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Gravitational counterterms and Becchi-Rouet-Stora symmetry

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All one-loop quantum gravitational corrections to the Einstein action, involving the graviton and the ghosts as bilinears, have been systematically determined in a class of Lorentz-covariant gauges and in a manner which is explicitly Becchi-Rouet-Stora invariant. The procedure explains why certain miraculous relations arise in previous analyses.

I. INTRODUCTION

There have been many one-loop calculations in quantum gravity, the most notable attempts being Refs. 1—4. Practically all these investigations have focused on the graviton and ghost bilinears (as indicators of the counterterms needed to renormalize the theory to first order in Newton's constant G) although isolated studies[5] of other Green's functions and a selective two-loop calculation should be mentioned.[6] These latter valiant efforts underline the algebraic difficulties of the task in quantum gravity.

From the outset it was recognized that gravitational Becchi-Rouet-Stora (BRS) symmetry[7] would provide strong constraints on the structure of the Green's functions and on the ensuing renormalizations. Yet every analysis so far, although appreciating the importance of BRS invariance, has contented itself with checking the BRS identities in a slightly *ad hoc* manner and only with reference to the Green's functions that were individually studied. In this paper we wish to systematically catalog the complete one-loop counterterms in a manner which takes advantage of and explicitly realizes the BRS symmetry of the quantum action. (The exercise obliges us to adopt dimensional regularization in order to preserve the BRS invariance.) We restrict ourselves to the bilinears involving ghosts and gravitons but also include the invariant sources. We did not have the strength to go beyond these terms as the calculations become notoriously difficult then.

Section II summarizes the BRS invariance properties of the action and the resulting identities among Green's functions; it was necessary to include this section to spell out the notation, if for no other reason. In Sec. III we categorize the possible counterterms and show how they are determined. The results appear in Sec. IV; comparison with earlier work shows why remarkable relations between different amplitudes have a simple explanation. The appendixes contain some of the relevant Feynman rules and diagrams plus the basic dimensionally continued Feynman integrals.

Our work represents the first step of a more ambitious undertaking: that of evaluating the three- and higher-point functions in a way which automatically respects gravitational BRS symmetry. No doubt, the same considerations apply to the conformal R^2 theories.

II. BRS SYMMETRY OF THE GRAVITATIONAL ACTION

We shall express all our covariants in terms of the metric density[7]

$$\tilde{g}^{\mu\nu}=\sqrt{-g}\; g^{\mu\nu}\equiv\eta^{\mu\nu}+Kh^{\mu\nu}\,,$$

where η is the Minkowski metric, $K^2=32\pi G$, and the associated graviton field is h. The entire Lagrangian including gauge fixing,[8] ghosts, and invariant sources is given in arbitrary $(2l)$ dimensions by

$$\begin{aligned}\mathscr{L}&=\mathscr{L}_g+\mathscr{L}_f+\mathscr{L}_I\\&=\left[\tilde{g}^{\rho\sigma}\tilde{g}_{\lambda\mu}\tilde{g}_{\kappa\nu}-2\delta^\sigma_\kappa\delta^\rho_\lambda\tilde{g}_{\mu\nu}-\frac{\tilde{g}^{\rho\sigma}\tilde{g}_{\mu\kappa}\tilde{g}_{\lambda\nu}}{2l-2}\right]\frac{\partial_\rho\tilde{g}^{\mu\kappa}\partial_\sigma\tilde{g}^{\nu\lambda}}{2K^2}+[\tilde{g}^{\mu\nu}(\partial_\mu B_\nu+\partial_\nu B_\mu)+\alpha\eta^{\mu\nu}B_\mu B_\nu]/2K^2\\&\quad+\partial_\mu\bar{C}_\nu D^{\mu\nu}{}_\lambda C^\lambda+K(I_{\mu\nu}D^{\mu\nu}{}_\lambda C^\lambda+I_\lambda C^\kappa\partial_\kappa C^\lambda)\end{aligned} \tag{1}$$

and it enjoys invariance under BRS transformations

$$\delta\tilde{g}^{\mu\nu}=KD^{\mu\nu}\lambda C^{\lambda},\quad \delta C^{\lambda}=KC^{\kappa}\partial_{\kappa}C^{\lambda}\ ,$$
$$\delta\bar{C}^{\lambda}=-B^{\lambda}/K,\quad \delta B_{\lambda}=\delta I_{\mu\nu}=\delta I_{\lambda}=0\ . \tag{2}$$

It is fairly well known that the gauge-fixing and source terms, $\mathscr{L}_f$ and $\mathscr{L}_I$, respectively, do not share the full gravitational gauge invariance (associated with coordinate transformations) possessed by the classical Einstein term $\mathscr{L}_g$. Therefore at the quantum level it is the BRS symmetry which is directly relevant.[9] Nevertheless, by ensuring that the physical subspace has a zero ghost charge one can prove that the unphysical degrees of g cancel out against the ghost fields and that the physical, on-shell S matrix becomes fully gauge invariant, which is all one could wish for.

The BRS identities satisfied by the Green's functions are derived by adding the usual source terms to (1), viz.,

$$\mathscr{L}_J=J_{\mu\nu}(\tilde{g}^{\mu\nu}-\eta^{\mu\nu})+\bar{C}_{\mu}J^{\mu}+\bar{J}_{\mu}C^{\mu}\ , \tag{3}$$

by making a variation on the vacuum functional $\exp(iW)$, and by noting the invariance of the functional measure and the existence of a regularization—specifically, dimensional—which properly defines the amplitudes. In this way after eliminating the B field one obtains

$$J_{\mu\nu}\frac{\delta W}{\delta I_{\mu\nu}}-\frac{\eta_{\mu\lambda}\partial_{\nu}}{\alpha K^2}\frac{\delta W}{\delta J_{\nu\lambda}}\partial_{\kappa}\frac{\delta W}{\delta I_{\mu\kappa}}-\bar{J}_{\mu}\frac{\delta W}{\delta I_{\mu}}=0 \tag{4}$$

using, in the process, the ghost equation of motion

$$J^{\mu}=\frac{\partial_{\nu}}{K}\frac{\delta W}{\delta I_{\mu\nu}}\ . \tag{5}$$

One then passes to the effective action

$$\tilde{\Gamma}=\Gamma+\int\eta_{\nu\lambda}\partial_{\mu}\tilde{g}^{\mu\nu}\partial_{\kappa}\tilde{g}^{\kappa\lambda}/2\alpha K^2\ ,$$
$$\Gamma\equiv W-\int[J_{\mu\nu}(\tilde{g}^{\mu\nu}-\eta^{\mu\nu})+\bar{C}_{\mu}J^{\mu}+\bar{J}^{\mu}C_{\mu}]\ , \tag{6}$$

so as to convert Eq. (4) into the equivalent statement

$$\frac{\delta\tilde{\Gamma}}{\delta\tilde{g}^{\mu\nu}}\frac{\delta\tilde{\Gamma}}{\delta I_{\mu\nu}}+\frac{\delta\tilde{\Gamma}}{\delta C^{\mu}}\frac{\delta\tilde{\Gamma}}{\delta I_{\mu}}=0\ . \tag{7}$$

Equation (7) is the fundamental BRS identity.

The purpose of our investigation is to extract the full one-loop contribution to $\tilde{\Gamma}$, at least for all of the bilinears and some of the source terms, so we make the loop expansion

$$\tilde{\Gamma}=\sum_{n=0}^{\infty}\Gamma^{(n)}=S+\Gamma^{(1)}+\Gamma^{(2)}+\cdots\ ,$$
$$S=\text{classical action}\ ,$$

substitute in (7), and obtain the BRS invariance property of the first quantum correction $\Gamma^{(1)}$,

$$0=\Delta\Gamma^{(1)}\equiv\left[\frac{\delta S}{\delta\tilde{g}^{\mu\nu}}\frac{\delta}{\delta I_{\mu\nu}}+\frac{\delta S}{\delta I_{\mu\nu}}\frac{\delta}{\delta\tilde{g}^{\mu\nu}}+\frac{\delta S}{\delta C^{\mu}}\frac{\delta}{\delta I_{\mu}}+\frac{\delta S}{\delta I_{\mu}}\frac{\delta}{\delta C^{\mu}}\right]\Gamma^{(1)}\ . \tag{8}$$

The general solution of (8) may be expressed in the usual way as the sum of a gauge-invariant part[10] (square of curvature, to this order) plus a BRS variation,

$$\Gamma^{(1)}=\sqrt{-g}\,(b_1R^2+b_2R_{\mu\nu}R^{\mu\nu})+\Delta G\ . \tag{9}$$

It only remains to catalog ΔG and extract the coefficients b_1,b_2 as well as those coefficients a_i occurring in G. We will find that not all the coefficients can be uniquely found without as well studying the higher-order functions (h^3,hR)—something which we have not done in this paper—or consulting other research. Consequently, we will have at first to content ourselves with a determination of seven independent a,b coefficient combinations as the price of our timidity. The situation has a parallel in Yang-Mills theory but, by contrast, there it is quite easy to state and calculate all possible higher-point counterterms so the complete evaluation can be carried through relatively painlessly. In gravity the problem assumes mammoth proportions and this is just another facet of the nonrenormalizability of the Einstein-Hilbert action. Still, we can pick the last two coefficients b_1,b_2 by leaning on the background-field calculations.[1]

III. CATEGORIZATION OF THE COUNTERTERMS

Let us first take note of the mass dimensions and ghost numbers associated with the various dynamical variables in the four-dimensional limit $(l\to 2)$. These are set out for quick inspection in Table I. Our BRS variation term ΔG in (9) has to be constructed out of the fields C^{λ}, $\bar{C}_{\lambda}$, I_{λ}, $I_{\mu\nu}$, and $Kh^{\mu\nu}$ and therefore the bilinear terms in G can only involve

$$\tilde{g}^{\mu\nu}\partial_{\mu}\bar{C}_{\nu},\quad \tilde{g}^{\mu\nu}I_{\mu\nu},\quad C^{\mu}I_{\mu}\ ,$$

because they are constrained to have total ghost number -1. Moreover, the one-loop pieces have to be associated with two derivatives, as well as two powers of K, with I and $\bar{C}$ in the particular combination

$$\tilde{I}_{\mu\nu}=I_{\mu\nu}+(\partial_{\mu}\bar{C}_{\nu}+\partial_{\nu}\bar{C}_{\mu})/2K\ ,$$

as one can discern in the original Lagrangian. The most general bilinear G therefore involves *seven* coefficients

TABLE I. Mass dimensions and ghost numbers of the fields and constants.

Quantity	Mass dimension	Ghost No.
$g,\tilde{g},S,\Gamma$	0	0
x,K	-1	0
∂,h,B	1	0
δ,C^{λ}	1	1
$\bar{C}_{\lambda}$	-1	1
$J_{\mu\nu}$	4	0
J^{λ}	3	1
$\bar{J}_{\lambda},I_{\mu\nu},G$	3	-1
I_{λ}	2	-2
$R_{\mu\nu}$	2	0

$$G=K^3(a_1\partial_\mu h^{\mu\lambda}\partial^\nu\tilde{I}_{\nu\lambda}+a_2\partial_\lambda h_\mu{}^\mu\partial^\nu\tilde{I}_\nu{}^\lambda+a_3\partial_\lambda h^{\lambda\mu}\partial_\mu\tilde{I}_\nu{}^\nu+a_4\partial^\lambda h^{\mu\nu}\partial_\lambda\tilde{I}_{\mu\nu}+a_5\partial^\lambda h_\mu{}^\mu\partial_\lambda\tilde{I}_\nu{}^\nu)$$
$$+K^2(a_6\partial_\mu C^\mu\partial^\nu I_\nu+a_7\partial^\mu C^\nu\partial_\mu I_\nu) \tag{10}$$

and produces the BRS-invariant counterterms

$$\Delta G=-K^2h^{\rho\sigma}\Delta_0{}^{-1}{}_{\rho\sigma\mu\nu}(a_1\partial^\nu\partial_\lambda h^{\lambda\mu}+a_2\partial^\mu\partial^\nu h_\lambda{}^\lambda+a_3\eta^{\mu\nu}\partial_\kappa\partial_\lambda h^{\kappa\lambda}+a_4\partial^2h^{\mu\nu}+a_5\eta^{\mu\nu}\partial^2h_\lambda{}^\lambda)$$
$$-K^3D^{\mu\nu}{}_\rho C^\rho(a_1\partial_\mu\partial^\lambda\tilde{I}_{\lambda\nu}+a_2\eta_{\mu\nu}\partial^\kappa\partial^\lambda\tilde{I}_{\kappa\lambda}+a_3\partial_\mu\partial_\nu\tilde{I}_\lambda{}^\lambda+a_4\partial^2\tilde{I}_{\mu\nu}+a_5\eta_{\mu\nu}\partial^2\tilde{I}_\lambda{}^\lambda)$$
$$-K^2[(\bar{C}_\lambda\partial_\kappa-I_{\kappa\lambda})D^{\kappa\lambda}{}_\mu+KI_\lambda\partial_\mu C^\lambda-KI_\mu C^\lambda\partial_\lambda](a_6\partial^\mu\partial_\nu C^\nu+a_7\partial^2C^\mu)-K^3C^\nu\partial_\nu C^\mu(a_6\partial_\mu\partial^\lambda I_\lambda+a_7\partial^2I_\lambda)\ . \tag{11}$$

This of course needs supplementing by the generally covariant terms

$$\sqrt{-g}\,(b_1R^2+b_2R_{\mu\nu}R^{\mu\nu})\ .$$

All quantities in (11) have been already defined except for $\Delta_0{}^{-1}$. This stands for the inverse graviton propagator without gauge fixing, viz., $\delta^2S/\delta h\,\delta h$.

It is quite obvious that if one wishes to find out the several a_i and b_1,b_2 it is necessary to examine (a) the graviton self-energy, (b) the $I_{\mu\nu}$-C^λ and $\bar{C}_\kappa$-C^λ graphs (which must be related to one another), and (c) the I_μ-C-C graphs, at the very least. In fact we shall find that, even then, there is insufficient information to determine all of the coefficients without making separate use of background-field determinations of b_1 and b_2.

IV. EVALUATION OF THE COUNTERTERMS

To start, convert the graviton part of (11) into momentum space. We have the inverse propagator (not yet gauge fixed)

$$\Delta_0{}^{-1}{}_{\rho\sigma\mu\nu}(k)=\tfrac{1}{2}k^2(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})-\frac{k^2}{2l-2}\eta_{\mu\nu}\eta_{\rho\sigma}-\tfrac{1}{2}(k_\mu k_\rho\eta_{\nu\sigma}+k_\mu k_\sigma\eta_{\nu\rho}+k_\nu k_\sigma\eta_{\mu\rho}+k_\nu k_\rho\eta_{\mu\sigma})\ . \tag{12}$$

Substituting in (11) we arrive at the h-h part of ΔG,

$$\Delta G\supset K^2h^{\kappa\lambda}(p)h^{\mu\nu}(-p)\left[a_1\left[-p_\kappa p_\lambda p_\mu p_\nu-\frac{p^2}{4(l-1)}(\eta_{\kappa\lambda}p_\mu p_\nu+\eta_{\mu\nu}p_\kappa p_\lambda)\right]\right.$$
$$+a_2\left[-\tfrac{1}{2}p^2(\eta_{\mu\nu}p_\kappa p_\lambda+\eta_{\kappa\lambda}p_\mu p_\nu)-\frac{p^4}{2(l-1)}\eta_{\kappa\lambda}\eta_{\mu\nu}\right]$$
$$+a_3\left[-\frac{p^2}{2(l-1)}(\eta_{\mu\nu}p_\kappa p_\lambda+\eta_{\kappa\lambda}p_\mu p_\nu)-2p_\kappa p_\lambda p_\mu p_\nu\right]$$
$$+a_4\left[\tfrac{1}{2}p^4(\eta_{\mu\kappa}\eta_{\nu\lambda}+\eta_{\mu\lambda}\eta_{\nu\kappa})-\frac{p^4}{2(l-1)}\eta_{\mu\nu}\eta_{\kappa\lambda}\right.$$
$$\left.-\tfrac{1}{2}p^2(\eta_{\mu\kappa}p_\nu p_\lambda+\eta_{\nu\lambda}p_\mu p_\kappa+\eta_{\mu\lambda}p_\nu p_\kappa+\eta_{\nu\kappa}p_\mu p_\lambda)\right]$$
$$\left.+a_5\left[-p^2(\eta_{\mu\nu}p_\kappa p_\lambda+\eta_{\kappa\lambda}p_\mu p_\nu)-\frac{p^4}{l-1}\eta_{\mu\nu}\eta_{\kappa\lambda}\right]\right]\ . \tag{13}$$

This has to be added to the R^2 contributions associated with

$$R_{\mu\nu}=\tfrac{1}{2}\left[\partial^2h_{\mu\nu}-\partial^\kappa\partial_\nu h_{\mu\kappa}-\partial^\kappa\partial_\mu h_{\kappa\nu}-\frac{\partial^2h_\kappa{}^\kappa\eta_{\mu\nu}}{2(l-1)}\right]+O(h^2)\ . \tag{14}$$

Combining the two contributions we arrive at the *entire* one-loop BRS-invariant, graviton counterterms:

$$\Gamma^{(1)}_{hh} \supset \tfrac{1}{2}K^2 h^{\kappa\lambda}(p)h^{\mu\nu}(-p)$$
$$\times\Bigg[(a_4+\tfrac{1}{4}b_2)[(\eta_{\kappa\mu}\eta_{\lambda\nu}+\eta_{\kappa\nu}\eta_{\lambda\mu})p^4+(\eta_{\kappa\lambda}p_\mu p_\nu+\eta_{\mu\nu}p_\kappa p_\lambda)p^2-(\eta_{\kappa\mu}p_\lambda p_\nu+\eta_{\lambda\nu}p_\kappa p_\mu+\eta_{\kappa\nu}p_\lambda p_\mu+\eta_{\lambda\mu}p_\kappa p_\nu)p^2]$$
$$+(b_1+\tfrac{1}{2}b_2-a_1-2a_3)\left[2p_\kappa p_\lambda p_\mu p_\nu+\frac{p^2}{2(l-1)}(\eta_{\kappa\lambda}p_\mu p_\nu+\eta_{\mu\nu}p_\kappa p_\lambda)\right]$$
$$+\left[\frac{b_1+b_2(1-l/2)}{2l-2}-a_2-a_4-2a_5\right]\left[p^2(\eta_{\mu\nu}p_\kappa p_\lambda+\eta_{\kappa\lambda}p_\mu p_\nu)+\frac{p^4}{l-1}\eta_{\kappa\lambda}\eta_{\mu\nu}\right]\Bigg]. \tag{15}$$

The fact that only three independent kinematic covariants occur in (15) is a statement that BRS invariance requires *two* independent relations to exist in the most general expansion of the graviton self-energy Π,

$$K^{-2}\Pi_{\kappa\lambda\mu\nu}\equiv p_\kappa p_\lambda p_\mu p_\nu t_1+p^4\eta_{\kappa\lambda}\eta_{\mu\nu}t_2+(\eta_{\kappa\mu}\eta_{\lambda\nu}+\eta_{\kappa\nu}\eta_{\lambda\mu})p^4 t_3+(\eta_{\kappa\lambda}p_\mu p_\nu+\eta_{\mu\nu}p_\kappa p_\lambda)p^2 t_4$$
$$+(\eta_{\kappa\mu}p_\lambda p_\nu+\eta_{\kappa\nu}p_\lambda p_\mu+\eta_{\lambda\nu}p_\kappa p_\mu+\eta_{\lambda\mu}p_\kappa p_\nu)p^2 t_5\ , \tag{16}$$

namely,

$$t_3+t_5\equiv 0,\quad \frac{t_1}{4(l-1)}+(l-1)t_2+t_3-t_4\equiv 0\ .$$

The first of these relations is explicitly obvious in the paper of Capper and Namazie[2] but the second (which is miraculously satisfied) is only implicit in their work. Specifically, we have the following connections between their coefficients and ours:

$$\begin{aligned}
\tfrac{1}{2}t_1&=b_1+\tfrac{1}{2}b_2-a_1-2a_3\ ,\\
\tfrac{1}{2}t_2&=\frac{b_1+b_2(1-l/2)}{4(l-1)^2}-\frac{(a_2+a_4+2a_5)}{2(l-1)}\ ,\\
\tfrac{1}{2}t_3&=-t_5=\tfrac{1}{8}b_2+\tfrac{1}{2}a_4\ ,\\
\tfrac{1}{2}t_4&=\frac{b_1+\tfrac{1}{2}b_2-\tfrac{1}{2}a_1-a_3}{2(l-1)}-\tfrac{1}{2}a_2-a_5\ .
\end{aligned} \tag{17}$$

All told then, the graviton self-energy offers a method of determining *three* of our nine coefficients.

Next we turn to the other bilinears. The most informative one is associated with the $I_{\mu\nu}$-C^λ transition element which can be read off from (11) after making the replacement

$$D^{\mu\nu}{}_\rho C^\rho=\partial^\mu C^\nu+\partial^\nu C^\mu-\eta^{\mu\nu}\partial\cdot C+O(h)\ . \tag{18}$$

In this manner we have the two-point function

$$\Delta G\supset K^3\{(a_1+2a_4+2a_7)\partial^\mu I_{\mu\nu}\partial^2 C^\nu+2[(l-1)a_2-a_6]\partial^\mu\partial^\nu I_{\mu\nu}\partial\cdot C+[a_4-a_3+2(l-1)a_5+a_6+a_7]\partial^2 I_\mu{}^\mu\partial\cdot C\}\ . \tag{19}$$

Here we need to perform a calculation from scratch to work out the associated self-energy,

$$-i\Sigma^{\mu\nu}{}_\lambda=2[(l-1)a_2-a_6]p^\mu p^\nu p_\lambda-\tfrac{1}{2}(a_1+2a_4+2a_7)(\delta^\mu_\lambda p^\nu+\delta^\nu_\lambda p^\mu)+p^2\eta^{\mu\nu}p_\lambda[a_4-a_3+(2l-2)a_5+a_6+a_7]\ . \tag{20}$$

See Appendix B for some relevant details. Our results are

$$a_2+(1-l)^{-1}a_6=\left[-\frac{1}{4}-\frac{\alpha(2l^2-6l+3)}{(2l-1)(2l-2)}\right]I ,$$

$$a_1+2a_4+2a_7=-\frac{\alpha}{2(2l-1)}I , \qquad (21)$$

$$a_1+2a_3+4(1-l)(a_2+a_5)+2a_6=\frac{(1-l)\alpha}{2l-1}I ,$$

and, comparing against (17), we observe that this supplies us with *two* new, independent coefficients. We get no further information from the ghost self-energy $\bar{C}$-C, merely reconfirmation of the last two combinations in (21), though it is always nice to have separate checks on the work.

The final contribution of interest to us is the I-C-C term, the last part of (11). It is not very difficult to extract the two coefficients a_6 and a_7 providing that we go to the zero-momentum limit at the I leg; this effectively corresponds to looking at source I independent of x and provides

$$\Delta G\supset K^3 I_\mu[a_6(C^\mu\partial^2\partial\cdot C-\partial\cdot C\partial^\mu\partial\cdot C)+2a_7C^\mu\partial^2\partial\cdot C] . \qquad (22)$$

An evaluation of the corresponding Feynman graph (Appendix B contains pertinent details) yields the next two coefficients:

$$a_6=-\frac{\alpha lI}{2l-1},\quad a_7=\frac{(5-2l)\alpha I}{8(2l-1)} . \qquad (23)$$

Without tackling the one-loop corrections to the higher-point functions—and we are understandably reticent about treating them—it is not possible to proceed further, not unless we make use of *other* results. Fortunately, the graviton self-energy was determined via the background-field method by 't Hooft and Veltman,[1] who arrived at the coefficients b_1 and b_2 of the (gauge-invariant) R^2 terms in a particular background gauge:

$$b_1=\frac{I}{120}+O(l-2),\quad b_2=\frac{7I}{20}+O(l-2) .$$

With these numbers at hand we attain our limited objective, inasmuch as we may extract all the coefficients within (9) and (11). These are summarized below in the four-dimensional limit $l\rightarrow 2$:

$$\begin{aligned}
a_1&=-(\tfrac{1}{2}+\tfrac{5}{4}\alpha+2\alpha^2)I ,\\
a_2&=-\tfrac{1}{4}(1+2\alpha)I ,\\
a_3&=-\tfrac{1}{6}(1+\tfrac{1}{4}\alpha)I ,\\
a_4&=(\tfrac{1}{4}+\tfrac{1}{2}\alpha+\alpha^2)I ,\\
a_5&=(\tfrac{1}{12}-\tfrac{1}{12}\alpha-\tfrac{1}{2}\alpha^2)I ,\\
a_6&=-\tfrac{2}{3}\alpha I ,\\
a_7&=\tfrac{1}{24}\alpha I ,\\
b_1&=I/120 ,\\
b_2&=7I/20 .
\end{aligned} \qquad (24)$$

However it is not clear that these special values of b_1,b_2 are the appropriate ones for arbitrary α given that the R^2 counterterms are gauge dependent,[11] unlike the Yang-Mills case. Thus (24) may only be strictly correct in the limit $\alpha\rightarrow 1$, although all the earlier formulas are, of course, exact for any α.

Perhaps others, endowed with more perseverance and stamina, will be able to include higher powers of the graviton field in (10), obtain the coefficients for these, and bring to completion the full BRS counterterms program, given our choice of gauge fixing. A parallel exercise can of course be initiated for other, non-Lorentz-covariant gauge choices.[12]

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APPENDIX A: FEYNMAN RULES

Denote the graviton propagator $\langle h_{\mu\nu}h_{\kappa\lambda}\rangle\equiv\Delta_{\mu\nu\kappa\lambda}$ by a wavy line. After the gauge-fixing choice (1),

$$k^2\Delta_{\kappa\lambda\mu\nu}(k)=\frac{1}{2}(\eta_{\kappa\mu}\eta_{\lambda\nu}+\eta_{\kappa\nu}\eta_{\lambda\mu})-(1+\alpha)\eta_{\kappa\lambda}\eta_{\mu\nu}$$
$$+\frac{(1+2\alpha)}{2k^2}(2\eta_{\kappa\lambda}k_\mu k_\nu+2\eta_{\mu\nu}k_\kappa k_\lambda-\eta_{\kappa\nu}k_\lambda k_\mu-\eta_{\kappa\mu}k_\lambda k_\nu-\eta_{\lambda\nu}k_\kappa k_\mu-\eta_{\lambda\mu}k_\kappa k_\nu) .$$

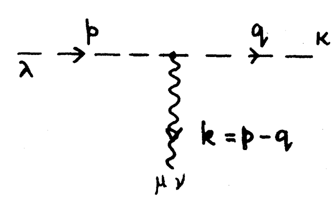

FIG. 1. The ghost-graviton-ghost vertex.

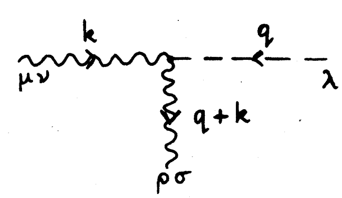

FIG. 2. The ghost-graviton-source vertex.

We make much use in our computations of the contractions

$$k^2\Delta^{\lambda}{}_{\lambda\mu\nu}(k)=(2-2l)[(1+\alpha)\eta_{\mu\nu}-(1+2\alpha)k_\mu k_\nu/k^2]\ ,$$

$$k^2\Delta^{\mu}{}_{\lambda\mu\nu}(k)=(l-1-2\alpha)\eta_{\lambda\nu}+(1-l)(1+2\alpha)k_\lambda k_\nu/k^2\ ,$$

$$k^2k^\kappa\Delta_{\kappa\lambda\mu\nu}(k)=\alpha(k_\lambda\eta_{\mu\nu}-k_\mu\eta_{\lambda\nu}-k_\nu\eta_{\lambda\mu})\ ,$$

$$q^\kappa q^\lambda q^\mu q^\nu\Delta_{\kappa\lambda\mu\nu}(k)=-\alpha q^4/k^2\ .$$

Denote the ghost propagator $\langle C^\mu\bar{C}_\nu\rangle=\Delta^\mu{}_\nu$ by a dashed line. From (1) and (18) we get quite simply

$$\Delta^\mu{}_\nu(k)=\delta^\mu_\nu/k^2\ .$$

The interactions can also be read off from (1).

The graviton-ghost vertex, $\langle h_{\mu\nu}C_\lambda\bar{C}^\kappa\rangle_{\rm amp}$, depicted in Fig. 1, is described by

$$\Gamma^\kappa{}_{\lambda\mu\nu}=\tfrac{1}{2}K[q_\lambda(k_\mu\delta^\kappa_\nu+k_\nu\delta^\kappa_\mu)+\delta^\kappa_\lambda(q_\mu p_\nu+q_\nu p_\mu)]\ ,$$

with momenta directed as shown. Let the solid-wavy line be associated with the source $I_{\rho\sigma}$; then the $\langle I_{\rho\sigma}h_{\mu\nu}C_\lambda\rangle_{\rm amp}$ vertex is obtained (Fig. 2) as

$$\Gamma_{\rho\sigma\mu\nu\lambda}=\tfrac{1}{2}iK^2[q_\mu(\eta_{\nu\rho}\eta_{\lambda\sigma}+\eta_{\nu\sigma}\eta_{\lambda\rho})+q_\nu(\eta_{\mu\rho}\eta_{\lambda\sigma}+\eta_{\mu\sigma}\eta_{\lambda\rho})-(q+k)_\lambda(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})]\ .$$

The only other interaction we shall require involves the source I_λ, represented by a solid line. With reference to Fig. 3 we have the vertex $\langle I_\lambda C^\mu C^\nu\rangle$ described by

$$\Gamma^{\mu\nu}{}_\lambda=iK(q^\nu\delta^\mu_\lambda-r^\mu\delta^\nu_\lambda)\ .$$

Notice the antisymmetry under ghost interchange $(q,\mu\leftrightarrow r,\nu)$.

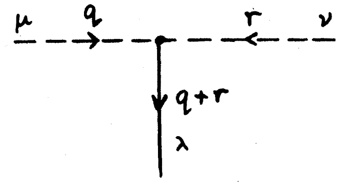

FIG. 3. The ghost-ghost-source vertex.

APPENDIX B: FEYNMAN INTEGRALS AND SELECTED GRAPHS

By making appropriate contractions, every integral we come across in our calculations can be expressed in terms of the two basic integrals:

$$I=-i\int\frac{d^{2l}k}{(2\pi)^{2l}}\frac{1}{k^2(q+k)^2\mu^{2l-2}}=\frac{\Gamma(2-l)\Gamma(l-1)\Gamma(l-1)}{(4\pi)^l\Gamma(2l-2)}\left[-\frac{q^2}{\mu^2}\right]^{l-2}\underset{l\to 2}{\sim}\frac{1}{16\pi^2(2-l)}$$

and

$$-i\int\frac{d^{2l}k}{(2\pi)^{2l}}\frac{q^2}{k^2(q+k)^4\mu^{2l-2}}=(3-2l)I\underset{l\to 2}{\sim}\frac{1}{16\pi^2(l-2)}\ .$$

Such integrals arise in the one-loop diagram for the I-C correction of Fig. 4, viz.,

$$\Sigma_{\rho\sigma}{}^\kappa(p)=K^3\int\frac{d^{2l}k}{(2\pi)^{2l}}\frac{\Delta^{\mu\nu\mu'\nu'}(k)}{(p-k)^2}[\eta_{\mu\rho}\eta_{\lambda\sigma}(p-k)_\nu+\eta_{\mu\sigma}\eta_{\lambda\rho}(p-k)_\nu-p_\lambda\eta_{\mu\rho}\eta_{\nu\sigma}][(p-k)^\kappa k_{\mu'}\delta^\lambda_{\nu'}+(p-k)_{\mu'}p_{\nu'}\eta^{\kappa\lambda}]\ .$$

when constructing the scalar invariants

$$p_\kappa p^\rho p^\sigma\Sigma_{\rho\sigma}{}^\kappa,\quad p_\kappa\eta^{\rho\sigma}\Sigma_{\rho\sigma}{}^\kappa,\quad p^\rho\delta^\sigma{}_\kappa\Sigma_{\rho\sigma}{}^\kappa\ .$$

They also come in the ghost self-energy which has been separately evaluated. We will not report anything about this calculation except to mention that the coefficients are in perfect agreement with the expected counterterm,

$$\tfrac{1}{2}K^2\{(a_1+2a_4+2a_7)\partial^2\bar{C}_\mu\partial^2C^\mu-[a_1+2a_3+(4-4l)(a_2+a_5)+2a_6]\partial\cdot\bar{C}\partial^2\partial\cdot C\}$$

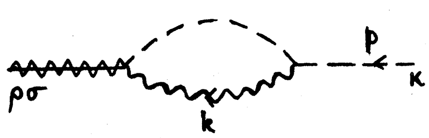

FIG. 4. The source-ghost quantum correction.

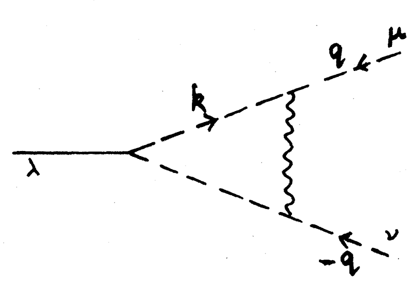

FIG. 5. The source-ghost-ghost correction.

that has been noted in (21). The last computation is associated with Fig. 5 and the three-point function.

It is enough for our purpose to set the l momentum equal to zero in order to find a_6 and a_7:

$$\Gamma_\lambda{}^{\mu\nu} \to -iK^3[(a_6+2a_7)q^2(q^\mu\delta^\nu_\lambda+q^\nu\delta^\mu_\lambda)+2a_6q^\mu q^\nu q_\lambda]$$

$$=K^3\int\frac{d^{2l}k}{(2\pi)^{2l}}[(q+k)_\rho\eta_{\sigma\lambda}+(q+k)_\sigma\eta_{\rho\lambda}]\frac{\Delta^{\alpha\beta\gamma\delta}(k)}{(q+k)^4}[(q+k)^\mu k_\gamma\delta^\rho_\delta-\eta^{\mu\rho}(q+k)_\gamma q_\delta]$$

$$\times[(q+k)^\nu k_\alpha\delta^\sigma_\beta-\eta^{\nu\sigma}(q+k)_\alpha q_\beta]\ ,$$

whereupon we form the independent invariants $\eta_{\mu\nu}q^\lambda\Gamma_\lambda{}^{\mu\nu}$ and $q_\mu\delta^\lambda{}_\nu\Gamma_\lambda{}^{\mu\nu}$ to arrive at the quoted coefficients (24). We have suppressed other details which are plainly irrelevant; there is sufficient information above for the interested reader to reproduce our results if he is so moved.

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[8]Here we adopt the usual Lorentz-invariant fixing term parametrized by a *single* constant α (no weight parameter).

[9]At the level of (2) indices are raised and lowered through η and

$$D^{\mu\nu}{}_\lambda V^\lambda \equiv +\tilde{g}^{\mu\nu}\partial\cdot V+\partial_\lambda\tilde{g}^{\mu\nu}V^\lambda-\tilde{g}^{\lambda\nu}\partial_\lambda V^\mu-\tilde{g}^{\mu\lambda}\partial_\lambda V^\nu\ .$$

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