixing angle θ_i of the octet O_i. The table below can also be used for $J/\psi \to PPV$ or $\eta_c \to VVP$ by appropriate change in labeling. For three identical octets, take $\theta_1 = \theta_2 = \theta_3$. Because of the fact that the interaction is symmetric under interchange of SU(3) indices, all other final-state couplings can be obtained by permuting the particles. As an example, for $\eta_c \to \rho\pi\phi$, we use the $\pi\pi\eta$ entry and put $\sin\theta_V = (\frac{1}{3})^{1/2}$ and $\cos\theta_V = (\frac{2}{3})^{1/2}$ for ideal mixing. This amplitude vanishes in the limit of nonet symmetry as it should. Note that when no sign is indicated, π stands for either $\pi^\pm$ or π^0, K for K^+ or K^0, and $\bar{K}$ for K^- or $\bar{K}^0$.

Decay mode	Coupling constant				
$\eta_c \to PPP$	g_s	C_{123}	C_{132}	C_{231}	f
$\pi\pi\pi$	0	0	0	0	0
$\pi^+K^-K^0$	$(\frac{1}{2})^{1/2}$	0	0	0	0
$\pi^-K^+\bar{K}^0$	$(\frac{1}{2})^{1/2}$	0	0	0	0
$\pi^0K^+K^-$	$\frac{1}{2}$	0	0	0	0
$\pi^0K^0\bar{K}^0$	$-\frac{1}{2}$	0	0	0	0
$\pi\pi\eta$	$(\frac{1}{3})^{1/2}c_3$	$-(\frac{2}{3})^{1/2}s_3$	0	0	0
$\pi\pi\eta'$	$(\frac{1}{3})^{1/2}s_3$	$+(\frac{2}{3})^{1/2}c_3$	0	0	0
$K\bar{K}\eta$	$-\frac{1}{2\sqrt{3}}c_3$	$-(\frac{2}{3})^{1/2}s_3$	0	0	0
$K\bar{K}\eta'$	$-\frac{1}{2\sqrt{3}}s_3$	$+(\frac{2}{3})^{1/2}c_3$	0	0	0
$\eta\eta\eta$	$-(\frac{1}{3})^{1/2}c_1c_2c_3$	$-(\frac{2}{3})^{1/2}c_1c_2s_3$	$-(\frac{2}{3})^{1/2}c_1s_2c_3$	$-(\frac{2}{3})^{1/2}s_1c_2c_3$	$-(\frac{2}{3})^{1/2}s_1s_2s_3$
$\eta\eta\eta'$	$-(\frac{1}{3})^{1/2}c_1c_2s_3$	$+(\frac{2}{3})^{1/2}c_1c_2c_3$	$-(\frac{2}{3})^{1/2}c_1s_2s_3$	$-(\frac{2}{3})^{1/2}s_1c_2s_3$	$+(\frac{2}{3})^{1/2}s_1s_2c_3$
$\eta\eta'\eta'$	$-(\frac{1}{3})^{1/2}c_1s_2s_3$	$+(\frac{2}{3})^{1/2}c_1s_2c_3$	$+(\frac{2}{3})^{1/2}c_1c_2s_3$	$-(\frac{2}{3})^{1/2}s_1s_2s_3$	$-(\frac{2}{3})^{1/2}s_1c_2c_3$
$\eta'\eta'\eta'$	$+(\frac{1}{3})^{1/2}s_1s_2s_3$	$+(\frac{2}{3})^{1/2}s_1s_2c_3$	$+(\frac{2}{3})^{1/2}s_1c_2s_3$	$+(\frac{2}{3})^{1/2}c_1s_2s_3$	$+(\frac{2}{3})^{1/2}c_1c_2c_3$

TABLE V. $J/\psi \to PV$ branching ratios. Statistical and systematic errors have been combined quadratically and a 5.8% systematic error common to all channels has been removed (from Ref. 2). An average value is used when the branching ratio has been measured in more than one final state.

Decay mode $J/\psi \to X$	Branching ratios in units of 10^{-3}
$\frac{1}{3}\rho\pi$	4.43 ± 0.44
$\frac{1}{2}(K^{*+}K^{-}+K^{*-}K^{+})$	2.71 ± 0.22
$\frac{1}{2}(K^{*0}\bar{K}^{0}+\bar{K}^{*0}K^{0})$	1.95 ± 0.29
$\omega\eta$	1.9 ± 0.34
$\omega\eta'$	0.40 ± 0.11
$\phi\eta$	0.68 ± 0.07
$\phi\eta'$	0.37 ± 0.06
$\rho^0\eta$	0.18 ± 0.04
$\rho^0\eta'$	<0.1 (90% C.L.)
$\omega\pi^0$	0.67 ± 0.12
$\phi\pi^0$	<0.013 (90% C.L.)

$\tilde{B}(J/\psi \to PV) \equiv B(J/\psi \to PV)/P_V$. The present upper limit on $B(J/\psi \to \omega f')$ yields $0.5 < R = g_1/g_8 < 1.5$ at the 90% confidence level.

B. Pseudoscalar-meson mixing

The following analysis is an extension of a previous study made by the Mark III Collaboration.[14] If nonet symmetry is assumed, the set of amplitudes given in Table II implies

$$\frac{\tilde{B}(J/\psi \to \omega\eta)}{\tilde{B}(J/\psi \to \rho^0\pi^0)} = \frac{\tilde{B}(J/\psi \to \rho^0\eta)}{\tilde{B}(J/\psi \to \omega\pi^0)} = |(\tfrac{1}{3})^{1/2}\cos\theta_P - (\tfrac{2}{3})^{1/2}\sin\theta_P|^2 \equiv X_\eta^2 \tag{3.3}$$

and

$$\frac{\tilde{B}(J/\psi \to \omega\eta')}{\tilde{B}(J/\psi \to \rho^0\pi^0)} = \frac{\tilde{B}(J/\psi \to \rho^0\eta')}{\tilde{B}(J/\psi \to \omega\pi^0)} = |(\tfrac{1}{3})^{1/2}\sin\theta_P + (\tfrac{2}{3})^{1/2}\cos\theta_P|^2 \equiv X_{\eta'}^2 . \tag{3.4}$$

Using $\theta_P = -10°$, we expect $X_\eta^2 = 0.505$ and $X_{\eta'}^2 = 0.495$. Although model and data agree fairly well on the value of X_η^2 (see Tables VII and VIII), there is a sharp disagreement on the value of $X_{\eta'}^2$.

When taken into account, a possible nonet-symmetry breaking yields the results presented in Table VIII. The discrepancy between model and data is reduced, but the χ^2 probability of the fit is still very small (2%), and the value of the nonet-symmetry-breaking parameter r lies outside the 90%-C.L. limit set in Sec. III A.

The squares of X_η and $X_{\eta'}$ in Eqs. (3.3) and (3.4) correspond to the nonstrange-quark content ($u\bar{u}$ and $d\bar{d}$) of the η and η', respectively. Similarly, the strange-quark contents are given by

$$Y_\eta^2 = |(\tfrac{2}{3})^{1/2}\cos\theta_P + (\tfrac{1}{3})^{1/2}\sin\theta_P|^2 , \tag{3.5}$$

$$Y_{\eta'}^2 = |(\tfrac{2}{3})^{1/2}\sin\theta_P - (\tfrac{1}{3})^{1/2}\cos\theta_P|^2 , \tag{3.6}$$

TABLE VI. Relative coupling strengths for the decays $J/\psi \to VT$. $R = g_1/g_8$ is equal to 1 if nonet symmetry is exact. Ideal mixing is assumed for the vector and tensor mesons.

Decay mode $J/\psi \to X$	Coupling constant g_8	g_M	g_E
ωf	$\frac{1}{3}+\frac{2}{3}R$	$(\frac{1}{3})^{1/2}$	$\frac{1}{3}$
$\omega f'$	$\frac{\sqrt{2}}{3}(1-R)$	0	0
ϕf	$\frac{\sqrt{2}}{3}(1-R)$	0	0
$\phi f'$	$\frac{2}{3}+\frac{1}{3}R$	$-\frac{2}{\sqrt{3}}$	$-\frac{2}{3}$

TABLE VII. Quark content of the η and η' determined from ratios of decay rates [Eqs. (3.3) and (3.4)], from Ref. 2.

Ratio	Coefficient
$\frac{\omega\eta}{\rho^0\pi^0}$	$X_\eta^2 = 0.48 \pm 0.10$
$\frac{\rho^0\eta}{\omega\pi^0}$	$X_\eta^2 = 0.29 \pm 0.09$
$\frac{\omega\eta'}{\rho^0\pi^0}$	$X_{\eta'}^2 = 0.13 \pm 0.04$
$\frac{\rho^0\eta'}{\omega\pi^0}$	$X_{\eta'}^2 < 0.21$ (90% C.L.)

TABLE VIII. $J/\psi \to VP$: results of the fits. The $J/\psi \to PV$ reduced branching ratios expressed in units of 10^{-3} are used as input. The angle ϕ_{PV} is the phase between the mass and electromagnetic breaking terms. The parameter $r = g_1/g_8$ is equal to one if nonet symmetry is exact. The different fits are described in the text.

Fit	Parameters	
Nonet symmetry Standard mixing	$g = 0.92 \pm 0.02$	$\phi_{PV} = 1.43 \pm 0.07$
	$g_M = 0.17 \pm 0.03$	$\theta_P = (-15.4 \pm 1.7)°$
	$g_E = 0.38 \pm 0.03$	
	Prob(χ^2) $< 10^{-11}$	
Standard mixing	$g_8 = 1.01 \pm 0.03$	$\phi_{PV} = 1.24 \pm 0.11$
	$g_M = 0.21 \pm 0.03$	$\theta_P = (-21 \pm 2)°$
	$g_E = 0.36 \pm 0.03$	$r = 0.72 \pm 0.05$
	Prob(χ^2) $= 0.02$	
Nonet symmetry Quark content	$g = 1.02 \pm 0.03$	$X_\eta = 0.63 \pm 0.05$
	$g_M = 0.24 \pm 0.04$	$Y_\eta = 0.80 \pm 0.12$
	$g_E = 0.44 \pm 0.03$	$X_{\eta'} = 0.36 \pm 0.05$
	$\phi_{PV} = 1.13 \pm 0.19$	$Y_{\eta'} = 0.69 \pm 0.11$
	Prob(χ^2) $= 0.17$	
Nonet symmetry Euler angles	$g = 1.03 \pm 0.03$	$\alpha = (-5.7 \pm 4.1)°$
	$g_M = 0.18 \pm 0.06$	$\beta = (48.8 \pm 3.7)°$
	$g_E = 0.43 \pm 0.03$	$\gamma = (20.4 \pm 3.4)°$
	$\phi_{PV} = 1.18 \pm 0.17$	
	Prob(χ^2) $= 0.24$	

and ${X_\eta}^2+{Y_\eta}^2={X_{\eta'}}^2+{Y_{\eta'}}^2=1$. However, if the η_8^0 and η_1^0 are mixed with another pseudoscalar state, e.g., $\iota(1440)$ or a $J^P=0^{-+}$ radial excitation, the relations (3.3)—(3.6) do not hold anymore. The result of a fit[15] in which the parameters X and Y are independently varied is given in Table VIII (the relevant amplitudes are given in Table IX). Although the quark contents of the η agree well with the values expected from a pseudoscalar mixing angle of $-10°$, the nonstrange-quark content of the η' is four times smaller than expected. Adding the nonstrange- and strange-quark content, we get

$$\begin{aligned} {X_\eta}^2+{Y_\eta}^2&=1.04\pm0.20\ , \\ {X_{\eta'}}^2+{Y_{\eta'}}^2&=0.61\pm0.16\ . \end{aligned} \tag{3.7}$$

This suggests that an additional component may be present in the η' wave function. In the following, we shall assume that this extra component is the $\iota(1440)$.

The mixing of the three pseudoscalar states can be described by three Euler angles α, β, and γ;

$$\begin{bmatrix} \eta \\ \eta' \\ \iota \end{bmatrix} = M \begin{bmatrix} \eta_8^0 \\ \eta_1^0 \\ \eta_0^0 \end{bmatrix}\ , \tag{3.8}$$

where

$$M=\begin{bmatrix} \cos\alpha\cos\gamma-\sin\alpha\cos\beta\sin\gamma & \sin\alpha\cos\gamma+\cos\alpha\cos\beta\sin\gamma & \sin\beta\sin\gamma \\ -\cos\alpha\sin\gamma-\sin\alpha\cos\beta\cos\gamma & -\sin\alpha\sin\gamma+\cos\alpha\cos\beta\cos\gamma & \sin\beta\cos\gamma \\ \sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta \end{bmatrix}\ .$$

The quark contents are given by

$${X_j}^2=|(\tfrac{2}{3})^{1/2}(a_{j2}+a_{j1}/\sqrt{2})|^2\ , \tag{3.9}$$

$${Y_j}^2=|(\tfrac{1}{3})^{1/2}(a_{j2}-a_{j1}\sqrt{2})|^2\ , \tag{3.10}$$

where $j=1,2,3$ for η,η', and $\iota(1440)$, respectively (the a_{ji} are the elements of the mixing matrix). The amount of η_0^0 component, ${Z_j}^2$, is equal to ${a_{j3}}^2$. The result of a fit which uses this parametrization is presented in Table VIII. The probability of the χ^2 is 24%. From this determination of the mixing angles, we get

$$\begin{aligned} &X_\eta=0.66\pm0.02,\ \ Y_\eta=-0.70\pm0.03,\ \ Z_\eta=0.26\pm0.05\ , \\ &X_{\eta'}=0.37\pm0.06,\ \ Y_{\eta'}=0.61\pm0.04,\ \ Z_{\eta'}=0.71\pm0.04\ , \\ &X_\iota=-0.65\pm0.04,\ \ Y_\iota=-0.37\pm0.02,\ \ Z_\iota=0.66\pm0.05\ , \end{aligned}$$

and predict

$$B(J/\psi\to\omega\iota)=|g+2g_M+g_E|^2{X_\iota}^2{k_\iota}^3 =(0.75\pm0.12)\times10^{-3}\ , \tag{3.11}$$

$$B(J/\psi\to\phi\iota)=|g-4g_M-2g_E|^2{Y_\iota}^2{k_\iota}^3 =(0.09\pm0.02)\times10^{-3}\ . \tag{3.12}$$

In addition, a standard SU(3) calculation yields[16]

$$\Gamma(\iota\to\gamma\gamma)=|(\tfrac{1}{3})^{1/2}(a_{31}+2\sqrt{2}a_{32})|^2\left[\frac{m_\iota}{m_{\pi^0}}\right]^3\Gamma(\pi^0\to\gamma\gamma) =15.9\pm2.2\ \text{keV}\ , \tag{3.13}$$

$$\Gamma(\iota\to\gamma\rho^0)=3{X_\iota}^2\left[\frac{k_{\rho^0}}{k_{\pi^0}}\right]^3\Gamma(\omega\to\gamma\pi^0) =2.9\pm0.4\ \text{MeV}\ , \tag{3.14}$$

$$\Gamma(\iota\to\gamma\omega)=3(\tfrac{1}{3}X_\iota)^2\left[\frac{k_\omega}{k_{\pi^0}}\right]^3\Gamma(\omega\to\gamma\pi^0) =0.31\pm0.04\ \text{MeV}\ , \tag{3.15}$$

$$\Gamma(\iota\to\gamma\phi)=3(\tfrac{2}{3}Y_\iota)^2\left[\frac{k_\phi}{k_{\pi^0}}\right]^3\Gamma(\omega\to\gamma\pi^0) =0.15\pm0.02\ \text{MeV}\ , \tag{3.16}$$

where k_ι, k_{ρ^0}, k_ω, k_ϕ, and k_{π^0} are the respective center-of-mass momenta. In this calculation, we have used the following values: $\Gamma(\pi^0\to\gamma\gamma)=7.84\pm0.57$ eV, $\Gamma(\omega\to\gamma\pi^0)=861\pm56$ keV, and $m_\iota=1460$ MeV/c^2. Evidence for the decay $\iota\to\gamma\rho^0$ with a width of 2.1 ± 0.6 MeV$\times B(\iota\to K\bar{K}\pi)$ had recently been obtained,[17] and an

TABLE IX. Relative coupling strengths for the decays $J/\psi\to\eta V$ and $J/\psi\to\eta' V$. X_η and Y_η correspond to the nonstrange- and strange-quark components of the η (and similarly for the η'). Nonet symmetry and ideal mixing are assumed ($g\equiv g_8=g_1$).

Decay mode $J/\psi\to X$	g	Coupling constant g_M	g_E
$\eta\phi$	Y_η	$-\frac{2}{\sqrt{3}}Y_\eta$	$-\frac{2}{3}Y_\eta$
$\eta\omega$	X_η	$\frac{1}{\sqrt{3}}X_\eta$	$\frac{1}{3}X_\eta$
$\eta\rho^0$	0	0	X_η
$\eta'\phi$	$Y_{\eta'}$	$-\frac{2}{\sqrt{3}}Y_{\eta'}$	$-\frac{2}{3}Y_{\eta'}$
$\eta'\omega$	$X_{\eta'}$	$\frac{1}{\sqrt{3}}X_{\eta'}$	$\frac{1}{3}X_{\eta'}$
$\eta'\rho^0$	0	0	$X_{\eta'}$

TABLE X. $J/\psi \to PP$ branching ratios. Statistical and systematic errors have been combined quadratically and a 5.8% systematic error common to all channels has been removed (from Ref. 2).

Decay mode $J/\psi \to X$	Branching ratio in units of 10^{-4}
$\pi^+\pi^-$	1.58 ± 0.23
K^+K^-	2.39 ± 0.29
$K_S K_L$	1.01 ± 0.16

upper limit on $\Gamma(\iota \to \gamma\gamma)$ of 8 keV$/B(\iota \to K\bar{K}\pi)$ at the 95% confidence level has been set.[18] To be compatible with the predictions, these results require $B(\iota \to K\bar{K}\pi) \simeq 0.6$. The expected $J/\psi \to \iota\omega$ branching ratio, $(0.75 \pm 0.12) \times 10^{-3}$, is substantial, and this decay mode should be observed in the samples of several $\times 10^6$ J/ψ decays collected by the Mark III and DM2 Collaborations.

C. J/ψ decays to two pseudoscalar mesons

The $J/\psi \to PP$ decay amplitudes have been discussed in Sec. II and presented in Table I. Nonet-symmetry breaking is irrelevant to these decay modes which do not involve η or η'. The observed branching ratios are given in Table X. Note that the $J/\psi \to K_S K_L$ decay rate is almost equal to that of the purely electromagnetic decay $J/\psi \to \pi^+\pi^-$, which indicates that the effects of mass and electromagnetic breaking are approximately equal. More quantitatively, a χ^2 fit to the data yields $g_M/g_E = 1.33 \pm 0.16$ (Table XI). The relative phase, ϕ_{PP}, between g_M and g_E is approximately equal to $\pi/2$ which implies the absence of an interference term. This value differs from the one obtained for ϕ_{PV} in a similar fit to the branching ratios $J/\psi \to PV$ by only 1.6σ. The significance of this observation is not clear to us at present.

D. η_c decays

Branching ratios for the decays $\eta_c \to VV$ and $\eta_c \to PPP$ have been measured recently by the Mark III Collaboration.[4] Unfortunately, the contribution of possible two-body intermediate states to the three-body final states is not known. This, when combined with the relatively small number of three-body decay modes observed to date, makes comparison with our model very difficult. Two $\eta_c \to VV$ decay modes have been measured, $\phi\phi$ and $K^{*0}\bar{K}^{*0}$, and upper limits have been obtained for the branching ratios $\eta_c \to \rho^0\rho^0$ and $\eta_c \to \omega\omega$ (Table XII). The ratios of reduced branching ratios,

$$\frac{\frac{1}{2}\tilde{B}(\eta_c \to K^{*0}\bar{K}^{*0})}{\tilde{B}(\eta_c \to \phi\phi)} = 0.22 \pm 0.13 , \tag{3.17}$$

TABLE XI. $J/\psi \to PP$: result of the fit. The $J/\psi \to PP$ reduced branching ratios expressed in units of 10^{-4} are used as input.

$g_M = 0.76 \pm 0.06$
$g_E = 0.57 \pm 0.05$
$\phi_{PP} = 1.57 \pm 0.17$

TABLE XII. $B(J/\psi \to \gamma\eta_c) \times B(\eta_c \to VV)$ product of branching ratios. Statistical and systematic errors have been combined quadratically, and a 5.8% systematic error common to all channels has been removed (from Ref. 4).

Decay mode $\eta_c \to VV$	Product of branching ratios in units of 10^{-4}
$\phi\phi$	1.0 ± 0.3
$K^{*0}\bar{K}^{*0}$	0.6 ± 0.3
$\rho^0\rho^0$	<0.6 (90% C.L.)
$\omega\omega$	<0.4 (90% C.L.)

$$\frac{\tilde{B}(\eta_c \to \rho^0\rho^0)}{\tilde{B}(\eta_c \to \phi\phi)} < 0.64 \quad (90\% \text{ C.L.}) , \tag{3.18}$$

$$\frac{\tilde{B}(\eta_c \to \omega\omega)}{\tilde{B}(\eta_c \to \phi\phi)} < 0.34 \quad (90\% \text{ C.L.}) , \tag{3.19}$$

show that the $\eta_c \to VV$ decay rate increases with the number of strange quarks in the final state. This SU(3)-breaking pattern is very different from the one observed in $J/\psi \to PV$ decays (see Sec. III B). To explain this difference, one must go beyond the considerations of this paper and discuss the dynamics of the decay. For example, consider the three different mechanisms relevant to the decay $\eta_c \to VV$ shown in Fig. 1. In order to understand Eq. (3.18), it would seem that the graph of Fig. 1(b) must dominate, as it implies[19]

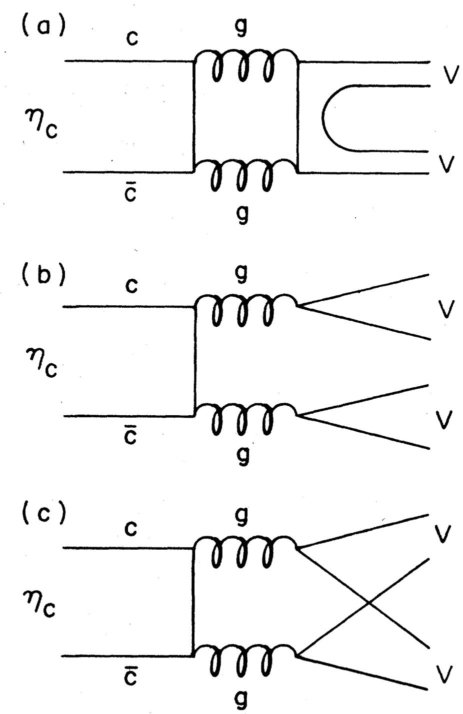

FIG. 1. Various mechanisms for $\eta_c \to VV$ decay. In (b), the produced mesons are color-octet states, but final-state interactions (i.e., exchange of soft gluons) are assumed to turn them into color singlets.

$$\frac{\tilde{B}(\eta_c\to\rho^0\rho^0)}{\tilde{B}(\eta_c\to\phi\phi)}=\frac{|\psi_\rho(0)|^4}{|\psi_\phi(0)|^4}=\left[\frac{2m_\rho{}^2\Gamma(\rho\to e^+e^-)}{9m_\phi{}^2\Gamma(\phi\to e^+e^-)}\right]^2$$
$$=0.48\pm0.07\ , \quad (3.20)$$

where $\psi_V(0)$ is the wave function at the origin of the vector meson. We have used $\Gamma(V\to e^+e^-)=16\pi\alpha^2 e_Q{}^2|\psi_V(0)|^2/m_V{}^2$ with $9e_Q{}^2=9$, 1, 2 for ρ, ω, and ϕ, respectively.[20] Similarly,

$$\frac{\tilde{B}(\eta_c\to\omega\omega)}{\tilde{B}(\eta_c\to\phi\phi)}=0.36\pm0.05\ , \quad (3.21)$$

$$\frac{\tilde{B}(\eta_c\to\omega\phi)}{\tilde{B}(\eta_c\to\phi\phi)}=0.60\pm0.05\ . \quad (3.22)$$

These results are consistent with the observed dominance of the $\phi\phi$ mode. On the other hand, the fact that $\eta_c\to K^{*0}\bar{K}^{*0}$ is seen at the rate given by Eq. (3.17) implies that the above picture is too simple. It is clear that the diagram of Fig. 1(b) does not contribute to the $K^*\bar{K}^*$ final state, whereas diagrams 1(a) and 1(c) can contribute. The three mechanisms can be treated in a purely phenomenological way by noting that while VV final states ($V=\rho,\omega,\phi$) can arise from all three diagrams, $K^*\bar{K}^*$ can only arise from Fig. 1(a) and 1(c), and $\omega\phi$ can arise only from Fig. 1(b). Eventually, the relative contributions of the three mechanisms could be determined experimentally once all the above modes are measured. A word of caution is in order here. Equation (3.20) assumes that there are no extraneous mass factors. This is not obvious. For example, a model given in Ref. 21 implies that the contribution to $\Gamma(\eta_c\to VV)$ from Fig. 1(b) scales as $[\Gamma(V\to e^+e^-)/m_V e_Q{}^2]^2$. This would imply that $\Gamma(\eta_c\to\rho^0\rho^0)>\Gamma(\eta_c\to\phi\phi)$ in contrast with the argument above and in contradiction to the data. Clearly, more theoretical work is needed to understand the SU(3)-breaking pattern exhibited by the $n_c\to VV$ decays.

IV. SUMMARY AND DISCUSSION

We would like to emphasize the inherent limitations of the work presented here. Even in the context of our model-independent analysis, we have only worked to first order in all potentially small parameters which describe deviation from nonet symmetry (the OZI rule), and deviation from SU(3) symmetry due to mass breaking ($m_s\neq m_u, m_d$) or electromagnetism. For example, in this approximation, we have neglected the effects of second-order OZI-rule-violating processes which Pinsky[2] found important in the decays $J/\psi\to\omega\eta'$ and $J/\psi\to\phi\eta'$. Another second-order effect of interest is one which is simultaneously first order in mass and electromagnetism SU(3) breaking. Nearly ten years ago, Isgur[22] showed that the prediction for $K^*\to\gamma K$ could be related to the magnetic moments of the quarks (which are related, in part, to the magnetic moment of the proton, neutron, and Λ in the usual way, but where the constituent mass splittings $m_s\neq m_u$, m_d are explicitly taken into account). In both cases, a specific model for the second-order effects is used. In principle, a totally general algebraic analysis of such an effect could be performed by treating the breaking by the appropriate spurion fields, and inserting them twice in an SU(3)-invariant way into the different interaction terms. Such an analysis would unfortunately result in too many new parameters to be useful. The advantage of model building here is obvious since it usually singles out one particular parameter as being the dominant one.

A second possible shortcoming of our analysis is the particular mixing scheme employed for the isoscalar pseudoscalar mesons. If the prediction for $\Gamma(\iota\to\gamma\gamma)$ and $\Gamma(\iota\to\gamma\rho^0)$ end up deviating significantly from the observed rates, this could imply that the mixing scheme needs to be more complicated. For example, we have omitted the first radial excitations of the η and η' from the analysis. Palmer, Pinsky, and Bender[23] have advocated that the $\zeta(1275)$ must be included in the pseudoscalar mixing analysis. It is fairly straightforward to use the results of this paper in conjunction with a given mixing scheme. An even more exotic possibility would be the existence of a long-lived light gluino (with mass <1 GeV/c^2). Such an object would bind with a gluon (or $q\bar{q}$) to form a long-lived neutron hadron[24] which could have been lost in the neutron background in previous experimental searches.[25,26] If such a gluino were present, a $\tilde{g}\tilde{g}$ bound state would exist[27] with the quantum numbers of the η and would strongly mix with the η, η', and ι. This new mixing scheme would require a new analysis, but we suspect that such a pseudoscalar can be ruled out by present J/ψ data.[28]

To go beyond the type of analysis presented in this paper, one would have to build models. This would involve dealing with dynamical questions which the present analysis purposely avoids. For example, the precise SU(3)-breaking pattern in η_c decays would depend on understanding the relative contributions of the various mechanisms shown in Fig. 1. Examples of this model-building approach have appeared in the literature.[2,29] It would be of interest to reconsider some of these approaches given the marked improvement in the data.

To summarize, we have presented a model-independent parametrization of J/ψ and η_c decays to two- and three-meson final states. Limits on nonet-symmetry breaking from electromagnetic interaction have been set using the $J/\psi\to\pi^0\omega$ and $J/\psi\to\pi^0\phi$ branching ratios: $0.74<r=g_{E,81}/g_{E,88}<1.4$ with a 90% confidence level (where $r=1$ implies no breaking). In $J/\psi\to K^+K^-$ and $J/\psi\to K_SK_L$ decays, the effects of electromagnetic and mass breaking of SU(3) are found to be of similar importance. The analysis of $J/\psi\to PV$ decay modes has provided evidence for mixing of η and η' with a third (or additional) pseudoscalar(s). From a fit to all $J/\psi\to PV$ branching ratios and assuming that the third pseudoscalar is the $\iota(1440)$, we have made predictions for the decay widths $\Gamma(J/\psi\to\iota\omega)$, $\Gamma(J/\psi\to\iota\phi)$, $\Gamma(\iota\to\gamma\gamma)$, and

$\Gamma(\iota\rightarrow\gamma V)$. The expected $J/\psi\rightarrow\iota\omega$ branching ratio is large and should be observable in the near future. A detailed study of these decays will provide important information regarding the mixing of the isoscalar-pseudoscalar mesons.

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