Supersymmetric composite models with preons in two hypercolor representations

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Supersymmetric composite models with preons in either of two hypercolor representations R_1 and R_2 such that $R_1 \times R_2 \times R_2 \supset 1$ are studied. Chirality preservation is investigated by using the 't Hooft anomaly-matching equations and the decoupling theorem. No solutions to these equations are found if R symmetry is assumed to survive supersymmetry breaking. Solutions exist if R symmetry is assumed to break, and the embedding of $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ in the chiral-symmetry group is considered.

I. INTRODUCTION

One of the fundamental problems encountered in composite models of quarks and leptons is the near masslessness of these particles when viewed from the hypercolor binding scale. This has led to consideration of how massless composite spin-$\frac{1}{2}$ particles can arise in a confining gauge theory (the relatively small quark and lepton masses then arising from perturbations). Three ways of achieving the above have been proposed. The first proposal is that the masslessness of composite spin-$\frac{1}{2}$ states arises because a chiral symmetry due to massless preons survives the binding process.[1] In Ref. 1 't Hooft provided a set of necessary conditions for chirality to be preserved: flavor anomaly matching and the decoupling of massive preons. If these conditions are *not* met then chiral symmetry is dynamically broken. If they *are* met then chirality may survive, though this is not assured. The other two proposals require a supersymmetric preon theory. One scheme postulates that the quarks and leptons are the Goldstone particles of broken supersymmetries.[2] The other suggests that quarks and leptons may be the supersymmetric partners of the (quasi-) Goldstone bosons arising from a dynamically broken flavor symmetry.[3] Even though the 't Hooft chirality protection mechanism does not necessarily rely on supersymmetry, there are arguments which suggest that supersymmetry may be efficient at preserving chirality.[4] Also since supersymmetry introduces spin-0 preons, more freedom can exist in solving the 't Hooft consistency conditions, both with the possibility of spin-$\frac{1}{2}$ two-body composites and with an increase in the number of preon configurations associated with a given set of flavor quantum numbers at the composite level.[5]

In this paper we will study the chirality protection mechanism via the 't Hooft consistency conditions, for supersymmetric composite models where the preons are in either of two hypercolor representations R_1 and R_2 such that $R_1 \times R_2 \times R_2 \supset 1$. This hypercolor hypothesis is of interest because of some results due to Bars[6] for nonsupersymmetric versions of these models. Bars showed that it is possible to obtain a generation structure for quarks and leptons within these schemes, while also ridding the low-energy spectrum of exotic $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ representations. These models also allow some flexibility; for example, various horizontal and technicolor symmetries may be introduced. Given these useful properties it is of interest to examine the supersymmetric versions of these theories, bearing in mind the discussion of the preceding paragraph.

Consider a supersymmetric gauge theory with two types of massless chiral superfields, one type in representation R_1 of the confining gauge group, the other in representation R_2. If R_1 and R_2 are complex then at the classical level we have the following chiral or flavor symmetry:

$$G_F^{(\mathrm{clas})} = \mathrm{SU}(N_1)_L \times \mathrm{SU}(M_1)_R \times \mathrm{SU}(N_2)_L \times \mathrm{SU}(M_2)_R \times \mathrm{U}(1)_{V_1} \times \mathrm{U}(1)_{A_1} \times \mathrm{U}(1)_{V_2} \times \mathrm{U}(1)_{A_2} \times \mathrm{U}_R(1)\,. \tag{1}$$

Here N_1 (N_2) and M_1 (M_2) are the number of left- and right-handed chiral superfields in R_1 (R_2), respectively. V_1 (V_2) and A_1 (A_2) refer to the vector and axial-vector number operators, respectively, for preons in R_1 (R_2). $\mathrm{U}_R(1)$ is the R symmetry of supersymmetry. In the quantum theory hypercolor instantons break $\mathrm{U}(1)_{A_1}$, $\mathrm{U}(1)_{A_2}$, and $\mathrm{U}_R(1)$ down to $\mathrm{U}(1)_{A'} \times \mathrm{U}_{R'}(1)$ where A' and $Q_{R'}$ are hypercolor anomaly-free linear combinations of A_1, A_2, and Q_R. The chiral symmetry G_F is then

$$G_F = \mathrm{SU}(N_1)_L \times \mathrm{SU}(M_1)_R \times \mathrm{SU}(N_2)_L \times \mathrm{SU}(M_2)_R \times \mathrm{U}(1)_{V_1} \times \mathrm{U}(1)_{V_2} \times \mathrm{U}(1)_{A'} \times \mathrm{U}_{R'}(1)\,. \tag{2}$$

If R_1 is real then the type-1 right-handed preons correspond to charge conjugates of type-2 left-handed preons and the chiral symmetry G_F' is

$$G_F' = \mathrm{SU}(N_1)_L \times \mathrm{SU}(N_2)_L \times \mathrm{SU}(M_2)_R \times \mathrm{U}(1)_{V_2} \times \mathrm{U}(1)_{A'} \times \mathrm{U}_{R'}(1)\,. \tag{3}$$

The structure of the chiral-symmetry group should now be clear for the cases where R_2 is real and where both are real.

Phenomenological considerations require that theories with broken supersymmetry be studied. In the context of the chirality protection mechanism we then need to know how broken supersymmetry alters (if at all) the chiral-symmetry group. A detailed investigation of this issue is beyond the scope of this paper. We shall assume that one of two things occur: either (i) chirality is unaffected or (ii) R symmetry does not survive the breakdown of supersymmetry.[5] We shall discover, as the authors of Ref. 5 found for other models, that the 't Hooft conditions are difficult to satisfy if R symmetry survives.

II. EXACT R SYMMETRY

Hypercolor instantons induce processes which break the Abelian charges in G_F in the following way:

$$\Delta n_{1L}=N_1 q(R_1)\nu_{\rm HC},\quad \Delta n_{1R}=-M_1 q(R_1)\nu_{\rm HC}\,,$$
$$\Delta n_{2L}=N_2 q(R_2)\nu_{\rm HC},\quad \Delta n_{2R}=-M_2 q(R_2)\nu_{\rm HC}\,, \tag{4}$$
$$\Delta Q_R=[q({\rm adj})-(N_1+M_1)q(R_1)-(N_2+M_2)q(R_2)]\nu_{\rm HC}\,.$$

Here the n_{iH} are number operators for preon superfields of type i ($i=1,2$) and chirality $H(=L,R)$. $\nu_{\rm HC}$ is the winding number of the hypercolor instanton. $q(R)$ is the quadratic anomaly of the representation R; that is,

$$\mathrm{Tr}[T_a(R)T_b(R)]=q(R)\delta_{ab}\,,$$

where $T_a(R)$ is the matrix representing the group generator labeled by a. It is normalized so that q (fundamental representation) $=1$. Equations (4) imply that the following charges are conserved:

$$V_1\equiv\frac{n_{1L}}{N_1}+\frac{n_{1R}}{M_1},\quad V_2\equiv\frac{n_{2L}}{N_2}+\frac{n_{2R}}{M_2}\,,$$
$$A'\equiv(N_1+M_1)q(R_1)(n_{2L}-n_{2R})-(N_2+M_2)q(R_2)(n_{1L}-n_{1R})\,, \tag{5}$$
$$Q_{R'}\equiv-(N_1+M_1)q(R_1)Q_R+[q({\rm adj})-(N_1+M_1)q(R_1)-(N_2+M_2)q(R_2)](n_{1L}-n_{1R})\,.$$

We first consider the cases where R_1 and R_2 are complex. The chiral properties of the preons are displayed in Table I. The composite state chiral properties are shown in Table II. Note that hypercolor-singlet spin-$\frac{1}{2}$ states occur in the products $R_1\times\bar{R}_1$ and $R_2\times\bar{R}_2$ as well as in $R_1\times R_2\times R_2$. These two-body spin-$\frac{1}{2}$ states are a feature of the supersymmetric theory that does not appear in the nonsupersymmetric models of Bars.[6] Note also the presence of the superhyperglueball $\bar{\lambda}A_\mu$.

The decoupling of composite states that contain a preon that may, conceptually, be made massive must now be ensured, as required by the theorem of Appelquist and Carazzone.[7] This implies relations among the 't Hooft indices as follows:[1]

$$\sum_{r\subset R} l(R)=0\,, \tag{6}$$

where r is a representation of the residual chiral-symmetry group after a preon superfield acquires a mass. Of course, the relations obtained may vary with the particular model under consideration. For example, if $N_1=1$ the $\mathrm{SU}(N_1)_L$ is no longer a nontrivial symmetry and so Eq. (6) *may* give different relations to when $N_1\geq 2$. If $N_1=0$ then $\mathrm{SU}(N_1)_L$ disappears along with the indices of states which contain a left-handed type-1 preon. So when the chiral-symmetry group is altered Eq. (6) must be examined afresh. Note that decoupling must be ensured both when a type-1 superfield gains a mass and when a type-2 superfield gains a mass. Of course if a particular type of preon only occurs with one chirality then a hypercolor-preserving mass term is impossible to construct owing to the complex nature of the representations and decoupling with respect to this preon becomes meaningless (e.g., if $N_1=0$ then type-1 preons remain massless). If both types of preons can become massive then the residual chiral symmetry $G_F^{(\rm res)}$ is

TABLE I. Flavor properties of preons when R symmetry survives supersymmetry breaking. Subscripts 1 and 2 refer to preons in hypercolor representations R_1 and R_2, respectively. Subscripts L and R refer to left-handed and right-handed preons, respectively. ψ denotes spin-$\frac{1}{2}$ preons and A denotes spin-0 preons. A (ψ,A) pair sharing the same subscripts are components of a single chiral superfield. λ labels the hypercolor gaugino while A_μ labels the hypercolor gauge boson.

Preon	$\mathrm{SU}(N_1)_L$	$\mathrm{SU}(M_1)_R$	$\mathrm{SU}(N_2)_L$	$\mathrm{SU}(M_2)_R$	V_1	V_2	A'	Q_R
ψ_{1L}	$\square$	1	1	1	$\frac{1}{N_1}$	0	$-(N_2+M_2)q(R_2)$	$q(\mathrm{adj})-(N_2+M_2)q(R_2)$
A_{1L}	$\square$	1	1	1	$\frac{1}{N_1}$	0	$-(N_2+M_2)q(R_2)$	$q(\mathrm{adj})-(N_1+M_1)q(R_1)-(N_2+M_2)q(R_2)$
ψ_{1R}	1	$\square$	1	1	$\frac{1}{M_1}$	0	$(N_2+M_2)q(R_2)$	$-q(\mathrm{adj})+(N_2+M_2)q(R_2)$
A_{1R}	1	$\square$	1	1	$\frac{1}{M_1}$	0	$(N_2+M_2)q(R_2)$	$-q(\mathrm{adj})+(N_1+M_1)q(R_1)+(N_2+M_2)q(R_2)$
ψ_{2L}	1	1	$\square$	1	0	$\frac{1}{N_2}$	$(N_1+M_1)q(R_1)$	$(N_1+M_1)q(R_1)$
A_{2L}	1	1	$\square$	1	0	$\frac{1}{N_2}$	$(N_1+M_1)q(R_1)$	0
ψ_{2R}	1	1	1	$\square$	0	$\frac{1}{M_2}$	$-(N_1+M_1)q(R_1)$	$-(N_1+M_1)q(R_1)$
A_{2R}	1	1	1	$\square$	0	$\frac{1}{M_2}$	$-(N_1+M_1)q(R_1)$	0
λ	1	1	1	1	0	0	0	$-(N_1+M_1)q(R_1)$
A_μ	1	1	1	1	0	0	0	0

TABLE II. Flavor properties and 't Hooft indices of composite states when R symmetry survives supersymmetry breaking. Here $q \equiv q(\text{adj})$, $q_1 \equiv (N_1+M_1)q(R_1)$, and $q_2 \equiv (N_2+M_2)q(R_2)$.

Composites	$SU(N_1)_L$	$SU(M_1)_R$	$SU(N_2)_L$	$SU(M_2)_R$	V_1	V_2	Q_R	A'	Index
$\psi_{1L}\psi_{2L}\psi_{2L}$	$\square$	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	1	$\frac{1}{N_1}$	$\frac{2}{N_2}$	$q+2q_1-q_2$	$2q_1-q_2$	l_{1+}, l_-
$\psi_{1L}A_{2L}A_{2L}$, $A_{1L}\psi_{2L}A_{2L}$	$\square$	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	1	$\frac{1}{N_1}$	$\frac{2}{N_2}$	$q-q_2$	$2q_1-q_2$	l_{2+}, l_{2-}
$\psi_{1L}\psi_{2R}\psi_{2R}$, $A_{1L}\psi_{2R}A_{2R}$	$\square$	1	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	$\frac{1}{N_1}$	$\frac{2}{M_2}$	$q-2q_1-q_2$	$-2q_1-q_2$	l_{3+}, l_{3-}
$\psi_{1L}A_{2R}A_{2R}$	$\square$	1	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	$\frac{1}{N_1}$	$\frac{2}{M_2}$	$q-q_2$	$-2q_1-q_2$	l_{4+}, l_{4-}
$A_{1L}\psi_{2L}A_{2R}$, $\psi_{1L}\psi_{2L}\psi_{2R}$, $\psi_{1L}A_{2L}A_{2R}$	$\square$	1	$\square$	$\square$	$\frac{1}{N_1}$	$\frac{1}{N_2}+\frac{1}{M_2}$	$q-q_2$	$-q_2$	l_5
$A_{1L}A_{2L}\psi_{2R}$	$\square$	1	$\square$	$\square$	$\frac{1}{N_1}$	$\frac{1}{N_2}+\frac{1}{M_2}$	$q-2q_1-q_2$	$-q_2$	l_6
$\psi_{1R}\psi_{2R}\psi_{2R}$	1	$\square$	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	$\frac{1}{M_1}$	$\frac{2}{M_2}$	$-q-2q_1+q_2$	$-2q_1+q_2$	l'_{1+}, l'_{1-}
$\psi_{1R}A_{2R}A_{2R}$, $A_{1R}\psi_{2R}A_{2R}$	1	$\square$	1	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	$\frac{1}{M_1}$	$\frac{2}{M_2}$	$-q+q_2$	$-2q_1+q_2$	l'_{2+}, l'_{2-}
$\psi_{1R}\psi_{2L}\psi_{2L}$, $A_{1R}\psi_{2L}A_{2L}$	1	$\square$	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	1	$\frac{1}{M_1}$	$\frac{2}{N_2}$	$-q+2q_1+q_2$	$2q_1+q_2$	l'_{3+}, l'_{3-}
$\psi_{1R}A_{2L}A_{2L}$	1	$\square$	$\square\square$, $\genfrac{}{}{0pt}{}{\square}{\square}$	1	$\frac{1}{M_1}$	$\frac{2}{N_2}$	$-q+q_2$	$2q_1+q_2$	l'_{4+}, l'_{4-}
$\psi_{1R}\psi_{2L}\psi_{2R}$, $\psi_{1R}A_{2L}A_{2R}$, $A_{1R}A_{2L}\psi_{2R}$	1	$\square$	$\square$	$\square$	$\frac{1}{M_1}$	$\frac{1}{N_2}+\frac{1}{M_2}$	$-q+q_2$	q_2	l'_5
$A_{1R}\psi_{2L}A_{2R}$	1	$\square$	$\square$	$\square$	$\frac{1}{M_1}$	$\frac{1}{N_2}+\frac{1}{M_2}$	$-q+2q_1+q_2$	q_2	l'_6
$\psi_{1L}\bar{A}_{1L}$	adj,1	1	1	1	0	0	q_1	0	a,a'
$\psi_{1L}\bar{A}_{1R}$	$\square$	$\square^*$	1	1	$\frac{1}{N_1}-\frac{1}{M_1}$	0	$2q-q_1-2q_2$	$-2q_2$	b
$\psi_{1R}\bar{A}_{1L}$	$\square^*$	$\square$	1	1	$-\frac{1}{N_1}+\frac{1}{M_1}$	0	$-2q+q_1+2q_2$	$2q_2$	c
$\psi_{1R}\bar{A}_{1R}$	1	adj,1	1	1	0	0	$-q_1$	0	d,d'
$\psi_{2L}\bar{A}_{2L}$	1	1	adj,1	1	0	0	q_1	0	e,e'
$\psi_{2L}\bar{A}_{2R}$	1	1	$\square$	$\square^*$	0	$\frac{1}{N_2}-\frac{1}{M_2}$	q_1	$2q_1$	f
$\psi_{2R}\bar{A}_{2L}$	1	1	$\square^*$	$\square$	0	$-\frac{1}{N_2}+\frac{1}{M_2}$	$-q_1$	$-2q_1$	g
$\psi_{2R}\bar{A}_{2R}$	1	1	1	adj,1	0	0	$-q_1$	0	h,h'
$\bar{\lambda}A_\mu$	1	1	1	1	0	0	q_1	0	x

$$G_F^{(\text{res})}=SU(N_1-1)_L\times SU(M_1-1)_R\times SU(N_2-1)_L\times SU(M_2-1)_R$$
$$\times U(1)_{V_1'}\times U(1)_{V_2'}\times U(1)_{A''}\times U_{R''}(1)\times U(1)_{F_1}\times U(1)_{F_2}\,, \tag{7}$$

where

$$V'_1 \equiv \frac{n'_{1L}}{N_1-1}+\frac{n'_{2R}}{M_1-1}, \quad V'_2 \equiv \frac{n'_{2L}}{N_2-1}+\frac{n'_{2L}}{M_2-1},$$
$$A'' \equiv (N_1+M_1-2)q(R_1)(n'_{2L}-n'_{2R})-(N_2+M_2-2)q(R_2)(n'_{1L}-n'_{1R}), \tag{8}$$
$$Q''_R \equiv -(N_1+M_1-2)q(R_1)Q_R+[q(\mathrm{adj})-(N_1+M_1-2)q(R_1)-(N_2+M_2-2)q(R_2)](n'_{1L}-n'_{1R}).$$

The n' are the number operators for the preon superfields that remain massless. F_1 and F_2 are the vector number operators for the massive preons.

As well as ensuring the decoupling of massive preons we must impose anomaly matching between the preons and candidate massless composite states:

$$\sum_{\substack{\text{preons}\\ \text{in } R}} A(R) = \sum_{\substack{\text{massless}\\ \text{composites}\\ \text{in } R'}} A(R'), \tag{9}$$

where $A(R)$ is the cubic anomaly of the representation R.

We now examine the various cases that exist within this framework.

A. $N_1,M_1,N_2,M_2 \geq 2$. Decoupling implies that

$$\begin{aligned} &a=b=c=d=e=f=g=h=0,\\ &l_{2\pm}=l'_{2\pm}=l_{4\pm}=l'_{4\pm}=l_6=l'_6=0,\\ &a'+e'=d'+h',\\ &l_{1-}=l_{3+}=-l'_{1+}=-l'_{3-}\equiv l,\\ &l_{1+}=l_{3-}=-l'_{1-}=-l'_{3+}\equiv l',\\ &l_5=-l'_5=-(l+l'),\\ &x \text{ unconstrained}. \end{aligned} \tag{10}$$

The anomaly-matching equations in this case lead to the result that $l=l'=0$, so that no interesting solutions are obtained. To see this only the following anomaly-matching equations are needed:

$$[\mathrm{U}_{R'}(1)]^2\mathrm{U}(1)_{V_2}:\ 0=(N_1-M_1)(N_2+M_2)q_1(l+l')+(N_1+M_1)[(N_2-M_2+2)l'+(N_2-M_2-2)l](q-q_2), \tag{11a}$$
$$[\mathrm{U}(1)_{A'}]^2\mathrm{U}(1)_{V_2}:\ 0=(N_1-M_1)(N_2+M_2)q_1(l+l')-(N_1+M_1)[(N_2-M_2+2)l'+(N_2-M_2-2)l]q_2. \tag{11b}$$

Here $q_1 \equiv (N_1+M_1)q(R_1)$, $q_2 \equiv (N_2+M_2)q(R_2)$, and $q \equiv q(\mathrm{adj})$. The three groups that are listed to the left of each equation refer to the corresponding axial-vector flavor currents which have a triangle anomaly associated with them. Now Eqs. (11a) and (11b) immediately imply that

$$(N_2-M_2+2)l'+(N_2-M_2-2)l=0. \tag{11c}$$

Using this result in either Eq. (11a) or (11b) implies that either $N_1=M_1$ or $l+l'=0$ or both. But the following,

$$[\mathrm{SU}(N_2)_L]^2\mathrm{U}(1)_{V_2}:\ d(R_2)=(N_1-M_1)[(N_2-M_2+4)l'+(N_2-M_2-4)l] \tag{11d}$$

shows that $N_1 \neq M_1$, where $d(R_2)$ is the dimension of R_2. Hence $l+l'=0$. But now Eq. (11c) gives that $l'-l=0$. Thus $l=l'=0$.

B. $N_1=M_1=1$; $N_2,M_2 \geq 2$. Decoupling gives the same relations as in Eq. (10). Thus Eqs. (11a) and (11b) immediately imply that there is no solution.

C. $N_1,M_1 \geq 2$; $N_2=M_2=1$. Decoupling implies that the only nonzero indices are a', e', d', h', and x so that the solutions are uninteresting.

So, thus far it has been shown that there are no useful solutions when $N_1,M_1,N_2,M_2>0$. We now consider the cases where one or two of these numbers are zero.

D. $M_1=0$; $N_1,N_2,M_2>0$. In this case the following indices do not exist: b, c, d, d', and all the l' indices. Decoupling requires that

$$\begin{aligned} &l_{2\pm}=l_{4\pm}=l_6=e=f=g=h=0,\\ &l_{1+}=l_{3-}\equiv l',\quad l_{1-}=l_{3+}\equiv l,\\ &l_5=-(l+l'),\quad a,x \text{ unconstrained}. \end{aligned} \tag{12}$$

The relevant anomaly-matching equations are

$$[\mathrm{U}_{R'}(1)]^2\mathrm{U}(1)_{V_2}:\ 0=(N_2+M_2)q_1(l+l')+[(N_2-M_2+2)l'+(N_2-M_2-2)l](q-q_2), \tag{13a}$$
$$[\mathrm{U}(1)_{A'}]^2\mathrm{U}(1)_{V_2}:\ 0=(N_2+M_2)q_1(l+l')-[(N_2-M_2+2)l'+(N_2-M_2-2)l]q_2. \tag{13b}$$

These equations imply that

$$(N_2-M_2+2)l'+(N_2-M_2-2)l=0. \tag{14}$$

Thus from Eqs. (13) we see that $l+l'=0$. But Eq. (14) now gives that $l'-l=0$. Hence $l=l'=0$ and there are no interesting solutions.

E. $N_1=0$; $M_1,N_2,M_2>0$. This case is similar to D.

F. $N_1,M_1,N_2>0$; $M_2=0$. The following indices do not exist in this case: $l_{3\pm},l_{4\pm},l'_{1\pm},l'_{2\pm},l_5,l_6,f,g,h,h'$. Decoupling shows that

$$l_{1+}+l'_{3+}=0,\ l_{1-}+l'_{3-}=0\ ,\ l_{2+}+l'_{4+}=0,\ l_{2-}+l'_{4-}=0\ ,\ a=b=c=d=0,\ a'=d'\ ,$$
$$e,e',x \text{ unconstrained}\ . \tag{15}$$

Two of the anomaly-matching equations immediately show that no solution is possible:

$$[\mathrm{U}(1)_{A'}]^2\mathrm{U}(1)_{V_1}\text{:}\ \ 0=(N_2+1)(l_{1+}+l_{2+})+(N_2-1)(l_{1-}+l_{2-})\ , \tag{16a}$$

$$[\mathrm{U}(1)_{V_1}]^2\mathrm{U}(1)_{A'}\text{:}\ \ -\left[\frac{1}{N_1}+\frac{1}{M_1}\right]q_2d(R_1)=\left[\frac{1}{N_1}-\frac{1}{M_1}\right]N_2[(N_2+1)(l_{1+}+l_{2+})+(N_2-1)(l_{1-}+l_{2-})]q_1$$
$$-\frac{1}{2}\left[\frac{1}{N_1}+\frac{1}{M_1}\right]N_2[(N_2+1)(l_{1+}+l_{2+})+(N_2-1)(l_{1-}+l_{2-})]q_2\ . \tag{16b}$$

Thus $d(R_1)=0$ which is unacceptable.

G. $N_1,M_1,M_2>0$; $N_2=0$. Same as F.

H. $M_1=M_2=0$; $N_1,N_2>0$. The anomaly-matching equations for this case which involve two $\mathrm{SU}(N_1)_L$ axial-vector currents imply that $d(R_1)=0$. Thus any solution must have $N_1=1$. The remaining equations are as follows (note that the only relevant indices are l_{1+}, l_{1-}, l_{2+}, l_{2-}, a', e, e', and x):

$$[\mathrm{SU}(N_2)_L]^3\text{:}\ \ d(R_2)=(N_2+4)(l_{1+}+l_{2+})+(N_2-4)(l_{1-}+l_{2-})\ , \tag{17a}$$

$$[\mathrm{SU}(N_2)_L]^2\mathrm{U}(1)_{A'}\text{:}\ \ q_1d(R_2)=[(N_2+2)(l_{1+}+l_{2+})+(N_2-2)(l_{1-}+l_{2-})](2q_1-q_2)\ , \tag{17b}$$

$$[\mathrm{SU}(N_2)_L]^2\mathrm{U}_{R'}(1)\text{:}\ \ q_1d(R_2)=[(N_2+2)(l_{1+}+l_{2+})+(N_2-2)(l_{1-}+l_{2-})][q+q_2]$$
$$+2[(N_2+2)l_{1+}+(N_2-2)l_{1-}+N_2e]q_1\ , \tag{17c}$$

$$[\mathrm{U}_{R'}(1)]^3\text{:}\ \ (q-q_2)^3d(R_1)+N_2q_1{}^3d(R_2)=\tfrac{1}{2}N_2(N_2+1)[(q+2q_1-q_2)^3l_{1+}+(q-q_2)^3l_{2+}]$$
$$+\tfrac{1}{2}N_2(N_2-1)[(q+2q_1-q_2)^3l_{1-}+(q-q_2)^3l_{2-}]$$
$$+q_1{}^3[a'+(N_2{}^2-1)e+e'+x+d(\mathrm{adj})]\ , \tag{17d}$$

$$[\mathrm{U}(1)_{A'}]^3\text{:}\ \ -q_2{}^3d(R_1)+N_2q_1{}^3d(R_2)=\tfrac{1}{2}N_2(N_2+1)(2q_1-q_2)^3(l_{1+}+l_{2+})$$
$$+\tfrac{1}{2}N_2(N_2-1)(2q_1-q_2)^3(l_{1-}+l_{2-})\ , \tag{17e}$$

$$[\mathrm{U}_{R'}(1)]^2\mathrm{U}(1)_{A'}\text{:}\ \ -(q-q_2)^2q_2d(R_1)+N_2q_1{}^3d(R_2)=\tfrac{1}{2}N_2(N_2+1)(2q_1-q_2)[(q+2q_1-q_2)^2l_{1+}+(q-q_2)^2l_{2+}]$$
$$+\tfrac{1}{2}N_2(N_2-1)(2q_1-q_2)[(q+2q_1-q_2)^2l_{1-}+(q-q_2)^2l_{2-}]\ , \tag{17f}$$

$$[\mathrm{U}(1)_{A'}]^2\mathrm{U}_{R'}(1)\text{:}\ \ q_2{}^2(q-q_2)d(R_1)+N_2q_1{}^3d(R_2)=\tfrac{1}{2}N_2(N_2+1)(2q_1-q_2)^2[(q+2q_1-q_2)l_{1+}+(q-q_2)l_{2+}]$$
$$+\tfrac{1}{2}N_2(N_2-1)(2q_1-q_2)^2[(q+2q_1-q_2)l_{1-}+(q-q_2)l_{2-}]\ . \tag{17g}$$

The following definitions have been used in Eqs. (17a)–(17g): $q\equiv q(\mathrm{adj})$, $q_1\equiv q(R_1)$, and $q_2\equiv N_2q(R_2)$. We have been unable to show that the above equations inevitably lead to physically or mathematically unacceptable consequences [such as $d(R_1)=0$, $q(R_1)=0$, and so on]. Thus a search was made to see if there were any complex representations R_1 and R_2 which obeyed Eqs. (17a)–(17g) as well as providing a nonzero integer value for N_2 while satisfying $R_1\times R_2\times R_2\supset 1$. The procedure used depended on the fact that all the quantities appearing in Eqs. (17a)–(17g) are integers (the normalization convention for the quadratic anomalies may be chosen to make them integers). A small computer program then examined all possible combinations of the quantities in the following ranges:

$$N_2=2,\ldots,20;\ q_1=1,\ldots,35;\ q_2=1,\ldots,35\ ;$$

$$l_{1+},l_{2+},l_{1-},l_{2-}=-5,\ldots,5;\ q=1,\ldots,35\ ,$$

and selected those which satisfied the equations for positive integers $d(R_1)$ and $d(R_2)$, and integers a', e, e', and x. Then $d(R_1),q(R_1),d(R_2),q(R_2),q(\mathrm{adj})$ were checked against a data file containing properties of group representations to see if they correspond to any actual group representations. If so then the requirement $R_1\times R_2\times R_2\supset 1$

was checked. The data file contained properties of representations of the groups SU(3) to SU(9) which had partition labels $\{\lambda_1\lambda_2\ldots\lambda_{N-1}\}$ for SU(N) constrained so that $\lambda_1 \leq 9$ and $\lambda_2 \leq 5$. This data file was obtained by using the program SCHUR developed at the University of Canterbury in Christchurch, New Zealand.[8] The result of this investigation was that no solution was found.

I. $N_1=N_2=0$; $M_1,M_2>0$. Same conclusions as for H.

J. $M_1=N_2=0$; $N_1,M_2>0$. As with case H the anomaly-matching equations which involve two $SU(N_1)_L$ axial-vector currents imply that $d(R_1)=0$. Thus any solution must have $N_1=1$. The remaining anomaly-matching equations are

$$[SU(M_2)_R]^3:\ d(R_2)=-(M_2+4)(l_{3+}+l_{4+})-(M_2-4)(l_{3-}+l_{4-})\ , \tag{18a}$$

$$[SU(M_2)_R]^2U(1)_{A'}:\ q_1 d(R_2)=-[(M_2+2)(l_{3+}+l_{4+})+(M_2-2)(l_{3-}+l_{4-})](2q_1+q_2)\ , \tag{18b}$$

$$\begin{aligned}[SU(M_2)_R]^2U_{R'}(1):\ q_1 d(R_2)=&[(M_2+2)(l_{3+}+l_{4+})+(M_2-2)(l_{3-}+l_{4-})](q-q_2)\\ &-2[(M_2+2)l_{3+}+(M_2-2)l_{3-}+M_2h]q_1\ ,\end{aligned} \tag{18c}$$

$$\begin{aligned}[U_{R'}(1)]^3:\ (q-q_2)^3q(R_1)+M_2q_1{}^3d(R_2)=&\tfrac{1}{2}M_2(M_2+1)[(q-2q_1-q_2)^3l_{3+}+(q-q_2)^3l_{4+}]\\ &+\tfrac{1}{2}M_2(M_2-1)[(q-2q_1-q_2)^3l_{3-}+(q-q_2)^3l_{4-}]\\ &+[-(M_2{}^2-1)q_1{}^3h-q_1{}^3h'+q_1{}^3x+q_1{}^3d(\mathrm{adj})]\ ,\end{aligned} \tag{18d}$$

$$\begin{aligned}[U(1)_{A'}]^3:\ -q_2{}^3d(R_1)+M_2q_1{}^3d(R_2)=&-\tfrac{1}{2}M_2(M_2+1)(2q_1+q_2)^3(l_{3+}+l_{4+})\\ &-\tfrac{1}{2}M_2(M_2-1)(2q_1+q_2)^3(l_{3-}+l_{4-})\ ,\end{aligned} \tag{18e}$$

$$\begin{aligned}[U_{R'}(1)]^2U(1)_{A'}:\ -(q-q_2)^2q_2d(R_1)+M_2q_1{}^3d(R_2)=&-\tfrac{1}{2}M_2(M_2+1)(2q_1+q_2)[(q-2q_1-q_2)^2l_{3+}+(q-q_2)^2l_{4+}]\\ &-\tfrac{1}{2}M_2(M_2-1)(2q_1+q_2)[(q-2q_1-q_2)^2l_{3-}+(q-q_2)^2l_{4-}]\ ,\end{aligned} \tag{18f}$$

$$\begin{aligned}[U(1)_{A'}]^2U_{R'}(1):\ (q-q_2)q_2{}^2d(R_1)+M_2q_1{}^3d(R_2)=&\tfrac{1}{2}M_2(M_2+1)(2q_1+q_2)^2[(q-2q_1-q_2)l_{3+}+(q-q_2)l_{4+}]\\ &+\tfrac{1}{2}M_2(M_2-1)(2q_1+q_2)^2[(q-2q_1-q_2)l_{3-}+(q-q_2)l_{4-}]\ .\end{aligned} \tag{18g}$$

A similar computational method to that used in the previous case was employed to search for explicit solutions. The following ranges for the integer variables were investigated:

$$M_2=2,\ldots,20;\ q_1=1,\ldots,30;\ q_2=1,\ldots,30\ ;$$
$$q=1,\ldots,30;\ l_{3+},l_{3-},l_{4+},l_{4-}=-5,\ldots,5\ .$$

No solution was found for the group representations considered above.

This concludes the investigation of the cases where complex representations R_1 and R_2 are involved.

We consider now the possibility that R_1 and/or R_2 are real representations. In these cases the right-handed preons which are in a real representation of the hypercolor gauge group can be charge conjugated into left-handed preons without altering the hypercolor representation they are in. For example, if R_1 is real then there are really N_1+M_1 left-handed preon chiral superfields and they have $SU(N_1+M_1)_L$ as their chiral-invariance group. Thus these models bear a similarity to the complex representation models which have preons of only one chirality in a hypercolor group representation. The difference is that now these preons may be given a Majorana mass and so their decoupling must be ensured. This will force the 't Hooft indices to satisfy relations which they were not required to do in the complex case.

If both R_1 and R_2 are real then the only indices which are not required to be identically zero by decoupling are those that refer to composites which are singlets under the non-Abelian sector of the chiral-symmetry group. These theories are uninteresting from the point of view of composite models of quarks and leptons and so will not be considered further.

If R_2 is real and R_1 complex then all the 't Hooft indices associated with three-body composites are forced to be identically zero by decoupling. As in the case in the previous paragraph the uninteresting 't Hooft indices survive. For the cases where either of N_1 and M_1 is zero and the other is greater than or equal to two, index a or d survives decoupling, as the preons of which they are composed cannot be given a mass. However, the anomaly-matching equation for $[SU(N_1)_L]^3$ or $[SU(M_1)_R]^3$ then implies that $d(R_2)=0$ since the real adjoint representation is anomaly free. (This is not true for $[SU(2)]^3$ since all flavor group representations will then be real and so the equation becomes $0=0$. However SU(2) cannot contain SU(3) color and so this case is uninteresting.)

Consider now the situation where R_1 is real and R_2 is complex. For similar reasons to those given above most of these models are uninteresting. There are two models, namely, when $N_1+M_1=1$ and where one of N_2 or M_2 is zero and the other is greater than or equal to 2, which fail for other reasons. If $M_2=0$ then the only indices that exist are $l_{1\pm}$, $l_{2\pm}$, a', e, e', and x. Decoupling requires that $l_{1+}+l_{2+}=l_{1-}+l_{2-}=0$. However the anomaly-matching equations for $[\mathrm{SU}(N_2)_L]^3$ are then Eq. (17a) and so $d(R_2)=0$. Thus we must set N_2 equal to 1, which renders the model uninteresting. Similar considerations apply for the $N_2=0$ case.

The conclusion is that no interesting models are obtained when R_1 and/or R_2 is a real representation.

III. BROKEN R SYMMETRY

In this section we assume that R symmetry does not survive the breaking of supersymmetry. The following anomaly-free charges will be used in this section:

$$V_1 \equiv \frac{n_{1L}}{N_1}+\frac{n_{1R}}{M_1}, \quad V_2 \equiv \frac{n_{2L}}{N_2}+\frac{n_{2R}}{M_2}, \tag{19}$$
$$Q \equiv (N_2-M_2)q(R_2)(n_{1L}+n_{1R}) - (N_1-M_1)q(R_1)(n_{2L}+n_{2R}) .$$

This choice of charges is the same as used by Bars[6] in his analysis of the nonsupersymmetric case. We treat the case where R_1 and R_2 are complex first.

Solving the 't Hooft constraints in the case of broken R symmetry is facilitated by the fact that a powerful supergroup technique may be used.[6,9] The details of this procedure may be obtained from Ref. 6. Here only an outline will be presented: Introduce a grading between N left-handed and M right-handed preons so that they are contained in the fundamental representation of $\mathrm{SU}(N/M)$. Classify composite states as higher representations of $\mathrm{SU}(N/M)$. Assign a 't Hooft index to each composite representation. To satisfy the decoupling constraints break the composite representations of $\mathrm{SU}(N/M)$ down to $\mathrm{SU}(N)\times\mathrm{SU}(M)\times\mathrm{U}(1)_V$ while respecting the left-right grading. To satisfy the anomaly-matching equations calculate them with respect to the supergroup classification but replace the trace in the cubic and quadratic anomalies by the supertrace. For this model the actual chiral symmetry G_F,

$$G_F=\mathrm{SU}(N_1)_L\times\mathrm{SU}(M_1)_R\times\mathrm{SU}(N_2)_L\times\mathrm{SU}(M_2)_R \times\mathrm{U}(1)_{V_1}\times\mathrm{U}(1)_{V_2}\times\mathrm{U}(1)_Q \tag{20}$$

is imbedded in $G_F^{(\mathrm{super})}$ where

$$G_F^{(\mathrm{super})}\equiv\mathrm{SU}(N_1/M_1)\times\mathrm{SU}(N_2/M_2)\times\mathrm{U}(1)_Q . \tag{21}$$

TABLE III. The classification of preons (R-symmetry broken case) under $G_F^{(\mathrm{super})}$.

Preons	$\mathrm{SU}(N_1/M_1)$	$\mathrm{SU}(N_2/M_2)$	Q
ψ_1	$\boxslash$	1	$(N_2-M_2)q(R_2)$
A_1	$\boxslash$	1	$(N_2-M_2)q(R_2)$
ψ_2	1	$\boxslash$	$-(N_1-M_1)q(R_1)$
A_2	1	$\boxslash$	$-(N_1-M_1)q(R_1)$

The classification of preons and composites under $G_F^{(\mathrm{super})}$ is indicated in Tables III and IV, respectively. Note that supersymmetry has caused the appearance of the two-body states $\psi_1\bar{A}_1$ and $\psi_2\bar{A}_2$, as well as increasing the number of preon configurations which correspond to given chiral properties in the three-body sector ($\psi_1 A_2 A_2$ and $A_1\psi_2 A_2$ as well as $\psi_1\psi_2\psi_2$). The calculations of the 't Hooft conditions now closely resemble those of Ref. 6. The decoupling constraints are implemented by decomposing the composite supergroup representations down to the actual chiral-symmetry group (20); while respecting the left-right grading:

$$\begin{aligned} a_+(\boxslash,\boxslash\boxslash) = {} & a_+(\Box,1;\Box\Box,1)_{1/N_1,2/N_2}-a_+(\Box,1;\Box,\Box)_{1/N_1,1/N_2+1/M_2} \\ & +a_+(\Box,1;1,\begin{smallmatrix}\Box\\\Box\end{smallmatrix})_{1/N_1,2/M_2}-a_+(1,\Box;\Box\Box,1)_{1/M_1,2/N_2} \\ & +a_+(1,\Box;\Box,\Box)_{1/M_1,1/N_2+1/M_2}-a_+(1,\Box;1,\begin{smallmatrix}\Box\\\Box\end{smallmatrix})_{1/M_1,2/M_2} , \end{aligned} \tag{22a}$$

$$\begin{aligned} a_-(\boxslash,\begin{smallmatrix}\boxslash\\\boxslash\end{smallmatrix}) = {} & a_-(\Box,1;1,\Box)_{1/N_1,1/M_1}-a_-(\Box,1;\Box,\Box)_{1/N_1,1/N_2+1/M_2} \\ & +a_-(\Box,1;1,\Box\Box)_{1/N_1,2/M_2}-a_-(1,\Box;\begin{smallmatrix}\Box\\\Box\end{smallmatrix},1)_{1/M_1,2/N_2} \\ & +a_-(1,\Box;\Box,\Box)_{1/M_1,1/N_2+1/M_2}-a_-(1,\Box;1,\Box\Box)_{1/M_1,2/M_2} , \end{aligned} \tag{22b}$$

$$\begin{aligned} \omega_1(\boxslash\boxslash\ 1) = {} & \omega_1(\mathrm{adj},1;1,1)_{0,0}-\omega_1(\Box^*,\Box;1,1)_{-1/N_1-1/M_1,0} \\ & +\omega_1(1,\mathrm{adj};1,1)_{0,0}-\omega_1(\Box,\Box^*;1,1)_{-1/N_1+1/M_1,0} \\ & +\omega_1(1,1;1,1)_{0,0} , \end{aligned} \tag{22c}$$

$$u_1(1,1)=u_1(1,1;1,1)_{0,0} . \tag{22d}$$

The notation used above is as follows: On the left-hand side (LHS) the first supertableaux refers to $\mathrm{SU}(N_1/M_1)$ and the second to $\mathrm{SU}(N_2/M_2)$. On the RHS the Young tableaux refer to $\mathrm{SU}(N_1)_L$, $\mathrm{SU}(M_1)_R$, $\mathrm{SU}(N_2)_L$, $\mathrm{SU}(M_2)_R$ from left to right. The subscripts are the V_1 charge and the V_2 charge in that order. Similar decompositions apply for the states associated with ω_2 and u_2.

TABLE IV. The classification and 't Hooft indices of composites (R-symmetry broken case) under $G_F^{(\mathrm{super})}$.

Composites	SU(N_1/M_1)	SU(N_2/M_2)	Q	Index
$\psi_1\psi_2\psi_2$	⧅	⧅⧅	$(N_2-M_2)q(R_2)-2(N_1-M_1)q(R_1)$	a_+
$\psi_1 A_2 A_2$				
$A_1\psi_2 A_2$	⧅	⧅ ⧅	$(N_2-M_2)q(R_2)-2(N_1-M_1)q(R_1)$	a_-
$\psi_1\bar{A}_1$	⧅⧅, 1	1	0	w_1,u_1
$\psi_2\bar{A}_2$	1	⧅⧅, 1	0	w_2,u_2

The anomaly-matching equations are as follows:

$$[\mathrm{SU}(N_1/M_1)]^3:\ d(R_1)=\tfrac{1}{2}\Delta_2(\Delta_2+1)a_+ + \tfrac{1}{2}\Delta_2(\Delta_2-1)a_-\ , \tag{23a}$$

$$[\mathrm{SU}(N_1/M_1)]^2\mathrm{U}(1)_Q:\ \Delta_2 d(R_1)=[\tfrac{1}{2}\Delta_2(\Delta_2+1)a_+ + \tfrac{1}{2}\Delta_2(\Delta_2-1)a_-]\left[\Delta_2-2\Delta_1\frac{q(R_1)}{q(R_2)}\right], \tag{23b}$$

$$[\mathrm{SU}(N_2/M_2)]^3:\ d(R_2)=\Delta_1(\Delta_2+4)a_+ + \Delta_1(\Delta_2-4)a_-\ , \tag{23c}$$

$$[\mathrm{SU}(N_2/M_2)]^2\mathrm{U}(1)_Q:\ -\Delta_1\frac{q(R_1)}{q(R_2)}d(R_2)=\Delta_1[(\Delta_2+2)a_+ + (\Delta_2-2)a_-]\left[\Delta_2-2\Delta_1\frac{q(R_1)}{q(R_2)}\right], \tag{23d}$$

$$[\mathrm{U}(1)_Q]^3:\ \Delta_1\Delta_2^3 d(R_1)-\Delta_1^3\Delta_2 d(R_2)\left[\frac{q(R_1)}{q(R_2)}\right]^3=[\tfrac{1}{2}\Delta_1\Delta_2(\Delta_2+1)a_+ + \tfrac{1}{2}\Delta_1\Delta_2(\Delta_2-1)a_-]\left[\Delta_2-2\Delta_1\frac{q(R_1)}{q(R_2)}\right]^3 . \tag{23e}$$

Here $\Delta_1\equiv N_1-M_1$ and $\Delta_2\equiv N_2-M_2$. We now analyze these equations.

Using (23a) in (23b) we see that $\Delta_1=0$ given that quadratic anomalies are never zero. Equation (23e) is then satisfied. However (23c) implies that $d(R_2)$ is zero, which is unacceptable. Thus either (N_2,M_2) equals (1,0) or (0,1) so that Eqs. (23c) and (23d) do not apply. This implies that $\Delta_2=\pm 1$. So the conclusion is that if SU(N_1/M_1) exists then $\Delta_2=\pm 1$, $d(R_1)=a_\pm$ using (23a). Furthermore to ensure the renormalizability of the theory the following must hold:

$$\Delta_1 A(R_1)+\Delta_2 A(R_2)=0\ , \tag{24}$$

so that in this case we must have that $A(R_2)=0$. Thus R_2 is a complex anomaly-free representation. The smallest such representation is the **16** of SO(10). Indeed in SO(10) we have that $\mathbf{16}\times\mathbf{16}=\mathbf{10}+\mathbf{120}+\mathbf{126}$ where **126** is a complex representation so that $R_1=\overline{\mathbf{126}}$ and $R_2=\mathbf{16}$ is a solution of the 't Hooft equations. However since $d(R_1)=a_\pm$ this requires very large and probably unphysical 't Hooft indices. So we consider this class of solutions no further.

For the conclusions of the preceding paragraph not to follow, it must be that (N_1,M_1) equals (1,0) or (0,1). Consider the case where $N_1=1$ and $M_1=0$. Equations (23a)–(23e) reduce to

$$d(R_2)=(\Delta_2+4)a_+ + (\Delta_2-4)a_-\ , \tag{25a}$$

$$-\frac{q(R_1)}{q(R_2)}d(R_2)=[(\Delta_2+2)a_+ + (\Delta_2-2)a_-]\left[\Delta_2-2\frac{q(R_1)}{q(R_2)}\right], \tag{25b}$$

$$\Delta_2^3 d(R_1)-\Delta_2 d(R_1)\left[\frac{q(R_1)}{q(R_2)}\right]^3=[\tfrac{1}{2}\Delta_2(\Delta_2+1)a_+ + \tfrac{1}{2}\Delta_2(\Delta_2-1)a_-]\left[\Delta_2-2\frac{q(R_1)}{q(R_2)}\right]^3 . \tag{25c}$$

Hypercolor anomaly cancellation implies that

$$A(R_1)+\Delta_2 A(R_2)=0\ . \tag{26}$$

These equations admit an infinite number of solutions. As in Ref. 6 it is convenient to classify possible solutions into three classes on the basis of Eq. (22): (i) $\Delta_2=0$, $A(R_1)=0$; (ii) $A(R_1)=A(R_2)=0$; (iii) $\Delta_2\neq 0$ and $A(R_1)=-\Delta_2 A(R_2)$. Cases (i) and (ii) require complex anomaly-free representations. As discussed by Bars,[6] although solutions do exist, asymptotic freedom must be sacrificed. For this reason only case (iii) is interesting.

Various interesting solutions to case (iii) were given in Ref. 6. In particular, two infinite classes of solutions exist:

$$(\mathrm{a})\ G_{\mathrm{HC}}=\mathrm{SU}(N);\ \ \Delta_2=N-4;\ \ a_+=1 \text{ and } a_-=0;\ \ R_1=\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)^*;\ \ R_2=\square\ , \tag{27}$$

$$(\mathrm{b})\ G_{\mathrm{HC}}=\mathrm{SU}(N);\ \ \Delta_2=N+4;\ \ a_+=0 \text{ and } a_-=1;\ \ R_1=(\square\square)^*;\ \ R_2=\square\ . \tag{28}$$

In the nonsupersymmetric case only one preon configuration, $\psi_1\psi_2\psi_2$, corresponds to the representation of the composite states under chiral symmetry. Thus one is led to consider 't Hooft indices which are zero or one in magnitude. This requirement may be relaxed in the supersymmetric theory since the scalar preons increase the multiplicity of preon configurations corresponding to a given three-body composite representation. A search was made for extra solutions. The range considered for Δ_2 and $a_\pm$ was as follows:

$$\Delta_2=-20,\ldots,20;\quad a_\pm=-5,\ldots,5\ .$$

The solutions that were found [excluding those belonging to the infinite classes in (27) and (28)] are displayed in Table V. Note that the search was restricted to the groups SU(3) to SU(7). The last column in Table V indicates whether or not a given solution may be asymptotically free in the hypercolor coupling constant. When a theory is asymptotically free, the upper bound that must be imposed on N_2+M_2 is given. Clearly only the asymptotically free solutions are interesting in this context. One should note that asymptotic freedom in supersymmetric theories places more stringent bounds on the number of preons than is the case in the corresponding nonsupersymmetric theories. This is because the fundamental scalars and the gauginos contribute to the Gell-Mann—Low β function of the renormalization-group equations.[10] We see from Table V that there is only one asymptotically free solution for Eqs. (25) and (26) which contains indices greater than one in the range considered.

The structure of the massless composite spectrum generally in these models may be obtained from Eqs. (22a)—(22d):

$$a_+(\Box\!\Box\,1)-(a_++a_-)(\Box,\Box)+a_+\Big(1,\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\Big)+a_-\Big(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},1\Big)+a_-(1,\Box\!\Box)$$
$$+\omega_2(\mathrm{adj},1)+\omega_2(1,\mathrm{adj})-\omega_2(\Box,\Box^*)-\omega_2(\Box^*,\Box)+\omega_2(1,1)+u_2(1,1)\ . \tag{29}$$

Consideration will now be given to embedding $SU(3)\times SU(2)\times U(1)$ in the chiral-symmetry groups of these theories. Only those embeddings which yield an anomaly-free $SU(3)\times SU(2)\times U(1)$ sector are physically relevant. There are two ways of systematically doing this. Consider $SU(N_2)_L\times SU(M_2)_R$. [$U(1)_{V_2}$,$U(1)_Q$ will be ignored in this section.] The first way relies on the anomalies of the left-handed preons canceling with the anomalies of the right-handed preons. In particular the following scenario is interesting. Let N_2 and M_2 be ≥ 5 so that $SU(N_2)$ and $SU(M_2)$ contain SU(5). Break $SU(N_2)_L\times SU(M_2)_R$ as follows:

$$SU(N_2)_L\to SU(5)_L\times G_L,\quad \Box_L\to(5_L,1)+(1,x)\ , \tag{30}$$

and

$$SU(M_2)_R\to SU(5)_R\times G_R,\quad \Box_R\to(5_R,1)+(1,y)\ , \tag{31}$$

where $x\equiv N_2-5$ and $y\equiv M_2-5$ are the dimensions of the representations of the residual groups G_L and G_R which appear in the decomposition. If we then do $SU(5)_L\times SU(5)_R\to SU(5)_{L+R}$, anomaly cancellation in the SU(5) sector is ensured. To see the ramifications of this embedding for the composite states we decompose the representations in (29) down to $SU(5)\times G_L\times G_R$:

$$(\Box\!\Box,1)\to(15,1,1)+(5,x,1)+[1,\tfrac{1}{2}x(x+1),1]\ , \tag{32a}$$
$$\Big(\begin{smallmatrix}\Box\\\Box\end{smallmatrix},1\Big)\to(10,1,1)+(5,x,1)+[1,\tfrac{1}{2}x(x-1),1]\ , \tag{32b}$$
$$(1,\Box\!\Box)\to(15,1,1)+(5,1,y)+[1,1,\tfrac{1}{2}y(y+1)]\ , \tag{32c}$$
$$\Big(1,\begin{smallmatrix}\Box\\\Box\end{smallmatrix}\Big)\to(10,1,1)+(5,1,y)+[1,1,\tfrac{1}{2}y(y-1)]\ , \tag{32d}$$
$$(\Box,\Box)\to(10,1,1)+(15,1,1)+(5,x,1)+(5,1,y)+(1,x,y)\ , \tag{32e}$$

TABLE V. Asymptotically free solutions to Eqs. (21a), (21b), (21c), and (22). This table does not include the two infinite series of solutions given in (23) and (24). $\{\lambda\}_1$ and $\{\lambda\}_2$ are the partition labels of R_1 and R_2, respectively. Note that $\overline{R}_1$ and $\overline{R}_2$ also give a solution if R_1 and R_2 give a solution.

Δ_2	a_+	a_-	$\{\lambda\}_1,d(R_1)$	$\{\lambda\}_2,d(R_2)$	Group	Asymptotic freedom
2	1	0	$\{31\},15$	$\{2^2\},6$	SU(3)	No
11	-1	3	$\{4\},15$	$\{2^2\},6$	SU(3)	No
6	1	0	$\{31\},45$	$\{2^3\},10$	SU(4)	No
-1	5	1	$\{1\},5$	$\{1^2\},10$	SU(5)	$N_2+M_2\leq 10$
6	1	0	$\{21^2\},45$	$\{1^3\},10$	SU(5)	No
11	1	0	$\{31\},105$	$\{2^4\},15$	SU(5)	No
11	1	0	$\{21^2\},105$	$\{1^4\},15$	SU(6)	No
17	1	0	$\{31\},210$	$\{2^5\},21$	SU(6)	No
19	0	1	$\{32^4\},120$	$\{1^2\},15$	SU(6)	No
17	1	0	$\{21^2\},210$	$\{1^5\},21$	SU(7)	No

$$(\text{adj},1)\to(24,1,1)+(\overline{5},x,1)+(5,\bar{x},1)+(1,x^2-1,1)+(1,1,1)\,, \tag{32f}$$

$$(1,\text{adj})\to(24,1,1)+(\overline{5},1,y)+(5,1,\bar{y})+(1,1,y^2-1)+(1,1,1)\,, \tag{32g}$$

$$(\square,\square^*)\to(1,1,1)+(24,1,1)+(\overline{5},x,1)+(5,1,\bar{y})+(1,x,\bar{y})\,, \tag{32h}$$

$$(\square^*,\square)\to(1,1,1)+(24,1,1)+(5,\bar{x},1)+(\overline{5},1,y)+(1,\bar{x},y)\,, \tag{32i}$$

$$(1,1)\to(1,1,1)\,. \tag{32j}$$

Since the two-body composites form real representations of $SU(N_2)_L\times SU(M_2)_R$ they can be given masses at this level and so would be expected to be significantly more massive than the three-body states. We consider only three-body states in what follows. So for the solutions with $a_+=1$, $a_-=0$ the massless spectrum at the $SU(5)\times G_L\times G_R$ level is

$$(15,1,1)_L+(5,x,1)_L+[1,\tfrac{1}{2}x(x+1),1]_L+(10,1,1)_R+(5,1,y)_L+[1,1,\tfrac{1}{2}y(y-1)]_L$$
$$+(15,1,1)_R+(10,1,1)_R+(5,x,1)_R+(5,1,y)_R+(1,x,y)_R\,. \tag{33}$$

A generation of quarks and leptons is contained in $5+\overline{10}$ of SU(5). Thus only one such generation appears in (33). There are, however, $x+y-1$ extra 5, s (of the required chirality). Mirror particles appear in (33) as well as exotics in the 15 of SU(5). Similar conclusions are obtained for the $a_+=0$, $a_-=1$ solutions.

The second way of embedding $SU(3)\times SU(2)\times U(1)$ in $SU(N_2)\times SU(M_2)$ is to break, say, $SU(N_2)$ to a group which has anomaly-free representations. As discussed by Bars,[6] interesting decompositions may be obtained by this method. These embeddings appear in the supersymmetric case as well, but supersymmetry seems to add nothing to their utility.

To complete the analysis we must examine the cases where either R_1 and R_2 is real, or both are real. In all these cases, however, the only indices that are ever possibly not zero after decoupling belong to two-body bound states. Thus these models are not interesting.

IV. CONCLUSION

We have examined in detail the possible massless bound-state spectra of supersymmetric composite models in which there are preons in two hypercolor representations. Using the decoupling theorem and the 't Hooft anomaly-matching equations it was shown that chiral-symmetry preservation is difficult to arrange for theories respecting supersymmetric R symmetry. Solutions to the equations were discovered for the cases where R symmetry is violated by supersymmetry breaking.

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