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Anomalous, chiral Lagrangians of pseudoscalar, vector, and axial-vector mesons generated from quark loops

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The authors' previously reported approach of producing the purely pseudoscalar Wess-Zumino anomalous action from a quark-loop expansion is extended to include, also, hadronic spin-one fields and the photon. A line of argument is presented which links QCD to an effective-Lagrangian model of quarks with nonlinearly transforming pseudoscalar mesons and other composite bosons. Several methods of modeling vector and axial-vector hadronic currents are reviewed, and a novel nonlinear treatment of the axial-vector currents is also introduced. The inclusion of the spin-one sector in the loop calculations is described. The loop calculations of three- and four-point functions are developed. $\omega \rightarrow 3\pi$ is calculated and compared to experiment and to the results of other works.

I. INTRODUCTION

The standard picture of the consequences of quantum chromodynamics (QCD) at low energy is generally regarded to be fairly represented by a chiral Lagrangian dominated by the would-be pseudoscalar Goldstone bosons in the zero-momentum limit.[1] A host of phenomenologically successful applications of this picture has accumulated over the years,[2] and a number of theoretical issues have been clarified, such as the QCD origin of the η' mass.[3] However, the abnormal-parity terms in the chiral Lagrangian, those involving the rank-four, totally antisymmetric, constant Lorentz tensor $\epsilon_{\alpha\beta\mu\nu}$, had been largely ignored or, at best, treated unsystematically,[4] until recently. Witten's reformulation[5] of the anomalous Wess-Zumino action[6] involving vector, axial-vector, and pseudoscalar fields, from which one can derive all of the low-energy-theorem content of the chiral anomalies, stimulated new interest in these abnormal-parity interactions. Two groups of authors[7,8] have reported a series of studies of the phenomenological application of the Wess-Zumino Lagrangian in the Witten form, and they find good agreement with data on processes involving weak and electromagnetic currents and on purely hadronic processes. These studies go a long way toward supporting the case for the importance in phenomenology of the chiral anomaly, a case which had been supported alone by the $\pi^0 \rightarrow \gamma\gamma$ example, a phenomenologically successful application which dates back to the discovery of the anomalies.

In the present paper, we extend our approach, reported earlier,[9] of generating the purely pseudoscalar Wess-Zumino action from a quark-loop expansion to include, also, hadronic spin-one fields and the photon. In Sec. II we present a line of argument which provides a QCD-based rationale for our effective-Lagrangian model of quarks in interaction with nonlinearly transforming pseudoscalar mesons and other composite hadronic bosons. This section amplifies on the remark in our previous work[9] that such an effective Lagrangian could be regarded as the consequence of "integrating out" the gluon degrees of freedom, and this development provides the background for Sec. III where we introduce the effective Lagrangian. In Sec. III we also review several methods of treating the vector and axial-vector hadronic currents and describe their inclusion into our loop calculation. We treat the case of linearly transforming spin-one fields and the case of nonlinearly transforming fields.[2,10] In the latter case, we outline a model for including the axial-vector as well as the vectors as nonlinearly transforming fields. The vector and axial-vector fields are clearly not chiral partners in such a treatment, and this parallels a scheme which we have reported previously where the scalars and pseudoscalars are treated as independent, nonlinearly transforming fields.[11]

In Sec. IV we discuss the loop calculation of three-point and four-point interactions, and we modify our loop-generated effective action by adding a Bardeen counterterm[12] in order to ensure vector-current conservation. The axial-vector, vector, vector (AVV) vertex and its role in the $\pi^0 \rightarrow \gamma\gamma$ calculation is discussed in this connection. The Bardeen counterterm must also be added to the anomalous action when calculated in the manner of Witten,[5] as shown by Kaymakcalan, Rajeev, and Schechter.[7] After the $\pi^0 \rightarrow \gamma\gamma$ discussion, the $PVAA$, $PPVA$, and $PPPV$ interactions are calculated and then the interaction terms relevant to $\omega \rightarrow 3\pi$ are presented. In each case, the linear- and nonlinear-model results are presented in parallel. It is remarked that vector-meson dominance in the anomalous action does not produce the current-algebra result for the $\gamma\pi^+\pi^-\pi^0$ vertex, as noticed by Rudaz[13] in connection with the calculations of Ref. 7. Also in Sec. IV, we briefly consider some points of phenomenology related to fixing the values of parameters in the completely nonlinear theory, and to subsequent calculation of the $\omega \rightarrow 3\pi$ rate. Some summarizing remarks are included in Sec. V. An appendix provides some details of our loop calculation of the four-point interactions.

II. QUARK PLUS HADRONIC-MESON EFFECTIVE LAGRANGIANS

We develop here a line of reasoning which leads one from the QCD Lagrangian to Lagrangians depending on quark degrees of freedom and composite-boson degrees of freedom. This "hybrid" type of Lagrangian is the basis for our fermion-loop evaluation of effective actions of the anomalous Wess-Zumino type.

Let us begin by noting that the general form of the action after gauge fixing and integrating over gluon fields and possible ghost fields in the QCD generating functional for connected Green's functions is

$$W=\int dx\,\mathscr{L}(x)=\int \psi(x)(i\partial\!\!\!/-m)\psi(x)dx + \mathrm{Tr}\sum_n \frac{1}{n!}G^c_{\nu_1\cdots\nu_n}(x_1,\ldots,x_n)\bar\psi(x_1) \times\Gamma_{\nu_1}\psi(x_1),\ldots,\bar\psi(x_m)\Gamma_{\nu_n}\psi(x_n) + \text{source terms}\,, \tag{1}$$

where color and flavor indices are implicit, Γ_ν are coupling matrices which involve internal symmetry and Dirac indices and $G^c_{\nu_1\cdots\nu_n}(x_1,\ldots,x_n)$ are connected pure Yang-Mills n-point functions. An integral $\int dx_1,\ldots,\int dx_n$ is understood. Except for the mass term, Eq. (1) is globally chiral symmetric, reflecting the chiral symmetry of the original QCD Lagrangian.

At this stage we can introduce the bilocal auxiliary fields $\eta(x,y)$ and $B(x,y)$, which are matrices in Dirac, color, and flavor space, by the following device.[14] We add the terms involving η and B to W as follows (source terms will be henceforth set to zero):

$$\begin{aligned}W[\psi,\bar\psi,\eta,B]=&\int \bar\psi(x)(i\partial\!\!\!/-m)\psi(x)dx\\&-\mathrm{Tr}\int dx\,dy\,\eta(x,y)[B(y,x)-\psi(y)\bar\psi(x)]\\&-\mathrm{Tr}\sum_n\frac{1}{n!}G^c_{\nu_1\cdots\nu_n}\\&\times\Gamma_{\nu_1}B(x_1,x_2)\cdots\Gamma_{\nu_n}B(x_n,x_1)\,,\end{aligned}\tag{2}$$

where color, flavor, and spin indices are implicit as before, and $\int dx_1,\ldots,\int dx_n$ is understood in the last term. The equations of motion for η and B (equations of constraint) are

$$\frac{\delta W}{\delta\eta(x,y)}=0=B(y,x)-\psi(y)\bar\psi(x) \tag{3}$$

and

$$\frac{\delta W}{\delta B(y,x)}=0=-\eta(x,y)-G^c_{\nu_1\nu_2}(x-y)\Gamma_{\nu_1}B(x,y)\Gamma_{\nu_2}+\cdots\,. \tag{4}$$

When Eqs. (3) and (4) are replaced in the equation for ψ,

$$(i\partial\!\!\!/-m)\psi(x)-\int dy\,\eta(x,y)\psi(y)=0\,, \tag{5}$$

one gets the same equation for ψ that would have been obtained directly from Eq. (1), thus showing that the actions of Eqs. (1) and (2) are equivalent.

The generating functional Z can be written as

$$\begin{aligned}Z=&\int D\psi D\bar\psi D\eta\, e^{W_1[\psi,\bar\psi,\eta]}\int DB\, e^{W_2[\eta,B]}\\ \equiv&\int D\psi D\bar\psi D\eta\, e^{W[\psi,\bar\psi,\eta]}\,.\end{aligned}\tag{6}$$

From Eqs. (2) and (6) we see that $W[\psi,\bar\psi,\eta]$ breaks into a term depending only on ψ and $\bar\psi$, one depending only linearly on ψ, $\bar\psi$, and η and one depending only on η, which is the general form we seek:

$$\begin{aligned}W[\psi,\bar\psi,\eta]=&\int dx\,\bar\psi(x)(i\partial\!\!\!/-m)\psi(x)\\&-\int\int dx\,dy\,\bar\psi(x)\eta(x,y)\psi(y)+W[\eta]\,.\end{aligned}\tag{7}$$

From inspection of Eqs. (3)—(7), it is apparent that $W[\eta]$ is chiral symmetric if the starting L was,[15] and that it will involve arbitrarily high orders of derivatives of η if the nonlocal terms contained in these expressions are expanded about a common point. Note here that one has the option of assigning the mass term $\bar\psi m\psi$ to the quark Lagrangian alone, partly to the quark Lagrangian and partly to $W[\eta]$, or entirely to $W[\eta]$ since one can write

$$\bar\psi m\psi=\mathrm{Tr}[mB(x,x)]\,,$$

which looks like a part of the last term in Eq. (2). It is consistent, for example, to split the $\bar\psi m\psi$ into a singlet mass term $m_0\bar\psi\psi$ and a flavor-symmetry-breaking piece $\mathrm{Tr}\,m_{\mathrm{SB}}B(x,x)$ which starts with $\mathrm{Tr}[m_{\mathrm{SB}}(M+M^\dagger)]$, where M is a spin-zero $(3,3^*)+(3^*,3)$ flavor representation, as elaborated below.

We turn to the explicit development of the terms $\int dx\int dy\,\bar\psi(x)\eta(x,y)\psi(y)$ about a common point to illustrate how the expansion of the effective Lagrangian into successively higher derivatives can unfold. Let us introduce variables z and t through $x=z+\frac{1}{2}t$ and $y=z-\frac{1}{2}t$ and rename $\eta(x,y)$ to be $\eta(z,t)$. We expand ψ and $\bar\psi$:

$$\begin{aligned}\psi(z-\tfrac{1}{2}t)&=\psi(z)-\tfrac{1}{2}t_\mu\partial^\mu\psi(z)+\cdots\,,\\ \bar\psi(z+\tfrac{1}{2}t)&=\bar\psi(z)+\tfrac{1}{2}t_\mu\partial^\mu\bar\psi(z)+\cdots\,,\end{aligned}\tag{8}$$

and we make the plausible assumption that the color-singlet part of $\eta(t,z)$ admits a bound-state type of factorization, namely,

$$\begin{aligned}\eta(z,t)=&M(z)F_0(t)+\partial_\mu M\,t^\mu F_1(t)\\&+M\,\partial\!\!\!/ M F_2(t)+\cdots\,,\end{aligned}\tag{9}$$

where the field M is a $(3^*,3)+(3,3^*)$ say, of a chiral flavor SU(3) group under which the ψ's transform as triplets. The functions F_i are singlets. In what follows, just the color-singlet part of η will be kept and only the flavor symmetry content will be considered. The higher-spin terms in the expansion Eq. (9) are not explicitly shown, so that our expressions stay manageable.

We expand the term in the action which is of interest in the loop calculation, finding

$$\begin{aligned}\int\int dx\,dy\,\bar\psi(x)\eta(x,y)\psi(y)=&\int\int dt\,dz[\bar\psi(z)+\tfrac{1}{2}t_\mu\partial^\mu\bar\psi(t)+\cdots]\\&\times[M(z)F_0(t)+\partial_\mu M\,t^\mu F_1(t)+M\,\partial\!\!\!/ M\,F_2(t)+\cdots]\,[\psi(z)-\tfrac{1}{2}t_\mu\partial^\mu\psi(z)+\cdots]\,,\end{aligned}\tag{10}$$

and we collect the leading terms in the form

$$\int\int dx\,dy\,\bar\psi(x)\eta(x,y)\psi(y)=\mu\int dz\,\bar\psi(z)M(z)\psi(z)-i\frac{\lambda}{2}\int dz\,\bar\psi(z)M(z)\partial M(z)\psi(z)-X\int dz\,\partial_\mu\bar\psi(z)M(z)\partial^\mu\psi(z)$$
$$+Y\int dz[-\bar\psi(z)\partial_\mu M(z)\partial^\mu\psi(z)+\partial^\mu\bar\psi(z)\partial_\mu M(z)\psi(z)]+\cdots\ , \qquad (11)$$

where

$$X\equiv\tfrac{1}{32}\int dt\,F_0(t)t^2,\quad Y\equiv\tfrac{1}{16}\int dt\,F_1(t)t^2\ ,$$
$$\mu=\int dt\,F_0(t),\quad i\frac{\lambda}{2}=\int dt\,F_2(t)\ ,$$

and where several terms vanish because $\int dt\,F_0(t)t_\mu=0$.

If $M(z)$ in Eq. (11) is identified as a $(3^*,3)+(3,3^*)$ of chiral SU(3) and is a function of a nonlinearly transforming pseudoscalar flavor octet of fields while ψ are flavor-triplet quark fields, then the first two terms in Eq. (11) are recognizable as two of the effective-Lagrangian terms which we considered in a previous work.[9] Obviously the expansion of Eq. (9) admits also the presence of other fields of various spin and parity corresponding to $Q\bar Q$ bound states other than the lowest mass pseudoscalars. We have isolated the latter for simplicity of discussion of the main ideas.

The preceding line of argument is provided to clarify the motivation and content of models and fermion-loop calculations based on these models which we present in the following sections.

III. QUARK-MESON CHIRAL MODELS

The basic structure of the models which we adopt is given by the following Lagrangian form:

$$\mathscr{L}=\bar\psi(x)[i\partial\!\!\!/-\mu M-g(V\!\!\!/+A\!\!\!/\gamma_5)]\psi(x)$$
$$+\mathscr{L}(D_\mu M,F^{\pm}_{\mu\nu})+\kappa_1\frac{M_V{}^2}{2}\mathrm{Tr}(V_\mu V^\mu+A_\mu A^\mu)\ , \qquad (12)$$

where the ψ's are quark fields, M is a $(3,3^*)+(3^*,3)$ chiral SU(3) flavor matrix of fields

$$M=\exp[2i\pi(x)\gamma_5/F_\pi]\ , \qquad (13)$$

with π the nonlinearly transforming, matrix pseudoscalar octet, $\pi=\lambda^a\pi^a/\sqrt{2}$. The (1,8) + (8,1) fields $V^{\pm}=V\pm A$ are similarly defined, $V^{\pm}=(V^{\pm})^a\lambda^a/\sqrt{2}$, where the SU(3) matrices are normalized according to $\mathrm{Tr}\lambda^a\lambda^b=2\delta^{ab}$. Completing the definitions, we have

$$D_\mu M=\partial_\mu M-igMV^+_\mu+igV^-_\mu M$$

and

$$F^{\pm}_{\mu\nu}=\partial_\mu V^{\pm}_\nu-\partial_\nu V^{\pm}_\mu+ig[V^{\pm}_\mu,V^{\pm}_\nu]\ .$$

The mass term for V and A and the locally invariant term $\mathscr{L}(D_\mu M,F^{\pm}_{\mu\nu})$ serve to renormalize the fermion-loop expansion which we discuss in the next section, and we adopt the form

$$\mathscr{L}(D_\mu M,F^{\pm}_{\mu\nu})=\frac{\kappa_2}{32}F_\pi{}^2\mathrm{Tr}D_\mu M^\dagger D^\mu M$$
$$+\kappa_3(-\tfrac{1}{4}\mathrm{Tr}F^+_{\mu\nu}F^{+\mu\nu}-\tfrac{1}{4}\mathrm{Tr}F^-_{\mu\nu}F^{-\mu\nu})\ . \qquad (14)$$

In the absence of fermion loops the κ_i's in Eqs. (12) and (14) would all be equal to one. Higher derivative terms and globally, but not locally, symmetric terms have not been included. The former limitation is made because we will be interested in the low-energy regime, and the latter is made with the application of the phenomenologically successful, field-current identity and vector-meson dominance of the electromagnetic current in mind.[16] As outlined in Sec. II, QCD implies a global chiral invariance for these mesonic terms in the low-energy effective Lagrangian, and our restriction to a form which is locally invariant except for the mass term introduces an additional, dynamical assumption. Globally invariant V, A, M abnormal-parity terms could be added arbitrarily otherwise. For example, $\mathrm{Tr}(V^{+\alpha}\partial^\beta M^\dagger\partial^\mu M\partial^\nu M^\dagger)\epsilon_{\alpha\beta\mu\nu}$ is globally, but not locally, chiral symmetric.

The spin-one fields can be introduced either linearly or nonlinearly, the latter case allowing one to include either a single vector field or a vector field and axial-vector field which are not chiral partners, along with the pseudoscalar field. We consider both cases, linear and nonlinear, below in order to illustrate the loop techniques for generating the terms in the anomalous action, which, besides the traditional π, ρ, K^*, etc., systems, might have application to composite systems with Goldstone modes which crop up at higher mass scales in composite pictures of weak Higgs and/or gauge-weak bosons and fermions.

Outline of the linear model

In the form shown in Eqs. (12) and (14), the Lagrangian has kinetic energy and mass terms which act as counterterms for divergent terms in the fermion-loop expansion.[12] In the linear case, where the fields V and A are independent, mixing between π and A is induced by the mass and kinetic-energy terms of the mesons, and diagonalization is achieved by introducing properly normalized fields $\tilde\pi$ and $\tilde A_\mu$, where

$$A_\mu\equiv\tilde A_\mu+\frac{g}{M_V{}^2}\tilde F_\pi D^V_\mu\tilde\pi+\cdots \qquad (15)$$

with

$$D^V_\mu\tilde\pi=\partial_\mu\tilde\pi-ig[V_\mu,\tilde\pi]$$

and

$$\tilde\pi=\pi\bigg/\left[1+\frac{g^2}{M_V{}^2}F_\pi{}^2\right]^{1/2},$$
$$\tilde F_\pi=F_\pi\bigg/\left[1+\frac{g^2}{M_V{}^2}F_\pi{}^2\right]^{1/2}=135\ \mathrm{MeV}\ .$$

In what follows we will drop the tilde symbol and all quantities will be understood to be properly normalized.

The set of transformation laws for the ψ, V, A, and M fields is defined as follows: (i) vector,

$$\psi(x)\to e^{-i\phi}\psi(x),\quad M\to e^{-i\phi}Me^{i\phi}\,,$$
$$(V\pm A)_\mu\to e^{-i\phi}(V\pm A)_\mu e^{i\phi}-\frac{i}{g}e^{-i\phi}\partial_\mu e^{i\phi}\,; \tag{16a}$$

(ii) axial vector,

$$\psi(x)\to e^{-i\gamma_5\theta}\psi(x),\quad M\to e^{i\gamma_5\theta}Me^{i\gamma_5\theta}\,,$$
$$(V\pm A)_\mu\to e^{\mp i\gamma_5\theta}(V\pm A)_\mu e^{\pm i\gamma_5\theta}-\frac{i}{g}e^{\mp i\theta\gamma_5}\partial_\mu e^{\pm i\gamma_5\theta}\,. \tag{16b}$$

The $\rho\pi\pi$ coupling constant evaluated on the ρ mass shell is, with the interactions shown in Eqs. (12) and (14) and after taking into account the renormalization of kinetic-energy and mass terms,

$$g_{\rho\pi\pi}=\sqrt{2}g\left[1-\tfrac{1}{2}g^2\frac{F_\pi{}^2}{M_V{}^2}\right], \tag{17a}$$

where

$$\Gamma(\rho\to\pi\pi)=\frac{2}{3}\frac{(g_{\rho\pi\pi})^2}{4\pi}\frac{1}{M_\rho{}^2}(M_\rho{}^2-4M_\pi{}^2)^{3/2}\,. \tag{17b}$$

Higher derivative terms, invariant under the local transformations (16a) and (16b) can modify the $g_{\rho\pi\pi}\leftrightarrow g$ relationship. For example, the term

$$\kappa_4\mathrm{Tr}(F^{+\mu\nu}D_\mu M^\dagger D_\nu M+F^{-\mu\nu}D_\mu MD_\nu M^\dagger)$$

has been included in phenomenological chiral Lagrangians discussed in the literature.[2,7]

The usual vector-meson-dominance prescription[16] calls for the introduction of a photon kinetic energy, $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$, where $F^{\mu\nu}\equiv\partial^\mu\Phi^\nu-\partial^\nu\Phi^\mu$ and Φ^μ is the electromagnetic potential, and for the replacement of V_μ by $V_\mu+(e/g)Q\Phi_\mu$ everywhere but in the mass term, which breaks the local gauge invariance. Here

$$Q=\frac{1}{3}\begin{bmatrix}2&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$$

is the quark charge matrix. A change of variable can be made to eliminate Φ_μ in the gauge-invariant part of the Lagrangian, and $V_\mu-(e/g)Q\Phi_\mu$, where V_μ is the hadronic vector field, appears in the mass term. A mixing term

$$-g^{-1}eM_V{}^2\Phi^\mu\mathrm{Tr}(V_\mu Q)$$

is induced. The mixing term and the hadronic interactions of V produce the Lagrangian for electromagnetic processes involving the hadrons.

Outline of the nonlinear model of V, A

In a nonlinear model for the V and A potentials which appear in the Lagrangians in Eqs. (12) and (14), we first consider the case of a single independent spin-one field in addition to π. One can express V and A in terms of a nonlinearly transforming vector field B_μ and the nonlinearly transforming pseudoscalar field π. We write

$$V_\mu-A_\mu\gamma_5\equiv UB_\mu U^\dagger-\frac{i}{g}U\,\partial_\mu\,U^\dagger\equiv V_\mu^-\,, \tag{18a}$$

$$V_\mu+A_\mu\gamma_5=U^\dagger B_\mu U-\frac{i}{g}U^\dagger\partial_\mu\,U\equiv V_\mu^+\,, \tag{18b}$$

where

$$U=e^{i\pi\gamma_5/F_\pi}=1+i\gamma_5\frac{\pi}{F_\pi}-\frac{1}{2}\frac{\pi^2}{F_\pi{}^2}+\cdots\,, \tag{19a}$$

and

$$U^\dagger=e^{-i\pi\gamma_5/F_\pi}=1-i\gamma_5\frac{\pi}{F_\pi}-\frac{1}{2}\frac{\pi^2}{F_\pi{}^2}+\cdots\,. \tag{19b}$$

The axial transformation properties of $V_\mu^\pm$ are

$$V_\mu^\pm\to e^{\mp i\theta\gamma_5}V_\mu^\pm e^{\pm i\theta\gamma_5}-\frac{i}{g}e^{\mp i\theta\gamma_5}\partial_\mu e^{\pm i\theta\gamma_5}\,, \tag{20}$$

as in Eq. (16b), and the vector transformations are those shown in Eq. (16a). The vector transformations of U and B_μ are linear, but the global axial-vector transformations are given by

$$B_\mu\to e^{iu}B_\mu e^{-iu}-\frac{i}{g}e^{iu}\partial_\mu e^{-iu}\,, \tag{21a}$$

and

$$U\to e^{iu}Ue^{i\theta\gamma_5},\quad U^\dagger\to e^{-i\theta\gamma_5}U^\dagger e^{-iu}\,, \tag{21b}$$

where u is a function of π and θ.

Expanding the expressions (18) and (19) and determining B_μ and A_μ in terms of V_μ and π, one finds that

$$B_u=V_\mu+\frac{i}{g}\frac{1}{2F_\pi{}^2}[\pi,D_\mu\pi]+\cdots\,, \tag{22a}$$

$$gA_\mu=\frac{1}{F_\pi}D_\mu\pi+\frac{1}{2F_\pi{}^3}\{\pi^2,D_\mu\pi\}+\cdots\,, \tag{22b}$$

where

$$D_\mu\pi=\partial_\mu\pi+ig[V_\mu,\pi]\,.$$

In this nonlinear example where there is only one independent spin-one field, the covariant derivative of M is identically zero:

$$D_\mu M=\partial_\mu M+igV_\mu^-M-igMV_\mu^+=0\,.$$

The kinetic energy of the pseudoscalar fields comes from the vector-meson mass term alone, while the $V\pi\pi$ coupling arises partly from this mass term and partly from the kinetic energy of the spin-one field. One finds that canonical normalization of the pseudoscalar kinetic energy requires

$$M_V{}^2/g^2F_\pi{}^2=1\,, \tag{23}$$

while

$$\mathscr{L}_{V\pi\pi}=ig\mathrm{Tr}V_\mu[\pi,\pi_\mu]+\frac{i}{2g^2F_\pi{}^2}\mathrm{Tr}(\Box V_\mu[\pi,\partial^\mu\pi])\,. \tag{24}$$

On the vector-meson mass shell, the coupling is then

$$g_{V\pi\pi}=g\left[1-\frac{M_V^2}{2g^2F_\pi^2}\right], \tag{25}$$

and comparing to the $\rho\to\pi\pi$ decay

$$g_{\rho\pi\pi}=g/\sqrt{2}\ , \tag{26}$$

where the condition $M_V^2/g^2F_\pi^2=1$, Eq. (23), has been used. This leads to

$$\frac{(g_{\rho\pi\pi})^2}{4\pi}\cong 1.3\ ,$$

while experimentally (27)

$$\frac{(g_{\rho\pi\pi})^2}{4\pi}\cong 3.0\ .$$

The constraint Eq. (23) and the subsequent results Eqs. (26) and (27) were pointed out long ago by Gasiorowicz and Geffen,[2] who concluded that one must introduce axial-vector partners for the vectors and implement the chiral symmetry with linearly transforming vector and axial-vector fields.

Model including nonlinearly transforming axial-vector

As an alternative to introducing chiral partners for the vector fields, one can include a term to produce an extra pseudoscalar-kinetic-energy contribution[17] and/or introduce an independent, nonlinearly transforming axial-vector field C_μ, whose mixing with the pseudoscalars introduces another parameter which modifies the relationships (23), (25), and (26). This idea of adding the axial-vector fields as independent, nonlinearly transforming fields rather than as chiral partners of the vectors is similar to that in our previous treatment[11] of the scalar fields as a nonlinearly transforming multiplet rather than as chiral partners of the pseudoscalars. We now outline the main features of a model where a nonlinearly transforming axial-vector field C_μ is introduced.

The axial-vector field transforms as

$$C_\mu\to e^{iu}C_\mu e^{-iu}\ , \tag{28}$$

under the global, axial transformations. The function u was defined in Eqs. (21a) and (21b). The vector transformation on C_μ is linear. The fields $C_\mu^\pm$ are defined by

$$C_\mu^-\equiv\gamma_5 UC_\mu U^\dagger,\ \ C_\mu^+\equiv-\gamma_5 U^\dagger C_\mu U\ , \tag{29}$$

with U as defined in Eqs. (21a) and (21b). The set of transformation laws for $C_\mu^\pm$, Eqs. (28) and (29) are then (i) vector,

$$C_\mu^\pm\to e^{-i\phi}C_\mu^\pm e^{i\phi}\ ; \tag{30a}$$

(ii) axial-vector,

$$C_\mu^\pm\to e^{\mp i\theta\gamma_5}C_\mu^\pm e^{\pm i\theta\gamma_5}\ . \tag{30b}$$

Our nonlinear Lagrangian model is then given by

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_{M_C}+\mathscr{L}_{CV}+\mathscr{L}_{\mathrm{KE}(C)}\ ,$$

where

$$\mathscr{L}_0=-\tfrac{1}{4}\mathrm{Tr}(V_{\mu\nu}^+V^{+\mu\nu}+V_{\mu\nu}^-V^{-\mu\nu})+\tfrac{1}{2}M_V^2\mathrm{Tr}(V_\mu V^\mu+A_\mu A^\mu)$$

with $V^\pm$ defined in Eqs. (18) and (22). The remaining terms are

$$\mathscr{L}_{M_C}=\tfrac{1}{2}M_C^2\mathrm{Tr}(C_\mu C^\mu)\ , \tag{31a}$$

$$\mathscr{L}_{\mathrm{KE}(C)}=-\tfrac{1}{4}\mathrm{Tr}(C_{\mu\nu}C^{\mu\nu})\ , \tag{31b}$$

$$\mathscr{L}_{CV}=\alpha\,\mathrm{Tr}(C_\mu^-V_\mu^-+C_\mu^+V^{+\mu})\ , \tag{31c}$$

where

$$C_{\mu\nu}=D_\mu C_\nu-D_\nu C_\mu,\ \ D_\mu C_\nu=\partial_\mu C_\nu-ig[B_\mu,C_\nu]\ .$$

Choosing the parameter α to be

$$\alpha=\pm\tfrac{1}{8}gF_\pi M_C(M_V^2/g^2F_\pi^2-1)^{1/2}\ , \tag{31d}$$

recalling that

$$A_\mu=\frac{1}{gF_\pi}D_\mu\pi+\cdots\ ,$$

Eq. (22b), and introducing $\mathscr{A}_\mu$ by the identification

$$C_\mu=\mathscr{A}_\mu+\frac{8\alpha}{gF_\pi M_C^2}D_\mu\pi\ ,$$

we can write the Lagrangian in diagonalized form as

$$\mathscr{L}=-\tfrac{1}{4}\mathrm{Tr}V_{\mu\nu}V^{\mu\nu}+\tfrac{1}{2}M_\rho^2\mathrm{Tr}V_\mu V^\mu+\tfrac{1}{2}\mathrm{Tr}D_\mu\pi D^\mu\pi+ig\left[\frac{M_V^2}{g^2F_\pi^2}-1\right]\mathrm{Tr}D_\mu\pi[V_\mu,\pi]+\tfrac{1}{2}M_C^2\mathrm{Tr}\mathscr{A}_\mu\mathscr{A}^\mu-\tfrac{1}{4}\mathrm{Tr}\mathscr{A}_{\mu\nu}\mathscr{A}^{\mu\nu}+\cdots\ . \tag{32}$$

The fourth term in Eq. (32) shows that we have paid a price—the loss of local gauge invariance in some interactions—for the introduction of a new "masslike" term (31c). The relationship between $g_{V\pi\pi}$ and g which follows from Eq. (32) is

$$g=\frac{1}{2F_\pi^2}\frac{M_V^2}{g_{V\pi\pi}}=3.75\ , \tag{33}$$

where $M_V=M_\rho=0.77$ GeV, $F_\pi=0.135$ GeV, and $g_{V\pi\pi}=g\rho\pi\pi/\sqrt{2}=4.35$ have been used in (33). We will return to the question of the value of g when we discuss $\omega\to3\pi$ in the following section. There we will describe the construction of the anomalous, effective action from the loop expansion in the presence of $V_\mu^\pm$ fields for both the linear and nonlinear cases.

IV. THE FERMION-LOOP EXPANSION AND THE ANOMALOUS ACTION

The loop expansion of the fermion determinant which follows from the Lagrangian of Eq. (12) produces the effective Lagrangian

$$\mathscr{L}_{\mathrm{loop}}=i\,\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\{(i\partial\!\!\!/-\mu)^{-1}[\mu(M-1)+gV\!\!\!/+gA\!\!\!/\gamma_5]\}^n\ . \tag{34}$$

$\mathscr{L}_{\rm loop}$ yields all of the abnormal-parity, minimum-derivative, finite terms in the Wess-Zumino effective Lagrangian, thus incorporating all of the phenomenological consequences of the chiral anomalies. Our loop calculations treat A and V symmetrically, so both the A and V currents based on Eq. (34) are anomalous. The counterterm which allows one to remove the anomalies in the vector current, which turns out to be necessary in order to implement vector-meson dominance of the electromagnetic current, is obtained by calculating the pure V and A anomalous part of $\mathscr{L}_{\rm loop}$ when $M=1$ is exactly the counterterm discussed first by Bardeen.[12] We develop this anomalous part of $\mathscr{L}_{\rm loop}$ when $M=1$ is exactly the counterterm discussed first by Bardeen.[12] We develop this point below in the framework of our loop approach to the anomalous action. This approach amplifies and complements the discussion of the need for such a counterterm in the work of Kaymakcalan, Rajeev, and Schechter[7] and of Gömm, Kaymakcalan, and Schechter,[7] who follow the method of Witten[5] in arriving at the form of the anomalous action, a method in which the underlying quark degrees of freedom never appear explicitly.

The counterterm to $\mathscr{L}_{\rm loop}$

A calculation of the simple $\pi^0\to\gamma\gamma$ vertex reveals the question quite clearly. If we calculate $\pi^0\to\gamma\gamma$ by introducing the electromagnetic covariant derivative in the purely pseudoscalar model discussed in our previous work, then the effective action is

$$\mathscr{L}_{\rm loop}(M,\phi)=i\,{\rm Tr}\sum_{n=1}^{\infty}\frac{1}{n}\{(i\partial\!\!\!/-\mu)^{-1}[\mu(M-1)+eQ\phi\!\!\!/]\}^n\,, \quad (35)$$

and the classic graph shown in Fig. 1 yields the familiar result

$$\mathscr{L}(\pi,\phi,\phi)=\frac{Ne^2}{16\pi^2F_\pi}\int dx\,{\rm Tr}[\pi(x)Q^2]\epsilon_{\alpha\beta\mu\nu}F^{\alpha\beta}_{(x)}F^{\mu\nu}(x)\,, \quad (36)$$

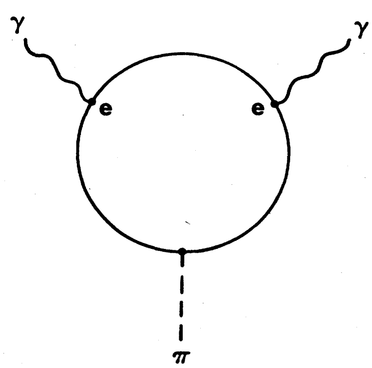

FIG. 1. The simple $\pi\to\gamma\gamma$ loop graph.

where $F^{\alpha\beta}=\partial^\alpha\Phi^\beta-\partial^\beta\Phi^\alpha$, N is the number of quark colors and Q is the quark charge matrix as defined in Sec. III.

Next, we include A_μ and V_μ. For purposes of the present argument, we do not have to do the $V_\mu\leftrightarrow\Phi_\mu$ mixing treatment outlined in Sec. III, but may simply include the gauge-covariant derivative with Φ_μ in $\mathscr{L}_{\rm loop}$, Eq. (34), which leads to

$$\mathscr{L}^{(\rm EM)}_{\rm loop}=i\,{\rm Tr}\sum_{n=1}^{\infty}\left\{(i\partial\!\!\!/-\mu)^{-1}\left[\mu(M-1)+g\left[V\!\!\!/+\frac{e}{g}Q\Phi\!\!\!/+A\!\!\!/\gamma_5\right]\right]\right\}^n, \quad (37)$$

where either

$$A_\mu\to A_\mu+\frac{g}{M_V{}^2}F_\pi(\partial_\mu\pi+\cdots),\ \text{linear model}\,,$$

or

$$A_\mu\to\frac{1}{gF_\pi}\partial_\mu\pi+\cdots,\ \text{nonlinear model}\,.$$

The problem is now clearly seen. There is a new pseudoscalar coupling $\sim\partial_\mu\pi$ and an additional term, shown in Fig. 2, which contribute, respectively,

$$\mathscr{L}'(\pi,\phi,\phi)=-\frac{Ne^2}{24\pi^2F_\pi}\begin{bmatrix}g^2F_\pi{}^2/M_V{}^2\\1\end{bmatrix}\times\int dx\,{\rm Tr}[\pi(x)Q^2]\epsilon_{\alpha\beta\mu\nu}F^{\alpha\beta}_{(x)}F^{\mu\nu}_{(x)}\,. \quad (38)$$

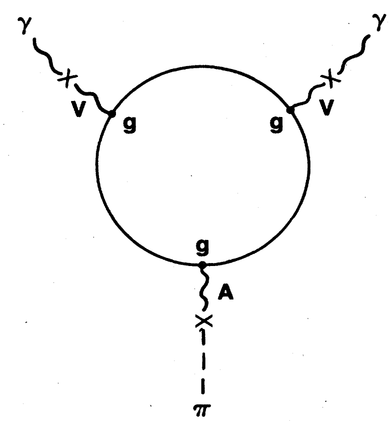

FIG. 2. The additional $\pi\to\gamma\gamma$ loop graph that arises from $\pi\leftrightarrow A$ mixing. This graph should be subtracted. The vector-dominance version of the graph is used for illustration. On the mass shell, the X factor in γ-V mixing, the V propagator, and the factor g all combine to produce a factor e, of course.

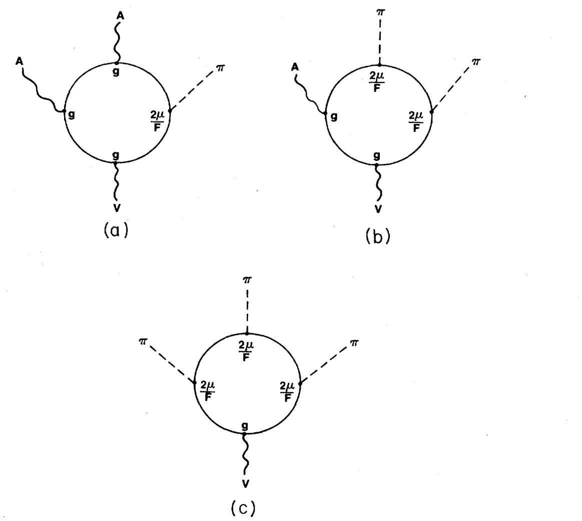

FIG. 3. The $VAA\pi$, $VA\pi\pi$, and $V\pi\pi\pi$ vertex loop graphs. Subtraction removes the $VAAA$ graph. Details of the computation of (a) are shown in the Appendix.

This comes about because the V and A current terms alone are producing an extra contribution from the anomalous AVV vertex,

$$\mathcal{L}(A,V,V)=-\frac{1}{12\pi^2}\epsilon_{\alpha\beta\mu\nu}\int dx\,\mathrm{Tr}(V^\beta\partial^\alpha V^\mu A^\nu-\partial^\alpha V^\beta V^\mu A^\nu)\ ,$$

which has anomalies in all of the legs, V as well as A. The nonlinear model, where A is the pion field to leading order, is especially interesting because

$$\mathcal{L}(\pi,\phi,\phi)+\mathcal{L}'(\pi,\phi,\phi)=\tfrac{1}{3}\mathcal{L}(\pi,\phi,\phi)\ ,$$

but this is not unexpected, since models in which the V and A currents are treated symmetrically are known to have anomalous AVV vertices with the anomalous divergence in each channel being one third of the value that it has when two channels are anomaly free and the third is anomalous.[18,19]

Our answers in Eq. (38) are the same, of course, when, as prescribed by vector-meson dominance (VMD) of the electromagnetic current, Φ is absorbed into a redefinition of V in Eq. (37) and then the mass-mixing term $-(e/g)\mathrm{Tr}Qv_\mu\Phi^\mu$ is used to compute $\pi\to\gamma\gamma$ from $\pi\to VV$ and $V\leftrightarrow\Phi$. The lesson of the above discussion is that, as pointed out by other authors,[7] the local, anomalous functional of V and A which results when the field $M=1$ in Eq. (34) should be subtracted from $\mathcal{L}_{\rm loop}$

$$\tilde{\mathcal{L}}_{\rm loop}=\mathcal{L}_{\rm loop}-\mathcal{L}_{\rm loop}(M=1,\ \text{anomalous})\ ,$$

and the resulting $\tilde{\mathcal{L}}_{\rm loop}$ used for phenomenological applications to hadronic processes. $\mathcal{L}_{\rm loop}$ alone is inconsistent with isospin conservation and, when VMD is implemented, with electromagnetic-current conservation. From the loop-diagram point of view, it simply means that all abnormal-parity terms, those proportional to $\epsilon_{\alpha\beta\mu\nu}$, which involve only A's and V's attached to the quark loop, should be omitted in calculating the anomalous action.

$V\pi\pi\pi$, $V\pi\pi A$, and $V\pi AA$ terms in the effective actions

We turn next to a discussion of the quartic terms $V\pi\pi\pi$, $V\pi\pi A$, and $V\pi AA$ in the effective action in order to illustrate the loop calculation more fully and to prepare the ingredients for the calculation of $\omega\to3\pi$ and related processes.

The relevant terms from the loop expansion are contained in the $n=4$ term,

$$\mathcal{L}_{\rm loop}=\frac{i}{4}\mathrm{Tr}\left[(i\partial\!\!\!/-\mu)^{-1}\left[\frac{2\mu i}{F_\pi}\pi+gV\!\!\!/+gA\!\!\!/\gamma_5\right]\right]^4 .$$

The calculation, which is outlined in the Appendix, produces the following effective-action terms corresponding to Figs. 3(a), 3(b), and 3(c), respectively:

$$\mathcal{L}(V\pi AA)=\frac{ig^3N}{2\pi^2F_\pi}\int dx\,\mathrm{Tr}[-\tfrac16\partial^\alpha\pi V^\beta A^\mu A^\nu-\tfrac13\pi\partial^\alpha V^\beta A^\mu A^\nu+\tfrac16\pi V^\alpha\partial^\beta A^\mu A^\nu-\tfrac16\partial^\alpha V^\beta\pi A^\mu A^\nu$$
$$-\tfrac16 V^\alpha\pi\partial^\beta A^\mu A^\nu-\tfrac16\partial^\alpha\pi A^\beta V^\mu A^\mu-\tfrac12\pi A^\alpha\partial^\beta V^\mu A^\nu]\epsilon_{\alpha\beta\mu\nu}\ , \qquad (39)$$

$$\mathcal{L}(V\pi\pi A)=\frac{ig^2N}{\pi^2F_\pi^{\,2}}\int dx\,\mathrm{Tr}(2\partial^\alpha\pi\partial^\beta\pi V^\mu A^\nu-2\partial^\alpha\pi V^\beta\partial^\mu\pi A^\nu+\pi\partial^\alpha\pi A^\beta\partial^\mu V^\nu-\partial^\alpha\pi\pi A^\beta\partial^\mu V^\nu)\epsilon_{\alpha\beta\mu\nu}\ , \qquad (40)$$

$$\mathcal{L}(V\pi\pi\pi)=-\frac{igN}{3\pi^2F_\pi^{\,3}}\int dx\,\mathrm{Tr}(V^\alpha\partial^\beta\pi\partial^\mu\pi\partial^\nu\pi)\epsilon_{\alpha\beta\mu\nu}\ . \qquad (41)$$

Equation (41) agrees with the low-energy theorem for $\gamma\to\pi\pi\pi$ in early work on the anomalies[6,20] when one replaces $V^\alpha\to-(e/g)Q\Phi^\alpha$ in the trace expression.

The expressions Eqs. (39)–(41) provide the contact term for the $V\to\pi\pi\pi$ interaction needed in $\omega\to3\pi$, for example. For illustration, let us compare the linear model with the nonlinear model with one independent field. The linear-model replacement,

$$A_\mu\to g\frac{F_\pi}{M_V^{\,2}}\partial_\mu\pi\ ,$$

yields

$$\mathscr{L}(V\pi\pi\pi) = -\frac{igN}{3\pi^2 F_\pi{}^3}\left[1-\frac{3g^2F_\pi{}^2}{M_V{}^2}+\frac{3}{2}\left[\frac{g^2F_\pi{}^2}{M_V{}^2}\right]^2\right] \times \int dx\, \mathrm{Tr}(V^\alpha \partial^\beta \pi \partial^\mu \pi \partial^\nu \pi)\epsilon_{\alpha\beta\mu\nu} \quad \text{(linear model)} . \qquad (42)$$

For the nonlinear model, the replacement

$$A_\mu \rightarrow \frac{1}{gF_\pi}\partial_\mu \pi$$

yields

$$\mathscr{L}(V\pi\pi\pi) = +\frac{igN}{6\pi^2 F_\pi{}^3}\int dx\, \mathrm{Tr}[V^\alpha \partial^\beta \pi \partial^\mu \pi \partial^\nu \pi]e_{\alpha\beta\mu\nu} \quad \text{(nonlinear model)} . \qquad (43)$$

When VMD is used with either (42) or (43) the resulting $\gamma \rightarrow 3\pi$ vertices are different from the chiral-algebra value,[20] a point already made by Rudaz[13] in connection with the work of Ref. 7.

This concludes our discussion of the loop evaluation of terms in the Wess-Zumino action when V and A fields are present along with the pseudoscalar, π field. This discussion expands our previous work on the loop evaluation of the pure π effective action, and supplements some of the theoretical consideration, like choice of counterterms, given by other authors[7] in the context of the Witten form[5] of the π, V, A action.

Several phenomenological considerations

The hadronic decay $\omega \rightarrow 3\pi$ can be extracted from Eqs. (42) and (43) and the ρ-pole contributions from the $\omega \rightarrow \rho\pi$, $\rho \rightarrow \pi\pi$ amplitudes. For example, assuming that the momentum-dependent terms of the $g_{\rho\pi\pi}$ vertex cancel one another on the mass shell, then $g_{\rho\pi\pi} = \sqrt{2}g$, and one obtains[7] $\Gamma_{\omega \rightarrow 3\pi} = 7.1$ MeV where the Particle Data Group lists 8.9 ± 0.3 MeV.

In the nonlinear-model case, we can evaluate the $\omega \rightarrow 3\pi$ width from Eq. (43) and the value of g given in (33) in combination with the ρ-pole terms. For this nonlinear model, we find $\Gamma_{\omega \rightarrow 3\pi} = 3.0$ MeV, phenomenologically a bit worse than the linear-model result. The failure of this value in comparison with experiment is probably what one should expect, given that the model contains some arbitrariness. For example, one could add a term $\lambda\, \mathrm{Tr} B_{\mu\nu}[C^\mu, C^\nu]$ which is locally vector-gauge invariant, to modify the momentum-dependent part of the $g_{v\pi\pi}$ vertex in a way analogous to that in the linear-model case. That such a term might be needed phenomenologically is indicated by the value $\Gamma_{A \rightarrow \rho\pi} \sim 1.0$ GeV which one calculates from the $A\rho\pi$ couplings contained in the Lagrangian (32). Adding a term $\lambda\, \mathrm{Tr} V_{\mu\nu}[C^\mu, C^\nu]$ permits one to adjust λ in order to bring down the $A \rightarrow \rho\pi$ width to the neighborhood of its experimental ($\lesssim 300$ MeV) value. Such detailed phenomenological questions go beyond our purpose here, which is to illustrate the construction of the phenomenological Lagrangian under various assumptions.

We should comment here that we have not included the axial-vector field C_μ in the covariant-derivative interaction with the fermions, Eqs. (12) and (34). As a consequence, the anomalous action terms contain only pseudoscalar and vector fields, and the photon-pseudoscalar low-energy theorems are the same as they are in the pure photon-pseudoscalar case, with no spin-one hadrons present.

Next, in the final section, we summarize our results and draw several conclusions.

V. SUMMARY AND CONCLUSIONS

We have extended our fermion-loop technique of generating the terms of the anomalous Wess-Zumino action to include also spin-one hadrons and the photon in addition to the pseudoscalar mesons. The latter alone had been treated in our previous work.[9] In Sec. II we developed a line of argument, culminating in the expansion Eq. (11), which indicates how an effective Lagrangian expressed in terms of quark and quark-composite degrees of freedom could appear after integrating out the gluon degrees of freedom. At this stage, the pseudoscalar mesons were assumed to transform nonlinearly, and are thus massless in the absence of quark mass terms which explicitly break the flavor chiral symmetry.

The developments of Sec. II elaborated on remarks in our previous work and provided background for our assumption of the Lagrangian forms Eqs. (12) and (14) of Sec. III and the consequent loop expansion of Eq. (34) in Sec. IV. Both linear and nonlinear treatments of the V and A fields, introduced in Eq. (12), were outlined in Sec. III. We started with the working hypothesis that the effective, spin-one Lagrangian is locally chiral symmetric except for the spin-one mass terms, thus ensuring the field-current identity and vector-meson dominance of the electromagnetic current in the manner of Kroll, Lee and Zumino, that Wess-Zumino terms are generated with unique relative coefficients solely from the loop expansion, and that globally invariant, abnormal-parity terms of the Wess-Zumino form are not present with arbitrary coefficients. However, we concluded in Sec. III that the phenomenology untenable relation $M_V{}^2/g^2F_\pi{}^2 = 1$, and thus $(g\rho\pi\pi)^2/4\pi = 1.3$, which is required in the simplest nonlinear model, necessitates modifications to the spin-one mass term in the nonlinear model. We described a model with a nonlinearly transforming axial-vector field which solves the phenomenological problem just mentioned, but in which the field-current identities are not satisfied.

In Sec. IV we first studied in detail the PVV vertices by our loop-expansion method. Our calculation treats all vertices symmetrically, as did Bardeen's original loop calculation[12] of non-Abelian anomalies, and we therefore needed to add his counterterm in order to ensure vector-current conservation. We obtained this counterterm by setting the field M equal to one in Eq. (34) and isolating abnormal-parity terms. The desired counterterm is the negative of the expression thus obtained. The standard $\pi^0 \rightarrow \gamma\gamma$ interaction was consequently obtained when vector-meson dominance was implemented. The role of the counterterm is rather transparent in the loop framework. Our discussion complements that of Kaymakcalan, Rajeev, and Schechter,[7] who have treated this problem in the context of the Witten[5] form of the anomalous action.

We illustrated our method with the calculation of the anomalous four-point interactions, shown graphically in Figs. 3, and the results are then applied to the $\omega\rightarrow 3\pi$ calculation. Putting in numbers, we found

$$\Gamma(\omega\rightarrow 3\pi)=3.0 \text{ MeV nonlinear model with nonlinear axial vector as well as vector} .$$

In the case of the nonlinear model with an extra, axial-vector field, the identification of the $S=0$, $I=1$ meson with the $A_1(1275)$ leads to $\Gamma_{A\rightarrow\rho\pi}\cong 1.0$ GeV, compared to $\Gamma_{\text{expt}}\cong 0.30$ GeV, which indicates that our phenomenological picture is not complete. However, we suggest that the nonlinear picture presented here could provide an alternative way of portraying the lowest-mass axial-vector-meson multiplet when more terms are added to the phenomenological Lagrangian.

In summary, our loop calculation of terms in the anomalous action shows the link between the underlying quark degrees of freedom and the composite field effective action. The color factor appears explicitly, gauge fields such as the photon, and "phenomenological flavor gauge fields" such as ρ, K^*, and ω, are simply introduced by covariant derivatives on the quarks, and the diagrammatic interpretation of individual terms in the action is convenient for isolating pieces which are of phenomenological interest.

Note added in proof. Recent papers which have come to our attention in which fermion degrees of freedom are made explicit in deriving effective Wess-Zumino-type actions are I. J. R. Aitchison and C. M. Fraser, Phys. Rev. D **31**, 2605 (1985); A. Dhar, R. Shanker, and S. R. Wadia, *ibid.* **31**, 3256 (1985). The methods of calculation and the applications discussed are different in these works than those in our work.

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APPENDIX

In this appendix we illustrate our procedure for calculating the terms in the anomalous action by focusing on the abnormal-parity terms with one pseudoscalar field (π), one vector field (V) and two axial-vector fields (A). The relevant pieces are contained in the $n=4$ term in the loop expansion, Eq. (34), and the result is presented in Eq. (39). The graphical form is shown in Fig. 3(a).

Denoting the vertex factor

$$\mu(M-1)+g(V+A\gamma_5)$$

by Γ below, we can write the effective four-point interaction as

$$S_\mu^{\text{eff}}=\frac{i}{4}\int\left[\prod_{i=1}^{4} d\bar{q}_i\right]\delta\left[\sum_{i=1}^{4} q_i\right]J(p_i;q_i,p)I(q_i)\,, \tag{A1}$$

where $d\bar{q}_i\equiv d^4q_i/(2\pi)^4$. The four propagator momenta p_i are expressed as $p_i=p+q_i$, where $\sum_{i=1}^4 q_i=0$, and $J(p_i;q_i,p)$ is a Jacobian factor. The integral $I(q_i)$ is given by

$$I(q_i)=\int d\bar{p}\left[\prod_{i=1}^{4}\frac{1}{(p+q_i)^2-\mu^2}\right](\not p+\not q_1+\mu)\times\tilde{\Gamma}(q_1-q_2)\cdots(\not p+\not q_4+\mu)\tilde{\Gamma}(q_5-q_1)\,, \tag{A2}$$

and the momentum-space vertices, $\tilde{\Gamma}(q)$, are defined as

$$\tilde{\Gamma}(q)=\int d^4x\, e^{iqx}\Gamma(x)\,.$$

Next we isolate contributions with one π, one V, and two A's, and we find four terms of each of the forms πVAA, πAVA, and πAAV. The leading term in the momentum expansion is obtained by setting $q_i=0$ in the propagator denominator in Eq. (A2), keeping terms odd in γ_5 and collecting all terms with one q_i power in the numerator from the expression

$$\begin{aligned}S_4^{\text{eff}}(\pi,V,A,A)=\frac{g^3 2i\mu}{F_\pi}\frac{i}{4}4\int\frac{d\bar{p}}{(p^2-\mu^2)^4}&\\ \times\int\left[\prod_i d\bar{q}_i\right]J(p_i;q_i,P)&\\ \times\text{Tr}\gamma_5\{(-\not p-\not q_1+\mu)\tilde{\pi}_{12}(\not p+\not q_2+\mu)\tilde{\not V}_{23}(\not p+\not q_3+\mu)\tilde{\not A}_{34}(\not p+\not q_4-\mu)\tilde{\not A}_{41}&\\ +(\not p+\not q_1+\mu)\tilde{\pi}_{12}(\not p+\not q_2-\mu)\tilde{\not A}_{23}(\not p+\not q_3+\mu)\tilde{\not V}_{34}(\not p+\not q_4+\mu)\tilde{\not A}_{41}&\\ +(-\not p-\not q_1+\mu)\tilde{\pi}_{12}(\not p+\not q_2+\mu)\tilde{\not A}_{23}(\not p+\not q_3-\mu)&\\ \times\tilde{\not A}_{34}(\not p+\not q_4+\mu)\tilde{\not V}_{41}\}\,,&\end{aligned}$$

where γ_5 has been moved to the first place in the trace and $\tilde{\pi}_{12}\equiv\tilde{\pi}(q_1-q_2)=\int d^4x\, e^{i(q_1-q_2)x}\pi(x)$, etc. Picking off terms with one q_i factor will lead to effective action terms with one derivative, and the expression in curly brackets in Eq. (A3) reduces to

$$\{\ \} = [\mu^3 \not{q}_1 - \mu(p^2+\mu^2)\not{q}_2 - \mu(p^2-\mu^2)(\not{q}_3+\not{q}_4)]\tilde{\pi}_{12}\tilde{\not{V}}_{23}\tilde{\not{A}}_{34}\tilde{\not{A}}_{41}$$
$$+[-\mu^3(\not{q}_1-\not{q}_2) - \mu(p^2-\mu^2)(\tilde{\not{q}}_3-\tilde{\not{q}}_4)]\tilde{\pi}_{12}\tilde{\not{A}}_{23}\tilde{\not{V}}_{34}\tilde{\not{A}}_{41}$$
$$+[\mu p^2 \not{q}_1 + \mu^3(\not{q}_1-\not{q}_2) + \mu(p^2-\mu^2)(\not{q}_3+\not{q}_4)]\tilde{\pi}_{12}\tilde{\not{A}}_{23}\tilde{\not{A}}_{34}\tilde{\not{V}}_{41} \ . \qquad \text{(A4)}$$

In order to write a local configuration-space functional, it is convenient to introduce variables $k_i = q_i - q_{i+1}$, $q_5 \equiv q_1$. The Jacobian factor is

$$J(q;k) = J^{-1}(p_i;q_i,p) \ ,$$

and the Jacobian factor from $p_i \to q_i$, p cancels that from $q_i \to k_i$. Next we substitute

$$q_1 = \frac{2k_1+k_2-k_4}{4}, \quad q_2 = \frac{(k_2-k_4-2k_1)}{4} \ ,$$
$$q_3 = -\frac{(k_4+2k_1+3k_2)}{4}$$

and evaluate the p integrals to get

$$\{\ \} = \frac{i}{16\pi^2\mu}[-\tfrac{1}{6}(\not{k}_2-\not{k}_4)\tilde{\pi}_1\tilde{\not{V}}_2\tilde{\not{A}}_3\tilde{\not{A}}_4$$
$$+(-\tfrac{1}{6}\not{k}_1+\tfrac{1}{2}\not{k}_3)\tilde{\pi}_1\tilde{\not{A}}_2\tilde{\not{V}}_3\tilde{\not{A}}_4$$
$$+\tfrac{1}{6}(\not{k}_2-\not{k}_4)\tilde{\pi}_1\tilde{\not{A}}_2\tilde{\not{A}}_3\tilde{\not{V}}_4] \ , \qquad \text{(A5)}$$

where $\tilde{\pi}_1 \equiv \tilde{\pi}(k_1)$, etc. Putting Eqs. (A3) and (A4) together and using, for example,

$$\int dx\, \pi(x)\partial \not{A} \not{A} \not{V} = -i \int d\bar{k}_1 d\bar{k}_2 d\bar{k}_3 \not{k}_2 \tilde{\pi}_1 \tilde{\not{A}}_2 \tilde{\not{A}}_3 \tilde{\not{V}}_4 \ ,$$

and performing the Dirac trace and rearranging terms, we obtain the result quoted in Eq. (39).

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