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Correlations in color and spin in multiparton processes

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We investigate the color and spin structure of multiparton processes, using a rigorous formalism for handling such new processes. The new ingredients are the double structure functions $\Gamma_{\Lambda L}(x_1,x_2,b_T)$'s which in addition to the joint x_1, x_2, and b_T distribution (b_T=relative transverse distance of the two incoming partons), carry information about the color (L) and spin (Λ) state of the diparton system. We also work out the cross section for these processes, which is a nontrivial generalization of the two-parton-initiated cross section.

I. INTRODUCTION

Composite hadron structure allows for the simultaneous participation to hard processes of more than one parton in the parent hadron (multiparton processes). The class of multiparton processes which are disconnected[1–3] at the Born level (Fig. 1) have become of increasing importance since the first observation, at the UA1 experiment[4] of four-jet events which are likely to be generated by a disconnected four-parton scattering. The new generation of machines such as the Superconducting Super Collider (SSC) will probe the very-small-x regions (x being the momentum fraction carried by the parton). In these regions the parton flux is becoming large ($\propto 1/x$) and one may expect multiparton processes to occur frequently.

Various authors[5–8] have so far roughly estimated the cross sections of such processes. The outcome of their computations[7,8] is that in the TeV region these processes are expected to be comparable in magnitude to the two-parton-initiated four-jets events ($\sigma \sim 10^{-32}$ cm^2 at $\sqrt{s} \simeq 1$ TeV to 10^{-28} cm^2 at $\sqrt{s} \simeq 20$ TeV).

In this paper we do not intend to pursue the study of multiparton processes in a phenomenological point of view (cf. Refs. 5—8) but rather to provide a rigorous and powerful computational method for handling such new processes. In a previous study[9] we have investigated the factorizability of the process depicted in Fig. 1. We have shown that the process remains disconnected to any leading order in α_s. We have also analyzed its spin and color structure when the initial diparton system is just a diquark system. The aim of the present paper is to generalize the result of Ref. 9 to the more complicated and realistic case where the initial diparton systems from hadron A and B may be any combination of quarks (q), antiquarks ($\bar{q}$), and gluons (g).

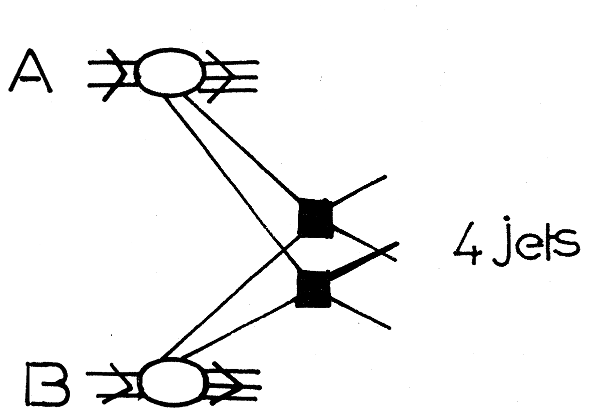

FIG. 1. Example of a disconnected multiparton process (four partons) generating four jets.

The cross section of multiparton processes is related to the forward elastic amplitude shown in Fig. 2. It has been shown in Ref. 9 that the cross section has the form

$$\sigma = \sum_{\substack{\text{diparton}\\ \text{species}}} \int \mathrm{Tr}\left[\Gamma_A(x_1,x_2,b_T)\frac{1}{2i\hat{s}_1}\mathrm{Disc}(T_1)\times\frac{1}{2i\hat{s}_2}\mathrm{Disc}(T_2)\Gamma_B(y_1,y_2,b_T)\right]\times dx_1dx_2dy_1dy_2d^2b_T\,. \quad (1)$$

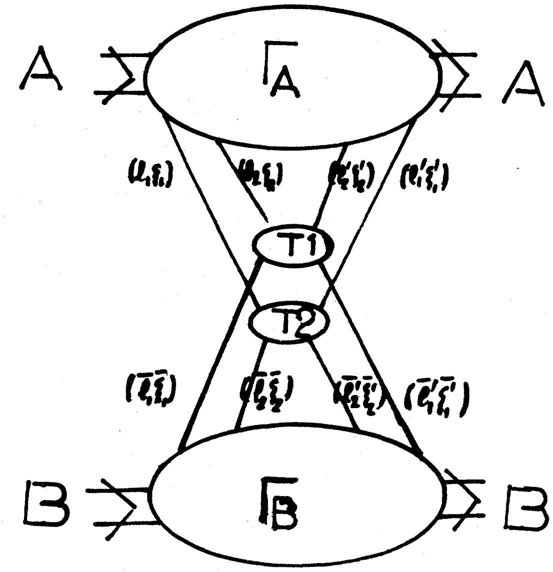

FIG. 2. The amplitude under investigation.

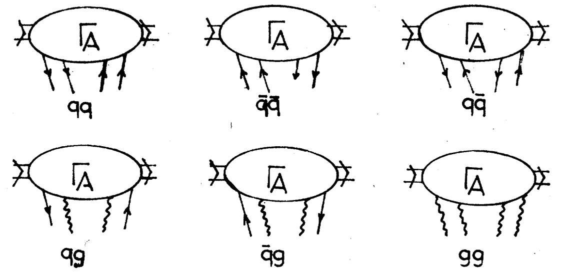

FIG. 3. The six different diparton systems in hadron A (B).

The sum $\sum$ is taken over the six different diparton systems sketched in Fig. 3, namely, (qq), $(\bar{q}\,\bar{q})$, $(\bar{q}q)$, (qg), $(\bar{q}g)$, (gg). The cut amplitude Γ_A (Γ_B) is a tensor with eight indices, i.e., $\Gamma_{A\,\xi_1\xi_2\xi_2'\xi_1'}^{\;l_1l_2l_2'l_1'}$. Upper indices refer to color and the lower ones to spin; more precisely we have

$$\left.\begin{aligned}(l_1l_2l_2'l_1')&\equiv(i_1,i_2,i_2',i_1')\\(\xi_1\xi_2\xi_2'\xi_1')&\equiv(\alpha_1\alpha_2\alpha_2'\alpha_1')\end{aligned}\right\}\text{ for }(qq),(\overline{qq}),(\overline{q}q)\ ,$$

$$\left.\begin{aligned}(l_1l_2l_2'l_1')&\equiv(iaa'i')\\(\xi_1\xi_2\xi_2'\xi_1')&\equiv(\alpha\mu\mu'\alpha')\end{aligned}\right\}\text{ for }(qg),(\overline{q}g)\ ,$$

$$\left.\begin{aligned}(l_1l_2l_2'l_1')&\equiv(a_1a_2a_2'a_1')\\(\xi_1\xi_2\xi_2'\xi_1')&\equiv(\mu_1\mu_2\mu_2'\mu_1')\end{aligned}\right\}\text{ for }(gg)\ .$$

The color indices (i) refer to quarks and antiquarks $(i=1,2,3)$ while (a) refer to gluons $(a=1,2,\ldots,8)$. α's and μ's are, respectively, Dirac and Lorentz indices.

x_1 (y_1) and x_2 (y_2) are the momentum fractions of the incoming partons and b_T is their relative transverse distance inside the hadron A (B) introduced by Paver and Treleani.[7] The amplitudes T_1 and T_2 describe the disconnected hard processes. They also carry color and spin indices, displayed in Fig. 4, which are to be contracted with those of Γ_A and Γ_B, whence the trace in formula 1. $(\hat{s}_1)^{1/2}$ and $(\hat{s}_2)^{1/2}$ are their center-of-mass energies.

The method we will develop consists in expanding the tensor $\Gamma_{A\,\xi_1\xi_2\cdots}^{\;l_1l_2\cdots}$ (Γ_B) for each diparton system in terms of projectors on invariant color and helicity subspaces. The coefficients of these projectors are necessarily positive definite and are newly defined double structure functions $\Gamma_{AL}(x_1,x_2,b_T)$. They describe, in addition to the joint x_1, x_2, b_T distribution the probability to find the diparton system in a given configuration of color $|L\rangle$ and helicity $|\Lambda\rangle$.

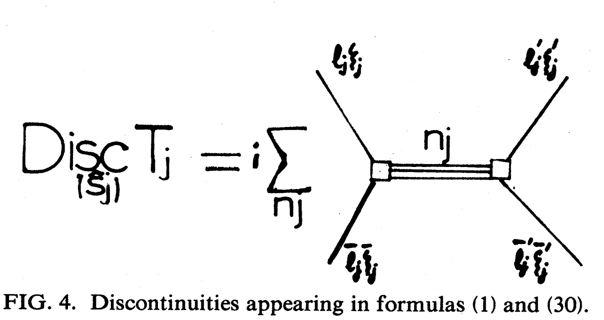

FIG. 4. Discontinuities appearing in formulas (1) and (30).

II. THE COLOR STRUCTURE

In this section we concentrate on the color structure of the tensor Γ_A (Γ_B) of formula 1. We first write $\Gamma^{l_1l_2l_2'l_1'}$ (subscript A and B, spin and flavor indices, are all understood) as

$$\Gamma^{l_1l_2l_2'l_1'}=\langle L_1'l_1'L_2'l_2'\,|\,\Gamma\,|\,L_1l_1L_2l_2\rangle\ , \tag{2}$$

where $\{\,|L_il_i\rangle,\ l_i=1,2,\ldots,n_{L_i}\}$ is the basis of the irreducible representation $\{L_i\}$ to which the incoming parton (i) belongs, n_{L_i} is its dimension. Γ is the matrix density in color space. The representations of interest are $\{L_i\}\equiv\mathbf{3}$, $\bar{\mathbf{3}}$, and $\mathbf{8}$, corresponding, respectively, to quarks, antiquarks, and gluons.

Consider the irreducible basis $|Ll\rangle$ defined by

$$|Ll\rangle=\sum_{l_1,l_2}\langle L_1l_1L_2l_2\,|\,Ll\rangle\,|L_1l_1\rangle\otimes|L_2l_2\rangle\ . \tag{3}$$

The most general form of the operator Γ in the new basis reads

$$\Gamma=\sum_{L,L'}\ \sum_{l,l'}^{n_L,n_{L'}}\Gamma_{L'l'Ll}\,|L'l'\rangle\langle Ll|\ . \tag{4}$$

Since color is conserved in Fig. 3, Γ commutes with all the generators of the $SU(3)_c$ group and therefore is proportional to the identity within each irreducible basis $\{\,|Ll\rangle,\ l=1,2,\ldots,n_L\}$ (Schur lemma):

$$\Gamma_{L'l'Ll}=\Gamma_L\delta_{L'L}\delta_{l'l}\ . \tag{5}$$

Putting this form into formula 4 we get

$$\Gamma=\sum_L\Gamma_L\sum_l|Ll\rangle\langle Ll| \tag{6a}$$

$$=\sum_L\Gamma_LP_L\ , \tag{6b}$$

where $P_L=\sum_l|Ll\rangle\langle Ll|$ is the projector on the representation $\mathbf{L}$. Color conservation thus reduces the number of independent quantities from $n_{L_1}\times n_{L_2}$ to just the number of irreducible representations $\mathbf{L}$ contained in the product $|L_1l_1\rangle\otimes|L_2l_2\rangle$. As a cut amplitude (Fig. 3), the operator Γ is positive definite, therefore the Γ_L's are positive quantities.

Using (6b), the matrix elements $\Gamma^{l_1l_2l_2'l_1'}$ of (2) read

$$\Gamma^{l_1l_2l_2'l_1'}=\sum_L\Gamma_L\frac{\langle l_1'l_2'\,|P_L|\,l_1l_2\rangle}{\mathrm{Tr}P_L}\ . \tag{7}$$

In this expression we have redefined the Γ_L's by dividing the projectors by their traces and by keeping the same notation for the Γ_L's. With this normalization Γ_L stands for the probability for the diparton system to be in the color configuration $|L\rangle$.

For the six cases of diparton systems considered, the Clebsch-Gordan decomposition reads

$$
\begin{aligned}
&(\mathrm{a})\ qq:3\otimes 3=\bar{3}\oplus 6\ ,\\
&\quad\ \overline{qq}:\bar{3}\otimes\bar{3}=3\oplus\bar{6}\ ,\\
&(\mathrm{b})\ q\bar{q}:3\otimes\bar{3}=1\oplus 8\ ,\\
&(\mathrm{c})\ qg:3\otimes 8=3\oplus 6\oplus 15\ ,\\
&\quad\ \bar{q}g:\bar{3}\otimes 8=\bar{3}\oplus\bar{6}\oplus\overline{15}\ ,\\
&(\mathrm{d})\ gg:8\otimes 8=1\oplus 8_s\oplus 8_a\oplus 10\oplus\overline{10}\oplus 27\oplus\ .
\end{aligned}
\tag{8}
$$

Therefore according to (7) we have to compute the projectors on the states appearing in the decomposition (8). We have worked out these projectors for each case. We list below the matrix elements of the different projectors, where $i=1,2,3$ denotes the quark/antiquark color, $a=1,2,\ldots,8$ that of the gluon, λ's are the Gell-Mann matrices, and f_{abc} and d_{abc} are, respectively, the totally antisymmetric and symmetric tensors of $\mathrm{SU}(3)_c$:

$$
\begin{aligned}
(\mathrm{a})\ &\langle i'j'\,|\,P_{\bar{3}(3)}\,|\,ij\,\rangle=\tfrac{1}{2}(\delta_{ii'}\delta_{jj'}-\delta_{ij'}\delta_{ji'})\ ,\\
&\langle i'j'\,|\,P_{6(\bar{6})}\,|\,ij\,\rangle=\tfrac{1}{2}(\delta_{ii'}\delta_{jj'}+\delta_{ij'}\delta_{ji'})\ ,
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
(\mathrm{b})\ &\langle i'j'\,|\,P_1\,|\,ij\,\rangle=\tfrac{1}{3}\delta_{ij}\delta_{i'j'}\ ,\\
&\langle i'j'\,|\,P_8\,|\,ij\,\rangle=\delta_{ii'}\delta_{jj'}-\tfrac{1}{3}\delta_{ij}\delta_{i'j'}\ ,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
(\mathrm{c})\ &\langle i'a'\,|\,P_{3(\bar{3})}\,|\,ia\,\rangle=\tfrac{3}{16}(\lambda^{a'}\lambda^{a})_{i'i}\ ,\\
&\langle i'a'\,|\,P_{6(\bar{6})}\,|\,ia\,\rangle=\tfrac{1}{2}\delta_{i'i}\delta_{a'a}-\tfrac{1}{4}(\lambda^{a}\lambda^{a'})_{i'i}-\tfrac{1}{8}(\lambda^{a'}\lambda^{a})_{i'i}\ ,\\
&\langle i'a'\,|\,P_{15(\overline{15})}\,|\,ia\,\rangle=\tfrac{1}{2}\delta_{i'i}\delta_{a'a}+\tfrac{1}{4}(\lambda^{a}\lambda^{a'})_{i'i}-\tfrac{1}{16}(\lambda^{a'}\lambda^{a})_{i'i}\ ,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
(\mathrm{d})\ &\langle a'b'\,|\,P_1\,|\,ab\,\rangle=\tfrac{1}{8}\delta_{ab}\delta_{a'b'}\ ,\\
&\langle a'b'\,|\,P_{8s}\,|\,ab\,\rangle=\tfrac{3}{5}d_{abe}d_{a'b'e}\ ,\\
&\langle a'b'\,|\,P_{8a}\,|\,ab\,\rangle=\tfrac{1}{3}f_{abe}f_{a'b'e}\ ,\\
&\langle a'b'\,|\,P_{10}\,|\,ab\,\rangle=\tfrac{1}{4}(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'}-\tfrac{2}{3}f_{abe}f_{a'b'e})+\tfrac{1}{4}i(d_{a'ae}f_{b'be}+d_{b'be}f_{a'ae})\ ,\\
&\langle a'b'\,|\,P_{\overline{10}}\,|\,ab\,\rangle=\tfrac{1}{4}(\delta_{aa'}\delta_{bb'}-\delta_{ab'}\delta_{ba'}-\tfrac{2}{3}f_{abe}f_{a'b'e})-\tfrac{1}{4}i(d_{a'ae}f_{b'be}+d_{b'be}f_{a'ae})\ ,\\
&\langle a'b'\,|\,P_{27}\,|\,ab\,\rangle=\tfrac{1}{2}(\delta_{aa'}\delta_{bb'}+\delta_{ab'}\delta_{ba'}-\tfrac{1}{4}\delta_{ab}\delta_{a'b'}-\tfrac{6}{5}d_{abe}d_{a'b'e})\ .
\end{aligned}
\tag{12}
$$

III. THE SPIN STRUCTURE

The spin structure of the partons inside the parent hadron is described by the spin tensor $\Gamma_{\xi_1\xi_2\xi_2'\xi_1'}$ where $\xi_1\xi_2\cdots$ are Lorentz indices for gluons and Dirac indices for quarks and antiquarks. In order to treat the spin structure in a fashion similar to the color, we convert from the ξ_i basis to the helicity one $|\,\lambda_i\,\rangle$. The advantage of the latter is that the total diparton helicity λ is approximately conserved, i.e., the λ nonconserving part of $\Gamma_A(\Gamma_B)$ is suppressed[3] by powers of $1/\sqrt{s}$ ($\sqrt{s}$ being the overall center-of-mass energy). This conservation rule, together with that of parity (for unpolarized beams) will play a role analogous to the color conservation in the preceding section. We therefore write $\Gamma_{\xi_1\xi_2}\cdots$ as

$$
\Gamma_{\xi_1\xi_2\xi_2'\xi_1'}=\sum_{\lambda_1\lambda_2\lambda_2'\lambda_4'}\langle\lambda_1'\lambda_2'\,|\,\bar{\Gamma}\,|\,\lambda_1\lambda_2\rangle
\times\varphi^*_{\xi_1}(\lambda_1)\varphi^*_{\xi_2}(\lambda_2)\varphi_{\xi_1'}(\lambda_1')\varphi_{\xi_2'}(\lambda_2')\ . \tag{13}
$$

$\varphi_{\xi_i}(\lambda_i)$ denotes the spin wave function of the incoming parton i with helicity λ_i ($\lambda_i=\pm$), i.e., $\varphi_{\xi_i}(\lambda_i)\equiv\epsilon_\mu(\lambda_i)$ for the gluon and $\varphi_{\xi_i}(\lambda_i)\equiv u_\alpha(\lambda_i)$ [$\bar{v}_\alpha(\lambda_i)$] for the quark [antiquark].

$\bar{\Gamma}$ is an operator in helicity space (matrix density) and therefore can be written as

$$
\bar{\Gamma}=\sum_{\lambda_1\lambda_2\lambda_2'\lambda_1'}\langle\lambda_1'\lambda_2'\,|\,\bar{\Gamma}\,|\,\lambda_1\lambda_2\rangle\,|\,\lambda_1'\lambda_2'\,\rangle\langle\,\lambda_1\lambda_2\,|\ . \tag{14}
$$

In general, $\bar{\Gamma}$ is nondiagonal in the $|\,\lambda_1\rangle\otimes|\,\lambda_2\rangle$ basis. However, due to helicity and parity conservation $\bar{\Gamma}$ is diagonal in the following basis:

$$
|\,\lambda\,\rangle\equiv|\,++\,\rangle;\,|\,--\,\rangle;\,\frac{|\,+-\,\rangle\pm|\,-+\,\rangle}{\sqrt{2}}\qquad[\text{cases (a),(b),(d)}]\ , \tag{15}
$$

$$
|\,\lambda\,\rangle\equiv|\,++\,\rangle;\,|\,--\,\rangle;\,|\,+-\,\rangle;\,|\,-+\,\rangle\quad[\text{case (c)}]\ . \tag{16}
$$

We thus write $\bar{\Gamma}$ in the new bases $|\,\lambda\,\rangle$ as a sum over the projectors P_λ associated to the state $|\,\lambda\,\rangle$:

$$
\bar{\Gamma}=\sum_\lambda\langle\,\lambda\,|\,\bar{\Gamma}\,|\,\lambda\,\rangle P_\lambda\ . \tag{17}
$$

Parity in addition requires that

$$
\begin{aligned}
&\langle\,++\,|\,\bar{\Gamma}\,|\,++\,\rangle=\langle\,--\,|\,\bar{\Gamma}\,|\,--\,\rangle\qquad[\text{cases (a),(b),(c),(d)}]\ ,\\
&\langle\,-+\,|\,\bar{\Gamma}\,|\,-+\,\rangle=\langle\,+-\,|\,\bar{\Gamma}\,|\,+-\,\rangle\quad[\text{case (c)}]\ .
\end{aligned}
\tag{18}
$$

Therefore, we can rewrite (17) as

$$\overline{\Gamma}=\sum_{\Lambda}\Gamma_{\Lambda}\frac{P_{\Lambda}}{\mathrm{Tr}P_{\Lambda}}\ , \tag{19}$$

where

$$P_{\Lambda}=(P_{|++\rangle}+P_{|--\rangle});(P_{|+-\rangle\pm|-+\rangle}) \quad [\text{cases (a),(b),(d)}]\ ,$$
$$P_{\Lambda}=(P_{|++\rangle}+P_{|--\rangle});(P_{|+-\rangle}+P_{|-+\rangle}) \quad [\text{case (c)}]\ . \tag{20}$$

As for color Γ_{Λ} is positive and is the structure function describing the diparton system in the helicity subspace Λ associated to P_{Λ}. We thus have only four kinds of projectors whose matrix elements are listed below:

$$\langle\lambda_1'\lambda_2'|P_{|++\rangle}+P_{|--\rangle}|\lambda_1\lambda_2\rangle=\tfrac{1}{2}(1+\lambda_1\lambda_2)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\ ,$$
$$\langle\lambda_1'\lambda_2'|P_{|+-\rangle}+P_{|-+\rangle}|\lambda_1\lambda_2\rangle=\tfrac{1}{2}(1-\lambda_1\lambda_2)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\ ,$$
$$\langle\lambda_1'\lambda_2'|P_{|+-\rangle+|-+\rangle}|\lambda_1\lambda_2\rangle=\tfrac{1}{2}(\delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'}-\lambda_1\lambda_2\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'})\ , \tag{21}$$
$$\langle\lambda_1'\lambda_2'|P_{|+-\rangle-|-+\rangle}|\lambda_1\lambda_2\rangle=\tfrac{1}{2}(\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}-\delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'})\ .$$

(In the above formulas, λ_i is twice the helicity for quarks and antiquarks.) Putting (19) into (13) we get the following form for the covariant tensor,

$$\Gamma_{\xi_1\xi_2\xi_2'\xi_1'}=\sum_{\Lambda}\sum_{\lambda_1\lambda_2\lambda_2'\lambda_1'}\Gamma_{\Lambda}\frac{\langle\lambda_1'\lambda_2'|P_{\Lambda}|\lambda_1\lambda_2\rangle}{\mathrm{Tr}P_{\Lambda}}\times\varphi^*_{\xi_1}(\lambda_1)\varphi^*_{\xi_2}(\lambda_2)\varphi_{\xi_1'}(\lambda_1')\varphi_{\xi_2'}(\lambda_2')\ . \tag{22}$$

Summing over the helicities λ_i,λ_i' in (22) we get

$$\Gamma_{\xi_1\xi_2\xi_2'\xi_1'}=\sum_{\Lambda}\Gamma_{\Lambda}\frac{(P_{\Lambda})_{\xi_1\xi_2\xi_2'\xi_1'}}{\mathrm{Tr}P_{\Lambda}}\ , \tag{23}$$

where the covariant (P_{Λ})'s are obtained from the noncovariant ones (21) by the substitutions (we use the convention of Bjorken and Drell;[10] also quark masses are neglected)

$$\delta_{\lambda\lambda'}\to\not p_{\alpha'\alpha}\ ,\qquad \lambda\delta_{\lambda\lambda'}\to\pm(\gamma_5\not p)_{\alpha'\alpha} \tag{24}$$

(with + for quarks and − for antiquarks of momentum p),

$$\delta_{\lambda\lambda'}\to -g_{\mu\mu'}\ ,\qquad \lambda\delta_{\lambda\lambda'}\to i\epsilon_{\sigma\mu\mu'\rho}k^{\rho}n^{\sigma} \tag{25}$$

(for gluons of momentum k), where $\epsilon_{\sigma\mu\mu'\rho}$ is the Levi-Civita totally antisymmetric tensor, and n is a spacelike four-vector such that $n\cdot k=-1$; a simple choice may be $n^{\sigma}=(0,\mathbf{k}/|\mathbf{k}|^2)$. The first three rules are straightforward. The fourth one is less trivial and involves the polarization sum

$$\sum_{\lambda}\lambda\epsilon_{\mu}(k,\lambda)\epsilon^*_{\mu'}(k,\lambda)\ .$$

The reader may check this substitution taking

$$\epsilon^{+}=\frac{1}{\sqrt{2}}(0,1,i,0)$$

and

$$\epsilon^{-}=\frac{1}{\sqrt{2}}(0,1,-i,0)\ .$$

The result of the substitution is listed below.

Cases (a), (b) (the lower sign corresponds to the $q\bar q$ system):

$$(P_{|++\rangle}+P_{|--\rangle})_{\alpha_1\alpha_2\alpha_2'\alpha_1'}=\tfrac{1}{2}[\not p_{\alpha_1'\alpha_1}\not p_{\alpha_2'\alpha_2}\pm(\gamma_5\not p)_{\alpha_1'\alpha_1}(\gamma_5\not p)_{\alpha_2'\alpha_2}]\ ,$$
$$(P_{|+-\rangle+|-+\rangle})_{\alpha_1\alpha_2\alpha_2'\alpha_1'}=\tfrac{1}{2}[\not p_{\alpha_1'\alpha_2}\not p_{\alpha_2'\alpha_1}\mp(\gamma_5\not p)_{\alpha_1'\alpha_1}(\gamma_5\not p)_{\alpha_2'\alpha_2}]\ , \tag{26}$$
$$(P_{|+-\rangle-|-+\rangle})_{\alpha_1\alpha_2\alpha_2'\alpha_1'}=\tfrac{1}{2}(\not p_{\alpha_1'\alpha_1}\not p_{\alpha_2'\alpha_2}-\not p_{\alpha_1'\alpha_2}\not p_{\alpha_2'\alpha_1})\ .$$

Case (c) (the lower sign corresponds to the $\bar q g$ system):

$$(P_{|++\rangle}+P_{|--\rangle})_{\alpha\mu\mu'\alpha'}=\tfrac{1}{2}[-g_{\mu\mu'}\not p_{\alpha'\alpha}\pm i\epsilon_{\sigma\mu\mu'\rho}k^{\rho}n^{\sigma}(\gamma_5\not p)_{\alpha'\alpha}]\ ,$$
$$(P_{|+-\rangle}+P_{|-+\rangle})_{\alpha\mu\mu'\alpha'}=\tfrac{1}{2}[-g_{\mu\mu'}\not p_{\alpha'\alpha}\mp i\epsilon_{\sigma\mu\mu'\rho}k^{\rho}n^{\sigma}(\gamma_5\not p)_{\alpha'\alpha}]\ . \tag{27}$$

Case (d):

$$(P_{|++\rangle}+P_{|--\rangle})_{\mu_1\mu_2\mu_2'\mu_1'}=\tfrac{1}{2}(g_{\mu_1\mu_1'}g_{\mu_2\mu_2'}-\epsilon_{\bar\sigma\mu_1\mu_1'\rho}\epsilon_{\sigma\mu_2\mu_2'\bar\rho}k^{\rho}k^{\bar\rho}n^{\sigma}n^{\bar\sigma})\ ,$$
$$(P_{|+-\rangle+|-+\rangle})_{\mu_1\mu_2\mu_2'\mu_1'}=\tfrac{1}{2}(g_{\mu_1\mu_2'}g_{\mu_2\mu_2'}+\epsilon_{\sigma\mu_1\mu_1'\rho}\epsilon_{\bar\sigma\mu_2\mu_2'\bar\rho}k^{\rho}k^{\bar\rho}n^{\sigma}n^{\bar\sigma})\ , \tag{28}$$
$$(P_{|+-\rangle-|-+\rangle})_{\mu_1\mu_2\mu_2'\mu_1'}=\tfrac{1}{2}(g_{\mu_1\mu_1'}g_{\mu_2\mu_2'}-g_{\mu_1\mu_2'}g_{\mu_2\mu_1'})\ .$$

Combining the color and the spin degrees of freedom we arrive at the final decomposition of the tensor $\Gamma^{l_1l_2l_2'l_1'}_{\xi_1\xi_2\xi_2'\xi_1'}$ which appears in (1):

$$\Gamma=\sum_{L\Lambda}\Gamma_{L\Lambda}\frac{P_L\otimes P_\Lambda}{(\mathrm{Tr}P_L)(\mathrm{Tr}P_\Lambda)}\,, \tag{29}$$

where $\Gamma_{L\Lambda}$ is the probability to find the diparton system in the color-helicity subspace $L\otimes\Lambda$, $P_L\otimes P_\Lambda$ is its corresponding projector with the traces $\mathrm{Tr}P_L=n_L$, $\mathrm{Tr}P_\Lambda=n_\Lambda$ (n_L and n_Λ, being, respectively, the dimensions of the subspace L and Λ). The matrix elements of these projectors are listed in (9)—(12) and (26)—(28). Finally the cross section of multiparton processes (1) have the general form

$$\sigma=\sum_{\substack{\text{diparton}\\ \text{species}}}\sum_{\substack{(\Lambda L)\\ (\Lambda' L')}}\int\Gamma^A_{\Lambda L}(x_1,x_2,b_T)\Gamma^B_{\Lambda' L'}(x_1',x_2',b_T)$$

$$\times\frac{\mathrm{Tr}\left[P_\Lambda\otimes P_L\left[\frac{1}{2i\hat{\rho}_1}\mathrm{Disc}T_1\right]\left[\frac{1}{2i\hat{\rho}_2}\mathrm{Disc}T_2\right]P_{\Lambda'}\otimes P_{L'}\right]}{(\mathrm{Tr}P_\Lambda)(\mathrm{Tr}P_L)(\mathrm{Tr}P_{\Lambda'})(\mathrm{Tr}P_{L'})}dx_1dx_2dx_1'dx_2'd^2b_T\,. \tag{30}$$

IV. CONCLUSION

In this paper we have elucidated the color and spin structure of multiparton processes where two partons from each hadron participate simultaneously to hard processes. We have introduced new structure functions $\Gamma_{\Lambda L}(x_1,x_2,b_T)$'s. The key procedure to define uniquely such $\Gamma_{\Lambda L}$'s is to expand the color-spin tensor $\Gamma^{l_1l_2\cdots}_{\xi_1\xi_2\cdots}$ in terms of projectors on invariant subspaces

$$L(\text{color})\otimes\Lambda(\text{spin})\,.$$

Conservation of color, helicity, and parity are needed for such an expansion.

The structure functions $\Gamma_{\Lambda L}(x_1,x_2,b_T)$'s thus defined are positive definite and therefore have a probabilistic meaning. They describe, apart from the joint (x_1,x_2,b_T) distribution, the way the two partons are correlated in color and spin inside the parent hadron. The formula that we have obtained for the multiparton cross section constitutes a nontrivial generalization of that for the two-parton-initiated processes. We have been able to write it in a simple form, with a universal structure for all flavors. Many other problems remain to be investigated; a particularly interesting one is the analysis of the evolution of multiparton distribution functions $\Gamma_{\Lambda L}$'s, which is not a trivial matter due to their mixing under radiative corrections.

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