Properties of phase transitions of the lattice SU(2) Higgs model

J. Jersák
Institut für Theoretische Physik E, Technische Hochschule Aachen, D-51 Aachen, Federal Republic of Germany

C. B. Lang
Institut für Theoretische Physik, Universität Graz, A-8010 Graz, Austria

T. Neuhaus
Institut für Theoretische Physik E, Technische Hochschule Aachen, D-51 Aachen, Federal Republic of Germany

G. Vones
Institut für Theoretische Physik, Universität Graz, A-8010 Graz, Austria
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We discuss the phase structure of the lattice-gauge—Higgs model for the gauge group SU(2) and Higgs fields in the fundamental representation with a dynamical radial mode. We find that the confinement-Higgs phase transition is of first order for small ϕ^4 coupling λ but weakens with increasing λ. Correlations of bound-state operators on the phase-transition point at $\lambda=0.5$, $\beta=2.25$ on a $8^3\times 16$ lattice show a scalar state with the mass of 0.2 inverse lattice spacings and a four-times-heavier vector isotriplet.

I. INTRODUCTION

The Monte Carlo (MC) simulation of the strongly interacting gauge theories formulated on a lattice[1] has been developed in recent years from the exploratory investigation of various properties to a powerful computational method in QCD. On the other hand, the study of electroweak interactions as a lattice gauge theory is still in the exploratory phase. The symmetry-breaking part of the standard model[2] consists of Higgs fields in the fundamental representation of SU(2); the fermions ride along without obvious influence on the breaking mechanism. The inherent left-right symmetry of lattice fermions gives rise to problems with the lattice formulation of the complete model.[3] However, a gauge theory with only bosons may be formulated and investigated on the lattice more or less straightforwardly. This essentially nonperturbative approach migh reveal restrictions on the mechanism of spontaneous generation of gauge-boson masses.

That the usual continuum formulation of this mechanism does not translate to the gauge-invariant formalism of the lattice gauge theory identically is obvious. The expectation values of gauge-noninvariant objects such as, e.g., $\langle\phi\rangle$ have to vanish (known as Elitzur's theorem[4] in this context). Furthermore, we expect from cluster expansions (cf. Ref. 5) and MC results (cf. the reviews of Ref. 6) that non-Abelian gauge groups such as SU(2) lead to confinement of fundamental charges even in the so-called Higgs region.[7] The Higgs fields themselves are not asymptotic states of the theory. However, the usual picture of symmetry breaking is not completely obsolete: the spectrum of the standard model [neglecting the U(1) group for this discussion] is reproduced by the bound states of the original Higgs fields. One finds scalars (isospin 0) and vectors (isospin-1 triplet) bound because of the confinement mechanism. MC simulations might provide information about possible mass ratios and excited states.

These and related questions may be studied in the framework of lattice theories coupling gauge fields with scalar matter fields. However, at present even our knowledge of the properties of the phase transitions of such models in the multidimensional space of coupling parameters is far from satisfactory. Our knowledge about the critical points and their structure as well as about suitable procedures to approach the continuum limit is still poor.

Here we present the results of a detailed MC study of the phase structure of the SU(2) lattice gauge field coupled to a scalar (Higgs) field in the fundamental representation of the gauge group. A suitable form of the action of this model with minimal number of parameters is

$$S=-\frac{\beta}{4}\sum_{p\in\Lambda}\mathrm{Tr}(U_p+U_p^\dagger)-\kappa\sum_{\substack{x\in\Lambda\\ \mu=1,\ldots,4}}(\phi_x^\dagger U_{x,\mu}\phi_{x+\mu}+\mathrm{H.c.})+\lambda\sum_{x\in\Lambda}(\phi_x^\dagger\phi_x-1)^2+\sum_{x\in\Lambda}\phi_x^\dagger\phi_x\ . \tag{1.1}$$

The radial degree of freedom $|\phi|$ is controlled by the quartic coupling λ. In the limit $\lambda\to\infty$ the length of the Higgs field freezes to $|\phi|=1$; this model has been studied by several authors.[8,9] The model (1.1) for finite λ was investigated earlier too,[10–12] but we extend these results in several ways.

(i) We include the region of negative β in our study and determine the phase diagram in the whole range of β for a specific value of $\lambda=0.01$. Although the interest in this region is marginal [e.g., presumably Osterwalder-Seiler (OS) positivity[5] does not hold there; the continuum limit has to be taken for positive β] it may prove helpful for a complete understanding of the lattice-system phase diagram. At negative β we find a frustration phase transition (PT) and a Higgs-type PT. For sufficiently low λ the confinement-Higgs PT extends from positive β to negative β and joins these two transitions. This feature has escaped an earlier investigation for larger λ (Ref. 10) and is analogous to the recently determined phase structure for the $Z(2)$ (Ref. 13) and U(1) (Refs. 14 and 15) lattice Higgs models for finite λ and negative β.

(ii) For positive β we perform an extensive study of the confinement-Higgs PT lines at several values of λ from 0.003 up to 1. Precise positions of the PT lines are given. For small λ (e.g., at 0.01) the transition is clearly of first order, weakening with increasing β. This is in agreement with the Coleman-Weinberg mechanism.[16]

(iii) The PT signal weakens also for given β but increasing λ and it becomes difficult to determine for $\lambda \simeq 1$. Therefore we investigate in detail and with high statistics the Higgs PT at one point in the (β,λ) plane, at $\lambda=0.5$, $\beta=2.25$. Varying κ and the lattice size we find significant finite-size effects and a very sharp peak in the specific heat in the interval $0.270<\kappa<0.271$ on a 16×8^3 lattice. We study the correlation properties of some bound-state operators (cf. also Ref. 12) and find a correlation length up to 5 lattice spacings within the peak region. This indicates that the system is close to criticality at this point.

In Sec. II the model (1.1) and some limiting cases are briefly discussed. Section III contains positions and properties of the PT lines, and in Sec. IV we concentrate on the PT between the confinement and the Higgs regime at one particular point. We pay due attention to the validity of our approximation of the SU(2) gauge group by its 120-element icosahedral subgroup $\tilde{Y}$. For very small λ this approximation distorts the system in the Higgs phase already at values of β far below the freezing PT of the pure $\tilde{Y}$ gauge-field system. This is discussed in Sec. V. The Appendix contains some technical remarks.

II. THE MODEL

The parametrization of the action is given in (1.1). We represent the complex Higgs doublet by a pair of variables (ρ,σ) where ρ is real and non-negative and σ is a SU(2) matrix in the fundamental representation. The measure becomes

$$d\phi^\dagger d\phi=\rho^3 d\rho\, d\sigma \,, \tag{2.1}$$

where $d\sigma$ is the SU(2)-invariant Haar measure. In these variables the Higgs part of the action reads

$$S_H=-\kappa \sum_{\substack{x\in\Lambda\\ \mu=1,\dots,4}} \rho_x\rho_{x+\mu}\mathrm{Tr}(\sigma_x^\dagger U_{x,\mu}\sigma_{x+\mu}) +\lambda\sum_{x\in\Lambda}(\rho_x{}^2-1)^2+\sum_{x\in\Lambda}\rho_x{}^2 \,. \tag{2.2}$$

Another familiar form is the straightforward discretization of the usual continuum action for the Higgs part,

$$\int d^4x \left[\sum_\mu (D_\mu\phi_x^c)^\dagger(D_\mu\phi_x^c)+m_c{}^2\phi^{c\dagger}\phi^c+\lambda_c(\phi^{c\dagger}\phi^c)^2\right] \to a^4\sum_{x\in\Lambda}\left[\sum_{\mu=1,\dots,4}(\phi_{x+\mu}^{c\dagger}U_{x,\mu}^\dagger-\phi_c^{c\dagger})(U_{x,\mu}\phi_{x+\mu}^c-\phi_x^c)/a^2+m_c{}^2\phi_x^{c\dagger}\phi_x^c+\lambda_c(\phi_x^{c\dagger}\phi_x^c)^2\right]. \tag{2.3}$$

The parameters are related to our notation

$$a\phi^c=\phi\sqrt{\kappa}\,,\quad \lambda_c=\lambda/\kappa^2\,,\quad (am_c)^2=(1-2\lambda-8\kappa)/\kappa\,. \tag{2.4}$$

In our parametrization there is an obvious symmetry $\kappa\leftrightarrow -\kappa$,[10] therefore it suffices to discuss only $\kappa\geq 0$.

Let us discuss some aspects of the phase structure (cf. also Ref. 10 for further details). For $\kappa=0$ the scalar fields decouple from the gauge system. For $\lambda\neq 0$ and $\kappa\to\infty$ the Higgs fields will be parallel in group space and one expects

$$\langle\phi^\dagger\phi\rangle\equiv\langle\rho^2\rangle\sim 4\kappa/\lambda$$

or

$$\langle\phi^{c\dagger}\phi^c\rangle\sim -m_c{}^2/2\lambda_c\,. \tag{2.5}$$

In the limit $\beta\to\infty$ one is left with the ϕ^4 spin system with a second-order PT and presumably mean-field critical exponents. If the results for one-component ϕ^4 theory extend to many-component theories we expect that the continuum limit is given by the long-distance behavior of a Gaussian fixed point, independent of the ϕ^4 coupling.[17] This describes a free theory; allowing for the gauge-field interactions (i.e., $\beta<\infty$) adds one relevant direction to the fixed point. One may expect that it is possible to define a sensible continuum theory of Higgs-gauge bound states. One construction of this limit would be to hold fixed certain dimensionless ratios of physical numbers obtained on the lattice. If this proves to be consistently possible the procedure defines trajectories approaching the relevant fixed point, presumably that at $\beta=\infty$ and $\kappa=\kappa_{\mathrm{crit}}$.

For $\lambda\to\infty$ the radial degree of freedom freezes and one is left with a fixed-length gauge-Higgs system often discussed in the literature.[7,6] For that model it is well known that the Higgs region is analytically connected with the confinement region.[5,7] This theory is not renormalizable.

For vanishing λ the model describes scalar QCD [for the SU(2) gauge group]. For $\beta=\infty$ the gauge field may be transformed to unity and one is left with the lattice theory of free bosons which may be solved explicitly. The critical point corresponding to the massless theory is at $\kappa=\frac{1}{8}$. Already for a finite lattice volume there are singularities for $\kappa \geq \frac{1}{8}$ with their positions depending on the type of boundary conditions and the lattice size. It has been pointed out[18] that even for $\beta=0$ such singularities should occur for any finite volume. This can be understood from the fact that the sum over all gauge-field configurations includes the configuration where all $U=1$. Performing the ρ integration for this gauge-field configuration produces the singularity at $\kappa=\frac{1}{8}$ even at $\beta<\infty$, e.g., at $\beta=0$. The same should happen for a set of configurations in the vicinity of $U=1$ which is of nonzero measure on a finite lattice.

In practice the Monte Carlo sampling of configurations provides an intrinsic regularization. The probability to produce such a dangerous U configuration is small and for any U configuration one does not really completely integrate the ρ field variables. Instead one samples one or a few ρ configurations before one changes again the U fields. So in the actual calculation there is a competition between the entropy due to the volume and the singularity-producing configurations. Because of the finite and actually relatively small number of MC iterations (several 1000 to be compared with e.g., $120^{N_{\text{links}}}$ possible U configurations for a discrete subgroup $\tilde{Y}$) the results are more like those for small but finite λ, at least as far as it concerns the discussed singularity. (A similar situation occurs in the MC realization of spontaneous breakdown of a global symmetry. In the simulation of the Ising system on a finite lattice one observes spontaneous magnetization even for vanishing external field although the rigorous expectation value is zero.)

In the numerical results one finds for $\lambda \simeq 0$ and small β no hint of a singularity at $\kappa=\frac{1}{8}$ but at a higher value of $\kappa=\kappa_{\text{crit}}(\beta)$ depending on β (see, e.g., Ref. 10 and Sec. III). This behavior is essentially alike for fermions and is continuously approached from the results for $\kappa_{\text{crit}}(\beta)$ from larger λ. We have performed MC simulations on lattices with 4^4, 6^4, 8^4 and $8^3\times 16$ sites. The updating is done with the usual Metropolis algorithm,[19] separately updating the gauge group and the radial degrees of freedom. Most of the information comes from thermal cycles at fixed β and λ, with 200—300 MC iterations at each step before increasing or decreasing the coupling κ throwing away the first 400 sweeps before we start to measure. We measure $\langle \phi^\dagger\phi\rangle$, $\langle \phi^\dagger U\phi\rangle$ and the plaquette observable. At some selected values of the couplings we performed long runs (several 10 000 sweeps) starting from hot and cold configurations in order to study the relaxation behavior and metastability. We also collected some data for the unnormalized specific-heat contribution $\partial_\kappa\langle \phi^\dagger U\phi\rangle$ and determined correlations of bound-state observables.

III. PHASE-TRANSITION LINES

We have performed extensive thermal cycles on a 6^4 lattice and determined the phase structure and the positions of the PT line for $\lambda=0.01$ in the whole (β,κ) plane. In Fig. 1 the positions of the hystereses are indicated. For that value of λ the Higgs PT line extends from $\beta=\infty$ down to negative β. For κ values above this line the ex-

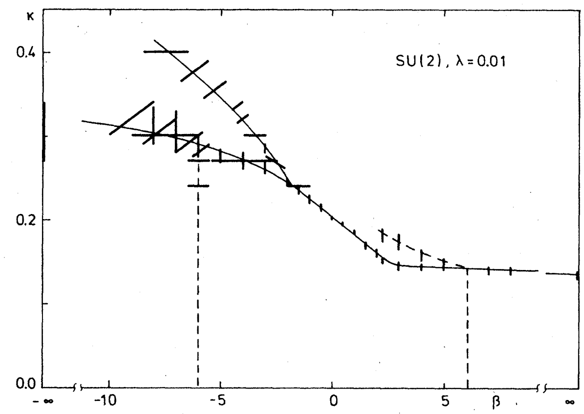

FIG. 1. The phase diagram for $\lambda=0.01$; for that value of λ the Higgs PT line extends from $\beta=\infty$ to $-\infty$. For $\beta<-2.5$ there is also a plaquette frustration PT. The dashed line indicates the onset of freezing effects due to the icosahedral approximation of SU(2).

pectation values of the length squared of the Higgs field

$$\langle \phi^\dagger\phi\rangle=\langle \rho^2\rangle\equiv\frac{1}{N_{\text{sites}}}\left\langle \sum_{x\in\Lambda}\rho_x{}^2\right\rangle \tag{3.1}$$

and the link product

$$\langle \phi^\dagger U\phi\rangle\equiv\frac{1}{N_{\text{links}}}\left\langle \sum_{\substack{x\in\Lambda\\ \mu=1,\dots,4}}\rho_x\rho_{x+\mu}\tfrac{1}{2}\text{Tr}(\sigma_x^\dagger U_{x,\mu}\sigma_{x+\mu})\right\rangle \tag{3.2}$$

grow proportional to κ/λ. We shall discuss this PT later.

To our knowledge the SU(2) Higgs model has not yet been studied for negative β; for this reason we describe the PT's found there in some detail. Close to $\beta=-2.5$ two first-order PT lines start and extend towards lower β in the (β,κ) plane. Figure 2 presents the thermal cycle of a run diagonal in the (β,κ) plane cutting both PT lines. The PT at lower κ is essentially the continuation of the Higgs PT at positive β values with similar behavior of Higgs field observables. Because of the $\beta\leftrightarrow-\beta$ symmetry for $\kappa=0$ it is possible to construct a completely frustrated gauge-field configuration[10] at $\beta=-\infty$. There the Higgs PT is observed at $\kappa=0.27$—0.34. The other PT at higher κ and $\beta<-2.5$ is the plaquette frustration transition where the plaquette mean value

$$\langle W_p\rangle=\frac{1}{N_{\text{plaq}}}\left\langle \sum_{p\in\Lambda}\tfrac{1}{2}\text{Tr}U_p\right\rangle \tag{3.3}$$

changes its signature. With decreasing β the line runs towards $\kappa\to\infty$.

We did not investigate these two PT lines for negative β for other values of λ since we expect a behavior similar to the U(1) Higgs model.[14,15] There the Higgs PT line from positive β extends for decreasing λ more and more towards negative β and it joins the other PT lines at sufficiently small λ.

We now turn to the confinement-Higgs PT at positive β. This transition is the lattice counterpart of the PT associated with the Higgs mechanism where the gauge bo-

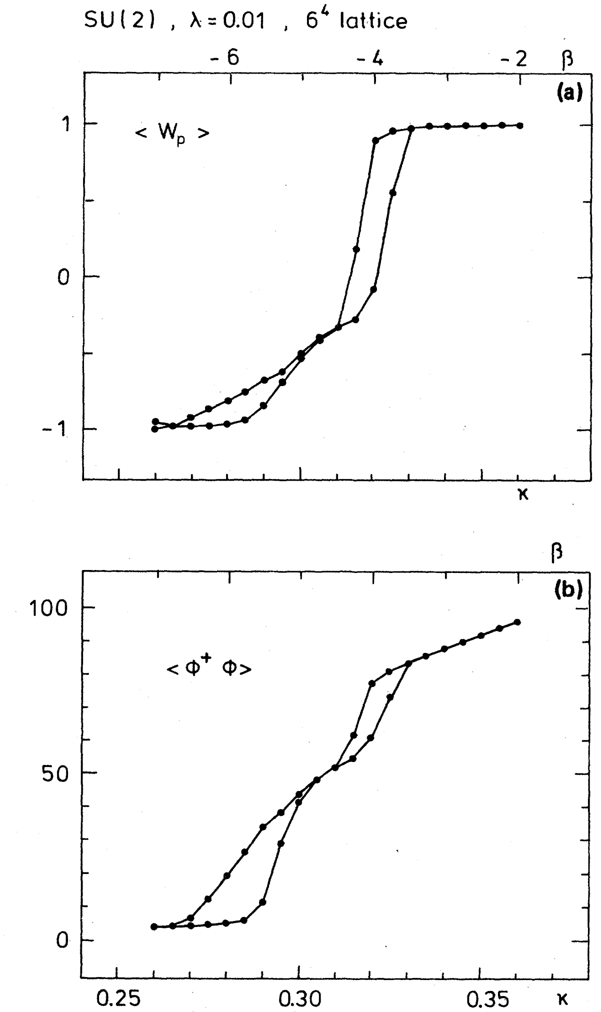

FIG. 2. Thermal cycles on a 6^4 lattice ($\lambda=0.01$) diagonal in the (β,κ) plane from $(-7,0.26)$ to $(-2,0.36)$ cutting both the Higgs PT and the plaquette frustration PT.

son acquires its mass due to the Higgs phenomenon. Any future investigation of the nonperturbative realization of the Higgs mechanism will concentrate on the vicinity of this PT; thus its position and properties should be known as precisely as possible. The critical value $\kappa_{\text{crit}}(\beta,\lambda)$ plays a role similar to the corresponding quantity in lattice QCD.

In Fig. 3 one finds the positions of the confinement-Higgs PT lines in the (β,κ) plane for various values of λ. They have been determined by hystereses found in thermal cycles for fixed λ and β on 6^4 and 8^4 lattices. We include also 4 points recently determined by Montvay.[12] Results for larger λ may be found in Ref. 10.

The PT lines start at $\beta=\infty$ at the PT's of the SU(2) lattice ϕ^4 model. For $\lambda\geq 0.1$ they have an end point at low positive β. This agrees with the prediction of a strip of analyticity connecting the confinement and the Higgs regions.[5,7] For β below this end point the PT transmutes into a crossover which rapidly disappears. However, as may be seen, e.g., in Fig. 1 for $\lambda=0.01$, for small enough

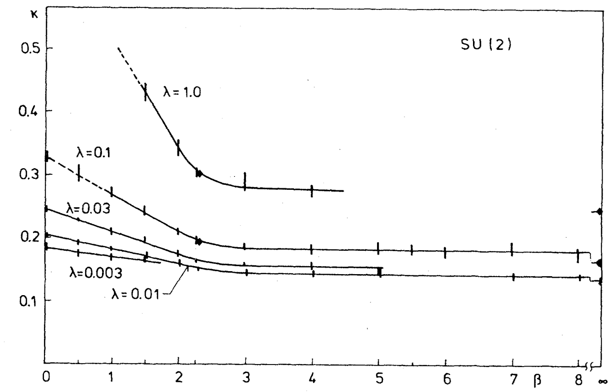

FIG. 3. Confinement-Higgs PT lines in the (β,κ) plane for various values of λ. The bars indicate the positions of hystereses found in thermal cycles on 6^4 and 8^4 lattices. Some points indicated by diamonds are from Ref. 12. Dashed lines indicate the approximate position of the PT line end points and the crossover behavior.

λ the confinement—Higgs PT line extends far into the negative β region where it joins other PT's.

Starting at $\lambda=\infty$ the overall position of the PT line shifts monotonically with decreasing λ towards lower κ. Figure 4 demonstrates this behavior for $\beta=2.25$. A similar curve for the U(1) Higgs model is not monotonic.[15]

Figure 5 illustrates the behavior of the mean link observable (3.2) at several values of β. For $\lambda=0.01$ an enormous hysteresis is found at $\beta=-2$ which weakens with growing β [Fig. 5(a)]. For somewhat larger $\lambda=0.03$ [Fig. 5(b)] almost no hysteresis signal is seen at $\beta=0$. This indicates the vicinity of an end point. With growing β the transition becomes stronger before it weakens again above $\beta>1.5$.

It is quite easy to convince oneself that the PT is of first order for $\lambda=0.01$ in the interval of finite β investigated. On the weakest of the hystereses shown in Fig. 5(a) at the point indicated by an arrow one clearly finds two coexisting phases (Fig. 6). Such a first order of the SU(2)

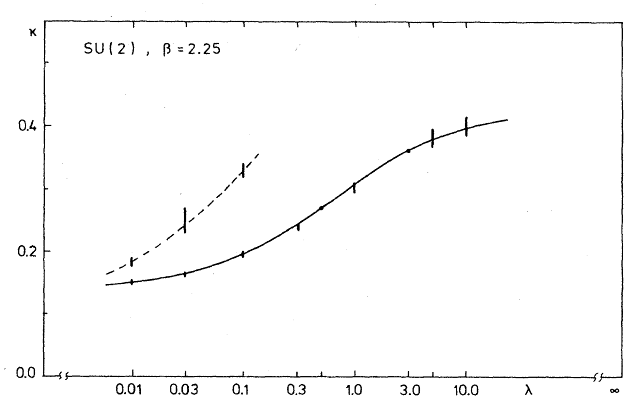

FIG. 4. Dependence of the confinement-Higgs PT in the (λ,κ) plane for $\beta=2.25$. The dashed line indicates the onset of the freezing effects due to the icosahedral approximation.

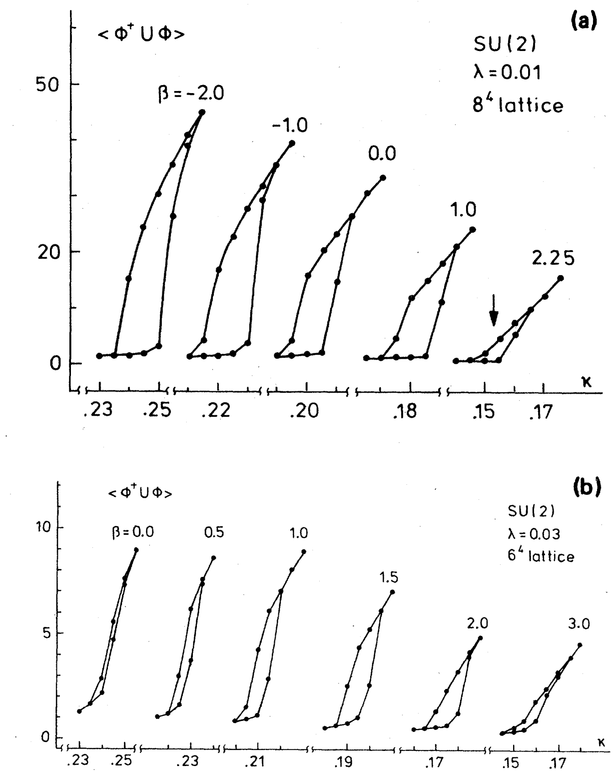

FIG. 5. Development of the hystereses at the confinement-Higgs PT with β for (a) $\lambda=0.01$ and (b) $\lambda=0.03$. At the point indicated by the arrow ($\lambda=0.01$, $\beta=2.25$) one still finds a clear signal for a two-phase structure (Fig. 6).

confinement-Higgs PT at small λ, observed also in Ref. 11, is in agreement with the Coleman-Weinberg conjecture.

Comparison of data at various values of λ shows, however, that the transition weakens with increasing λ. In Ref. 10 higher values of λ were investigated and, e.g., for $\lambda=0.5$ no first-order signal on an 8^4 lattice was found. It is a challenge to find out whether and at which value of λ the transition becomes critical. This requires a careful high-statistics study which we have performed as yet only at one point.

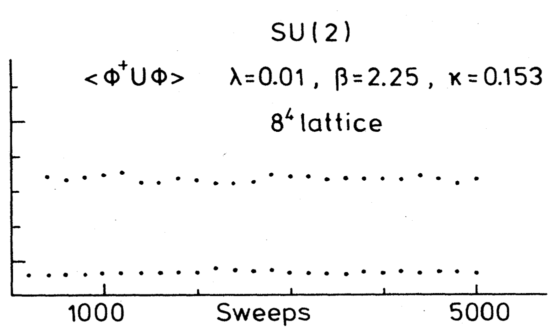

FIG. 6. Long runs at $\lambda=0.01$, $\beta=2.25$, $\kappa=0.153$ on an 8^4 lattice with cold and hot starts demonstrating the coexistence of two phases.

IV. HIGGS PHASE TRANSITION AT $\lambda=0.5$ and $\beta=2.25$

This point was chosen because thermal cycles in κ on 6^4 and 8^4 lattices indicated critical behavior. However, in long runs at fixed κ on a 6^4 lattice we also found a weak signal of two coexistent phases. A detailed investigation of the PT at this point could provide a hint about a possible change of the order of PT with λ. For this reason we performed a massive calculation of correlation functions on an $8^3\times 16$ lattice. During this calculation and in the course of the search for the PT point we have also collected some data on fluctuations of the link observable on 4^4, 6^4, and $8^3\times 16$ lattices.

In Fig. 7 we show the values of the link observable in a small interval in κ in the vicinity of the PT. Figure 8 exhibits the specific-heat contribution of this observable:

$$\partial_\kappa\langle\phi^\dagger U\phi\rangle = N_{\text{links}}(\langle[\phi^\dagger U\phi]^2\rangle - \langle\phi^\dagger U\phi\rangle^2)\,. \tag{4.1}$$

These data have been obtained in runs of about 100 000 sweeps for size 4^4, 10 000–30 000 for 6^4, and 15 000–30 000 for $8^3\times 16$. Insufficient data on 6^4 and the asymmetry of the largest lattice do not allow us to perform a finite-size scaling analysis. Nevertheless, both figures reveal strong finite-size dependence varying from the very broad structure on 4^4 to an extremely narrow peak in the interval $0.270<\kappa<0.271$ on $8^3\times 16$, in contrast with Ref. 20. This sharpening specific-heat peak indicates a substantial contribution of long-distance correlations.

Correlations over long distances contain information on the spectrum of asymptotic states of the theory (cf. also the recent results in Ref. 12). A measurement of several correlation functions was our reason to choose the asymmetric lattice size $8^3\times 16$. Treating the long direction as time direction allows the determination of bound-state propagators over somewhat larger distances than on a symmetric lattice. We are still collecting data for correlation functions at various points in the coupling-constant space and these results will be published in due time. Here we show two correlation functions determined at $\lambda=0.5$, $\beta=2.25$, and $\kappa=0.2703$, right inside the narrow specific-heat peak. The correlation functions are defined

$$\Gamma(t)\equiv\left\langle\sum_{\mathbf{x}}O(\mathbf{x},0)\sum_{\mathbf{y}}O(\mathbf{y},t)\right\rangle-\left\langle\sum_{\mathbf{x}}O(\mathbf{x},0)\right\rangle^2\,, \tag{4.2}$$

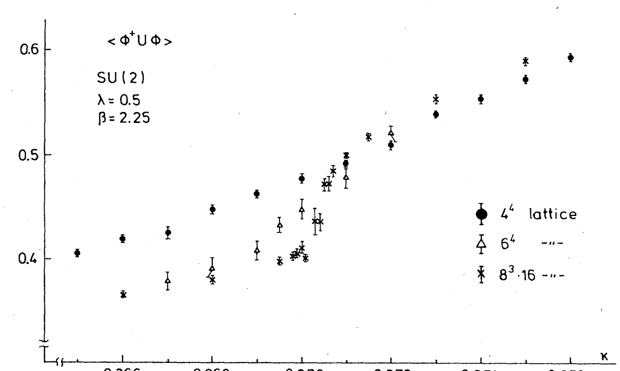

FIG. 7. Values of $\langle\phi^\dagger U\phi\rangle$ in the vicinity of the Higgs PT at $\lambda=0.5$ and $\beta=2.25$.

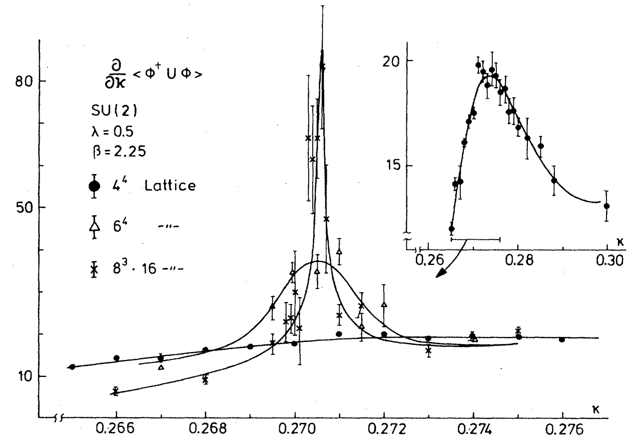

FIG. 8. The contribution of the link observables to the specific heat with respect to κ (4.1) in the vicinity of the Higgs PT at $\lambda=0.5$ and $\beta=2.25$. The range of κ covered in the main plot is indicated in the insert for the data on a 4^4 lattice. The curves are drawn to guide the eye and represent fits by a five-parameter function as in Ref. 15.

for the operators

$$\rho_x{}^2 \text{ and } \sum_{\mu=1}^{3}\rho_x\rho_{x+\mu}\mathrm{Tr}(\sigma_x^\dagger U_{x,\mu}\sigma_{x+\mu}) \text{ singlet } 0^{++}\ , \tag{4.3}$$

$$\rho_x\rho_{x+\mu}\mathrm{Tr}(\sigma\sigma_x^\dagger U_{x,\mu}\sigma_{x+\mu}) \text{ triplet } 1^{--}\ , \tag{4.4}$$

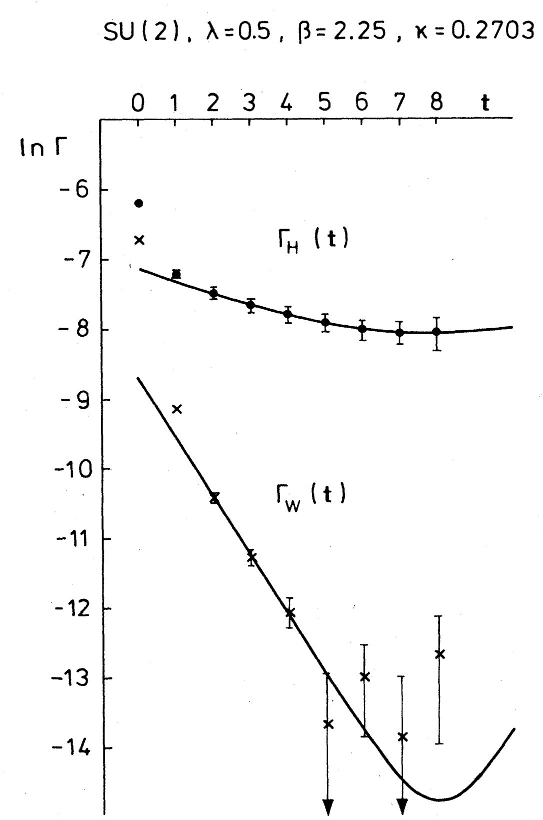

FIG. 9. Correlation functions in the Higgs-boson and the gauge-boson channels for $\lambda=0.5$, $\beta=2.25$, and $\kappa=0.2703$. The curves represent fits according to (4.5).

where σ denotes the Pauli matrices.

These operators contribute to the bound-state objects with quantum numbers 0^{++} (isoscalar) and 1^{--} (isovector), respectively. The masses obtained in a fit to the exponential decay

$$\Gamma(t)=A\{\exp(-Mt)+\exp[-M(16-t)]\} \tag{4.5}$$

correspond therefore to the Higgs boson and the gauge vector-boson particles realized as bound states of the system. Both correlation functions and the fits (to the points $t=2,\ldots,8$) are shown in Fig. 9. In the Higgs-boson channel we find a very slow decay and are able to identify the signal up to the symmetry point ($t=8$). The dimensionless mass values are

$$\begin{aligned} am_H&=0.20\pm0.03\ ,\\ am_W&=0.85\pm0.08\ . \end{aligned} \tag{4.6}$$

The magnitude of the correlation length in the Higgs channel $1/m_H=5a$ indicates that the system is almost critical. (What more can we expect on a finite lattice of the size $8^3\times16$?) We conclude that the confinement-Higgs transition at $\lambda=0.5$ and $\beta=2.25$ is either already of second order or still of first order, although very weak. A resolution of the remaining ambiguity and the localization of a tricritical point (where the order of the PT changes) will be a very difficult task [cf. the experience with tricritical points in pure U(1) lattice gauge theory[21]].

V. ICOSAHEDRAL APPROXIMATION OF THE SU(2) GROUP

Since the early times of MC studies the approximation of the SU(2) group by its icosahedral subgroup $\tilde{Y}$ has been considered excellent.[22,6] The motivation for its use is the substantial saving in storage and computer processing (CP) time. For the pure gauge system the freezing transition related to the discreteness of the group occurs at $\beta=6$, sufficiently above the β values of interest, where MC calculations are being performed. Inclusion of matter fields changes the situation. The hopping term [proportional to κ in (1.1)] increases the action gap of $\tilde{Y}$ in the Higgs phase. Therefore at $\kappa>0$ a deviation of the results from the full SU(2) group may arise already at smaller β.

Such a behavior was observed already in Ref. 8 for $\lambda=\infty$. We have followed the freezing PT line, where the results for $\tilde{Y}$ start to deviate from those for SU(2), for $\lambda=0.01$ and 0.03. The lines for $\kappa=0$ start at $\beta=6$ and remain at this value until the Higgs PT line is crossed. Then they bend sharply towards lower values of β. The freezing line for $\lambda=0.01$ is indicated by a dashed curve in Fig. 1.

Whereas the freezing transition is clearly identified near $\beta=6$, it becomes obscured towards smaller β. In Fig. 10 it is still clearly seen in both observables, the plaquette and the link, at $\beta=4$ but only crossoverlike at $\beta=2.25$. Experience with similar structures for the $Z(N)$ subgroups of U(1) in lattice Higgs models[15] suggests a substantial difference of results from a discrete subgroup and a full group: it starts unconspicuously at the freezing transition but grows with κ.

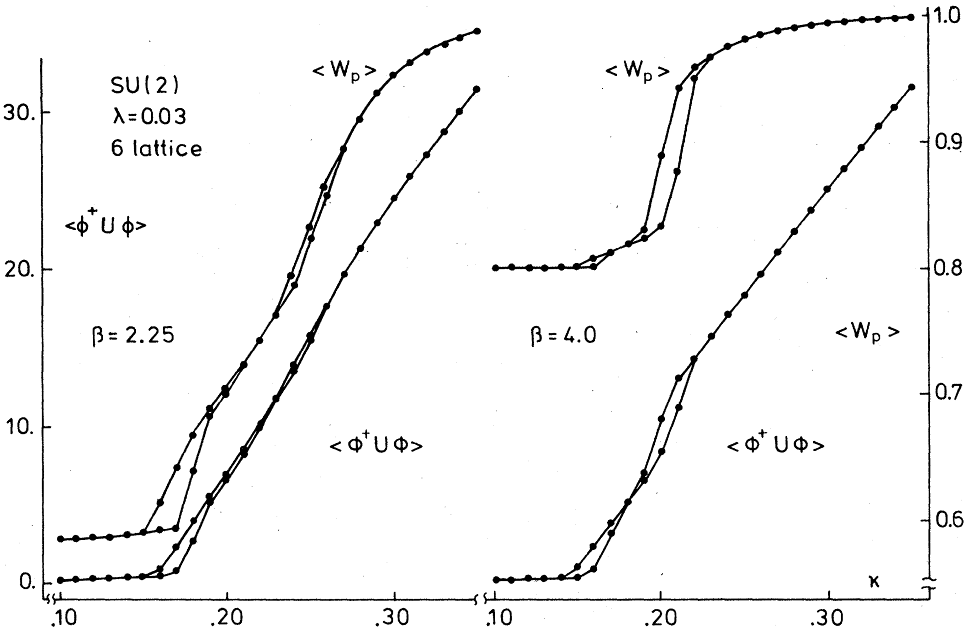

FIG. 10. Thermal cycles at $\beta=2.25$ and $\beta=4.0$ crossing both the Higgs PT and the icosahedron freezing line.

For this reason one has to use the $\tilde{Y}$ approximation in the Higgs phase with caution, especially for small λ. Fortunately, we learn from Fig. 4 that there remains a reasonably broad strip between the Higgs PT and the freezing (dashed line), where the approximation by $\tilde{Y}$ is justified.

ACKNOWLEDGMENTS

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APPENDIX: TECHNICAL DETAILS

Our calculations have been done mostly on a CYBER 205 utilizing about 50 CP hours. Some results have been obtained on other machines (CYBER 175, UNIVAC 1100/81). The vector algorithms suitable for the CYBER 205 are quite involved, therefore we have tested our programs by comparison of the results with those obtained by independently written programs for scalar computers.

During these tests we have observed a remarkable sensitivity of the SU(2) Higgs program (actually, $\tilde{Y}$) on the random-number generator used. The Control Data Corporation generator RANF (based on the congruence method) causes on the $8^3\times 16$ lattice a shift of the specific-heat peak by $\Delta\kappa=0.003$ towards smaller κ that is three times its width. Similar disturbing effects of RANF have been observed by other authors[23] in an Ising model simulation. A possible curve is additional shuffling of the pseudo-random-numbers generated. We followed the proposal of Ref. 23 in our calculations.

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