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Note on *CP* nonconservation in $K \to \pi\pi$

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A semiphenomenological upper bound on the value $\phi_{00}-\phi_{+-}$ is derived and discussed.

On the basis of unitarity arguments it is known that the phases ϕ_{+-} and ϕ_{00} of ratios η_{+-} and η_{00} should not be too different. It is also known that, e.g., Wolfenstein's model[1] predicts these phases to be the same. If the experimental values[2] $\phi_{+-}=(44.5\pm1.5)^\circ$ and $\phi_{00}=(54.1\pm6.2)^\circ$ are taken too literally, one could argue that the present data are not in favor of the superweak theory. What is the situation in the standard model with the Kobayashi-Maskawa (KM) mixing matrix? How large a phase difference can this model tolerate? While the basic formalism which allows an answer to this question was introduced more than 20 years ago,[4] most often it was used in the analyses of absolute values, e.g., $|\eta_{00}/\eta_{+-}|$, and not phase angles. In this note a model-independent relation that gives an upper bound on $\phi_{00}-\phi_{+-}$ is derived. It is hoped that with greater precision in new experiments phase angles might provide some additional insight into *CP* nonconservation.

Following the notation of Ref. 5, one can write the probability amplitudes in $K\to\pi\pi$ decays as

$$\begin{aligned}
\mathscr{A}(K^{\pm}) &= \pm(\tfrac{3}{2})^{1/2}\,\mathrm{Re}a_2 e^{i\delta_2}\ ,\\
\mathscr{A}(K^L_{00}) &= 2N_{\tilde\epsilon}[\,(\tfrac{1}{3})^{1/2}(\tilde\epsilon\,\mathrm{Re}a_0+i\,\mathrm{Im}a_0)e^{i\delta_0}\\
&\quad -(\tfrac{2}{3})^{1/2}(\tilde\epsilon\,\mathrm{Re}a_2)e^{i\delta_2}]\ ,\\
\mathscr{A}(K^S_{00}) &= 2N_{\tilde\epsilon}[\,(\tfrac{1}{3})^{1/2}(\mathrm{Re}a_0+i\tilde\epsilon\,\mathrm{Im}a_0)e^{i\delta_0}\\
&\quad -(\tfrac{2}{3})^{1/2}(\mathrm{Re}a_2)e^{i\delta_2}]\ ,\\
\mathscr{A}(K^L_{+-}) &= 2N_{\tilde\epsilon}[\,(\tfrac{2}{3})^{1/2}(\tilde\epsilon\,\mathrm{Re}a_0+i\,\mathrm{Im}a_0)e^{i\delta_0}\\
&\quad +(\tfrac{1}{3})^{1/2}(\tilde\epsilon\,\mathrm{Re}a_2)e^{i\delta_2}]\ ,\\
\mathscr{A}(K^S_{+-}) &= 2N_{\tilde\epsilon}[\,(\tfrac{2}{3})^{1/2}(\mathrm{Re}a_0+i\tilde\epsilon\,\mathrm{Im}a_0)e^{i\delta_0}\\
&\quad +(\tfrac{1}{3})^{1/2}(\mathrm{Re}a_2)e^{i\delta_2}]\ .
\end{aligned}\tag{1}$$

(I am using the convention in which $\mathrm{Im}a_2=0$.) The parameter $\tilde\epsilon$ describes $K^0\bar K^0$ mixing, $N_{\tilde\epsilon}=2(1+|\tilde\epsilon|^2)^{-1/2}$, and a_0 (a_2) are amplitudes for $I=0$ ($I=2$) final states.

From experimental values,[3] one easily finds

$$1 >> \frac{\mathrm{Re}a_2}{\mathrm{Re}a_0} >> \left(\frac{\mathrm{Re}a_2}{\mathrm{Re}a_0}\right)^2,\ |\tilde\epsilon|,\left|\frac{\mathrm{Im}a_0}{\mathrm{Re}a_0}\right|\ .\tag{2}$$

Note that this relation does not depend on the precise values of measured decay rates but rather on their order of magnitude. By defining[5]

$$\epsilon=\frac{\tilde\epsilon\,\mathrm{Re}a_0+i\,\mathrm{Im}a_0}{\mathrm{Re}a_0+i\tilde\epsilon\,\mathrm{Im}a_0},\quad \omega=\frac{\mathrm{Re}a_2}{\mathrm{Re}a_0+i\tilde\epsilon\,\mathrm{Im}a_0}\ ,\tag{3}$$

one can reexpress the relevant ratios $\eta=\mathscr{A}(K^L)/\mathscr{A}(K^S)$ as

$$\eta_{00}=\frac{\epsilon-\sqrt2\tilde\epsilon\omega e^{i\Delta}}{1-\sqrt2\omega e^{i\Delta}},\quad \eta_{+-}=\frac{\sqrt2\epsilon+\tilde\epsilon\omega e^{i\Delta}}{\sqrt2+\omega e^{i\Delta}}\ .\tag{4}$$

In (4), $\Delta=\delta_2-\delta_0$. If the small real parameters in (2) are denoted as

$$\sqrt2\rho=\mathrm{Re}a_2/\mathrm{Re}a_0,\quad \tau=\mathrm{Im}a_0/\mathrm{Re}a_0\ ,\tag{5}$$

then to a good approximation the quantity ϵ is characterized by

$$|\epsilon|\approx[(\mathrm{Re}\tilde\epsilon)^2+(\mathrm{Im}\tilde\epsilon+\tau)^2]^{1/2},\quad \tan\Phi_\epsilon\approx\frac{\mathrm{Im}\tilde\epsilon+\tau}{\mathrm{Re}\tilde\epsilon}\ .\tag{6}$$

(Note also that $\rho\tau\approx|\epsilon'|$.) Constraints (2) simplify now the equations for the absolute values and phases ϕ_{00} and ϕ_{+-} of ratios η in (4). With some algebra, neglecting double-suppressed terms, one obtains the expressions

$$\begin{aligned}
|\eta_{+-}| &= |\epsilon|\left[1+\frac{\rho\tau}{|\epsilon|}\sin(\Delta-\Phi_\epsilon)\right]\ ,\\
|\eta_{00}| &= |\epsilon|\left[1-2\frac{\rho\tau}{|\epsilon|}\sin(\Delta-\Phi_\epsilon)\right]\ ,\\
\tan\phi_{+-} &= \tan\Phi_\epsilon\left[1-\frac{\rho\tau}{|\epsilon|\cos\Phi_\epsilon\sin\Phi_\epsilon}\cos(\Delta-\Phi_\epsilon)\right]\ ,\\
\tan\phi_{00} &= \tan\Phi_\epsilon\left[1+2\frac{\rho\tau}{|\epsilon|\cos\Phi_\epsilon\sin\Phi_\epsilon}\cos(\Delta-\Phi_\epsilon)\right]\ .
\end{aligned}\tag{7}$$

The consequence of (7) is

$$\begin{aligned}
\tfrac13\tan(\phi_{00}-\phi_{+-}) &= \frac{\rho\tau}{|\epsilon|}\cos(\Delta-\Phi_\epsilon)\ ,\\
\tfrac16(1-|\eta_{00}/\eta_{+-}|^2) &= \frac{\rho\tau}{|\epsilon|}\sin(\Delta-\Phi_\epsilon)\ .
\end{aligned}\tag{8}$$

[In (8), the terms of the order ρ^2 were neglected.] Since the left-hand side of the second equation in (8) is simply $\mathrm{Re}(\epsilon'/\epsilon)$, one finally obtains[6]

$$\mathrm{Re}\frac{\epsilon'}{\epsilon}=\frac13\tan(\phi_{00}-\phi_{+-})\tan(\Delta-\Phi_\epsilon)\ .\tag{9}$$

It is expression (9) which gives the upper bound on the value of the phase difference $\phi_{00}-\phi_{+-}$. Indeed, by combining the values $\Delta=(-41.4\pm8.1)^\circ$ (from Ref. 7) and $\Phi_\epsilon=(43.7\pm0.2)^\circ$ (see, e.g., Ref. 8) with the recent result,[9] $|\mathrm{Re}(\epsilon'/\epsilon)|\lesssim0.010$, one finds

$$|\tan(\phi_{00}-\phi_{+-})|\leqslant0.007,\quad |\phi_{00}-\phi_{+-}|\leqslant0.4^\circ\ .\tag{10}$$

The smaller the value for $\mathrm{Re}(\epsilon'/\epsilon)$, the more stringent bounds on $\phi_{00}-\phi_{+-}$ are obtained. For example, the values $\mathrm{Re}(\epsilon'/\epsilon)=-0.0046$ and $\Delta=-41.4°$, give $\phi_{00}-\phi_{+-}\approx 0.07°$. Note that only with $\Delta\approx\Phi_\epsilon$ (mod π), which seems to be excluded by experiments, can one have both small $\mathrm{Re}(\epsilon'/\epsilon)$ and relatively large $\phi_{00}-\phi_{+-}$.

On the basis of the above analysis it is clear that with the amplitudes as given by expression (1), one can hardly accommodate the phase difference much bigger than a fraction of a degree.[10] In other words, the currently measured value[2] ($\phi_{00}-\phi_{+-}=12.6°\pm 6.2°$) with a range of one standard deviation disfavors the superweak and, e.g., the standard model in the same way. It is also true that within *two* standard deviations one can still have $\phi_{00}\approx\phi_{+-}$, and therefore the apparent inconsistency is not too disturbing at present.

The measurement of ϕ_{00} is one of the most difficult experimental tasks. Still, it is possible that in one of the future experiments[8] more precise results are obtained, and the inconsistency resolved. If, however, the phase difference $\phi_{00}-\phi_{+-}$ repeatedly comes out to be larger than allowed by (10), a clear signal for a new phenomenon (e.g., the violation of *CPT* symmetry[11]) will be in our hands.

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[2]J. H. Christenson *et al.*, Phys. Rev. Lett. **43**, 1209 (1979); **43**, 1212 (1979); for a world average, see also Ref. 3.

[3]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[4]T. T. Wu and C. N. Yang, Phys. Rev. Lett. **13**, 380 (1964); see also L. Wolfenstein, Nuovo Cimento **42A**, 17 (1966); M. Gourdin, Nucl. Phys. **B3**, 207 (1967).

[5]L.-L. Chau, Phys. Rep. **95**, 1 (1983). This work contains an extensive list of references on the subject.

[6]The reader can easily find several different ways, some even simpler than the one described here, leading to Eq. (9). Similarly, one can show that, e.g., $\mathrm{Im}\epsilon'/\epsilon=-\tan(\phi_{00}-\phi_{+-})/3$.

[7]The weighted average for $\delta_2-\delta_0$ was taken from T. J. Devlin and J. O. Dickey, Rev. Mod. Phys. **51**, 237 (1979).

[8]B. Winstein, in *Proceedings of the 11th International Conference on Neutrino Physics and Astrophysics, Dortmund, 1984,* edited by K. Kleinknecht and E. A. Paschos (World Scientific, Singapore, 1985), p. 627; in this reference a brief review of future experiments on direct *CP* nonconservation in the *K* system can be found.

[9]J. K. Black *et al.*, Phys. Rev. Lett. **54**, 1628 (1985); R. H. Bernstein *et al.*, *ibid.* **54**, 1631 (1985).

[10]Note that relation (9) is obtained assuming that pions form an exact isospin triplet, and also neglecting all radiative corrections. It is very difficult to take isospin breaking and radiative corrections into account. Although in principle they can play an important role at this level, it is believed (see, e.g., Ref. 4) that ratios and differences of relevant quantities remain mostly unaffected.

[11]See, e.g., V. V. Barmin *et al.*, Nucl. Phys. **B247**, 293 (1984).