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Magnetic moments of heavy quarks and leptons

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Theoretical arguments suggest that if quarks and leptons are composite objects, the heavy species have relatively large anomalous magnetic moments. We analyze some possible experiments in which anomalous magnetic moments of heavy quarks and leptons could be measured; the angular distribution of heavy-fermion pair production in electron-positron annihilation turns out to be sensitive to the existence of anomalous moments. According to a crude analysis, nonvanishing anomalous magnetic moments of heavy quarks and leptons are compatible with the presently available data from SLAC and DESY.

I. INTRODUCTION

Magnetic moments of "elementary" particles contain important physical information about their interactions and, possibly, about their constituents. Unfortunately, the classic spin-precession method of measuring magnetic moments is applicable only to the light charged leptons (e,μ), but not to τ and certainly not to quarks because they are confined inside hadrons. *In principle,* one could extract information about the magnetic moments of quarks, e.g., by measuring the magnetic moments of hadrons and computing hadronic moments from QCD: however, at present, the necessary calculational tools are unavailable; moreover, it is impractical to measure directly the moments of hadrons containing heavy quarks.

The measurement of magnetic moments of heavy quarks and leptons would be of importance, however, for the following reason. If, as suspected by many, quarks and leptons are composite objects, there should be structural contribution to their magnetic moments. Generalizing an argument of Shaw, Silverman, and Slansky,[1] we argued some time ago[2] that if quarks and leptons are built up of light subconstituents confined by some non-Abelian gauge field, the expression of their anomalous moments should be of the form

$$\kappa \simeq \kappa_L + \frac{\mu}{\Lambda}\left[\frac{m}{\Lambda}\right]^2 f\left[\frac{\mu}{\Lambda},\frac{m}{\Lambda}\right], \tag{1}$$

where Λ is the characteristic energy scale of the "new physics;" m and μ stand for some average mass of the constituents and of the composite quark or lepton, respectively, whereas κ_L is the contribution of low-energy interactions (below the scale Λ) to the anomalous moment. The function f represents, in essence, a "sidewise" dispersion integral, evaluated on the mass shell of the composite state; in Ref. 2 it was set equal to unity. It should be kept in mind, however, that this was merely an admission of one's ignorance of the dynamics involved. In any case, one expects the structural contribution to grow with the mass of the composite state: while the factor μ/Λ in Eq. (1) is essentially of a kinematical origin (due to the conventional normalization of the Pauli amplitude F_2 in the electromagnetic vertex), the magnitude of the dispersion integral is also expected to increase with growing μ/Λ. Thus, if the characteristic scale Λ is of the order of 1 TeV, there may be substantial structural contributions to the magnetic moments of heavy quarks and leptons. We argue in particular that the low-energy contribution κ_L should not be too large: in the case of τ leptons, it is of the order of α/π (the lowest-order QED contribution), whereas for b and t quarks it is probably of the order of $\alpha_s/\pi \simeq 0.03$, from the lowest-order QCD diagram. Thus, the measurement of the magnetic moments of heavy quarks and leptons may provide a clue about their composite nature at energies considerably below the characteristic scale.

In this paper we address the question whether it is possible to measure the magnetic moments of quarks and/or leptons in experiments which can be performed today or in the near future. In principle, one can think of either deep-inelastic lepton scattering or high-energy e^+e^- annihilation as a possible access to confined quarks.[3] We find, however, that deep-inelastic scattering is unsuitable for our purposes; by contrast, e^+e^- annihilation is found to offer a real possibility of measuring the magnetic moments of both quarks and leptons.

The plan of this paper is the following. In Sec. II we discuss the problem of introducing phenomenological couplings of the gauge bosons to fermions within the framework of the standard model of electroweak interactions. In Sec. III we analyze the possibility of measuring magnetic moments in deep-inelastic scattering, whereas e^+e^- annihilation is dealt with in Sec. IV. The results are discussed in Sec. V. Some computational information is collected in the Appendix.

II. PHENOMENOLOGICAL ELECTROWEAK COUPLINGS

Whether or not quarks, leptons, or even some gauge bosons are composite objects, the standard model of electroweak interactions is at least a good effective low-energy theory. Therefore, in order to look for effects of compositeness, we use a phenomenological parametrization of the matrix elements of the currents occurring in that model;

however, in doing so, we retain the relationship between the gauge fields introduced in the Lagrangian and the physical vector-meson states. No matter by what mechanism the group $SU(2)_L \times U(1)$ is broken, the mixing scheme between the neutral component of the SU(2) triplet and the U(1) gauge bosons remains. In what follows, we work out the formulas for the electrically neutral couplings only: there is no mixing for charged currents, so the procedure is even simpler there. In terms of the physical fields, the effective interaction Lagrangian with neutral vector mesons is

$$L_{\rm int} = -ie\left[j_\mu^E A^\mu + \frac{2}{\sin 2\theta_W}(j_\mu^3 - \sin^2\theta_W j_\mu^E)Z^\mu\right]$$
$$\equiv -ie(j_\mu^E A^\mu + j_\mu^Z Z^\mu)\ , \quad (2.1)$$

where j_μ^E and j_μ^3 stand for the electromagnetic current and the third component of the weak-isospin current, respectively. In parametrizing the matrix elements of the currents, it is convenient to decompose the latter into vector and axial-vector components: in this way, one can easily make sure that the couplings respect *CP* invariance. We write the matrix elements between states of spin $\frac{1}{2}$ as

$$\langle p'|j_\mu^E|p\rangle = Q\langle p'|v_\mu|p\rangle\ ,$$
$$\langle p'|j_\mu^Z|p\rangle = \frac{T_3}{\sin 2\theta_W}\langle p'|a_\mu|p\rangle + \frac{2}{\sin 2\theta_W}(\tfrac{1}{2}T_3 - \sin^2\theta_W Q)\langle p'|\tilde{v}_\mu|p\rangle\ , \quad (2.2)$$

where T_3 and Q are the values of the third component of the weak isospin and the charge (in units of the charge of the proton) of the target. By extracting these factors, the normalization of the form factors of the vector $(v_\mu, \tilde{v}_\mu)$ and axial-vector (a_μ) currents is independent of the internal quantum numbers of the target. The parametrization of such matrix elements has been given elsewhere.[4] In essence, both the vector and axial-vector matrix elements contain two invariant amplitudes, one is of electric, the other of magnetic type. In addition, the matrix element of the axial-vector current contains an irrotational ("longitudinal") piece; the contribution of the latter is, however, small, due to the fact that in each process considered there is a pair of light leptons present.[4] Therefore, we consider the solenoidal ("transverse") contributions to each matrix element only. Neglecting some trivial kinematic factors, one has[5]

$$\langle p'|v_\mu|p\rangle = \left[\frac{q_\mu(p\cdot q)}{q^2} - p_\mu\right]v_E(q^2) + \frac{1}{\Lambda}\epsilon_{\mu\rho\alpha\beta}q^\alpha p^\beta\gamma^\rho\gamma_5 v_M(q^2)\ ,$$
$$\langle p'|a_\mu|p\rangle = \left[\frac{q_\mu(p\cdot q)}{q^2} - p_\mu\right]\gamma_5 a_E(q^2) + \frac{1}{\Lambda}\epsilon_{\mu\rho\alpha\beta}q^\alpha p^\beta\gamma^\rho a_M(q^2) + (\text{irrotational part})\ , \quad (2.3)$$

where $q_\mu = p'_\mu - p_\mu$ and $v_E(0) = a_E(0) = 1$, $v_M(0) = 1 + \kappa_V$, $a_M(0) = 1 + \kappa_A$, each κ being an "anomalous moment." In this way, one is dealing with altogether three "magnetic moments," viz., one for the electromagnetic vertex and two for the neutral weak vertex.

III. DEEP-INELASTIC LEPTON SCATTERING

We describe the calculation for the case of inelastic electron scattering only; the results are similar for charged currents. The dominant contribution to the deep-inelastic structure functions comes from photon exchange, with *small* corrections due to the γZ^0 interference term. As a first approximation, we neglect the contribution of Z^0 exchange and we work within the framework of the naive parton model as described, e.g., in the book by Halzen and Martin.[6] On denoting the longitudinal-momentum distribution function of any given quark species i by $F_i(u)$, where the parton four-momentum p_μ is taken to be proportional to the four-momentum of the target, viz., $p_\mu = uP_\mu$, a simple calculation leads to the following expression of the structure functions describing unpolarized scattering:

$$2MW_1 = \int_0^1 du \sum_i F_i(u)Q_i^2 v_{Mi}^2 \delta(u-x)\ ,$$
$$\nu W_2 = \int_0^1 du \sum_i F_i(u)Q_i^2 \frac{v_{Ei}^2 + \tau v_{Mi}^2}{1+\tau} u\delta(u-x)\ , \quad (3.1)$$

where the notation of Sec. II has been used; M is the target mass, $\tau = -q^2/4\Lambda^2$; otherwise the notation is standard.

What is relevant for the below-TeV experiments of the near future is the "low"-momentum-transfer limit ($\tau \ll 1$, but $-q^2 \gg \Lambda_{\rm QCD}{}^2$) of these expressions. On the basis of dispersion theory, we argue that it may be dangerous to approximate $v_{E,M}$ by their values at $q^2=0$ because these amplitudes have to obey kinematic constraints. It is safer to approximate the Dirac and Pauli amplitudes, f_1 and f_2, related to $v_{E,M}$ by

$$v_E(q^2) = f_1(q^2) - \tau f_2(q^2)\ ,$$
$$v_M(q^2) = f_1(q^2) + f_2(q^2)\ , \quad (3.2)$$

which are free kinematic singularities and constraints. On setting $f_{1,2}(q^2) \simeq f_{1,2}(0)$, with $f_1(0)=1$, $f_2(0)=\kappa$, we obtain

$$2MW_1 = \sum_i F_i(x)Q_i^2(1+\kappa_i)^2\ ,$$
$$\nu W_2 = x\sum_i F_i(x)Q_i^2(1+\kappa_i^2\tau)\ . \quad (3.3)$$

We read off from these formulas that Bjorken scaling and the Callan-Gross relation are violated if quarks have anomalous moments. This is not unexpected; nevertheless, the result is somewhat disappointing for the following reasons.

(i) Light quarks are not expected to have large anomalous moments; thus the scaling violation arising from light quarks is extremely small. On neglecting terms of $O(\kappa^2)$, scaling is obeyed and one ends up with a miniscule violation of the Callan-Gross relation due to the presence of a term proportional to $\sum F_i Q_i{}^2 \kappa_i$ in W_1, which is, however, hard to detect.

(ii) Even if heavy quarks (c,b,t) have larger anomalous moments as conjectured in Sec. I, their effect is suppressed because they are not abundant in a normal target: $F_i(x) \ll 1$ for $i=c,b,t$.

We conclude that unpolarized deep-inelastic lepton scattering (the most common type of experiment one can perform) is unsuitable for a search of compositeness through the detection of anomalous moments. The situation is not likely to be very different for polarized beam, polarized target experiments, due to (ii). Other effects of compositeness may, however, be detectable, see Burges and Schnitzer for a detailed analysis.[7]

IV. ELECTRON-POSITRON ANNIHILATION

This process can overcome the shortcomings of deep-inelastic lepton scattering for two reasons.

(i) One can directly produce pairs of heavy quarks and leptons through their couplings to γ and Z; this process is easier to analyze than, e.g., inclusive processes of the type $ep \to eq\bar{q} + X$, where q is some heavy quark.

(ii) Due to the kinematics of the process, one can, at least in principle, adjust the center-of-mass-system (c.m.s.) energy so that γZ interference is substantial. Given the fact that electric and magnetic transitions (of the same multipolarity) are of opposite parities, the interference term is expected to be substantially affected by the presence of anomalous moments. Thus, one hopes to be able to detect the presence of anomalous moments by measuring, for instance, the forward-backward asymmetry of the angular distributions.

In principle, such a calculation is straightforward, although its details are somewhat tedious. Neglecting anomalous moment contributions at the electron vertices, one computes the differential cross section corresponding to the diagrams shown in Fig. 1. The heavy-fermion vertices are parametrized in a way discussed in Sec. II. Making again the same low-q^2 approximation as discussed in the preceding section, the invariant differential cross section can be expressed in terms of four unknown numerical parameters, viz., three anomalous moments and the normalization scale, Λ. The final formula can be expressed in a closed form. However, the analytic expression is lengthy and complicated despite the fact that we made the usual—and justifiable—simplifications; in particular, we approximated the propagator of Z^0 by a free propagator with a complex pole at the position, $q^2 = M_Z{}^2 - iM_Z\Gamma_Z$. The main source of complications is the kinematics: in producing heavy fermions, one cannot be sure *a priori* that their masses are negligible. In particular, we are looking for effects which are, presumably, small. Therefore corrections of the order of $M_f{}^2/q^2$, where M_f stands for the mass of the fermion produced, can be easily of the same order of magnitude as the anomalous moment effects we are seeking to extract from the data.

Given these facts, we introduce some simplifications in harmony with the previous approximations and transform our analytic expression into a form in which the deviations from the predictions of the standard model are clearly displayed.

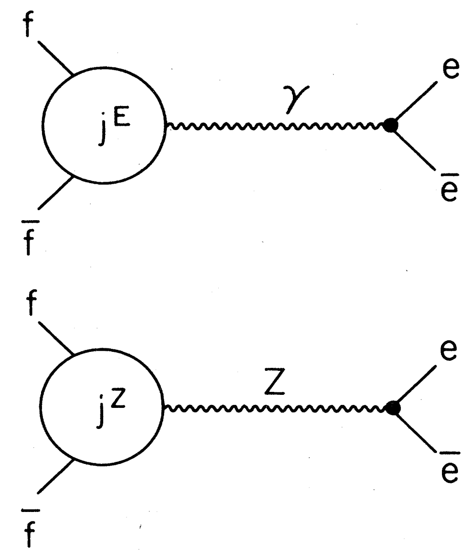

FIG. 1. Lowest-order diagrams contributing to the process $e^+e^- \to f\bar{f}$. The parametrization of the heavy-fermion vertices is described in Sec. II; the electron vertex is assumed to be given by the standard model of electroweak interactions.

(1) We notice that there are terms in the differential cross section proportional to $\kappa^2 q^2/\Lambda^2$ reflecting, as expected, the nonrenormalizable nature of couplings of the Pauli type. With any reasonable value of the anomalous moments, these terms should be small, up to the mass of Z^0, since Λ is expected to be of the order of 1 TeV; therefore we omit them.

(2) The differential cross section in the c.m.s. is of the form

$$\frac{d\sigma}{d\Omega} = a_0 + a_1\cos\theta + a_2\cos^2\theta \ ,$$

where the coefficients a_0, a_1, a_2 are functions of the c.m.s. energy and of the unknown parameters. There is not much hope of extracting small effects by measuring absolute magnitudes of differential cross sections due to the limited accuracy of such measurements. Therefore, we concentrate on the *angular distribution:* we divide the above expression by a_0. We obtain, obviously, an expression of the form

$$\frac{dN}{d\Omega} = 1 + A_1\cos\theta + A_2\cos^2\theta \ .$$

Furthermore, having already neglected terms of $O(\kappa^2)$, it is reasonable to linearize the coefficients A_1 and A_2 in the parameters κ. (This is permissible: there are no cancellations, a_0 does not vanish if we put some or all of the anomalous moments equal to zero, etc.)

(3) Upon performing these steps, we observe that all terms proportional to κ are multiplied by a factor M_f/Λ, corresponding to the fact that the Pauli coupling in the

matrix elements causes one unit of chirality flip. Therefore, the normalization mass Λ can be eliminated by rescaling the parameters κ to their conventional normalization, viz., to a Pauli term of the form $i\kappa\sigma_{\mu\nu}q^{\nu}/2M_f$.

After performing all these steps, we computed the coefficients in the angular distribution. We found that the angular distribution is very insensitive to the presence of Pauli-type couplings at the Z^0 vertex: the coefficients of the weak anomalous moments are, typically, some two to three orders of magnitude smaller than those of the electromagnetic ones. On the one hand, this is a welcome simplification because only one parameter (the electromagnetic anomalous moment) has to be determined from the data. On the other hand, the result is somewhat disappointing: composite models of the future are expected to give relationships between the various nonminimal couplings: it appears that those may not be easily testable. In view of this result, weak anomalous moments are henceforth neglected. Anomalous moments, κ, without a subscript, refer to electromagnetic moments with the Pauli coupling normalized to M_f.

We present our results in such a form that deviations from the standard model appear as *multiplicative factors,* viz., we write

$$\frac{dN}{d\Omega} = 1+A_1(E)[1+\kappa B_1(E)]\cos\theta + A_2(E)[1+\kappa B_2(E)]\cos^2\theta \ , \quad (4.1)$$

where $E=\sqrt{q^2}$ is the c.m.s. energy and $A_{1,2}$ give the results of the standard model. The various coefficients A and B are displayed in Figs. 2 through 5 for the production of τ, c, b, and t pairs; the input data are summarized in Table I. While in the case of τ production the interpretation is obvious, some more caution is needed for the case of quark-pair production due to confinement effects. As it is customary, we interpret Eq. (3.1) as the angular distribution of the jet axes in two jet events. A glance at Figs. 2 through 5 shows that *mass effects are not entirely negligible:* for instance, in the case of b production at energies available at SLAC PEP and DESY PETRA, the naive formula (with the mass of the produced fermion neglected) contains an error of the order of 20% in A_2. The customary procedure consists of determining the electroweak interference term as a test of the standard model by fitting the angular distribution to the function:

$$\frac{dN}{d\Omega} = 1+A_1\cos\theta+\cos^2\theta \ , \quad (4.2)$$

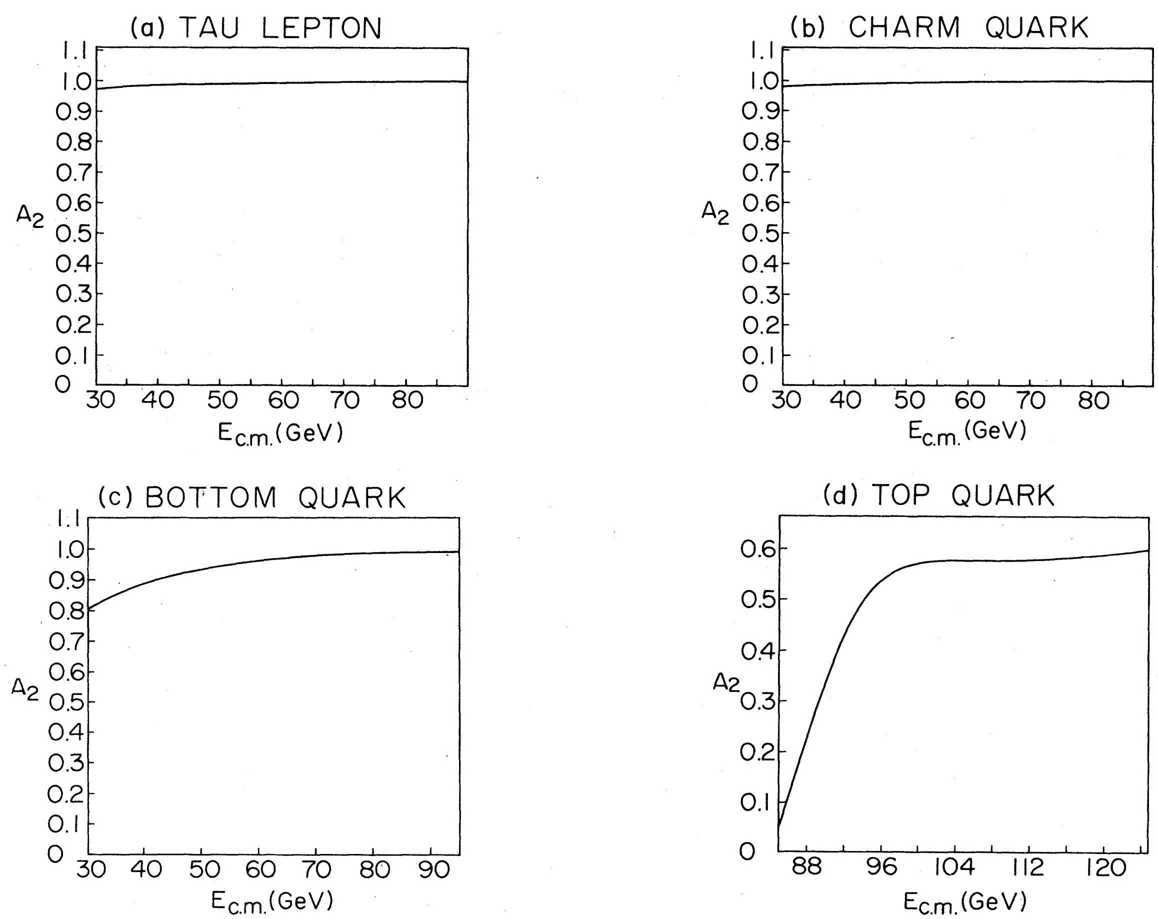

FIG. 2. Predictions of the standard model for the coefficient A_2 of $\cos^2\theta$ in the angular distribution of the process $e^+e^-\rightarrow f\bar{f}$. (a)—(d) give the results for τ, c, b, and t pair production, respectively.

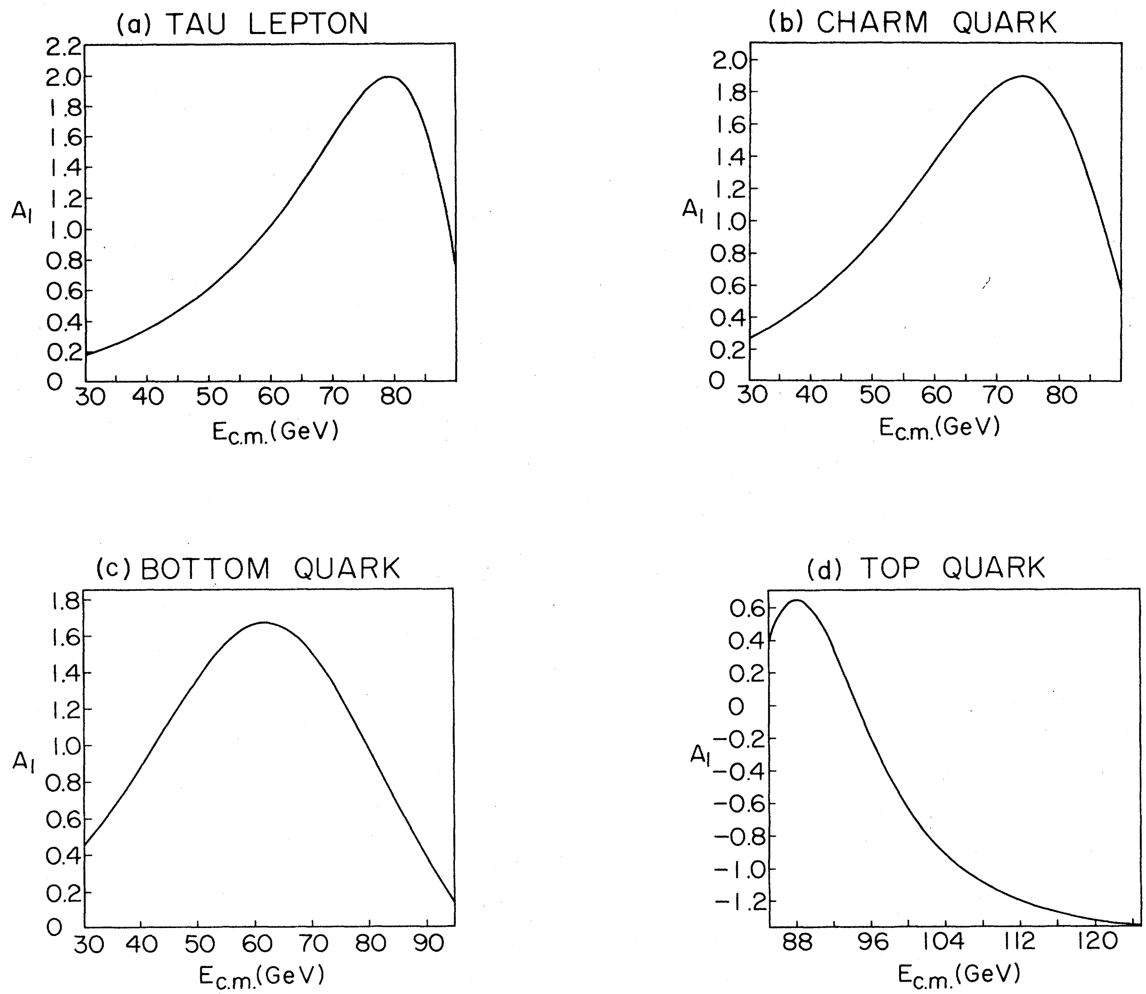

FIG. 3. Predictions of the standard model for the coefficient A_1 of $\cos\theta$ (the interference term between γ and Z^0) in the angular distribution of the process $e^+e^-\to f\bar{f}$. (a)—(d) give the results for τ, c, b, and t pair production, respectively.

whereas, according to our results, the form valid at PEP energies is, approximately (setting $\kappa=0$),

$$\frac{dN}{d\Omega}=1+A_1\cos\theta+0.8\cos^2\Theta \ . \tag{4.3}$$

Depending on the fitting procedure used, this affects the determination of A_1 in various ways. For instance, if just the forward-backward asymmetry is measured, the coefficient A_1 determined from the naive formula (4.2) is about 15% too high. Therefore the use of the correct formula is mandatory in a search for, presumably small, deviations from the standard model. The only exception appears to be the case of t quark production. If indeed the top mass is around 40 GeV, both the threshold factors and the Z^0 propagator are rapidly varying functions in an energy region which will be accessible at the Stanford Linear Collider. Consequently, the presence of even a small anomalous moment will dramatically affect the energy dependence of the interference term.

V. DISCUSSION

The analysis reported here shows that anomalous moments of heavy quarks and leptons may be successfully extracted from a study of angular distributions in electron-positron annihilation.[8] The method proposed in this paper appears to have the advantage that no accurate measurements of absolute magnitudes of differential cross sections are needed. There are alternatives, of course; in particular, Silverman and Shaw were able to extract an upper limit on the anomalous moment of τ from data on the total cross section of τ production.[9] In our opinion, however, it will be difficult to sharpen a method based on an absolute measurement of cross sections to the point of extracting actual values of the magnetic moments, whereas methods independent of absolute normalizations, such as the one proposed here, stand a better chance as event samples increase.

In order to illustrate the method proposed, we analyzed

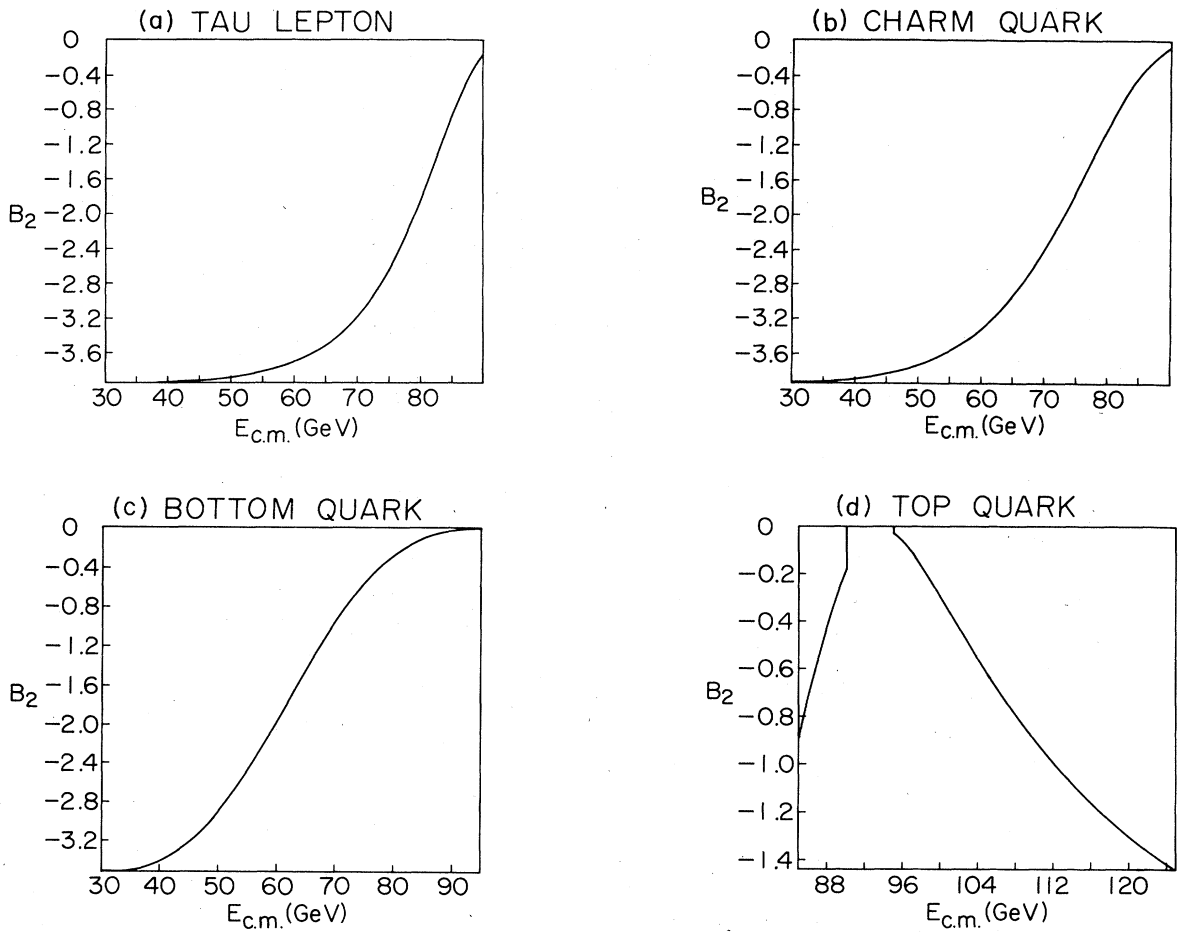

FIG. 4. Corrections to the standard model prediction of the angular distribution in the process $e^+e^- \to f\bar{f}$ due to an anomalous electromagnetic dipole moment of the heavy-fermion pair produced: corrections to the coefficient of $\cos^2\theta$. (a)—(d) display the coefficient B_2 of κ for τ, c, b, and t pair production, respectively [see Eq. (4.1)].

a sample of data on the forward-backward asymmetry of τ pairs and tagged c and b jets. The data were taken from Naroska's review.[10] In view of the very large errors, we analyzed the data in a rather crude way: we neglected any deviation of the coefficient of $\cos^2\theta$ from unity (although as mentioned before, this is quite a dangerous approximation for b quarks) and extracted the value of κ from the forward-backward asymmetry. The measured values of the forward-backward asymmetry do not agree exactly with the prediction of the standard model, hence, not surprisingly, we obtained nonvanishing values for the anomalous moments. (We made no attempt at estimating systematic errors.) Our results obtained at $E_{\text{c.m.}}=29$ GeV and $E_{\text{c.m.}}=34.5$ GeV are compatible with each other and with the upper limit given in Ref. 9 within the errors; they are displayed in Table II. Due to the large statistical (and unknown systematic) errors, all values of κ listed in Table II are compatible with $\kappa=0$. Nevertheless, one can discern a trend in the results. For instance, the

TABLE I. Input parameters for heavy-fermion (f) pair production, $e^+e^- \to f\bar{f}$.

Particle	Mass (GeV)	Width (GeV)
τ	1.784	0
c	1.5	0
b	5.0	0
t	42.0	0
Z^0	93.2	2.0

Weinberg angle, $\sin^2\theta_W=0.234$
Fermi coupling constant, $G_F=1.166\times10^{-5}$ GeV^{-2}

TABLE II. Anomalous magnetic moments of heavy quarks and leptons from data in Ref. 10. Errors are statistical. Anomalous moments of quarks are normalized to "current-algebra masses," cf. Table I.

Particle	Anomalous magnetic moment κ $E_{\text{c.m.}}=29$ GeV	$E_{\text{c.m.}}=34.5$ GeV	Average
τ	0.42 ± 0.36	0.35 ± 0.21	0.39 ± 0.30
c	-1.13 ± 1.53	-0.31 ± 0.38	-0.72 ± 1.1
b	-0.47 ± 0.49	-0.20 ± 0.33	-0.34 ± 0.42

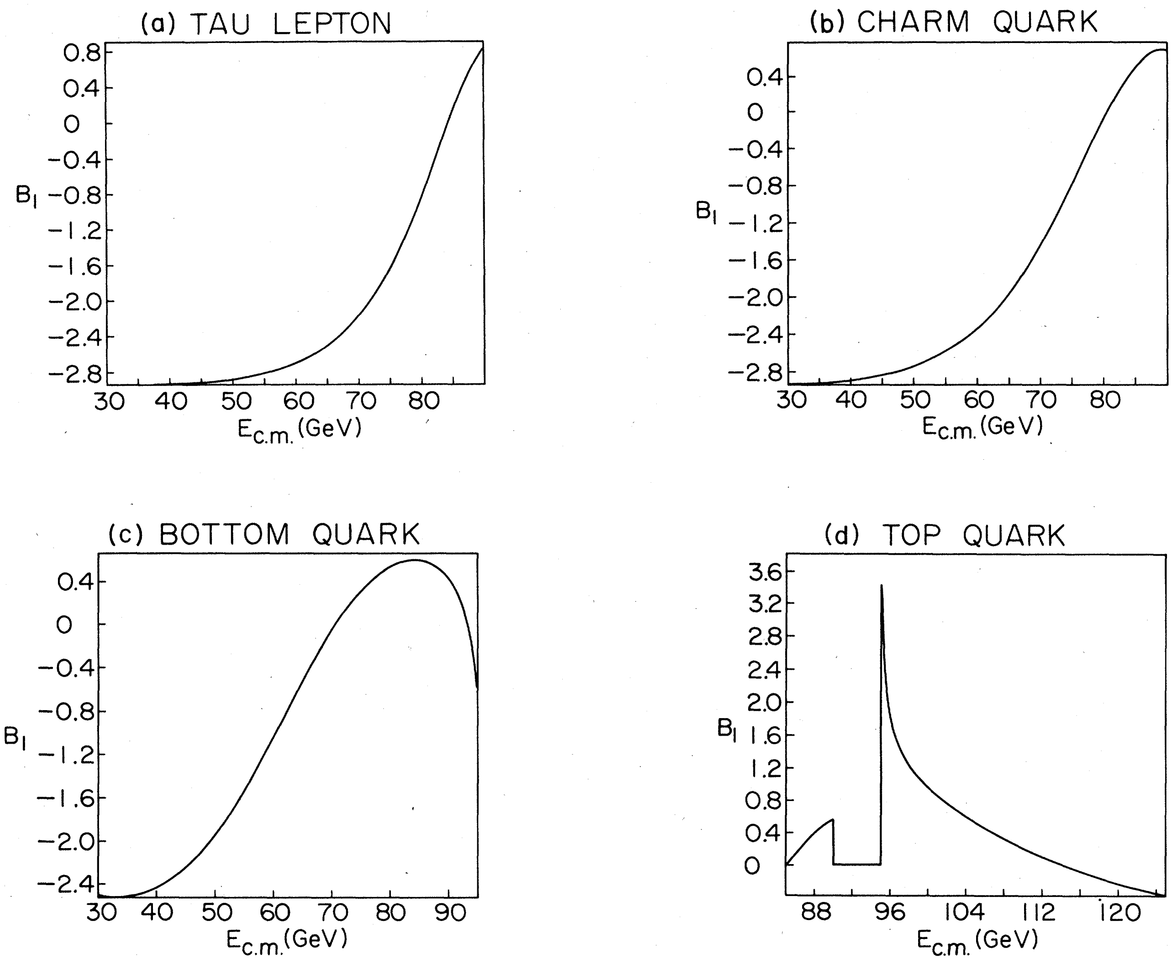

FIG. 5. Corrections to the standard model prediction of the angular distribution in the process $e^+e^-\rightarrow f\bar{f}$ due to an anomalous electromagnetic dipole moment of the heavy-fermion pair produced: corrections to the coefficient of $\cos\theta$. (a)—(d) display the coefficient B_1 of κ for τ, c, b, and t pair production, respectively [see Eq. (4.1)].

anomalous moments deduced from both the PEP and PETRA data are of the same sign; moreover, the numerical values are compatible with each other within about one standard deviation. We cannot at this time exclude the possibility that all deviations of the data from the predictions of the standard model are due to either statistical or to some systematic errors. Should, however, the effect turn out to be real, this may be the first indication of the compositeness of quarks and leptons.

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APPENDIX

The purpose of this appendix is to summarize formulas needed in the analysis of $e\bar{e}$ annihilation data. As discussed in Sec. IV, the effect of anomalous moments at the Z^0 vertices is small; henceforth, only electromagnetic moments with their conventional normalization are retained. In what follows, the mass of the electron is neglected; the mass of the fermion produced is denoted by m. Various factors at the electron and heavy-fermion vertices are distinguished by superscripts e and f, respectively, see Fig. 1. The four-momentum squared of the vector boson (the c.m. energy squared) is denoted by s. The fine-structure constant is denoted by α.

We begin by introducing the normalized couplings at the various vector and axial-vector vertices and a dimensionless quantity proportional to the ratio of the photon and Z^0 propagators.

Define

TABLE III. Sample program designed to generate numerical values of the coefficients in the angular distribution for c quarks.

```
C
C     THIS FORTRAN PROGRAM CALCULATES THE
C     PARAMETERS AS1,AS2,B1,B2 FOR THE CHARM
C     QUARK
C
C
      DIMENSION AS1(900),AS2(900),B1(900),B2(900)
      COMMON I,Q,XM,CVE,CAE,CVF,CAF,AS1,AS2,B1,B2
C
C     SET PHYSICAL PARAMETERS FOR THE QUARK
C
      XM=1.5                    ! MASS
      CAF=0.5                   ! AXIAL COUPLING CONSTANT
      CVF=0.19                  ! VECTOR COUPLING CONSTANT
      Q=2./3.                   ! CHARGE
C
C     COUPLING CONSTANTS FOR THE ELECTRON
C
      CVE=-0.03
      CAE=-0.5
C
C     FINDS AS1,ETC. FOR ENERGIES BETWEEN
C     30.0 and 90.0 GeV IN INCREMENTS OF 0.1
C
C     I is 10 TIMES ENERGY
C
      DO 1 I=300,900,1
      CALL ABB
1     CONTINUE
      STOP
      END
C
C     THE SUBROUTINE CALCULATES THE PARAMETERS AT A GIVEN
C     ENERGY
C
      SUBROUTINE ABB
      DIMENSION AS1(900),AS2(900),B1(900),B2(900)
      COMMON I,Q,XM,CVE,CAE,CVF,CAF,AS1,AS2,B1,B2
      TQL=I
      QL=TQL/10.                !QL IS THE ENERGY
      XSN=SQRT(2.0)*1.2705E-4*93.2**2*QL**2
      SD1=QL**2-93.2**2
      SD2=2.0*93.2
      SD=SD1**2+302**2
```

$$C_V = T_3 - \sin^2\theta_W Q \,,$$
$$C_A = T_3 \tag{A1}$$

(where T_3 and Q stand for the third component of the weak isospin and charge, respectively), and

$$R = \frac{G_F M_Z{}^2\sqrt{2}}{s - M_Z{}^2 + i\Gamma_Z M_Z}\,\frac{s}{4\pi\alpha}\,, \tag{A2}$$

where G_F is the Fermi coupling constant. The differential cross section of the process $e\bar{e}\to f\bar{f}$ in the c.m.s. can be written in the form

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[1-\frac{4m^2}{s}\right](A+\kappa C)(1+\alpha_1\cos\theta + d_2\cos^2\theta)\,. \tag{A3}$$

This expression has to be multiplied by an overall factor of 3 for quarks, in order to take color into account; however, only the coefficients α_1 and α_2 are relevant for the angular distribution (4.1). The expressions of α_1 and α_2 are then linearized in κ, as explained in Sec. IV. We have

TABLE III. (*Continued*).

```
C
C       Re(r) AND MAGNITUDE OF r SQUARED ARE CALCULATED
C
        RER=XSN*SD1/SD
        RSQ=XSN**2/SD
C
C       FORMULAS FOR THE COEFFICIENTS...
C
        A11=4.*XM**2/QL**2+1.0
        A1=Q**2*A11
        A2=2.0*Q*RER*CVE*A11*CVF
        A31=CVE**2+CAE**2
        A32=CVF**2+CAF**2
        A33=CVF**2-CAF**2
        A34=A32/16.0+XM**2*A33/(4.0*QL**2)
        A3=16.0*RSQ*A31*A34
        A=A1+A2+A3
C
        B=4.0*Q**2+4.0*Q*RER*CVE*CVF
C
        C11=1.0-4*XM**2/QL**2
        C1=Q**2*C11
        C2=2.0*Q*RER*C11*CVE*CVF
        C3-RSQ*A31*C11*A32
        C=C1+C2+C3
C
        D=8.0*RSQ*CVE*CAE*SQRT(C11)*CAF*CVF
C
        E=4.0*RSQ*CVE*CAE*SQRT(C11)*CAF*CVF
C
        E=4.0*Q*RER*CAE*SQRT(C11)*CAF
C
        AS1(I)=(D+E)/A
        B1(I)=E/(D+E)-B/A
        AS2(I)=C/A
        B2(I)=B/A
C
        RETURN
        END
```

$$\alpha_1=\frac{A(E+F)+\kappa[AF-C(E+F)]}{A^2}\,,$$

$$\alpha_2=\frac{AD-\kappa CD}{A^2}\,. \tag{A4}$$

From here the coefficients A_1 through B_2 of Eq. (4.1) can be easily extracted. The quantities A through F are given by the expressions

$$\begin{aligned}
A&=Q^2\left[1+\frac{4m^2}{s}\right]+2Q(\mathrm{Re}R)C_V^eC_V^f\left[1+\frac{4m^2}{s}\right]+4\,|R\,|^2(C_V^{e^2}+C_V^{e^2})(C_V^{f^2}+C_A^{e^2})\left[C_V^{f^2}+C_A^{f^2}+\frac{m^2}{s}(C_V^{f^2}-C_A^{f^2})\right],\\
C&=4(Q^2+Q\,\mathrm{Re}RC_V^eC_V^f)\,,\\
D&=\left[1-\frac{4m^2}{s}\right][Q^2+2Q\,\mathrm{Re}RC_V^eC_V^f+|\,R\,|^2(C_V^{e^2}+C_A^{e^2})(C_V^{f^2}+C_A^{f^2})]\,,\\
E&=8\,|\,R\,|^2C_V^eC_A^eC_V^fC_A^f\left[1-\frac{4m^2}{s}\right]^{1/2},\qquad F=4Q\,\mathrm{Re}RC_A^eC_A^f\left[1-\frac{4m^2}{s}\right]^{1/2}.
\end{aligned} \tag{A5}$$

A sample program designed to generate numerical values of the coefficients in the angular distribution is given in Table III. The program described generates the angular distribution for c quarks; other distributions are generated by appropriate changes in the input.

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[1]G. L. Shaw, D. Silverman, and R. Slansky, Phys. Lett. **94B**, 57 (1980).

[2]S. Kovesi-Domokos and G. Domokos, Phys. Lett. **103B**, 229 (1981).

[3]Other experiments, e.g., hadronic production of heavy quarks seem to be too difficult to analyze in view of the uncertainty of our knowlege of the long-distance behavior of QCD; thus, we restrict our attention to the potentially cleanest experiments.

[4]G. Domokos, S. Kovesi-Domokos, and E. Schonberg, Phys. Rev. D **4**, 2115 (1971); S. Kovesi-Domokos and G. Domokos, *ibid.* **24**, 2866 (1981).

[5]Throughout Sec. II we suppress helicity indices carried by plane-wave solutions of the free Dirac equation as well as indices of any internal symmetry. The relative scale of electric and magnetic amplitudes is chosen to be the characteristic energy rather than the inverse Compton wavelength of the target. Such a distinction is rather irrelevant as long as one considers a composite state or a size not very large or not very small on the scale of its own Compton wavelength, e.g., nucleons made up of quarks. If quarks and leptons are composite objects, however, their charge radii are much smaller than their Compton wavelengths [see S. J. Brodsky and S. D. Drell, Phys. Rev. D **22**, 2236 (1980)]. In such cases, our normalization convention allows one to obtain low-q^2 expansions of the various amplitudes in a straightforward way, without the necessity of paying a great deal of attention to cancellations between quantities of comparable magnitude and opposite signs. Any observable result can be, of course, expressed in a manner independent of the normalization conditions adopted.

[6]F. Halzen and A. D. Martin, *Quarks and Leptons* (Wiley, New York, 1984), Chap. 9.

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[8]A preliminary account of our findings was reported at the Irvine Conference on Composite Models in Particle Physics, 1984 (unpublished).

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