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Unique chiral three-preon model of quarks and leptons

J. M. Gipson
Physics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

Y. Tosa
Theory Division T-8, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

R. E. Marshak
Physics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
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A unique chiral three-preon model of quarks and leptons is deduced without referring to the resulting composite states. This model is based on the gauged $E_6 \otimes SO(10)$ preon symmetry where E_6 and SO(10) are the metacolor and color-flavor groups, respectively. The model has several notable features. The anomalies of the underlying theory are reproduced by pseudo-Goldstone bosons which acquire masses on the SO(10) scale. The model predicts three generations of ordinary quarks and leptons when SO(10) descends to the standard group through $SU(4)_C \times SU(2)_L \times SU(2)_R$ but not through $SU(5) \times U(1)$. It can explain the correct ordering of the gauge hierarchy in terms of the preonic structure of the Higgs composites. The model predicts a large number of exotic fermionic composites at relatively low energies.

I. INTRODUCTION

The recent detection of the t quark[1] completes the third generation of quarks and leptons and lends even greater attractiveness to the hypothesis of a common origin of the three generations of quarks and leptons (i.e., preon models). It is generally agreed that if quarks and leptons have a composite structure, the mass scale Λ_{MC} of the binding force (which we call "metacolor") must be at least 1 TeV and probably substantially higher.[2] The resulting condition $\Lambda_{MC} \gg M_{q,l}$ is very different from its analog in QCD, namely, $\Lambda_{QCD} \sim M_{baryon}$ (Λ_{QCD} is the mass scale), and places a major constraint on preon-model building. A successful preon theory must be able to predict at least three generations of *massless* quarks and leptons on the *metacolor* scale, in contradistinction to the quark theory where the hadron masses are comparable to the QCD scale.

Many different versions of the preon model have been proposed to deal with this basic problem,[3] ranging from purely fermionic to fermion-plus-boson models, from models with elementary to those with composite gauge bosons, from supersymmetric models to those which are not. The most conservative approach is to model the preon theory after the quark theory by using three fermionic preons, elementary gauge bosons, and no supersymmetry. This approach was pursued by two of the authors (Y.T. and R.E.M.),[4] starting with the *gauged* preon group G_{MC} (metacolor) $\otimes G_{CF}$ (color-flavor). A systematic search was carried out in that paper (which we will refer to as the TM paper) for a three-preon model satisfying a number of fairly general conditions and basically only one candidate preon model was identified.

Recently, an additional constraint has been placed on three-fermionic-preon models of quarks and leptons by Weingarten, Nussinov, and Witten,[5] which hereafter is referred to as the WNW constraint. In particular, they have shown that the mass inequality for bound states,

$$M(\bar{q}\gamma_5 q) \leq kM(qqq) , \tag{1.1}$$

must hold in any vectorlike theory (vectorlike with respect to the binding force). In Eq. (1.1), $M(\bar{q}\gamma_5 q)$ is the mass of the lightest (nonsinglet) meson, $M(qqq)$ is the mass of the lightest baryon, q is a spin-$\frac{1}{2}$ constituent, and k is a constant of the order of unity. This mass inequality requires composite $J=\frac{1}{2}$ fermions to be accompanied by less massive composite charged pseudoscalars. Since there is no evidence for charged $J=0$ bosons as light as quarks and leptons, this result eliminates vectorlike three-fermion preon models of quarks and leptons. As a consequence, it has mistakenly been argued that all three-fermion preon models of quarks and leptons are killed by the WNW constraint.

However, this conclusion is not warranted since the proof of the WNW mass inequality depends on an essential property that holds for any vectorlike but not always for chiral theories, namely the doubling of the Dirac-operator eigenvalues[6] Thus, if a three-fermion preon model is based on the chiral coupling of the preons to the metagluons, it is possible to escape the WNW constraint. Furthermore, if there exist no two-preon condensates (which is natural in chiral theories as we discuss in Sec. III), then the inequality (1.1) loses its meaning. Therefore, chiral three-fermion preon models can still survive. In particular, the model found in the TM paper, i.e., $E_6 \otimes G_{422}$ [$\equiv SU(4) \times SU(2)_L \times SU(2)_R$], with two (left-handed) preon representations [$(27;\bar{4},2,1) + (27;4,1,2)$], survives the WNW test. However, this model was uncovered through the requirement that at least three generations of quarks and leptons should emerge out of three-preon composites. That is, we had to refer to the resulting composites in order to fix the model, as had been the practice.

In this paper, we take a more heuristic approach in the sense that we look for a chiral three-fermion preon model *without* referring to the resulting composites. As a first step in this direction, we employ the group-theoretical method. Dynamical questions, e.g., mass scales, etc., will be left for future investigation. This exercise would only be of academic interest if the resulting model did not agree even qualitatively with the real world. However, it turns out that the unique chiral three-preon model that we find has many promising features, as will be seen.

We begin by considering the preon world with all gauge couplings but the metacolor turned off. The symmetry is then $G_{MC}\otimes G_{CF}$ where the metacolor group G_{MC} is gauged, but G_{CF} is a global color-flavor symmetry, part of which is subsequently gauged. We use the following mild assumptions:

1. Quarks and leptons are three-preon composites.
2. The (massless) preon transforms under a single irreducible representation (irrep) of a simple metacolor group.
3. The metacolor sector is asymptotically free (ASF).
4. The metacolor sector is anomaly-free (ANF).
5. The preon transforms under a single irrep of the gauged subgroup of G_{CF}.

Assumptions 1—4 uniquely fix the metacolor group to be E_6 with its representation (rep) **27** (or its conjugate). Assumption 5 fixes the gauged subgroup of G_{CF} as SO(10) with the rep **16** (or its conjugate). The derivation of these results is carried out in Sec. II.

In Sec. III, we examine the phenomenology of this unique $E_6\otimes$SO(10) model in some detail. The model predicts the correct ordering of the gauge hierarchy. All composite fermions are massless on the metacolor scale. With an additional assumption, the meta-Pauli principle, the model predicts precisely three generations of ordinary quarks and leptons as well as numerous exotics. Another interesting prediction of the $E_6\otimes$SO(10) preon model is that the composite SO(10) group should descend through the Pati-Salam group[7] $SU(4)\times SU(2)_L\times SU(2)_R$, but not the "Georgi-Glashow" path[8] SU(5)×U(1). At the end of Sec. III we briefly discuss the related $E_6\otimes G_{422}$ model previously identified in the TM paper. Some concluding remarks are given in Sec. IV.

II. SEARCH FOR $G_{MC}\otimes G_{CF}$

A. What is G_{MC}?

First, we must clarify the distinction between vectorlike and chiral theories. For convenience, our massless preons are all left handed. (Recall that any theory can be written entirely in terms of left-handed fermions.) Suppose a preon transforms under a rep (R,N) of $G_{MC}\otimes G_{CF}$. The theory is said to be vectorlike (with respect to G_{MC}) if, for every preon which transforms as R, there exists another preon which transforms as $\bar{R}$ so that the preon Ψ is $(R,N)+(\bar{R},N)$. One consequence of this form is that the eigenvalues of the Dirac operator come in pairs. The largest global (chiral) symmetry of the model is then $U(N)_L\times U(N)_R$. All such preon models are ruled out by the WNW constraint.[5] In contrast, in chiral theories, such pairing is not mandatory.

For chiral theories, we will restrict ourselves to the simplest situation in which the preon transforms under a single rep of the metacolor group so that $\Psi=(R,N)$. First, we show that R must be complex. If R is "real" and N even, the model is vectorlike and the WNW constraint eliminates this possibility. If N is odd, one can always divide the preon Ψ into two parts, Ψ_1 and Ψ_2, where $\Psi_1=(R,N=2n)$ and $\Psi_2=(R,1)$. Then, quarks and leptons must be made out of either Ψ_2 alone or a mixture of both, but not Ψ_1 alone, since the WNW constraint applies to the Ψ_1 sector. However, since the division of Ψ is arbitrary, one can always choose Ψ_2 such that at least some of the quarks and leptons are made up only of Ψ_1's. This model is then ruled out by the WNW constraint. Consequently, G_{MC} is limited to those groups which have complex reps. In particular, if we limit ourselves to a simple group, then the choice is only $G_{MC}=SU(n)$ $(n\geq 3)$, $SO(4n+2)$ $(n\geq 2)$, or E_6.

Next, the ASF constraint on G_{MC} must be considered, if preons exist at all. Let us consider for simplicity the case where R is an irrep. Then, the group $SU(n)$ $(n\geq 3)$ is excluded, because single complex irreps with no anomalies are very rare in $SU(n)$ and, indeed, their dimensions are so large as to violate the ASF (Ref. 9). For $SO(4n+2)$ $(n\geq 2)$, the spinor reps are the smallest complex reps. The ASF restricts the number of color-flavors N to be less than $11\times n\times 2^{4-2n}$. Thus, for SO(10), $N<22$; for SO(14), $N<9$, etc. For other reps, N is smaller, since the fermion contribution to the β function is proportional to the second-order index, which is proportional to the dimension of R. For three-fermion preon models of quarks and leptons, the rep is further restricted by the singlet condition, i.e., $R\times R\times R$ must contain a singlet. For those reps with congruence classes, (1,1), (1,3), (0,2) of $SO(4n+2)$ $(n\geq 2)$ (Ref. 10), this is impossible. For those complex reps, belonging to (0,0), this is incompatible with the ASF. Thus, $SO(4n+2)$ $(n\geq 2)$ is excluded for three-fermion preon models. The remaining candidate, E_6, contains exactly one complex rep which satisfies the ASF and the singlet condition,[4] namely **27** (or its conjugate). In this case, $N<22$. Summarizing, if G_{MC} is simple and R is a single complex irrep, then the *only* solution obeying the ASF, ANF, and singlet conditions is

$$G_{MC}=E_6 \text{ with } R=27 \text{ and } N<22\ . \qquad (2.1)$$

B. What is G_{CF}?

The largest global group for chiral theories is $U(N)$. This may be broken, as happens in vectorlike theories. A simple criterion exists for deciding whether the global symmetry G_{CF} is broken or not, namely the 't Hooft anomaly-matching condition.[11] If the anomalies associated with composites and preons do not match, then G_{CF} must be broken (but the reverse does not always hold). For vectorlike theories, Weingarten[12] has shown that $U(N)_L\times U(N)_R$ must be broken. Vafa and Witten[13] have shown that the subgroup $U(N)_{L+R}$ is not broken. In chiral theories, one can find solutions where the anomalies match[14] if one allows several flavors and/or several reps for the metacolor group. However, if one restricts oneself to a single irrep, $\Psi=(R,N)$, and fermionic composites consist of three fermionic preons, then no solution exists for $N\geq 3$, as 't Hooft has shown.[11] Consequently, $G_{CF}=U(N)$ must break into some subgroup of $U(N)$, H_{CF}, where the anomalies match.

Once G_{CF} is broken, massless Goldstone bosons must appear.[15] In vectorlike theories, these Goldstone bosons can be made heavy by introducing mass terms for preons in the fundamental Lagrangian which explicitly break the G_{CF} symmetry[16] (but do not break G_{MC}). Furthermore, once G_{CF} is broken into the vector subgroup, the composites usually acquire masses of the order of Λ_{vector}, even without explicit masses for preons. In chiral three-fermion preon models of quarks and leptons, one cannot introduce explicit mass terms to make Goldstone bosons heavy, since such terms are forbidden by the gauge invariance of G_{MC}. Hence, we must use gauge-boson radiative corrections to make Goldstone bosons heavy[17] and so we must at least gauge a subgroup H'_{CF}, of the unbroken global symmetry H_{CF} (we may have $H_{CH}=H'_{CF}$). Here, it is *necessary* to gauge a subgroup of the global symmetry. (On the contrary, if the anomaly matched, then it would be a mystery why a subgroup of the global symmetry had to be gauged.) Radiative corrections to the masses of fermionic composites can still be avoided, if the gauged subgroup is *chiral,* or there exists an unbroken discrete symmetry to forbid masses to the composites.[18]

Once H'_{CF} is gauged, preons have quantum numbers with respect to this group and thus must be anomaly-free (ANF) for the consistency of the gauge theory. The preon rep for the gauge group H'_{CF} must be complex; otherwise, composites will be "real" and the survival hypothesis tells us that they become heavy.[19] The constraints on H'_{CF} and its rep are very strong: the group must have a complex rep (of dimension less than 22) and it must be ANF. If the preons transform under a *single irreducible rep,* then H'_{CF} is SO(10) and the rep is spinor or antispinor. We do not have to assume that H'_{CF} is simple to prove this. If it is simple, the result is obvious, since E_6 is eliminated by the ASF condition. If $H'_{CF}=G_1\times G_2$ [neither of G_i is U(1)], we assume that the rep transforms nontrivially under both groups, i.e., $N=(r_1,r_2)$. Since it must be complex, one of the groups must be complex. The ASF and ANF conditions uniquely pick up this group to be SO(10) with a spinor (or antispinor) rep. The ASF condition ($N<22$) tells us that the other rep must be trivial, since $\dim N=\dim r_1\times\dim r_2$. Since SU(16) has no subgroup that contains SO(10) and has a 16-dimensional rep, it follows that $H_{CF}=H'_{CF}=SO(10)$. Consequently, as long as we limit ourselves to a single complex irreducible representation of H'_{CF}, the choice is uniquely fixed without referring to the resulting composites; that is to say

$$G_{CF}=U(16),\quad H_{CF}=H'_{CF}=SO(10)\ ,$$

and (2.2)

$$N=\mathbf{16}\ (\text{or}\ \overline{\mathbf{16}})\ .$$

The choice of taking spinor or antispinor is fixed by how we embed the standard group, $SU(3)_C\times SU(2)_L\times U(1)$, in SO(10). See Sec. III and the Appendix. Note that preons and composites share the same gauged anomaly-free SO(10) symmetry. Consequently, our unique chiral preon group is $E_6\otimes SO(10)$ with $\Psi=(27,\overline{16})$.

If we loosen the condition that the preons transform under a single irreducible rep of H'_{CF}, there are more choices. For example, any subgroup of SO(10) is a solution: e.g., $H'_{CF}=SU(5)$ and $N=\mathbf{1}+\mathbf{5}+\overline{\mathbf{10}}$ (or its conjugate), or $H'_{CF}=G_{422}$ with $N=[(\bar{4},2,1)+(4,1,2)]$ (or its conjugate) are solutions. However, if we consider the conditions imposed on the three-preon composites by the family structure (see Sec. III), the first possibility is ruled out while the second possibility survives. Thus, the model $E_6\otimes G_{422}$ can also serve as a chiral preon model of quarks and leptons. Note that this choice was the sole survivor in the TM paper.[4] The reason why $E_6\otimes SO(10)$ was rejected in that paper will be explained in Sec. III. While the uniqueness of the $E_6\otimes SO(10)$ model stems from the use of one complex irrep, it is interesting to inquire whether other solutions with two irreps exist and, if so, whether they give the correct family structure for ordinary quarks and leptons. This analysis is carried out in the Appendix and we find that while we can identify several other chiral preon models with two complex irreps, none of them, except $E_6\otimes G_{422}$, meets the family-structure test. Thus, we must refer to the resulting composite states to fix a model if we use more than one irrep of the gauged subgroup of G_{CF} for the preons.

III. COMPOSITE MODEL BASED ON $E_6\otimes SO(10)$

We turn now to a closer examination of the unique $E_6\otimes SO(10)$ chiral preon model[20] and conclude the section with a brief comparison between our previous model $E_6\otimes G_{422}$ and this model. The gauge-invariant Lagrangian for the preon has the global symmetry

$$SU(16)\times Z_{16}\ , \tag{3.1}$$

where the naive U(1) symmetry is broken by the metacolor instanton and is reduced to the discrete Z_{16} symmetry. Note that the gauge invariance of $E_6\otimes SO(10)$ prevents a bare mass term for the preon. As we have mentioned in the previous section, the symmetry SU(16) is broken into SO(10), where the anomaly matches trivially. Associated with the breaking are $(255-45)=210$ Goldstone bosons which transform as an irreducible rep **210** under SO(10). These acquire masses on the SO(10) scale via radiative corrections.[17]

A. Composite scalars

At the composite level, metacolor is presumed to be hidden: all composites transform as metacolor singlets. We shall begin our discussion by examining the composite scalars. First of all, one should note that there are no two-preon metacolor singlets, which are Lorentz scalars, since the only candidate $\bar{P}\not{D}P$ vanishes by the equation of motion. At the four-preon level, there exist composite scalars. With the omission of the Lorentz structure, these can be written as

$$\bar{P}\bar{P}PP \sim \mathbf{1}+\mathbf{45}+\mathbf{54}+\mathbf{210}+\cdots \; . \tag{3.2}$$

All of these composites are invariant under the discrete symmetry Z_{16}. Hence, even if these scalars acquire vacuum expectation values (VEV's), the discrete symmetry is unbroken. We have indicated only the lower-dimensional reps and their multiplicities are omitted. It is noteworthy that these scalar composites contain terms which transform as singlets under all five independent U(1) generators of SO(10): λ_3, λ_8, T_{3L}, T_{3R}, and $(B-L)$. Thus, if any of these acquires a VEV, the rank is preserved. Other terms, which are singlets under SU(3) color and $U(1)_{EM}$, can break the rank. However, it turns out (after a brute-force calculation) that they always break *both* $SU(2)_L$ and $SU(2)_R$ [with or without breaking $U(1)_{B-L}$]. Furthermore, these cannot give masses to composite fermions, since they belong to the congruence class (0,0), but not (0,2), of SO(10) (Ref. 10). Consequently, only those scalars, which do not break the rank, can be used phenomenologically.

At the six-preon level, the only composite scalars are of the form

$$(P^tCP)(P^tCP)(P^tCP) \tag{3.3}$$

and their complex conjugates. One might think that one could form preon scalars out of three P's and three $\bar{P}$'s. All such terms would have to consist of contractions of $\bar{P}\sigma_\mu P$ or $\bar{P}[\sigma_\mu,\sigma_\nu]P$. A careful examination indicates that they all vanish. The six-preon composites are not invariant under Z_{16}. Therefore, if any of these acquires a VEV, the discrete symmetry is broken. The SO(10) decomposition of the six-preon composites is[21]

$$PPPPPP \sim \mathbf{10}+\mathbf{120}+\mathbf{126}+\mathbf{210}'+\cdots \; . \tag{3.4}$$

Again we have given only the lower-dimensional reps and have neglected their multiplicities. It turns out after a long and tedious examination that *none* of the six-preon condensates transform as singlets under all five commuting generators of SO(10). Therefore, if a six-preon composite scalar acquires a VEV, it must reduce the rank of the group.

In the descent of SO(10) to $SU(3)_C \times U(1)_{EM}$ ($\equiv G_{31}$), one commonly has one (or more) stages of symmetry breaking in which the rank of the group is preserved, followed by stages in which the rank of the group is broken. If the Higgs bosons are indeed composite scalars, then our $E_6 \otimes SO(10)$ preon model must be able to explain the patterns of symmetry breaking and the Higgs quantum numbers to implement them. A typical chain obtained from the ordinary grand unified SO(10) model is as follows:[22]

$$SO(10)\underset{\mathbf{210}}{\rightarrow}G_{422}\underset{\mathbf{45}}{\rightarrow}G_{3122}\;[\equiv SU(3)_C\times U(1)_{B-L}\times SU(2)_L\times SU(2)_R]$$
$$\underset{\mathbf{126}}{\rightarrow}G_{321}\;[\equiv SU(3)_C\times SU(2)_L\times U(1)_Y]\underset{\mathbf{10}}{\rightarrow}G_{31}\;, \tag{3.5}$$

where the Higgs reps are shown underneath the arrows. From the analysis of the previous paragraph, it follows that the first two stages of symmetry breaking in Eq. (3.5) must be achieved by the four-preon scalar composites, while the remaining two stages can be accomplished by six-preon scalar composites. It is not unreasonable to assume that the four-preon composites bind at a higher scale than the six-preon composites,[23] thus leading to the correct ordering of the gauge hierarchy.

B. Composite fermions

We now consider the composite fermions which are composed of three preons and are metacolor singlets. The mass term for these composites would appear as

$$(PPP)^tC(PPP) \tag{3.6}$$

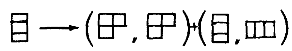
which violates both the discrete symmetry Z_{16} and the SO(10) gauged symmetry. The reason why it breaks the SO(10) symmetry is obvious, since the multiplication of six **16**'s of SO(10) belongs to the congruence class (0,2), but not (0,0), to which a singlet belongs.[10] These two symmetries keep the fermionic composites massless on the metacolor scale. [On the other hand, the mass terms for bosonic (scalar, vector) composites are not forbidden by these two symmetries.] The composite fermions acquire masses spontaneously through Yukawa couplings of the form

$$(PPP)^tC(PPP)(\bar{P}\bar{P}\bar{P}\bar{P}\bar{P}\bar{P}) \; . \tag{3.7}$$

Once the six-preon composite scalars get VEV's, then both the discrete symmetry and the SO(10) symmetry are broken and the fermions acquire masses of the order of the VEV of the six-preon composite scalar times some constant. Since the mass terms for chiral fermions must break both $SU(2)_L$ and $SU(2)_R$ [or $U(1)_R$], the mass scale for the VEV of six-preon composite scalars must be of the order of $M(W_L)$ or $M(W_R)$. Hence, all the masses of fermionic composites (including exotics) will be of that order. That is, in chiral gauge theories, the masses of chiral fermions (including exotics) should be accessible in the not-too-distant future.

Now, we look into the family structure of composite fermions. By the Pauli principle, the composite wave function must be totally antisymmetric under $E_6 \otimes SU(16)\times SU(2)_{L\text{-spin}}$ (the L-spin notation reminds us that all our Weyl spinors are left handed). As is well known, in QCD, one must make a stronger assumption in order to reproduce the hadron spectrum,[4] namely, that the composites are antisymmetric under $SU(3)_C \times SU(2N)$ (where N is the number of flavors) although the Pauli principle only requires that the antisymmetricity is for the product $SU(3)_C \times SU(N) \times SU(2)$. In QCD the symmetry $SU(2N)$ emerges out of the $SU(N)$ flavor symmetry and the nonrelativistic SU(2) [=SO(3)] spin symmetry. Consequently, we make the analogous assumption that in our model, the three-preon composites transform under the totally antisymmetric rep of SU(32) (meta-Pauli principle). (Note that the singlet of E_6 is in the totally symmetric part of the multiplication of three 27's.)

The breaking of the totally antisymmetric rep of SU(32) into $SU(16)\times SU(2)_{L\text{-spin}}$ is

$(RRR)_A =$ 

$$4960\rightarrow(1360,2)+(560,4)\ . \tag{3.8}$$

The first term on the right-hand side of Eq. (3.8) gives the spin-$\frac{1}{2}$ left-handed fermions, and the second term the spin-$\frac{3}{2}$ left-handed fermions.[24] As Weinberg and Witten have observed,[24] *massless* spin-$\frac{3}{2}$ particles (even massless spin-1 particles) are inconsistent at the level of the global symmetry G_{CF}. However, once part of G_{CF} is gauged, they can arise without any inconsistencies, in the same way that massless spin-1 gauge bosons are allowed. Consequently, these spin-$\frac{3}{2}$ particles do not have masses of the order of the metacolor scale, but of the order of the scales in the gauged SO(10) group; from the argument above, this implies of the order of $M(W_L)$ or $M(W_R)$. Here, we are using G_{CF} only as the tool for applying the meta-Pauli principle. The actual contents are gauged SO(10) particles. The SO(10) content of these SU(16) reps is

$$\mathbf{1360}\rightarrow\mathbf{16}+\mathbf{144}+\mathbf{1200}\ , \tag{3.9}$$

$$\mathbf{560}\rightarrow\mathbf{560}\ , \tag{3.10}$$

where we have embedded **16** of SU(16) in $\overline{\mathbf{16}}$ of SO(10). Evidently, the $E_6\otimes SO(10)$ preon model predicts a large number of composite fermions at relatively low energy.[25] Dynamical considerations may reduce the number of exotic fermions in the preon case. We note that no mirror fermions exist.

We have three SO(10) irreps of spin-$\frac{1}{2}$ and a single irrep of spin-$\frac{3}{2}$ fermions. If we now identify the 16 rep of SO(10) with an ordinary fermion family, our model yields just one family. This was the reason why $E_6\otimes SO(10)$ was rejected in the TM paper.[4] In the present approach, the question of family structure is posed in a more subtle fashion: Are there any intermediate stages of the breaking of SO(10) where we can identify at least three families of ordinary quarks and leptons? The answer to this question clearly depends on the path of descent from SO(10).

In the breaking of SO(10) to the standard model, there are two possibilities: either the group passes through $SU(5)\times U(1)$ as an intermediate step, or it passes through G_{422} [see Eq. (3.5)]. In traditional SO(10) grand unified theory where the ordinary fermions consist of three copies of 16's, both possibilities lead to three families of quarks and leptons. This is in marked contrast to what happens in our model, where the particle spectrum depends on what path one takes. In our model, only the path through G_{422} gives rise to precisely three copies of (4,2,1) and $(\bar{4},1,2)$, i.e., three families of ordinary quarks and leptons, in addition to exotics. On the other hand, the path through $SU(5)\times U(1)$ yields only one family plus exotics. Consequently, new gauge bosons, in addition to exotic fermions, flourish in the "desert" below 10^{15} GeV. Let us prove these statements.

First, we show that the path through $SU(5)\times U(1)$ produces only one family. The breaking of the three reps **16**, **144**, **1200** of SO(10) is as follows:

$$\begin{aligned}
\mathbf{16}&\rightarrow\mathbf{1}(-5)+\bar{\mathbf{5}}(3)+\mathbf{10}(-1)\ ,\\
\mathbf{144}&\rightarrow\bar{\mathbf{5}}(3)+\mathbf{10}(-1)+\mathbf{5}(7)+\mathbf{15}(-1)+\mathbf{24}(-5)+\mathbf{40}(-1)+\overline{\mathbf{45}}(3)\ ,\\
\mathbf{1200}&\rightarrow\bar{\mathbf{5}}(3)+\mathbf{10}(-1)+\mathbf{5}(7)+\overline{\mathbf{10}}(11)+\mathbf{15}(-1)+\mathbf{24}(-5)+\mathbf{40}(-1)+\overline{\mathbf{40}}(9)+\mathbf{45}(7)+\overline{\mathbf{45}}(3)+\overline{\mathbf{45}}(3)\\
&\quad+\mathbf{50}(7)+\overline{\mathbf{75}}(-5)+\mathbf{126}(-5)+\mathbf{175}(-1)+\mathbf{210}(-1)+\overline{\mathbf{280}}(3)\ ,
\end{aligned} \tag{3.11}$$

where the U(1) quantum number is given in parentheses. If the U(1) symmetry were not broken, we could end up having three $\bar{\mathbf{5}}(3)+\mathbf{10}(-1)$. Note also that only one singlet exists. However, U(1) must be broken and thus the use of the survival hypothesis[19] leads to the result that the light fermions are only $\mathbf{1}+\bar{\mathbf{5}}+\mathbf{10}+\mathbf{10}+\cdots$, where the ellipses denote the higher-dimensional particles. Hence, we have only one family plus exotic fermions for the path $SO(10)\rightarrow SU(5)\times U(1)$.

In contrast, if SO(10) breaks into G_{422}, each of the three spin-$\frac{1}{2}$ reps contains a single generation of ordinary quarks and leptons; to wit,

$$\begin{aligned}
\mathbf{16}&\rightarrow(4,2,1)+(\bar{4},1,2)\ ,\\
\mathbf{144}&\rightarrow(4,2,1)+(\bar{4},1,2)+(\bar{4},3,2)+(4,2,3)+(\overline{20},2,1)+(20,1,2)\ ,\\
\mathbf{1200}&\rightarrow(4,2,1)+(\bar{4},1,2)+(\bar{4},3,2)+(4,2,3)+(\overline{20},2,1)+(20,1,2)+(\overline{20}',2,1)+(20',1,2)\\
&\quad+(\overline{20},4,1)+(20,1,4)+(\overline{20},2,3)+(20,3,2)+(\overline{36},1,2)+(36,2,1)+(36,2,3)+(\overline{36},3,2)\ .
\end{aligned} \tag{3.12}$$

Note that $(4,2,1)+(\overline{4},1,2)$ is the single generation of quarks and leptons. Although a large number of exotics is predicted, these exotics belong to the same congruence class of quarks and leptons for G_{422}. Thus, we do not have mirror fermions. These exotic fermions will populate the mass scale around $M(W_L)$ and $M(W_R)$. For spin-$\frac{3}{2}$ fermions, we have:

$$\mathbf{560}\to(4,2,1)+(\overline{4},1,2)+(\overline{4},3,2)+(4,2,3)+(4,4,1)+(\overline{4},1,4)+(\overline{20},2,1)+(20,1,2)+(\overline{20},2,3)+(20,3,2)+(36,2,1)+(\overline{36},1,2)\ . \tag{3.13}$$

Hence, there exists one family of spin-$\frac{3}{2}$ quarks and leptons, plus many exotics.[26]

Now, we turn our attention to the mass matrix of spin-$\frac{1}{2}$ particles. The diagonal couplings for each generation can be given by both Higgs reps **10** and **126**, while the off-diagonal generation-connecting couplings are

$$\begin{aligned}&\phi_{10}\times\mathbf{16}\times\mathbf{144}\ ,\\&\phi_{126}\times\mathbf{144}\times\mathbf{1200}\ ,\\&\phi_{120}\times\mathbf{16}\times\mathbf{144},\ \ \phi_{120}\times\mathbf{16}\times\mathbf{1200}\ ,\\&\phi_{120}\times\mathbf{144}\times\mathbf{1200}\ ,\end{aligned} \tag{3.14}$$

where ϕ_{126} is responsible for the breaking of $SU(2)_R$ or $U(1)_R$, ϕ_{10} is responsible for the last stage of symmetry breaking, and ϕ_{120} is the (antisymmetric) Higgs rep that connects generations. The values of Yukawa couplings, in principle, can be determined by the dynamics of preons. The explicit evaluation is a formidable task. We can speculate that the Yukawa couplings are in proportion to the second-order indices of the fermion reps for the diagonal part.[27] Then, the mass increases with the dimension, which in turn leads to the identification of **16**, **144**, and **1200** as the first, second, and third generations, respectively. If the off-diagonal couplings are dominated by the Higgs **10** (as is usually assumed), then neither **16** nor **144** can couple to **1200**. Thus, one would expect from the first interaction in Eq. (3.14) that the first two generations mix with each other but not with the third generation. These qualitative results are consistent with the known phenomenology for ordinary quarks and leptons. Detailed calculations of the masses and mixing angles on the basis of the $E_6\otimes SO(10)$ model are under way.

We conclude this section with some remarks about the difference between the $E_6\otimes SO(10)$ and $E_6\otimes G_{422}$ models under the following headings: (1) Goldstone bosons; (2) gauge hierarchy; and (3) masses of composite fermions.

(1) With regard to the Goldstone bosons, the $E_6\otimes G_{422}$ model predicts 234 $(=255-21)$ Goldstone bosons (not 210) with the quantum numbers

$$\mathbf{234}=(1,1,1)+(15,1,1)+2(6,2,2)+(15,3,1)+(15,1,3)+(10,2,2)+(\overline{10},2,2)\ . \tag{3.15}$$

The difference of 24 between G_{422} and SO(10) comes from one of the (6,2,2) terms. Note that while in $E_6\otimes SO(10)$, 210 $(=255-45)$ Goldstone bosons belong to a single irrep of SO(10) and thus all of them acquire masses on the SO(10) scale, in $E_6\otimes G_{422}$, Eq. (3.15) tells us that we must deal separately with a singlet Goldstone boson with respect to G_{422}. However, this would not be a problem since the singlet should behave as a "Majoron," which couples very weakly to matter.[28]

(2) The attractive features of the ordering of the gauge hierarchy in the $E_6\otimes SO(10)$ model are replicated in the $E_6\otimes G_{422}$ model. Thus, a four-preon composite scalar like (15,1,1) breaks G_{422} into G_{3122} [see Eq. (3.5) for notation] without reducing the rank of the group, whereas a six-preon composite scalar like (1,2,2) breaks G_{321} into G_{31}, reducing the rank by one.

(3) When we come to the problem of the masses of composite fermions, the $E_6\otimes SO(10)$ model seems capable of providing a more "natural" explanation than the $E_6\otimes G_{422}$ model for the large mass ratios among the three generations of ordinary quarks and leptons. In the former model, the rapidly increasing dimensionalities of the SO(10) reps may offer a simple explanation of the increasingly larger average masses of the three generations. In the latter model, we end up with exactly the same three generations of ordinary quarks and leptons and thus we must seek another explanation of the mass ratios. One possibility is to differentiate $(\overline{4},2,1)$ $(\equiv T)$ and (4,1,2) $(\equiv V)$ in the $E_6\otimes G_{422}$ model. In that case it has been shown[4] that TTT (together with VVV) gives rise to one generation and TVV (together with TTV) gives rise to two generations. By applying an argument of Nussinov's,[29] it seems possible to show that $M(TVV)\geq M(TTT)$. This mass inequality, together with the 2×2 mass matrix associated with the TVV composites, could still lead to a satisfactory mass ordering of the three generations. The comparison between the $E_6\otimes SO(10)$ and $E_6\otimes G_{422}$ models becomes more subtle when we look at the exotic fermions that they predict. In fact, the G_{422} quantum numbers of the exotic fermions are exactly the same for the two models.[4] The only difference arises from the identification of which exotic composite belongs to which generation, a nuance which conceivably could be checked by experiment.

IV. CONCLUDING REMARKS

In this paper, we have reexamined the choice of a possible three-fermion preon model of quarks and leptons, by taking account *ab initio* of the mass-inequality constraint on vectorlike composite theories and by referring to the composite states in a minimal way. The constraint found by Weingarten-Nussinov-Witten[5] can be circumvented by insisting on *chiral* coupling of preons to metagluons. This approach leads uniquely to E_6 as the metacolor group because it is the only simple group with a single complex anomaly-free irrep that contains a singlet in the three-preon composite rep and can be asymptotically free. This is to be contrasted with QCD where two basic complex irreps for quarks, **3** and $\overline{\mathbf{3}}$ of SU(3), are responsible for its vectorlike character and ensure its freedom from anomalies. (Note that we use Weyl fermions.) An important difference between the SU(3) color and the E_6 metacolor groups also shows up in connection with the application of the generalized Pauli principle to three-fermion composites: $SU(3)_C$ contributes an antisymmetric part to the color singlet, while E_6 contributes a symmetric part to the metacolor singlet. The choice of E_6 as the metacolor group seems inescapable, in the sense that it is very natural to use one complex irrep for a chiral theory and two conjugate complex irreps for a vectorlike theory.

The choice of the color-flavor symmetry G_{CF} also turns out to be remarkably restrictive under some general conditions: that the metacolor group is asymptotically free and the gauged sector of G_{CF} is free from anomalies. If we limit ourselves to a single complex irrep of the gauged subgroup of G_{CF}, then we are compelled to choose the gauged subgroup to be SO(10) with $N=\overline{\mathbf{16}}$. This conclusion follows without using any input from the observed family structure of quarks and leptons.

If we enlarge the number of complex irreps of the gauged color-flavor group to two *and* take account of the observed family structure of the quarks and leptons, we are led to G_{422} with $N=[(\overline{4},2,1)+(4,1,2)]$ as the only viable possibility (see the Appendix). Since G_{422} is a (maximal) subgroup of SO(10) and $\overline{\mathbf{16}}$ decomposes into $(\overline{4},2,1)+(4,1,2)$, this model is evidently a close relative of the $E_6\otimes SO(10)$ model and, indeed, makes very similar predictions (as noted at the end of Sec. III).

What are the properties of the $E_6\otimes SO(10)$ preon model that sustain the hope that we may be on the right track to obtaining a solution to the generation problem for ordinary quarks and leptons? These encouraging properties are the following: (1) A good reason exists why a subgroup of the global color-flavor symmetry group must be gauged. (2) All Goldstone bosons [**210** of SO(10)] arising from the breaking of the global SU(16) color-flavor symmetry down to SO(10) become pseudo-Goldstone bosons via SO(10) gauge-boson radiative corrections and thus acquire masses of the order of the SO(10) scale. Hence, the Goldstone-boson problem is tractable in this model and there is no need to satisfy the 't Hooft anomaly-matching condition at the global SU(16) level; it suffices to do so at the SO(10) level. (3) The chiral nature of the model implies that there are no metacolor-singlet two-preon composites and so no contributions to quark and lepton masses. [This fact also affects the decays of exotic fermions[26]—in contrast, in QCD, the high-mass states almost always decay into low-mass states plus two-quark scalar composites (pions, etc.).] It turns out that even the four-preon composites are incapable of giving masses to quarks and leptons. Only the six-preon composites and beyond are capable of giving masses. This is the first clue to understanding the order of the gauge hierarchy and may help to explain why the masses of composite quarks and leptons are much lighter than the metacolor scale. (4) The generalization of the Pauli principle to the metacolor degree of freedom leads to three spin-$\frac{1}{2}$ irreps of SO(10), namely, **14**, **144**, and **1200**. When these reps are decomposed in terms of the G_{422} quantum numbers [the $SU(5)\times U(1)$ path must be rejected], one is able to identify precisely three generations of ordinary quarks and leptons and a large number of exotic fermions [which are "internally" excited in the quantum numbers of $SU(2)_L$, $SU(2)_R$, and/or SU(4)]. Note that there are no mirror fermions. Since our approach is totally group-theoretical, dynamical considerations may reduce the number of exotic fermions. This may alleviate any potential conflict in the renormalization group analysis. (5) Because of the chiral structure, all the masses of the exotic fermions will be of the order of $M(W_L)$ or $M(W_R)$. Thus, if they do exist, they should be seen in the not-too-distant future. (6) The occurrence of each generation of ordinary quarks and leptons corresponds to each of the three increasingly larger dimensional reps of SO(10). This feature offers the hope of explaining the substantial increases in mass as one moves from one generation to the next. This model even contains within it a mechanism that decouples the third generation from the first two, a phenomenon which seems to be showing up at least in the quark sector.

The observations (1)—(6) spelled out above make it clear that the $E_6\otimes SO(10)$ group passes a number of important tests. However, the ultimate usefulness of the model will obviously depend on its ability to make reasonably good quantitative predictions. In order to do so, we must perform *dynamical* calculations, although we are far from understanding the dynamics of chiral theories. Once these are done, we should be able to answer the haunting questions: (1) the values of the mass scales in the theory; (2) the fermion mass matrix and the mixing angles; (3) mass inequalities among composites. The $E_6\otimes SO(10)$ model must still pass the test of low-energy phenomenology.

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APPENDIX A: DEDUCTION OF THE GAUGED GROUP H'_{CF} WITH TWO COMPLEX IRREPS

The case of one complex irrep for H'_{CF} led to the unique choice $H'_{CF}=SO(10)$ with $N=16$, independent of considerations about the family structure of the fermionic composites. In the case of *two* complex irreps, there are initially several choices, but the family-structure argument eliminates all but one, namely G_{422} [a maximal subgroup of SO(10)]. Hereafter, for convenience, we do not distinguish between H_{CF} and H'_{CF}.

We begin by considering all possible combinations of two complex irreps of various groups with their dimension less than 22 (the ASF condition). To avoid arbitrariness, we assume that H_{CF} does not contain any U(1) groups as factors.

First, consider the case when H_{CF} is simple. Since we assume that the representation is complex, ASF restricts the group to be $SU(n)$ $(n\geq 3)$ or SO(10). Representations that can be used are **5**, **10** of SU(5); **6**, **15** of SU(6); **n** of $SU(n)$ $(n\geq 7)$; and **16** of SO(10). Of course, their conjugates can also be used. The way to decide which one to choose depends on how we embed the gauged group and how we identify the ordinary fermions. The trivial two irreducible reps with no anomaly are $N=n+\bar{n}$ of SU(n) ($n\geq 3$), but these are real. The nontrivial combination has only one solution: $N=\bar{\mathbf{5}}+\mathbf{10}$ (or its conjugate) of SU(5). The combination of $\bar{\mathbf{6}}$ and $\mathbf{15}$ is not allowed because of the anomaly. Thus, for a simple $H_{\rm CF}$, the only solution is:

$$H_{\rm CF}={\rm SU}(5) \text{ and } N=\bar{\mathbf{5}}+\mathbf{10} \text{ (or its conjugate)} . \qquad \text{(A1)}$$

For the case where $H_{\rm CF}$ is a product of two simple groups, i.e., $H_{\rm CF}=G_1\times G_2$, we have two possible quantum-number assignments: one is $N=(r_1,1)+(1,r_2)$ and the other is $N=(r_{11},r_{12})+(r_{21},r_{22})$. Since we assume that the reps are complex, one of G_i must be SU(n) ($n\geq 3$) or SO(10), using the ASF condition. First, discuss the case where $N=(r_1,1)+(1,r_2)$. If $G_1={\rm SU}(n)$ ($n\geq 3$), then a complex rep r_1 is not anomaly-free, since its dimension is less than 22. Thus, one must take $\mathbf{16}$ (or its conjugate) of SO(10). Then, the dimension of r_2 is limited by 5. Since SU(n) ($n\geq 3$) is not allowed by the ANF condition and the other simple groups do not have reps smaller than 6, G_2 is fixed as SU(2). Thus, the solution is

$$H_{\rm CF}={\rm SO}(10)\times{\rm SU}(2)$$

and (A2)

$$N=(16,1)+(1,n)\ (n=2,3,4,5) .$$

For the case where $N=(r_{11},r_{12})+(r_{21},r_{22})$, the dimension of r_{ij} is limited by 8, using the ASF condition. Thus, we cannot use SO(10) as one of G_i. One of G_i must be SU(n) ($n\geq 3$). For $G_1={\rm SU}(3)$, one cannot use both 3 and 6, because of $N=\dim r_{11}\times\dim r_{12}+\dim r_{21}\times\dim r_{22}<22$ (ASF) and the ANF condition. Thus, the solution for SU(3)×SU(3) does not exist, although real solutions do exist, i.e., $(3,3)+(\bar{3},\bar{3})$ or $(\bar{3},3)+(3,\bar{3})$. The case where $H_{\rm CF}={\rm SU}(3)\times G_2$, with G_2 a real group, is not allowed, since the solutions would be real by the ANF condition on SU(3). For $G_1={\rm SU}(n)$ ($n\geq 4$), n can only be 4 or 5 by the ASF condition, since the dim of the G_2 rep is greater than or equal to 2. However, for $G_1={\rm SU}(4)$ or SU(5), because of the ANF condition, solutions do not exist, although real ones do exist, i.e., $(n,2)+(\bar{n},2)$ with $n=4$ or 5 and $G_2={\rm SU}(2)$. This covers the possibility $H_{\rm CF}=G_1\times G_2$.

For the case where $H_{\rm CF}=G_1\times G_2\times G_3$, the natural quantum-number assignment is $N=(r_{11},r_{12},1)+(r_{21},1,r_{22})$. Since the total dimension N is equal to $\dim r_{11}\times\dim r_{12}+\dim r_{21}\times\dim r_{22}$, $\dim r_{ij}$ is limited to 8. Since the anomaly must cancel and the rep must be complex, G_1 must be SU(n) ($n\geq 3$). The solutions are:

$$H_{\rm CF}={\rm SU}(3)\times{\rm SU}(2)\times{\rm SU}(2) \text{ and } N=(3,n,1)+(\bar{3},1,n)\quad (n=2,3) ,$$

$$H_{\rm CF}={\rm SU}(4)\times{\rm SU}(2)\times{\rm SU}(2) \text{ and } N=(4,2,1)+(\bar{4},1,2) , \qquad \text{(A3)}$$

$$H_{\rm CF}={\rm SU}(5)\times{\rm SU}(2)\times{\rm SU}(2) \text{ and } N=(5,2,1)+(\bar{5},1,2) .$$

Another quantum-number assignment is $N=(r_{11},r_{12},r_{13})+(r_{21},r_{22},r_{23})$. In this case, since the rep is assumed to be complex, the total dimension N exceeds 21 (the smallest is 24). Thus, the ASF condition forbids this. Similarly, the possibility of $H_{\rm CF}$ being the product of more than three groups is not allowed by the ASF condition, using two irreps. Thus, we have exhausted all possible solutions with N being two complex irreps and satisfying the ASF and ANF conditions. Initially then we have more choices than for the case of one complex irrep. We next examine each candidate $H_{\rm CF}$ with two irreps to ascertain whether it is capable of producing a minimum of three generations of ordinary quarks and leptons.

APPENDIX B: FAMILY STRUCTURES FOR GROUPS WITH TWO COMPLEX IRREPS

First, we look at the solution (A1) where $H_{\rm CF}={\rm SU}(5)$ and $N=\bar{\mathbf{5}}+\mathbf{10}$. (The conjugate-rep case is easily done.) We use the Dynkin notation for reps and employ the survival hypothesis to count the number of light fermions.

The composites in this model are as follows:

$$\begin{aligned}
\bar{\mathbf{5}}\times\bar{\mathbf{5}}\times\bar{\mathbf{5}}&=(0003)_S+2(0011)_M+(0100)_A ,\\
\mathbf{10}\times\mathbf{10}\times\mathbf{10}&=[(0101)+(0300)]_S\\
&\quad+2[(1000)+(0101)+(1110)]_M\\
&\quad+[(0020)+(2001)]_A ,\\
\bar{\mathbf{5}}\times\bar{\mathbf{5}}\times\mathbf{10}&=(0000)+2(1001)+(0110)+(0102) ,\\
\bar{\mathbf{5}}\times\mathbf{10}\times\mathbf{10}&=2(0010)+2(1100)\\
&\quad+(0002)+(1011)+(0201) ,
\end{aligned} \qquad \text{(B1)}$$

where the subscript indicates the symmetry property, i.e., S (symmetric), M (mixed), or A (antisymmetric). If we regard one family to be $\mathbf{5}+\overline{\mathbf{10}}$ of SU(5) [(1000) + (0010) in the Dynkin notation], or its conjugate [(0001) + (0100)], there exists no family structure. The use of the meta-Pauli principle to diminish the number of available spin-$\frac{1}{2}$ composites does no good. The reason is that the meta-Pauli principle yields only (0011) from three $\bar{\mathbf{5}}$'s and one [(1000) + (0101) + (1110)] from three $\mathbf{10}$'s. Thus, we have only one $\mathbf{5}$ and two $\mathbf{10}$'s. Hence, we must give up the idea of family identification at the SU(5) scale. What happens if we identify families at the standard-model scale, i.e., G_{321}? By using the survival hypothesis to pair off (2,3) with $(2,\bar{3})$ and (1,3) with $(1,\bar{3})$, where we use the [SU(2), SU(3)] quantum numbers, we are left with $7(2,\bar{3})+5(1,3)+13(2,1)+10(1,1)$ without the meta-Pauli principle and $7(2,\bar{3})+5(1,3)+7(2,1)+7(1,1)$ with the meta-Pauli principle. Thus, the solution (A1) does not yield the family structure at any scale.

Next, we look at the solution (A2) where $H_{\rm CF}={\rm SO}(10)\times{\rm SU}(2)$ with $N=(16,1)+(1,n)$ ($n=2,3,4,5$). Possible composites are known from the following SO(10) quantum numbers:

$$16\times16\times16=[(10001)+(00003)]_S +2[(00010)+(10001)+(00101)]_M +[(01001)]_A ,$$

$$16\times16=[(10000)+(00002)]_S+[(00100)]_A , \quad (B2)$$

$$16=(00001) .$$

The family structure at the SO(10) scale is given by 2(00010) and n^2(00001), i.e., (n^2-2) **16**'s without the meta-Pauli principle. With the meta-Pauli principle, we have one (00010) and n^2 (00001), i.e., (n^2-1) **16**'s. Since n can be 2, 3, 4, or 5, the number of families are 2, 7, 14, or 23 without the meta-Pauli principle and 3, 8, 15, or 24 with the meta-Pauli principle. However, since the preon $(1,n)$ essentially plays the role of a generation quantum number, the model is not interesting.

Lastly, we look at the solutions (A3) where $H_{CF}=SU(n)\times SU(2)\times SU(2)$ ($n=3,4,5$). However, these solutions have been discussed in the TM paper[4] and only $n=4$ yields the right family structure.

Summarizing, we have shown that only $H_{CF}=SU(4)\times SU(2)\times SU(2)$ with $(\bar{4},2,1)+(4,1,2)$ is a viable chiral three-preon composite model of quarks and leptons when we allow two complex irreps for H_{CF}.

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