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Pre-equilibrium emission of lepton pairs from oscillating quark-antiquark plasma

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The production rates of lepton pairs from an oscillating $q\bar{q}$ plasma are calculated and compared with those expected from a plasma in local thermal equilibrium and from the Drell-Yan mechanism. It is found that for a reasonable choice of the parameters of the model, the pre-equilibrium rate may dominate over the thermal and Drell-Yan rates for the dilepton mass ≥ 2 GeV.

I. INTRODUCTION

There is nowadays a growing belief that a quark-gluon plasma could be formed in the collisions of ultrarelativistic heavy ions.[1] One of the most promising ways to investigate the properties of this possible new state of matter seems to be the measurement of lepton pairs produced by $q\bar{q}$ annihilation in the mass region below 3—5 GeV (Refs. 2—8). Because of their large mean free path, lepton pairs, once formed, escape from the interaction region, thus carrying out the information on the conditions which prevail at various stages of the collision.

Very recently, Hwa and Kajantie[8] have calculated the rate of emission of lepton pairs from the plasma formed in the central rapidity region of a heavy-ion collision. Their calculation follows the by-now standard scenario[7] which assumes local thermodynamic equilibrium and Bjorken's hydrodynamics[9] for the longitudinal expansion. The resulting rates exceed by more than one order of magnitude the rate expected from the simple Drell-Yan mechanism. This, together with an analysis of transverse-momentum effects[7,8] confirms the idea that indeed lepton-pair production could be used as a possible signature of the formation of a thermalized quark-gluon plasma.

It should be emphasized however that the early stages of the collision which possibly lead to thermal equilibrium are rather poorly understood. In this respect it is useful to study the lepton-pair production in situations where local thermodynamic equilibrium is not achieved. This is what we consider in the present paper, using a simple model of an oscillating quark-antiquark plasma which has been proposed recently.[10] This model describes a system of quarks and antiquarks interacting with a classical gluon field.[11] It reproduces on the average Bjorken's hydrodynamics which renders meaningful a comparison of our results with those of Refs. 7 and 8. We find that the resulting lepton-pair-production rates depend sensitively on the value of the chromoelectric field strength at the initial moment of plasma formation, a quantity at present rather poorly known. Nevertheless, we show that, for a strong enough (but hopefully not unrealistic) field, the pre-equilibrium rates we calculate may dominate over the thermal and Drell-Yan rates. We find this result encouraging for further investigations of pre-equilibrium plasma.

The paper is organized as follows. In Sec. II we present a space-time description of an oscillating $q\bar{q}$ plasma, according to the model of Ref. 10. The formula for the rate of lepton-pair production is given in Sec. III. The numerical estimates are discussed in Sec. IV. Our conclusions are listed in the last section. In Appendix A we present an explicit solution of the Heinz equations for oscillating plasma in the limit of vanishing quark mass. In Appendix B the formula for the lepton-pair rate is derived.

II. THE OSCILLATING QUARK-GLUON PLASMA

The model of plasma which we use in this paper has been described in detail in Ref. 10. In this section, we shall therefore only outline its main features and present the solution which is used in the calculation of the lepton-pair production.

The model describes an assembly of colored quarks and antiquarks following classical trajectories and interacting with a classical gluon field. The properties of the system may be calculated from the quark distribution function $n(\mathbf{x},t;\mathbf{p})$. We have in mind the description of a plasma formed in a central collision of two heavy ions. Let z denote the longitudinal coordinate, that is, the coordinate along the collision axis. We assume that $n(\mathbf{x},t;\mathbf{p})$ depends only on z and the linear momentum and does not depend on the transverse coordinates. An approximate dependence on transverse momenta will be introduced in Sec. III [see Eq. (3.4)].

In general the quark distribution function is a matrix in color space which is assumed here to be diagonal. Furthermore, we shall only consider here the one-color approximation of Ref. 10. In this case, the quark distribution function $n(\mathbf{x},t;\mathbf{p})$ is given by a single function $G(u,w)$,

$$n(\mathbf{x},t;\mathbf{p})=\tfrac{1}{2}G(u,w)\ , \tag{2.1}$$

where u and w are the boost-invariant variables

$$u \equiv t^2 - z^2 , \tag{2.2a}$$

$$w \equiv P_z t - Ez . \tag{2.2b}$$

The equations of motion for $G(u,w)$, derived in Ref. 10 admit a solution of the form

$$G(u,w) = G(u_0, w + \lambda H(u)) , \tag{2.3}$$

where λ is a coupling constant, taken to be qual to 1 in subsequent calculations, and $H(u)$ is directly related to the gluon field. More precisely,

$$\frac{dH}{du} = \frac{1}{2\sqrt{3}} F_{03} , \tag{2.4}$$

where F_{03} denotes the longitudinal component of the color-electric field. Following Bjorken,[9] we assume boost-invariant initial conditions specified at some proper time $\tau_0 = \sqrt{u_0}$ referred to as the "formation time" (see Fig. 1). Specifically we take

$$G(u_0, w) = d\delta(w) = d\delta(P_z t - Ez) . \tag{2.5}$$

Since the distribution function of antiquarks $\overline{G}(u,w)$ is related to $G(u,w)$ by [10]

$$\overline{G}(u,w) = G(u, -w) , \tag{2.6}$$

it follows from (2.5) that quarks and antiquarks have the same initial distribution function. The initial conditions need to be complemented by specifying the value of the color-electric field at proper time τ_0.

The state of the system at time $t = \tau_0$ is illustrated in Fig. 2(a). The function $H(u)$ in Eq. (2.3) is an oscillating function of u whose period and amplitude depend upon the initial conditions. Typically, both period and amplitude increase with the strength of the initial color-electric field and decrease with the initial density of quarks and antiquarks. These oscillations of $H(u)$ induce oscillations of the quark and antiquark currents, as well as of the color-electric field. An illustration of the state of the system at a time $t_1 > \tau_0$ is given in Figs. 2(b) and 2(c).

It is interesting to calculate from $G(u,w)$ the quark density and current. Defining

$$N^\mu \equiv \int dP \, P^\mu G(u,w) \tag{2.7}$$

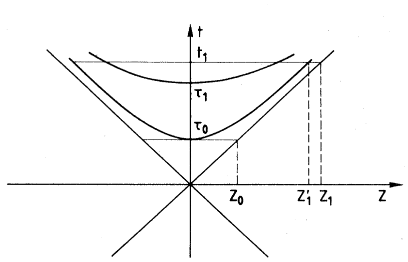

FIG. 1. Space-time diagram. The two colliding heavy ions are assimilated to flat disks supposed to move on the light cone. The hyperbolas correspond to lines of constant "proper time" $\tau = \sqrt{u}$. The hyperbola labeled τ_0 corresponds to the formation of quarks and antiquarks. The quarks and antiquarks which are created at a point z,t on this hyperbola have velocity z/t.

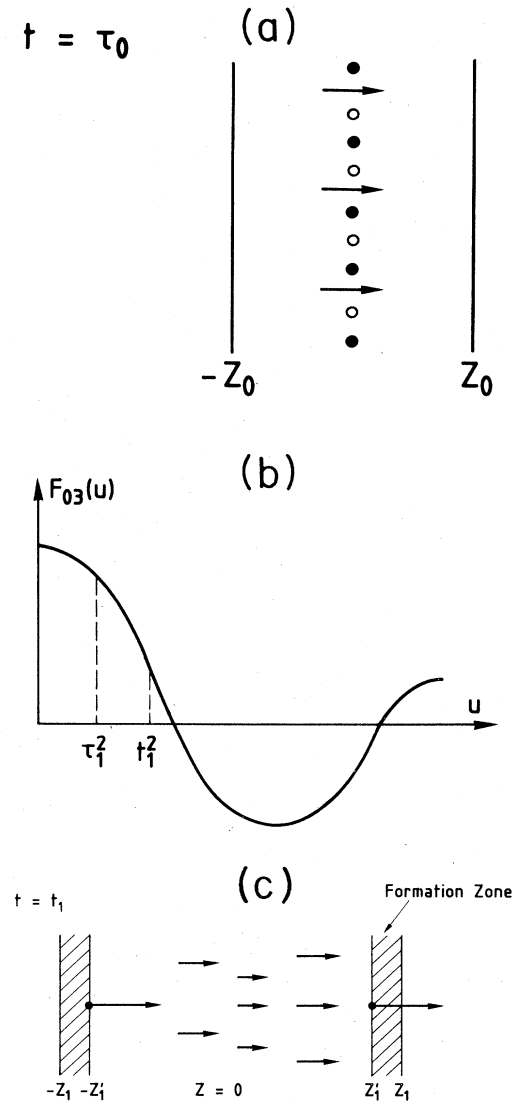

FIG. 2. (a) State of the system at time $t = \tau_0$. The two nuclei have moved a distance $2z_0 = 2\tau_0$ away from each other. Quarks (●) and antiquarks (○) appear in equal number at $z = 0$ with zero velocity. A color-electric field is also generated and is indicated by the arrows. (b) Characteristic behavior of the color-electric field at $z = 0$. For the definition of τ_1 and t_1 see Fig. 1. (c) State of the system at time $t_1 > \tau_1 > \tau_0$ (see Fig. 1). At that time, the nuclei have moved to $\pm z_1 = \pm t_1$. The front of the "formation zone" (indicated by the dashed areas) is in z_1'. Quarks and antiquarks appear in z_1' with the same density as in $z = 0$ and time $t = \tau_0$. They have velocity z_1'/t_1. Note that in an interval of time dt, those quarks move by $(z_1'/t_1)dt$ while the front of the formation zone moves by $(t_1/z_1')dt > dt > (z_1'/t_1)dt$. The strength of the color electric field in z_1' equals that at $t = \tau_0$ and $z = 0$. According to Fig. 2(b), it decreases toward the center since u increases from $z = z_1'$ to $z = 0$, $t - t_1$ being fixed.

and performing the calculation as indicated in Ref. 10 we find

$$N^0=\frac{d}{2}\frac{\gamma}{\tau}-\frac{z}{6\lambda}\frac{d^2H}{du^2}\ , \tag{2.8a}$$

$$N^3=\frac{d}{2}\frac{\gamma}{\tau}\frac{z}{t}-\frac{t}{6\lambda}\frac{d^2H}{du^2}\ , \tag{2.8b}$$

where $\tau=(t^2-z^2)^{1/2}$ and $\gamma=(1-z^2/t^2)^{-1/2}$. For $z=0$, these formulas reduce to

$$N^0=\frac{d}{2}\frac{1}{t},\ \ N^3=-\frac{t}{6\lambda}\frac{d^2H}{du^2}\ . \tag{2.8c}$$

Thus, at $z=0$, N^0, the quark density per unit volume d^3x decreases as $1/t$ while N^3, the quark current oscillates. The uniform decrease of the quark density results from the longitudinal expansion and is identical to what is obtained in Bjorken's hydrodynamics.[9] The oscillating part of the current is a new feature of the model. Note that these oscillating parts have opposite signs for the antiquark density and current. Therefore the total quark + antiquark density and current have a smooth hydrodynamic behavior, as already shown in Ref. 10.

III. RATE OF PAIR PRODUCTION FROM OSCILLATING PLASMA

Starting from the very definition of the cross section we derive the rate for the production of a lepton pair in the collision of quarks and antiquarks whose densities per unit volume of phase space $d^3x\,d^3p$ are, respectively, $n(\mathbf{x},\mathbf{p})$ and $\overline{n}(\mathbf{x},\mathbf{p})$:

$$\frac{dN}{d^4x}=\frac{dN}{d^3x\,dt}=v_{12}n(\mathbf{x},\mathbf{p})\overline{n}(\mathbf{x},\overline{p})\hat{\sigma}\,d^3p\,d^3\overline{p}\ . \tag{3.1}$$

In this formula, v_{12} is the relative velocity of the colliding quark and antiquark and

$$\hat{\sigma}=\frac{8\pi\alpha^2}{9}\sum_q e_q{}^2\delta^4(P+\overline{P}-Q)d^4Q \tag{3.2}$$

is the cross section for producing the lepton pair with four momentum Q, $\sum_q e_q{}^2$ is the sum over quark charges squared, which, for two flavors (u,d), equals $\frac{5}{9}$. Neglecting quark masses and integrating over the longitudinal momenta of the quarks and antiquarks we obtain

$$\frac{dN}{d^4x\,d M^2dy\,d^2q_\perp}=\frac{8\pi\alpha^2}{9}\sum_q^2 e_q{}^2\int\frac{d^2p_\perp d^2\overline{p}_\perp n(\mathbf{x},\mathbf{p})\overline{n}(\mathbf{x},\overline{\mathbf{p}})\delta^2(\mathbf{x}_\perp+\mathbf{p}_\perp-\mathbf{q}_\perp)}{4[(M^2+2\mathbf{p}_\perp\overline{\mathbf{p}}_\perp)^2-4\mathbf{p}_\perp{}^2\overline{\mathbf{p}}_\perp{}^2]^{1/2}}\ , \tag{3.3}$$

where M and y are, respectively, the mass and the rapidity of the lepton pair.

To proceed further, one has to specify the form of the densities $n(\mathbf{x},\mathbf{p})$ and $\overline{n}(\mathbf{x},\overline{\mathbf{p}})$. The dependence on the longitudinal variables is determined by the solution of Boltzmann-Vlasov equations, as specified in the previous section [Eqs. (2.1)—(2.6)]. Following the same approach, we assume that there is no dependence of transverse spatial coordinates $\mathbf{x}_\perp$. For the dependence on transverse momenta we take a Gaussian form

$$d(p_\perp)=\frac{D}{\pi\mu^2}e^{-\mathbf{p}_\perp{}^2/\mu^2}\ . \tag{3.4}$$

This choice is selected only for its simplicity. Since the solutions of Boltzmann-Vlasov equations presented in Sec. II are essentially one-dimensional, we cannot pretend to describe precisely the dependence on transverse momentum anyway. Thus the form (3.4) serves only for illustration. We discuss only the rate integrated over transverse momentum of the lepton pair which should not be very sensitive to the exact form of transverse momentum distribution of quarks and antiquarks.[12]

Using Eqs. (2.1)—(2.6) and (3.4) one can integrate the formula (3.3) with the result

$$\frac{dN}{dM^2dy}=\frac{\pi\alpha^2\sum e_q{}^2}{18}\frac{D^2S}{\mu^2M^2}\times\int_{u_0}^{u_{\max}}\frac{du}{u}\frac{e^{-(M^2-4P^2)/2\mu^2}}{(2-4P^2/M^2)^{1/2}}\ . \tag{3.5}$$

Here $S\approx\pi R_A{}^2$ is the transverse size of the (smaller) nucleus, $P=P(u)$ is the longitudinal momentum of the quark (at $z=0$) given by

$$P(u)=\lambda H(u)/\sqrt{u}\ . \tag{3.6}$$

and the integration is performed only for such values of u for which

$$M^2\geq 4P^2(u)\ . \tag{3.7}$$

The derivation of the Eq. (3.5) is given in the Appendix B. This is the final formula which was used for numerical estimates, as presented in the next section.

Our estimates were compared with those expected from plasma in thermodynamical equilibrium which were calculated according to the formula[7,8]

$$\frac{dN}{dM^2dy}=\frac{\alpha^2}{2\pi^{5/2}}\frac{1}{\pi R_A{}^2}\left[0.22\frac{dN_\pi}{dy}\right]^2\times\left[\frac{1}{M^4}\int_{M/T_i}^{M/T_c}dz\,z^{7/2}e^{-z}+\frac{25}{T_c{}^4}\sqrt{M}\,/T_c e^{-M/T_c}\right], \tag{3.8}$$

where T_c is the critical temperature and T_i is the initial temperature calculated from the relation

$$T_i^3\tau_0 = 0.22\frac{1}{\pi R_A^2}\frac{dN_\pi}{dy} . \qquad (3.9)$$

Here dN_π/dy is the density of pions per unit of rapidity.

IV. NUMERICAL ESTIMATES AND DISCUSSION

The rate as given by Eq. (3.5) depends explicitly on three parameters D, S, and μ. Furthermore, through the function $P(u)$ it depends implicitly on initial conditions, i.e., initial time τ_0 and value of the color field strength F_{03} at $\tau=\tau_0$.

The parameters D, S, and μ are relatively easy to control. Indeed, as seen from Eqs. (2.5), (2.7), and (3.4), DS is simply the density of partons (quarks + antiquarks) per unit rapidity. It is thus simply proportional to the density of pions per unit of rapidity:

$$DS = r\frac{dN_\pi}{dy} . \qquad (4.1)$$

The proportionality factor r depends on details of hadronization process. We shall consider $r=2$ (simple $q\bar{q}$ recombination into pions) and $r=1$ (intermediate formation of resonances).

The parameter μ can be estimated in a similar way by observing that

$$DS\langle p_\perp\rangle = \frac{\sqrt{\pi}}{2}DS\,\mu = \frac{dE}{dy} , \qquad (4.2)$$

where dE/dy is the energy produced per unit of rapidity (at $y\approx 0$). It follows that μ is related to the average momentum of produced pions by the formula

$$\mu \approx \frac{2}{r\sqrt{\pi}}\langle p_\perp\rangle_\pi . \qquad (4.3)$$

We thus see that D and μ can be determined from inclusive measurements of pion production up to a factor ~ 2.

The situation is much less clear with respect to F_{03} and τ_0. To restrict somehow the field strength F_{03} at the initial time we use an estimate based on a simple model in which the production of quarks and antiquarks occurs via tunneling of $q\bar{q}$ pairs in a uniform chromoelectric field.[13–15] This leads to[10]

$$F_{03}(\tau_0) = 4\pi\mu^2/\lambda \simeq 10\mu^2 . \qquad (4.4)$$

Note that in this model, the strength of the chromoelectric field increases with increasing transverse momentum. It is very hard however to ascertain the reliability of this relation.

Regarding the values of the initial time τ_0, we allow variations $0.1f \le \tau_0 \le 1f$, hoping that this covers most of the possible reasonable choices.

The rates we calculate are compared with the rates expected from plasma in thermal equilibrium with critical temperature related to observed transverse momentum of pions by the formula

$$\langle p_\perp\rangle_\pi = \frac{3\pi}{4}T_c . \qquad (4.5)$$

All the calculations are performed for central collisions of ^{16}O beam colliding with a heavy target. Thus we take

$$S \approx \pi(R_{\rm O^{16}})^2 \simeq 20\ \text{fm}^2 . \qquad (4.6)$$

For dN_π/dy we adopted the value predicted (approximately) by the additive quark model,[16] i.e.,

$$\frac{dN_\pi}{dy} \simeq 3\times 16\left.\frac{dN_\pi}{dy}\right|_{pp} \approx 48\times 3 = 144 . \qquad (4.7)$$

We have checked that our conclusions do not change if this value is changed by $\sim 30\%$.

We also compare the pre-equilibrium rates with those expected from the Drell-Yan formula which, for a central AB collision, reads[8]

$$\frac{dN}{dM^2dy} = \frac{4\pi\alpha^2}{9M^4}2\sum_q e_q^2\frac{AB}{\pi R_B^2}[S^N(0)]^2 , \qquad (4.8)$$

where R_B is the radius of the target (i.e., the largest of the two colliding nuclei) and $S^N(0)$ is the number of sea quarks per unit rapidity in a nucleon, at rapidity $y=0$, which was taken to be 0.2. The numerical results presented correspond to ^{16}O-^{238}U collisions.

In Fig. 3 we show the various rates calculated for $\mu=500$ MeV, $r=2$ (and thus $T_c=375$ MeV), and for two values of τ_0. One sees that at masses below ~ 3 GeV the thermal contribution dominates. At larger masses, however, the oscillating plasma produces much more lepton pairs than the plasma in equilibrium. The Drell-Yan contribution is consistently smaller, as already stated in Refs. 7 and 8.

In Fig. 4(a) similar plots, this time for $\mu=225$ MeV and $r=2$, which corresponds to $T_c=170$ MeV and $\langle p_\perp\rangle_\pi \sim 400$ MeV are shown. One sees another interesting feature of the pre-equilibrium contribution, namely, the existence of upper limits on masses of lepton pairs which

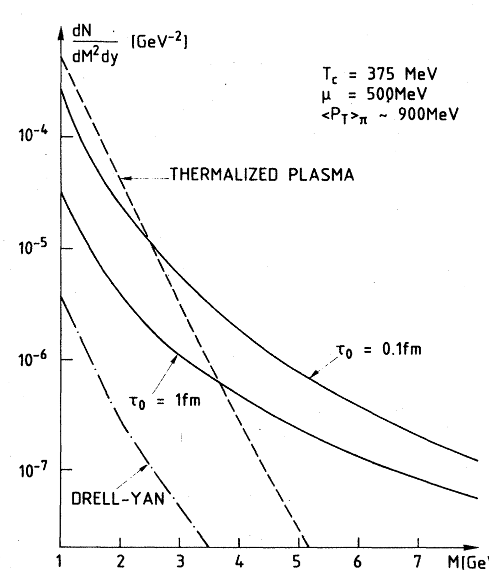

FIG. 3. Diplepton rates from oscillating plasma (solid curves), from a thermalized plasma (dashed curve), and from Drell-Yan mechanism (dashed-dot curve).

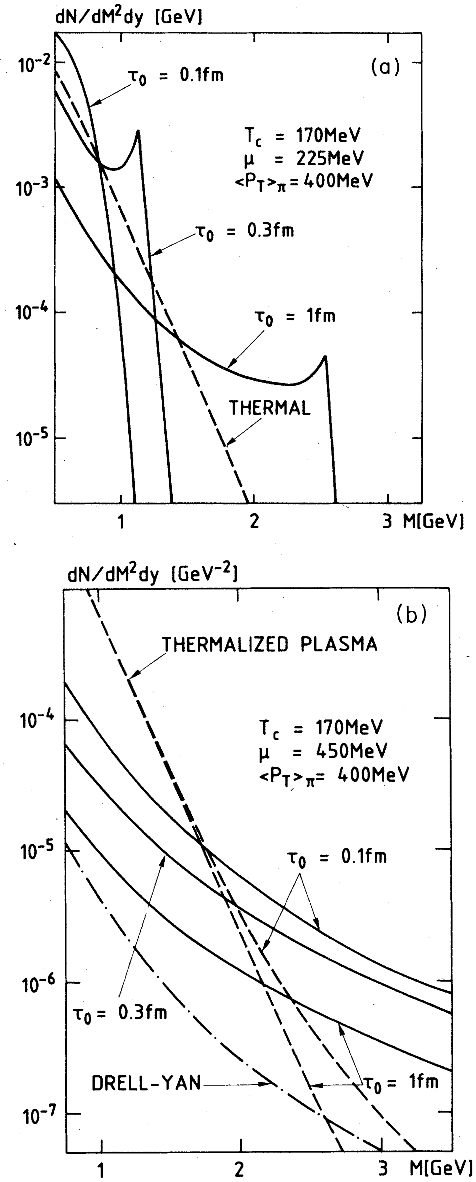

FIG. 4. (a) Same as in Fig. 3, for different values of the parameters T_c and μ. (b) Same as (a), for a different value of μ.

can be produced by oscillating plasma. Indeed, since the plasma oscillations are damped (due to the longitudinal expansion) the longitudinal momenta of quarks and antiquarks are limited and so also is the mass of the lepton pairs [transverse momenta are very effectively cut off by Eq. (3.4)]. The upper limit on quark (antiquark) longitudinal momentum depends strongly on the ratio $F_{03}(\tau_0)/D$ [see Appendix A, Eq. (A7)] and thus, because of Eq. (4.4), on the ratio μ^2/D. Consequently, for small μ, the contribution from oscillating plasma does not extend to large masses.

Finally, in Fig. 4(b) we present calculations for $\mu = 450$ MeV and $r = 1$ which again correspond to $T_c = 170$ MeV. Since μ [and thus $F_{03}(\tau_0)$] is large enough, the situation is similar to that shown in Fig. 3. The rate from the oscillating plasma neatly dominates over the thermal one for $\mu \geq 2$ GeV. The Drell-Yan production is systematically lower in the range of masses we have considered.

We see from these results that for dilepton masses below the maximum value $M_{\max}$ the obtained pre-equilibrium spectra do not vary drastically within the range of parameters we consider. One sees from Figs. 3 and 4 that the pre-equilibrium rates dominate thermal and Drell-Yan rates for 2 GeV $\leq M \leq M_{\max}$. The character of the dilepton spectrum depends crucially on the value of $M_{\max}$ which, as shown in Appendix A, is basically determined by the chromoelectric field strength F_{03} at the initial moment τ_0. Unfortunately, $F_{03}(\tau_0)$ cannot be confidently determined and Eq. (4.4), which we use in the present paper, should be considered only as a very first guess. Nevertheless, since the strength of the chromoelectric field is expected to increase with increasing average transverse momentum of quarks,[15] we conclude that the possibility of observing the lepton pairs produced by the pre-equilibrium plasma is greatly enhanced in events with hadron transverse momenta larger than typically 300 MeV.

V. CONCLUSIONS

We have estimated the rates of dilepton production from a $q\bar{q}$ plasma created in central collisions of two heavy ions. The plasma was described by a model developed recently[10] which takes into account, albeit in an approximate way, color oscillations. Our work has an exploratory character and does not pretend to give an accurate description of the pre-equilibrium dynamics. Nevertheless, our results suggest that the pre-equilibrium phenomena may have observable consequences. In particular we found that, for strong enough chromoelectric field at the initial time of plasma formation, the expected pre-equilibrium rates for dilepton production with masses $M > 2$ GeV are substantially higher than the rates predicted for a thermalized plasma as well as those arising from the Drell-Yan process. This result implies that measurements of dilepton production in the mass region $2 < M < 6$ GeV, for events with relatively large transverse momentum of produced hadrons, may provide interesting information on the behavior of $q\bar{q}$ plasma, even before it reaches thermodynamic equilibrium. Of course one should stress again that, in view of the simplicity of our model and the uncertainties in the initial conditions, our calculation can be regarded only as a first step in the investigation of pre-equilibrium phenomena. In particular, the strong sensitivity of our results to the initial value of the chromoelectric field (which is not well understood at present) introduces large uncertainties in the predicted rates. The results which we obtained, however, encourage further theoretical work in this domain. It would be most important to improve understanding of the origin and intensities of the color field spanned between the receding nuclei.[13–15,17] Furthermore, a better description of transverse momentum distribution of the produced pairs is clearly desirable. This can be achieved, however, only after the model of plasma shall be able to treat realistically the transverse degrees of freedom.

ACKNOWLEDGMENTS

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APPENDIX A: EXPLICIT SOLUTION OF THE FIELD EQUATION IN THE LIMIT OF VANISHING QUARK MASS

In the case where $m=0$, the equation for $H(u)$ takes the form [see Eq. (4.1) of Ref. 10]

$$\ddot{H}(u)=-\frac{\kappa}{u}\frac{H}{|H|}=(-)^{k+1}\frac{\kappa}{u},\quad \kappa=\frac{\lambda d}{12}, \tag{A1}$$

where the overdots denote derivatives with respect to u and the index k ($k=0,1,2,\ldots$) labels the various domains of u where H has a given sign. To be specific, H vanishes and changes sign at the points $u=u_{2k}$, while $\dot{H}(u)$ vanishes at the points $u=u_{2k+1}$. $H(u)$ has a definite sign for $u_{2k}<u<u_{2k+2}$, positive if k is even, negative if k is odd. The solution for $u_{2k}<u<u_{2k+2}$ reads

$$H(u)=(-)^{k+1}\kappa\left[u\left[\ln\frac{u}{u_{2k+1}}-1\right]-u_{2k}\left[\ln\frac{u_{2k}}{u_{2k+1}}-1\right]\right]. \tag{A2}$$

The points u_{2k+1} may be determined from the relation

$$u_{2k+1}u_{2k+3}=u_{2k+2}^{\;2} \tag{A3}$$

with u_1 given by

$$u_1=u_0\exp\frac{\dot{H}(u_0)}{\kappa}\equiv u_0\exp\frac{\dot{H}}{\kappa}. \tag{A4}$$

The points u_{2k+2} where $H=0$ are obtained from Eq. (A2) which implies

$$u_{2k+2}\left[\ln\frac{u_{2k+2}}{u_{2k+1}}-1\right]=u_{2k}\left[\ln\frac{u_{2k}}{u_{2k+1}}-1\right]. \tag{A5}$$

The momentum of a quark is related to $H(u)$ by [see Eq. (3.6)]

$$P(u)=\frac{-\lambda H(u)}{\sqrt{u}}. \tag{A6}$$

It is approximately maximum when $H(u)$ is maximum, that is, when $u=u_1$. We then have

$$P_{\max}\approx|P(u_1)|=\kappa\tau_0 e^{\dot{H}/2\kappa}[1-(1+\dot{H}/\kappa)e^{-\dot{H}/\kappa}]. \tag{A7}$$

APPENDIX B: FORMULA FOR RATE OF LEPTON-PAIR PRODUCTION IN OSCILLATING PLASMA

We start from a well-known expression for the rate per unit volume of space-time d^4x (see, e.g., Refs. 7 and 8):

$$\frac{dN}{d^4x\,dM^2dy\,d^2q}=\frac{1}{4}\frac{8\pi\alpha^2}{9}\sum_q e_q^{\;2}\int\frac{d^2p_1d^2p_2\delta^{(2)}(\mathbf{q}-\mathbf{p}_1-\mathbf{p}_2)}{[(M^2+2\mathbf{p}_1\mathbf{p}_2)^2-4p_1^{\;2}p_2^{\;2}]^{1/2}}n\bar{n}, \tag{B1}$$

where $\mathbf{q}$, $\mathbf{p}_1$, and $\mathbf{p}_2$ denote transverse momenta of the pair, the quark and the antiquark, respectively. Furthermore, n and $\bar{n}$ are densities of quarks and antiquarks per unit volume in phase space, related to G and $\bar{G}$ of Ref. 10 by the formula (2.1).

Using the solution of Ref. 10 we thus obtain

$$G=D\delta(w_1-\tau P(u))F(\mathbf{p}_1), \tag{B2}$$

$$\bar{G}=D\delta(w_2+\tau P(u))F(\mathbf{p}_2), \tag{B3}$$

where $P(u)=-\lambda H(u)/\tau$, with $\tau^2\equiv u$, is given by the solution of field equations, as discussed in Sec. II. $F(\mathbf{p})$ is the distribution of transverse momenta of quarks (antiquarks) [cf. Eq. (3.4)]:

$$F(\mathbf{p})=\frac{1}{\pi\mu^2}e^{-p^2/\mu^2} \tag{B4}$$

with $\mu^2\equiv\langle p^2\rangle$ being the average of squared transverse momentum. The mass of the quarks shall be neglected.

Using Eqs. (B1)—(B4) we thus have

$$\frac{dN}{d^4x\,dM^2dy\,d^2q}=\frac{\pi\alpha^2}{18}\sum e_q^{\;2}D^2\delta(M\tau\sinh(y-\eta))I \tag{B5}$$

with

$$I\equiv\int\frac{d^2p_1d^2p_2\delta^{(2)}(\mathbf{q}-\mathbf{p}_1-\mathbf{p}_2)F(\mathbf{p}_1)F(\mathbf{p}_2)}{[(M^2+2\mathbf{p}_1\mathbf{p}_2)^2-4p_1^{\;2}p_2^{\;2}]^{1/2}}\times\delta(w_1-\tau P(u)), \tag{B6}$$

where we have used the relation

$$w_1+w_2=\tau M\sinh(y-\eta) \tag{B7}$$

with

$$t=\tau\cosh\eta,\quad z=\tau\sinh\eta. \tag{B8}$$

The integral I is invariant with respect to Lorentz boosts in the z direction. It can be evaluated most easily in the frame where the longitudinal momentum of the lepton pair vanishes. In this frame we have

$$\begin{aligned}4M_\perp^{\;2}|P_L|^2&=M_\perp^{\;4}-2(p_1^{\;2}+p_2^{\;2})M_\perp^{\;2}+(p_1^{\;2}-p_2^{\;2})^2\\&=[M_\perp^{\;2}+2(\mathbf{p}_1\cdot\mathbf{p}_2)]^2-4p_1^{\;2}p_2^{\;2},\end{aligned} \tag{B9}$$

where

$$M_\perp{}^2 = M^2 + q^2 \ . \tag{B10}$$

It is also convenient to introduce the variables

$$\mathbf{p}_\pm = \mathbf{p}_1 \pm \mathbf{p}_2, \quad d^2p_1 d^2p_2 = \tfrac{1}{4} d^2p_+ d^2p_- \ . \tag{B11}$$

Since in this frame of reference the contributions to the cross section (B5) come only from the point $\eta = y = 0$ we have $w_1 = p_L \tau$. Consequently, we obtain

$$I = \frac{1}{4} \frac{1}{\mu^4 \pi^2} \frac{e^{-q^2/2\mu^2}}{2W} \times \int d^2p_- \frac{e^{-p_-{}^2/2\mu^2}}{|P_L|} \delta(P_L \tau - \tau P(u)) \ , \tag{B12}$$

where we have used the formula

$$p_1{}^2 + p_2{}^2 = \tfrac{1}{2}(p_+{}^2 + p_-{}^2) \tag{B13}$$

and integrated off one δ function (which implies $\mathbf{p}_+ = \mathbf{q}$).

The integration over d^2p_- can also be performed taking advantage of the other δ function. Using the formula

$$p_1{}^2 - p_2{}^2 = \mathbf{p}_+ \cdot \mathbf{p}_- = \mathbf{q} \cdot \mathbf{p}_- \equiv |\mathbf{q}|\,|\mathbf{p}_-| \cos\phi \ , \tag{B14}$$

where ϕ is the angle between $\mathbf{q}$ and $\mathbf{p}_-$, we obtain

$$\tfrac{1}{2} \int d\phi \, \delta(\tau P_L - \tau P(u)) = \frac{8 M_\perp{}^2 |P_L|}{M\tau} \frac{1}{[(p_-{}^2 - a)(b - p_-{}^2)]^{1/2}} \tag{B15}$$

with

$$a \equiv M^2 - 4P^2(u) \geq 0, \quad b = \frac{M_\perp{}^2}{M^2} a \quad \text{and} b \geq p_-{}^2 \geq a \ .$$

Consequently we have

$$I = \frac{1}{\pi^2 \mu^4} \frac{e^{-q^2/2\mu^2}}{M\tau} \int_b^a dp^2 \frac{e^{-p^2/2\mu^2}}{[(p^2 - a)(b - p^2)]^{1/2}} = \frac{1}{\pi^2 \mu^4} \frac{e^{-q^2/2\mu^2}}{M\tau} \pi \exp\left[- \left[2 + \frac{q^2}{M^2} \right] \frac{a}{4\mu^2} \right] I_0 \left[\frac{q^2 a}{4M^2 \mu^2} \right], \tag{B16}$$

where I_0 is the Bessel function of the second kind. Finally, substituting (B16) into Eq. (B5) and performing integrations over $d^4x = d^2s \, \tau \, d\tau \, d\eta = S\tau \, d\tau \, d\eta$ we obtain

$$\frac{dN}{dM^2 dy \, d^2q} = \frac{\alpha^2 \sum e_q{}^2}{36} \frac{D^2 S}{\mu^4 M^2} e^{-q^2/2\mu^2} \times \int \frac{du}{u} \exp\left[- \left[2 + \frac{q^2}{M^2} \right] \frac{a}{4\mu^2} \right] I_0 \left[\frac{q^2 a}{4M^2 \mu^2} \right], \tag{B17}$$

where the integration is to be performed over the region where

$$M^2 \geq 4P^2(u) \ . \tag{B18}$$

Equation (B17) can also be integrated over d^2q. Using the formula

$$\int_0^\infty dx \, e^{-\lambda x} I_0(\nu x) = \frac{1}{(\lambda^2 - \nu^2)^{1/2}} \tag{B19}$$

we obtain

$$\frac{dN}{dM^2 dy} = \frac{\pi \alpha^2 \sum e_q{}^2}{18} \frac{D^2 S}{\mu^2 M^2} \int \frac{du}{u} \frac{e^{-a/2\mu^2}}{(2 - 4P^2/M^2)^{1/2}} \ . \tag{B20}$$

It is interesting to consider the limit $\mu \to 0$. In this case $F(\mathbf{p}) \to \delta^{(2)}(\mathbf{p})$ and we obtain

$$\frac{dN}{dM^2 dy} \underset{\mu \to 0}{\longrightarrow} \frac{\pi \alpha^2 \sum e_q{}^2}{36} \frac{D^2 S}{M^3 u P'(u^*)} \ . \tag{B21}$$

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[1]See, e.g., *Quark Matter 84*, proceedings of the Fourth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Helsinki, 1984, edited by K. Kajantie (Springer, New York, 1985).

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[16]See, e.g., A. Bialas, in *Quark Matter Formation and Heavy Ion Collisions* (Ref. 6), p. 139.

[17]It may be possible to relate $F_{03}(\tau_0)$ to the gluon density in colliding nuclei [R. Peschanski (private communcation)].