Multiparton processes: An application to the double Drell-Yan mechanism

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We investigate multiparton processes in the framework of quantum chromodynamics. The spin and color, as well as the factorizability of the process, have been analyzed in detail. The analysis shows that the existence of connecting gluons does not spoil factorizability in the leading-logarithm approximation. We also find that the spin and color degrees of freedom play an important role in the description of multiparton processes. As a consequence, hadrons are described by a set of newly defined structure functions. We apply the general formalism to the double Drell-Yan mechanism. We find that the above-mentioned structure functions are convoluted with two completely uncorrelated single Drell-Yan cross sections.

I. INTRODUCTION

There is by now an ever increasing experimental evidence which confirms the QCD—parton-model picture of high-energy and high-p_T reactions based on *single* hard collisions between elementary pointlike constituents. Composite hadron structure, however, allows for new kinds of subprocesses to occur already at the naive parton level in the case of hadron-hadron collisions, namely, multiparton processes (Fig. 1). These subprocesses are powerlike corrections to the leading QCD terms of the form $(1/s)^p$ when all the kinematical invariants t_i $(i=1,2,\ldots,n)$ are large, with t_i/s fixed, $\sqrt{s}$ being the center-of-mass energy. However in the kinematical region where some of the t_i's, although much larger than any hadronic mass scale, are much smaller than s, the terms of order $(1/t_i)^p$ might not be so negligible.[1] In addition these subprocesses are linked to multiparticle distributions[2] $G(x_1,x_2,\ldots,x_n)$ which carry a wider amount of information on the hadronic bound states.

A subclass of multiparton processes, namely, the disconnected processes, as has been stressed in Ref. 2 are less suppressed than predicted by the naive counting rules with respect to any connected process involving the same number of constituents. That is why they are much more important than the connected ones. They are, however, poorly investigated as far as spin and color structure and factorizability are concerned. An example of a multiparton process, i.e., the inclusive double scattering, has been analyzed by Paver and Treleani[2] in the spinless and colorless case.

The aim of this paper is to study disconnected multiparton processes in the QCD framework. In dealing with radiative corrections to disconnected multiparton processes, one has to consider two distinct classes of gluons. We call nonconnecting (NC) gluons those which correct each hard process separately as in single parton processes [Fig. 2(a)] and connecting (C) gluons those which *connect* the various hard processes [Fig. 2(b)]. This distinction is necessary, for NC gluons and C gluons have different kinematical properties as we will see in Sec. III. The existence of C gluons apparently causes trouble for the factorizability of the process, but we show that in the leading-logarithm approximation (LLA) they do not. We also show that the spin and color degrees of freedom introduce six newly defined structure functions for each hadron. These structure functions provide information

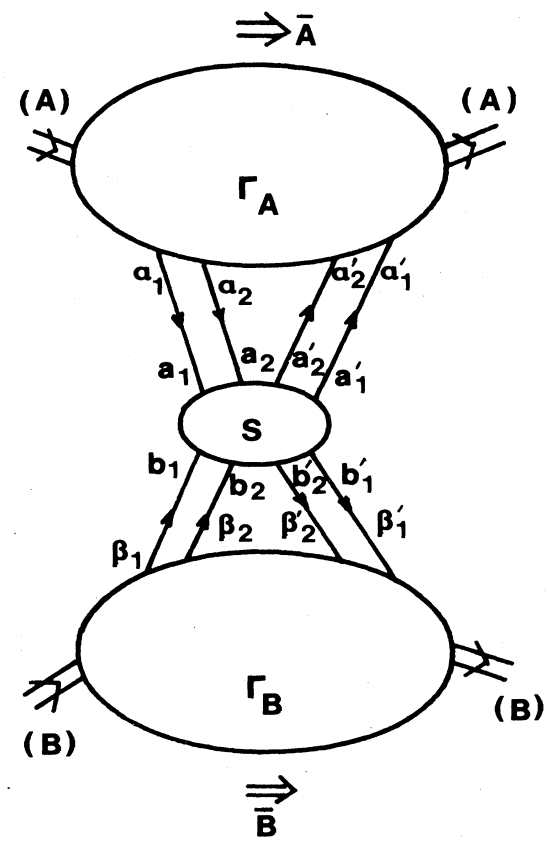

FIG. 1. The diagram corresponding to the amplitude A, the discontinuity of which gives the cross section of a multiparton process. Arrows indicate the momentum flow. The hard blob S is disconnected (up to radiative corrections).

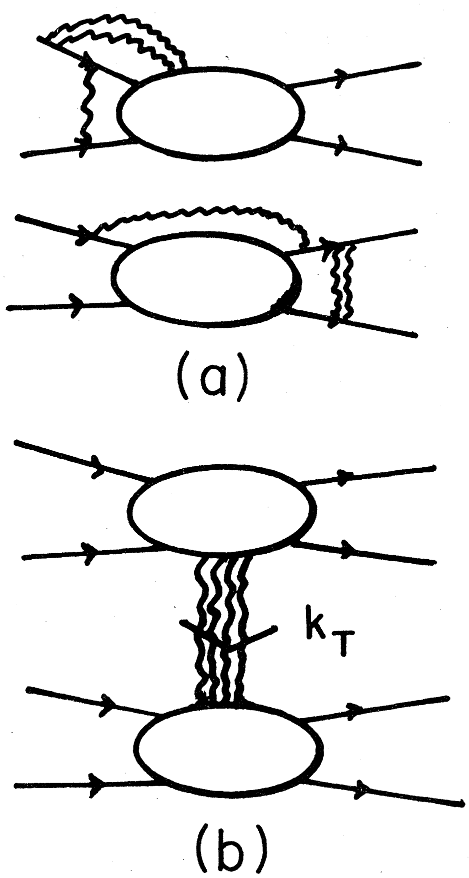

FIG. 2. (a) An example of NC gluons. (b) An example of C gluons; $k_T = \sum_i k_{Ti} \sim R^{-1}$.

not only on the longitudinal-momentum-fraction distributions of the partons, but also on the way the incoming-parton color and spin are correlated inside the parent hadron. These structure functions depend also on the relative transverse distance b_T separating, inside the hadron, the two incoming partons. We will recover this feature using a more direct method than that of Ref. 2. The general description will then be applied to the simplest example of a multiparton process, namely, the double Drell-Yan (DDY) mechanism. Goebel, Scott, and Halzen[3] have estimated the DDY cross section and concluded that it is at the limit of observability.

The paper is organized as follows. In Sec. II we carry out the study of disconnected processes without making reference to any specific reaction. Section III will be devoted to connecting gluons and show that they do not affect the factorizability at LLA. Finally in Sec. IV we analyze the DDY process. We work out the cross section $d\sigma/dQ_1dQ_2$ using our above-mentioned structure functions and show that these are convoluted with two completely uncorrelated single Drell-Yan cross sections.

II. FORMALISM OF THE DOUBLE SCATTERING

We study in this section the amplitude A associated with the diagram in Fig. 1. There are six independent internal momentum variables; we choose them as $\sigma,\Sigma,\bar{A},\bar{B},\sigma',\Sigma'$ with

$$\begin{aligned}
\sigma &= \frac{a_1-a_2}{2}\,, \\
\Sigma &= \frac{b_1-b_2}{2}\,, \\
\sigma' &= \frac{a_1'-a_2'}{2}\,, \\
\Sigma' &= \frac{b_1'-b_2'}{2}\,, \\
\bar{A} &= P_A - a_1 - a_2\,, \\
\bar{B} &= P_B - b_1 - b_2\,.
\end{aligned} \tag{2.1}$$

The amplitude A according to Fig. 1 can be written as (the α's are Dirac indices, color indices are understood for the moment)

$$\begin{aligned}
A = \int & \frac{d^4\sigma}{(2\pi)^4}\frac{d^4\Sigma}{(2\pi)^4}\frac{d^4\bar{A}}{(2\pi)^4}\frac{d^4\bar{B}}{(2\pi)^4}\frac{d^4\sigma'}{(2\pi)^4}\frac{d^4\Sigma'}{(2\pi)^4} \\
& \times \Gamma_{A\alpha_1\alpha_2\alpha_2'\alpha_1'}(\sigma,\bar{A},\sigma') S^{\beta_1\beta_2\beta_2'\beta_1'}_{\alpha_1\alpha_2\alpha_2'\alpha_1'}(\sigma,\Sigma,\bar{A},\bar{B},\sigma',\Sigma') \\
& \times \Gamma_{B\beta_1\beta_2\beta_2'\beta_1'}(\Sigma,\bar{B},\Sigma')\,.
\end{aligned} \tag{2.2}$$

The disconnected hard amplitude S possesses an extra δ function which we factorize so that S takes on the form

$$\begin{aligned}
& S^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}(\sigma,\Sigma,\bar{A},\bar{B},\sigma',\Sigma') \\
&\quad = \hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}(\sigma,\Sigma,\bar{A},\bar{B},\sigma',\Sigma')(2\pi)^4\delta^{(4)}(\sigma+\Sigma-\sigma'-\Sigma')\,.
\end{aligned} \tag{2.3}$$

To define the longitudinal fractions of momenta carried by the partons of hadron A, we introduce the lightlike four-vector η_A in terms of which we write the Sudakov decomposition

$$\sigma_\mu = XP_{A\mu} + \frac{\sigma^2+\sigma_T{}^2}{2X}\eta_{A\mu} + \sigma_{T\mu} \tag{2.4}$$

with

$$P_A{}^2 \simeq \eta^2 = \eta_A\cdot\sigma_T = P_A\cdot\sigma_T = 0,\quad P_A\cdot\eta_A \simeq 1\,.$$

One can take for η_A, for instance, $\eta_A \simeq P_B/P_A\cdot P_B$. Similarly,

$$\begin{aligned}
\Sigma_\mu &= YP_{B\mu} + \frac{\Sigma^2+\Sigma_T{}^2}{2Y}\eta_{B\mu} + \Sigma_{T\mu}\,, \\
\bar{A}_\mu &= X_{\bar{A}}P_{A\mu} + \frac{\bar{A}^2+\bar{A}_T{}^2}{2X_{\bar{A}}}\eta_{A\mu} + \bar{A}_{T\mu}\,, \\
\bar{B}_\mu &= Y_{\bar{B}}P_{B\mu} + \frac{\bar{B}^2+\bar{B}_T{}^2}{2Y_{\bar{B}}}\eta_{B\mu} + \bar{B}_{T\mu}\,,
\end{aligned} \tag{2.5}$$

and the same for primed variables σ',Σ'. Here $\eta_B \simeq P_A/P_A\cdot P_B$.

We write the δ function (2.3) as

$$\delta^{(4)}(\sigma+\Sigma-\sigma'-\Sigma')$$
$$=\delta^{(2)}(\sigma_T+\Sigma_T-\sigma'_T-\Sigma'_T)\delta(XP_{A_+}-X'P_{A_+})\times\delta(YP_{B_-}-Y'P_{B_-})\times 2$$
$$=\frac{2}{s}\delta^{(2)}(\sigma_T+\Sigma_T-\sigma'_T-\Sigma'_T)\delta(X-X')\delta(Y-Y')\,, \tag{2.6}$$

where $\sqrt{s}$ is the center-of-mass energy and $P_{A_+}P_{B_-}\simeq s$. The transverse component $\delta^{(2)}(\sigma_T+\Sigma_T-\sigma'_T-\Sigma'_T)$ mixes the upper and lower parts and it is singular for collinear interacting partons. In order to achieve factorization of the upper, middle, and lower parts, we use the integral representation of the δ function,

$$\delta^{(2)}(\sigma_T+\Sigma_T-\sigma'_T-\Sigma'_T)=\int e^{-ib_T\cdot(\sigma_T+\Sigma_T-\sigma'_T-\Sigma'_T)}\frac{d^2b_T}{(2\pi)^2}\,. \tag{2.7}$$

The parameter b_T, being the coordinate conjugate to the relative transverse momentum σ_T, is interpreted as the relative transverse distance, within the hadron, which separates the two incoming partons. This b_T dependence is a characteristic of disconnected processes. Since the hard amplitude $\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}$ is no longer singular in the collinear direction, one can Taylor-expand it around XP_{A_μ}, YP_{B_μ}, $X_{\bar{A}}P_{A_\mu}$, $X'P_{A_\mu}$, $Y'P_{B_\mu}$, $Y_{\bar{B}}P_{B_\mu}$, thus

$$\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}(\sigma,\Sigma,\bar{A},\bar{B},\sigma',\Sigma')=\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}(X,Y,X_{\bar{A}},Y_{\bar{B}},P_A,P_B)+\left[\frac{\partial\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}}{\partial\sigma_\mu}\right](\sigma_\mu-XP_{A\mu})+\cdots\,. \tag{2.8}$$

We shall retain only the first term of the expansion which depends solely on the X components of the parton's momenta. The remaining terms correspond to higher-power corrections[4] in $1/s$.

To integrate over the off-shell momentum squared and the transverse momenta of the incoming partons, keeping the X components fixed, we introduce the identity

$$\int d^4\sigma=\int d^4\sigma\, dX\,\delta(X-\sigma\cdot\eta_A) \tag{2.9}$$

and similarly for the other variables. Therefore the amplitude A in Eq. (2.2) takes on the form

$$A=\frac{1}{2s}\int dx_1dx_2dy_1dy_2d^2b_T\Gamma_{A\alpha_1\alpha_2\alpha'_2\alpha'_1}(x_1,x_2,b_T,\eta_A,P_A)\times\hat{S}^{\beta_1\beta_2\beta'_2\beta'_1}_{\alpha_1\alpha_2\alpha'_2\alpha'_1}(x_1,x_2,y_1,y_2,P_A,P_B)\times\Gamma_{B\beta_1\beta_2\beta'_2\beta'_1}(y_1,y_2,b_T,\eta_B,P_B)\,, \tag{2.10}$$

where

$$\Gamma_{A\alpha_1\alpha_2\alpha'_2\alpha'_1}(x_1,x_2,\dots)=4\pi\int\frac{d^4\sigma}{(2\pi)^4}\frac{d^4\bar{A}}{(2\pi)^4}\frac{d^4\sigma'}{(2\pi)^4}\delta(X-\sigma\cdot\eta_A)\delta(X_{\bar{A}}-\bar{A}\cdot\eta_A)\times\delta(X-\sigma'\cdot\eta_A)\Gamma_{A\alpha_1\alpha_2\alpha'_2\alpha'_1}(\sigma,\bar{A},\sigma')\times e^{-ib_T\cdot(\sigma_T-\sigma'_T)} \tag{2.11}$$

and similarly for $\Gamma_{B\beta_1\beta_2\beta'_2\beta'_1}$. The integrations over X' and Y' have been performed using the last δ function in Eq. (2.6) and we have converted in formula (2.10) to the variables x_1,x_2 and y_1,y_2 which are the momentum fractions corresponding, respectively, to a_1,a_2 and b_1,b_2 in Fig. 1. The total cross section is obtained from the amplitude A by the formula

$$\sigma_D=\frac{1}{2s}\underset{(s)}{\mathrm{Disc}}A\,. \tag{2.12}$$

Taking the discontinuity of A in the variable s, the cross section reads

$$\sigma_D=\frac{1}{4s^2}\int dx_1dx_2dy_1dy_2d^2b_T\tilde{\Gamma}_{A\alpha_1\alpha_2\cdots}(x_1,x_2,b_T,\dots)\times\underset{(\hat{s})}{\mathrm{Disc}}\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}\tilde{\Gamma}_{B\beta_1\beta_2\cdots}(y_1,y_2,b_T,\dots)\,, \tag{2.13}$$

where $\tilde{\Gamma}_{A\alpha_1\alpha_2\cdots}(x_1,x_2\cdots)$ is a cut amplitude in the variable $\bar{A}^2$ and similarly for $\tilde{\Gamma}_B$ where the variable is $\bar{B}^2$. The discontinuity in the hard amplitude $\hat{S}^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}$ is taken over $\hat{s}=(x_1+x_2)(y_1+y_2)s$.

In the next section we will concentrate on the spin and color structure of the cut amplitude $\tilde{\Gamma}_{\alpha_1\alpha_2\cdots}$.

A. The spin structure of the cut amplitude $\tilde{\Gamma}_{\alpha_1\alpha_2\alpha'_2\alpha'_1}$

In order to investigate the spin structure of Eq. (2.13), one has to expand the tensor $\tilde{\Gamma}_{\alpha_1\alpha_2\alpha'_2\alpha'_1}$ in the basis of the 16-γ matrices for each pair of indices $(\alpha_1\alpha_2)$ and $(\alpha'_1\alpha'_2)$. For massless quarks and to leading order in $1/s$, we have the general expansion:

$$\tilde{\Gamma}_{\alpha_1\alpha_2\alpha'_2\alpha'_1}(x_1,x_2,p,\eta,b_T)=G_D\not{p}_{\alpha_1\alpha'_1}\not{p}_{\alpha_2\alpha'_2}+G_E\not{p}_{\alpha_1\alpha'_2}\not{p}_{\alpha_2\alpha'_1}+G_{D5}(\not{p}\gamma_5)_{\alpha_1\alpha'_1}(\not{p}\gamma_5)_{\alpha_2\alpha'_2}+G_{E5}(\not{p}\gamma_5)_{\alpha_1\alpha'_2}(\not{p}\gamma_5)_{\alpha_2\alpha'_1}+\text{higher-power corrections}\,. \tag{2.14}$$

Terms proportional to b_T as well as η terms are not leading[4] since $\dim b_T = \dim\eta = -1$. Furthermore, there is only an even number of γ_5 matrices, due to parity conservation. It might appear at first sight that the four tensors are independent. This is not so; to see it, one converts to the helicity basis using the identities

$$\not{p}_{\alpha_1\alpha_1'} = \sum_{\lambda\lambda'} \delta_{\lambda\lambda'} u_{\alpha_1}(p,\lambda)\bar{u}_{\alpha_1'}(p,\lambda')$$

and (2.15)

$$(\not{p}\gamma_5)_{\alpha_1\alpha_1'} = \sum_{\lambda\lambda'} \lambda\delta_{\lambda\lambda'} u_{\alpha_1}(p,\lambda)\bar{u}_{\alpha_1'}(p,\lambda') \, .$$

The quantum number λ is twice the helicity and takes on the values $\lambda = \pm 1$. Therefore $\widetilde{\Gamma}$ takes on the form

$$\widetilde{\Gamma}_{\alpha_1\alpha_2\alpha_2'\alpha_1'}(x_1,x_2,\dots) = \sum_{\lambda,\lambda'} (G_D \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} + G_E \delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'} + G_{D5}\lambda_1\lambda_2\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}$$
$$+ G_{E5}\lambda_1\lambda_2\delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'}) u_{\alpha_1}(x_1 p,\lambda_1) u_{\alpha_2}(x_2 p,\lambda_2)\bar{u}_{\alpha_1'}(x_1 p,\lambda_1')\bar{u}_{\alpha_2'}(x_2 p,\lambda_2') \, . \tag{2.16}$$

Now the four tensors in formula (2.16) are not all independent. They are related by the identity

$$(1+\lambda_1\lambda_2)\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} = (1+\lambda_1\lambda_2)\delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'} \, . \tag{2.17}$$

The quantities G_D, G_E, G_{D5}, G_{E5} are, however, not all positive definite. In order to write $\widetilde{\Gamma}$ in terms of well-defined structure functions, we express the above-mentioned tensors in terms of the projectors

$$P_{(|++\rangle,|--\rangle)}, \quad P_{(|+-\rangle+|-+\rangle)}, \quad P_{(|+-\rangle-|-+\rangle)} \, , \tag{2.18}$$

which are, respectively, the projectors on the subspace spanned by the states $(|++\rangle, |--\rangle)$, on the state $(|+-\rangle+|-+\rangle)/\sqrt{2}$, and on $(|+-\rangle-|-+\rangle)/\sqrt{2}$. These are the irreducible states of the tensorial space $|\lambda_1\lambda_2\rangle = |\lambda_1\rangle \otimes |\lambda_2\rangle$. The matrix elements of the above projectors are given by

$$\langle\lambda_1\lambda_2 | P_{(|++\rangle,|--\rangle)} | \lambda_1'\lambda_2'\rangle = \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\frac{(1+\lambda_1\lambda_2)}{2} \, ,$$
$$\langle\lambda_1\lambda_2 | P_{(|+-\rangle+|-+\rangle)} | \lambda_1'\lambda_2'\rangle = \frac{\delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'} - \lambda_1\lambda_2\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}}{2} \, , \tag{2.19}$$
$$\langle\lambda_1\lambda_2 | P_{(|+-\rangle-|-+\rangle)} | \lambda_1'\lambda_2'\rangle = \frac{\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} - \delta_{\lambda_1\lambda_2'}\delta_{\lambda_2\lambda_1'}}{2} \, .$$

One may invert the above expressions (2.19) and write down the tensors of Eq. (2.16) in terms of the three independent projectors. Thus one can write down the cut amplitude $\widetilde{\Gamma}_{\alpha_1\alpha_2\alpha_2'\alpha_1'}$ in terms of the above projectors,

$$\widetilde{\Gamma}_{\alpha_1\alpha_2\alpha_2'\alpha_1'}(x_1,x_2,b_T,p) = \sum_{\lambda_1\lambda'} (\tfrac{1}{2}G_1\langle\lambda_1\lambda_2 | P_{(|++\rangle,|--\rangle)} | \lambda_1'\lambda_2'\rangle + G_2\langle\lambda_1\lambda_2 | P_{(|+-\rangle+|-+\rangle)} | \lambda_1'\lambda_2'\rangle$$
$$+ G_3\langle\lambda_1\lambda_2 | P_{(|+-\rangle-|-+\rangle)} | \lambda_1'\lambda_2'\rangle) u_{\alpha_1} u_{\alpha_2} \bar{u}_{\alpha_1'} \bar{u}_{\alpha_2'} \, . \tag{2.20}$$

The G_α's ($\alpha = 1,2,3$) are the probabilities to find two quarks within the proton in the helicity state $|\alpha\rangle$ corresponding to the projector $P_{(\alpha)}$. The normalization factor $\frac{1}{2}$ comes from $\mathrm{Tr}P_{(|++\rangle,|--\rangle)} = 2$. The structure functions G_α's being now properly defined, one can use the matrix elements (2.19) and perform the summation over the helicities to recover a well-defined expansion in $\not{p}$ and $\not{p}\gamma_5$. Thus we arrive at the final expansion of the tensor $\widetilde{\Gamma}_{\alpha_1\alpha_2}\dots$ on the physical basis for Dirac indices:

$$\widetilde{\Gamma}_{\alpha_1\alpha_2\alpha_2'\alpha_1'}(x_1,x_2,b_T,p) = \{ \tfrac{1}{4}G_1[\not{p}_{\alpha_1\alpha_1'}\not{p}_{\alpha_2\alpha_2'} + (\not{p}\gamma_5)_{\alpha_1\alpha_1'}(\not{p}\gamma_5)_{\alpha_2\alpha_2'}]$$
$$+ \tfrac{1}{2}G_2[\not{p}_{\alpha_1\alpha_2'}\not{p}_{\alpha_2\alpha_1'} - (\not{p}\gamma_5)_{\alpha_1\alpha_1'}(\not{p}\gamma_5)_{\alpha_2\alpha_2'}] + \tfrac{1}{2}G_3(\not{p}_{\alpha_1\alpha_1'}\not{p}_{\alpha_2\alpha_2'} - \not{p}_{\alpha_1\alpha_2'}\not{p}_{\alpha_2\alpha_1'})\} \, . \tag{2.21}$$

Now going back to the formula for the cross section σ_D (2.13) and inserting the expansion (2.21) we get an expression which can be written in a matrix form:

$$\sigma_D = \int dx_1 dx_2 dy_1 dy_2 d^2b_T G_A^\dagger(x_1,x_2,b_T)\widetilde{\sigma} G_B(y_1,y_2,b_T) \, , \tag{2.22}$$

where $G_A^\dagger \equiv (G_{A_1}, G_{A_2}, G_{A_3})$, similarly for G_B and $\widetilde{\sigma}$ is a three-by-three matrix

$$\widetilde{\sigma} = \begin{bmatrix} \widetilde{\sigma}_{11} & \widetilde{\sigma}_{12} & \widetilde{\sigma}_{13} \\ \widetilde{\sigma}_{21} & \widetilde{\sigma}_{22} & \widetilde{\sigma}_{23} \\ \widetilde{\sigma}_{31} & \widetilde{\sigma}_{32} & \widetilde{\sigma}_{33} \end{bmatrix} \, . \tag{2.23}$$

The components $\widetilde{\sigma}_{\alpha\alpha'}$ are linear combinations of elementary cross sections $\sigma_{DD}, \sigma_{EE}, \sigma_{DE}, \sigma_{ED}$, for instance,

$$\tilde{\sigma}_{11}=2\sigma_{DD} ,$$
$$\tilde{\sigma}_{22}=4(\sigma_{DD}+\sigma_{EE}-\sigma_{ED}-\sigma_{DE}) , \quad (2.24)$$
$$\tilde{\sigma}_{12}=2(2\sigma_{DE}-\sigma_{DD}) .$$

The cross sections σ_{KL} $(K,L=D,E)$ are the elementary cross sections associated with definite helicity diagrams. The notation D (direct) and E (exchange) refer to whether the incoming quarks a_1,a_2,b_1,b_2 in Fig. 1 keep their initial helicities or exchange them after the hard interaction (the same notation will be used for color). We sketch, in Fig. 3, for a particular color projection, the four helicity diagrams corresponding to different helicity projections. In these diagrams the loops with arrows indicate the helicity flow.

B. The color structure of the cut amplitude $\tilde{\Gamma}^{i_1i_2i_2'i_1'}$

Here the indices (i) are color indices and Dirac indices are understood. The cut amplitude has the following form,

$$\tilde{\Gamma}^{i_1i_2i_2'i_1'}=\sum_n \bar{\psi}^{(n)}_{i_1'i_2'}\psi^{(n)}_{i_1i_2}$$
$$=\sum_n \langle p,\bar{n}\,|\,i_1'i_2'\rangle\langle i_1i_2\,|\,p,\bar{n}\rangle . \quad (2.25)$$

In this notation, we keep only the color indices and omit all the remaining quantum numbers. n refers to the spectator system and $\bar{n}$ is its charge conjugate. p refers to the incoming hadron A or B.

The quarks belong to the three-dimensional representation of SU(3) and since $3\otimes3=\bar{3}\oplus6$, one can expand the two-quark state $|\,i_1i_2\rangle$ in the irreducible basis $|\,\bar{3}\rangle,|\,6\rangle$, i.e.,

$$|\,i_1i_2\rangle=\sum_{a=\bar{3},6}\langle a\,|\,i_1i_2\rangle\,|\,a\rangle . \quad (2.26)$$

Therefore the cut amplitude takes on the following form

$$\tilde{\Gamma}^{i_1i_2i_2'i_1'}=\sum_{a=\bar{3},6}\tilde{G}^{a}\langle i_1i_2\,|\,a\rangle\langle a\,|\,i_1'i_2'\rangle . \quad (2.27)$$

Since $\tilde{\Gamma}^{i_1i_2i_2'i_1'}$ is color singlet, only the projectors on the states $|\,\bar{3}\rangle$ and $|\,6\rangle$ appear in the sum. To rewrite $\tilde{\Gamma}$ with normalized G's we divide the projectors by their traces. Thus we arrive at

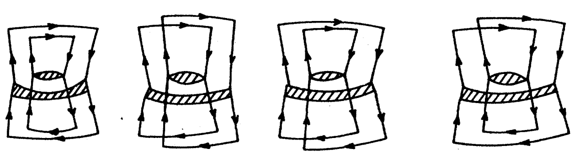

FIG. 3. The four helicity projections defining the cross sections σ^{IJ}_{KL}, $a=\sigma^{IJ}_{DD}$, $b=\sigma^{IJ}_{EE}$, $c=\sigma^{IJ}_{DE}$, $d=\sigma^{IJ}_{ED}$ where the loops indicate the helicity flow.

$$\tilde{\Gamma}^{i_1i_2i_2'i_1'}=\frac{2}{N^2-N}G^1\langle i_1i_2\,|\,P_{(\bar{3})}\,|\,i_1'i_2'\rangle$$
$$+\frac{2}{N^2+N}G^2\langle i_1i_2\,|\,P_{(6)}\,|\,i_1'i_2'\rangle \quad (2.28)$$

with N the number of color, $N=3$ for SU(3).

Here G^1 and G^2 stand, respectively, for the probabilities to find two quarks within the proton in the color states $|\,\bar{3}\rangle$ and $|\,6\rangle$ and $P_{(\bar{3})},P_{(6)}$ are their corresponding projectors. Their matrix elements are given by the following expressions

$$\langle i_1i_2\,|\,P_{(\bar{3})}\,|\,i_1'i_2'\rangle=\frac{\delta_{i_1i_1'}\delta_{i_2i_2'}-\delta_{i_1i_2'}\delta_{i_2i_1'}}{2} ,$$
$$\langle i_1i_2\,|\,P_{(6)}\,|\,i_1'i_2'\rangle=\frac{\delta_{i_1i_1'}\delta_{i_2i_2'}+\delta_{i_1i_2'}\delta_{i_2i_1'}}{2} . \quad (2.29)$$

Now putting the spin and the color all together we get six independent structure functions $G^i_\alpha(x_1,x_2,b_T)$ $(i=1,2;\ \alpha=1,2,3)$ which are the probabilities to find two quarks within the proton in the helicity states $(|++\rangle$ or $|--\rangle)$, $(|+-\rangle+|-+\rangle)/\sqrt{2}$ or $(|+-\rangle-|-+\rangle)/\sqrt{2}$ and in the color state $|\,\bar{3}\rangle$ or $|\,6\rangle$.

Finally the cross section σ_D takes on the form

$$\sigma_D=\int dx_1dx_2dy_1dy_2d^2b_TG^{\dagger}_A(x_1,x_2,b_T)\tilde{\sigma}G_B(y_1,y_2,b_T) \quad (2.30)$$

with $G^{\dagger}_A\equiv(G^1_{A_1},G^2_{A_1},G^1_{A_2},G^2_{A_2},G^1_{A_3},G^2_{A_3})$ and $\tilde{\sigma}^{ii'}_{\alpha\alpha'}$ $(\alpha,\alpha'=1,2,3;\ i,i'=1,2)$ is a six-by-six matrix generalizing that of Eq. (2.23). The upper indices in both $\tilde{\sigma}^{ii'}_{\alpha\alpha'}$ and G^i_α refer to color and the lower ones to spin. These cross sections are linear combinations of the elementary cross section σ^{IJ}_{KL} $(I,J,K,L=D,E)$ generalizing those introduced in Eq. (2.24). They correspond to definite color and helicity diagrams. For a given helicity flow, e.g., σ^{IJ}_{DD}; these cross sections are sketched in Fig. 3 where now the loops indicate the color flow.

So far the spin and color analysis leading to formula (2.30) is quite general and $\tilde{\sigma}$ may describe any hard disconnected process (e.g., double Drell-Yan, double scattering and so on), so there is a set of structure functions and a set of hard cross sections to compute in order to write down a hadronic cross section. Until now all the attempts to evaluate the double scattering cross section limited themselves to the direct term and this amounts to consider only the component σ^{DD}_{DD} represented in Fig. 3(a), whereas all the exchange terms represented by the remaining components are neglected. This is valid of course only for an order-of-magnitude estimate. We will see in the last section that the basis in spin and color on which we have expanded $\tilde{\Gamma}$ diagonalizes the matrix $\tilde{\sigma}$ for the double Drell-Yan process. We will also show that all the components σ^{IJ}_{KL} can be expressed in terms of the direct one, i.e., σ^{DD}_{DD}, and furthermore that σ^{DD}_{DD} factorizes as $\sigma^{DD}_{DD}=\sigma_1\sigma_2$ where σ_1 and σ_2 are the cross sections for two uncorrelated single Drell-Yan mechanisms.

III. PROPERTIES OF CONNECTING GLUONS (C)

At any order in α_s, say, $(\alpha_s)^n$ a diagram with only NC gluons gives the same correction as in single parton processes. Therefore the collinear NC gluons yield a logarithmic term of the form[5] $(\alpha_s \ln Q)^n$ in the LLA. In fact this is manifest, for instance, in the planar gauge of Ref. 5 where only ladder-type diagrams contribute at the LLA, and where the integration over the transverse momenta of the gluons gives a factor

$$\alpha_s{}^n I_{\rm NC}(Q^2) \propto \alpha_s{}^n \int^{Q^2} \frac{d^2k_{1T}}{k_{1T}{}^2} \cdots \frac{d^2k_{nT}}{k_{nT}{}^2} \propto (\alpha_s \ln Q^2)^n \tag{3.1}$$

(Q being a mass scale of the hard process).

The purpose of this section is to show that collinear C gluons instead yield no $\ln Q$ factor and therefore are not leading with respect to NC gluons.

First we note an important "kinematical" constraint, namely, that in a given diagram the *sum* of the transverse momenta of C gluons

$$k_T = \sum_{i=1}^{n} k_{Ti} \tag{3.2}$$

has a cutoff of the order of $1/R$ irrespective of the number of these gluons (R is the hadronic size; $R \simeq 1$ fm). As can be seen for instance by inspection of Fig. 2(b), momentum conservation requires

$$k_T = a_{1T} + b_{1T} - a'_{1T} - b'_{1T} \ . \tag{3.3}$$

Therefore, since $a_{1T}, a'_{1T}, b_{1T}, b'_{1T}$ are bounded by the hadronic wave functions (Γ_A and Γ_B in Fig. 1), k_T must be of the order of $1/R$. This constraint reduces the available phase space of the C gluons and this is crucial for the following. The possible leading contribution of C gluons to the amplitude comes from the collinear ones. At order $(\alpha_s)^n$ it is of the form

$$\alpha_s{}^n I_{\rm C}(Q^2) \propto \int_{\lambda^2}^{Q^2} \frac{d^2k_{1T}}{k_{1T}{}^2} \cdots \frac{d^2k_{nT}}{k_{nT}{}^2} \exp\left[-b\left[\sum_{i=1}^{n} k_{Ti}\right]^2\right] \times \alpha_s{}^n \ . \tag{3.4}$$

In formula (3.4) we have introduced an infrared cutoff ($\lambda \simeq R^{-1}$) to decouple long-wavelength gluons. The Gaussian form in the integrand comes from the hadronic vertices in Fig. 1 and accounts for the condition (3.2). The choice of a Gaussian form for the softness of the wave function is a matter of convenience. It is shown in the Appendix that

$$I_{\rm C}(Q^2) \underset{Q^2 \to \infty}{\sim} \text{const} \ . \tag{3.5}$$

That is to say, the contribution of C gluons yield no $\ln Q$ factor. Therefore owing to the results (3.1) and (3.5), the diagrams where one or more C gluons are exchanged are not leading, order by order, with respect to those which contain *no* C gluons.

Thus we can make the following conclusion: any multiparton process which is disconnected at the parton level, remains disconnected to any leading order in α_s. Therefore if there is any violation of factorization, it cannot come from connecting gluons. The question of whether or not nonconnecting gluons will spoil factorization[6] will be the object of a forthcoming study.

For the moment we conjecture that factorization does hold and that the structure functions of formula (2.30) should be replaced by evolving structure functions $G^i_\alpha(x_1,x_2,b_T,Q_1,Q_2)$, where Q_1 and Q_2 are typical masses associated to the two disconnected subprocesses. For the double Drell-Yan mechanism Q_1 and Q_2 stand for the masses of the lepton pairs.

IV. APPLICATION TO THE DOUBLE DRELL-YAN PROCESS

We first generalize to the double Drell-Yan process the single Drell-Yan formula which relates the cross section $d\sigma/dQ$ to $\hat{\sigma}$, $\hat{\sigma}$ standing for the annihilation cross section of the partons into a photon of mass Q.

For the case of two distinct lepton pairs, e.g., $(e^+e^-, \mu^+\mu^-)$ (Fig. 4) we get the simple generalization

$$\frac{d\sigma}{dQ_1{}^2 dQ_2{}^2} = \left[\frac{\alpha}{3\pi}\right]^2 \frac{1}{Q_1{}^2 Q_2{}^2} \times \int G_A^\dagger(x_1,x_2,b_T,Q_1,Q_2)\tilde{\sigma}_{(\rm Born)} \times G_B(y_1,y_2,b_T,Q_1,Q_2)dx_1dx_2dy_1dy_2 \ . \tag{4.1}$$

(This formula is valid under our conjecture of factorization stated above.) In Eq. (4.1), the matrix $\tilde{\sigma}$ [cf. formula (2.30)] is the disconnected cross section to produce two photons of masses Q_1 and Q_2. The factor $(\alpha/3\pi)^2/Q_1{}^2Q_2{}^2$ comes from integration over angular distributions of the two lepton pairs. $\tilde{\sigma}_{(\rm Born)}$ is a linear combination of $\sigma^{IJ}_{KL(\rm Born)}$ $(I,J,K,L = D,E)$ which have been introduced in Sec. II and represented in Fig. 3.

At first sight, one might think that there is no simple relations between the various elementary cross sections σ^{IJ}_{KL}. This is, however, not true. In fact, we have the relationship

$$\sigma^{IJ}_{KL(\rm Born)} = W^{IJ}_{KL}\,\sigma^{DD}_{DD(\rm Born)} \ , \tag{4.2}$$

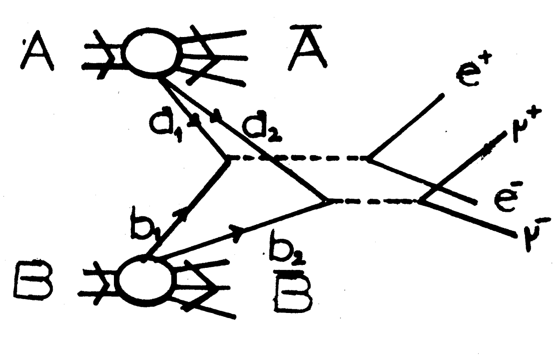

FIG. 4. The double Drell-Yan process with two different pairs (e^+e^-) and $(\mu^+\mu^-)$.

where W_{KL}^{IJ} are simple numerical constants. The matrix W has the form

$$W=\begin{bmatrix} 1 & \frac{1}{N} & \frac{1}{2} & \frac{1}{2N} \\ \frac{1}{N} & 1 & \frac{1}{2N} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2N} & \frac{1}{2} & \frac{1}{2N} \\ \frac{1}{2N} & \frac{1}{2} & \frac{1}{2N} & \frac{1}{2} \end{bmatrix} . \tag{4.3}$$

To work out W, one computes the components $\sigma_{KL(\mathrm{Born})}^{IJ}$. They involve the following traces (we omit the subscript Born in the following)

$$\begin{aligned} &\sigma_{DD}^{IJ} \propto C^{IJ}(\mathrm{Tr}\gamma_\mu \not{p}_A \gamma^\mu \not{p}_B)(\mathrm{Tr}\gamma_\nu \not{p}_A \gamma^\nu \not{p}_B) , \\ &\sigma_{EE}^{IJ} \propto C^{IJ}(\mathrm{Tr}\gamma_\mu \not{p}_A \gamma^\nu \not{p}_B)(\mathrm{Tr}\gamma_\nu \not{p}_A \gamma^\mu \not{p}_B) , \\ &\sigma_{ED}^{IJ}=\sigma_{DE}^{IJ} \propto C^{IJ}\mathrm{Tr}\gamma_\mu \not{p}_A \gamma^\mu \not{p}_B \gamma_\nu \not{p}_A \gamma^\nu \not{p}_B , \end{aligned} \tag{4.4}$$

where the coefficients C^{IJ} have the values

$$\begin{aligned} &C^{EE}=C^{DD}=N^2 , \\ &C^{ED}=C^{DE}=N . \end{aligned} \tag{4.5}$$

The above traces are easy to compute, and give the following relations

$$\begin{aligned} &\sigma_{DE}^{IJ}=\sigma_{ED}^{IJ}=\sigma_{EE}^{IJ}=\frac{\sigma_{DD}^{IJ}}{2}, \quad I,J=D,E , \\ &\sigma_{KL}^{DE}=\sigma_{KL}^{ED}=\frac{\sigma_{KL}^{DD}}{N}, \quad \sigma_{KL}^{EE}=\sigma_{KL}^{DD}, \quad K,L=D,E . \end{aligned} \tag{4.6}$$

Thus we succeed in writing, for the double Drell-Yan process, all the color and spin projections σ_{KL}^{IJ} in terms of only one, i.e., σ_{DD}^{DD}. Moreover, since connecting gluons are neglected σ_{DD}^{DD} factorizes as

$$\sigma_{DD}^{DD}=\sigma_1\sigma_2 , \tag{4.7}$$

where σ_1 and σ_2 are two uncorrelated single Drell-Yan cross sections which are computed in the usual way.

To compute the hadronic cross section Eq. (4.1) we come back to $\tilde{\sigma}_{\alpha\alpha'}^{ii'}$ which are linear combinations of σ_{KL}^{IJ} whose coefficients are fixed by the Γ expansions Eq. (2.21) and Eq. (2.28); an example has been given in the case of spin in Eq. (2.24). Using the matrix W, we can rewrite $\tilde{\sigma}_{\alpha\alpha'}^{ii'}$ in terms of $\sigma_1\sigma_2$ only and find that $\tilde{\sigma}_{\alpha\alpha'}^{ii'}$ is a diagonal matrix in the physical basis we have chosen for the expansion of the Γ's,

$$\tilde{\sigma}_{\alpha\alpha'}^{ii'}=C^i\delta_{ii'}\delta_{\alpha\alpha'}\sigma_1\sigma_2 \qquad (i=1,2;\alpha=1,2,3;\ \text{no sum over } i) \tag{4.8}$$

with

$$\begin{aligned} &C^1=\frac{4N}{N-1} , \\ &C^2=\frac{4N}{N+1} . \end{aligned} \tag{4.9}$$

Going back to formula (4.1) the cross section for the double Drell-Yan process takes on the final and very simple form

$$\frac{d\sigma}{dQ_1{}^2dQ_2{}^2}=\left[\frac{\alpha}{3\pi}\right]^2\frac{1}{Q_1{}^2Q_2{}^2}\sum_{\substack{i=1,2\\ \alpha=1,2,3}}\int C^iG_{A\alpha}^i(x_1,x_2,b_T,Q_1,Q_2)G_{B\alpha}^i(y_1,y_2,b_T,Q_1,Q_2) \times\sigma_{1(\mathrm{Born})}\sigma_{2(\mathrm{Born})}dx_1dx_2dy_1dy_2d^2b_T . \tag{4.10}$$

V. CONCLUSION

In this paper we have analyzed disconnected multiparton processes in the context of the QCD parton model. Particular attention has been given to the spin and color structure, as well as to the factorizability of the process. We have shown that connecting gluons which apparently lead to violation of factorization have in fact no influence at the leading-logarithm approximation. We have also shown that the spin and color degrees of freedom introduce six newly defined structure functions $G_\alpha^i(x_1,x_2,b_T)$. These are the probabilities to find the diquark system in the color representation i and in the helicity state α. The application of our formalism to the double Drell-Yan process has led us to write the cross section in a very simple and diagonal form: each G_α^i of hadron A is convoluted with the corresponding G_α^i of hadron B times the two completely uncorrelated single Drell-Yan subprocesses.

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APPENDIX

We propose to show the asymptotic behavior [cf. formula (3.5)]. This amounts to showing that the following integral converges at infinity:

$$I=\int_0^\infty \frac{d^2k_{1T}}{k_{1T}^2+\lambda^2}\cdots\frac{d^2k_{nT}}{k_{nT}^2+\lambda^2}\exp\left[-b\left[\sum_{i=1}^n k_{iT}\right]^2\right]. \tag{A1}$$

(We will omit the subscript T in what follows.) Let us write

$$\frac{1}{k^2+\lambda^2}=\int_0^\infty e^{-t(k^2+\lambda^2)}dt\ . \tag{A2}$$

Therefore I takes on the form

$$I=\int_0^\infty dt_1\cdots dt_n\exp\left[-\lambda^2\sum_{i=1}^n t_i\right]\times\int_0^\infty d^2k_1\cdots d^2k_{1n}\times\exp\left[-\sum_{i=1}^n t_ik_i^2-b\left[\sum_{i=1}^n k_i\right]^2\right]. \tag{A3}$$

Using the x and y components of k_i, i.e.,

$$k_i^2=k_{ix}^2+k_{iy}^2\ , \tag{A4}$$

we arrive at

$$I=\int_0^\infty dt_1\cdots dt_n\exp\left[-\lambda^2\sum_{i=1}^n t_i\right]\left\{\int_0^\infty dk_{ix}\cdots dk_{nx}\exp\left[-\sum_i(t_i+b)k_{ix}^2-b\sum_{\substack{i,j\\ i\neq j}}k_{ix}k_{jx}\right]\right\}^2. \tag{A5}$$

The Gaussian integral in curly brackets is easy to compute:

$$\int_0^\infty dk_{1x}\cdots dk_{nx}\exp\left[-\sum(t_i+b)k_{ix}^2-b\sum_{\substack{i,j\\ i\neq j}}k_{ix}k_{jx}\right]\propto\det{}^{-1/2}A(t_1,t_2,\ldots,t_n,b)\ , \tag{A6}$$

where

$$\det A(t_1,t_2,\ldots,t_n,b)=\begin{vmatrix} t_1+b & b & \cdots & b\\ b & t_2+b & \cdots & b\\ \vdots & \vdots & \ddots & \vdots\\ b & b & \cdots & t_n+b\end{vmatrix}=(t_1t_2\cdots t_n)\left[1+b\left[\frac{1}{t_1}+\cdots+\frac{1}{t_n}\right]\right]. \tag{A7}$$

Inserting the result (A7) into formula (A5) we get

$$I=\int_0^\infty\frac{dt_1\cdots dt_n\exp\left[-\lambda^2\sum t_i\right]}{t_1\cdots t_n\left[1+b\left[\frac{1}{t_1}+\cdots+\frac{1}{t_n}\right]\right]}\lesssim\int_0^{\lambda^{-2}}\frac{dt_1\cdots dt_n}{t_1\cdots t_n\left[1+b\left[\frac{1}{t_1}+\cdots+\frac{1}{t_n}\right]\right]}\ . \tag{A8}$$

According to formula (A2) we are now interested in the small-t_i region where I has the form

$$I\sim\int_0^{\lambda^{-2}}\frac{dt_1\cdots dt_n}{t_1\cdots t_n\left[\frac{1}{t_1}+\cdots+\frac{1}{t_n}\right]}\ . \tag{A9}$$

Introducing the extra variable $T=\sum_{i=1}1/t_i$ with $T>n\lambda^2$ since $1/t_i>\lambda^2$,

$$I\sim\int_{n\lambda^2}^\infty\frac{dT}{T}\int_0^{\lambda^{-2}}\frac{dt_1\cdots dt_n}{t_1\cdots t_n}\delta\left[T-\sum_{i=1}^n\frac{1}{t_i}\right]$$

$$=\int_{n\lambda^2}^\infty\frac{dT}{T^2}\int_0^{\lambda^{-2}}\frac{dt_1\cdots dt_n}{t_1\cdots t_n}\delta\left[1-\sum_{i=1}^n\frac{1}{Tt_i}\right]. \tag{A10}$$

putting $z_i=1/Tt_i$, we have

$$\frac{\lambda^2}{T}<z_i<1\ . \tag{A11}$$

The integral I takes the final form

$$I\sim\int_{n\lambda^2}^\infty\frac{dT}{T^2}\int_{\lambda^2/T}^1\frac{dz_1\cdots dz_n}{z_1\cdots z_n}\delta\left[1-\sum_{i=1}^n z_i\right]$$

$$=\int_{n\lambda^2}^\infty\frac{dT}{T^2}\int_{\lambda^2/T}^1\frac{dz_1\cdots dz_{n-1}\theta(1-z_1-\cdots-z_{n-1})}{z_1\cdots z_{n-1}(1-z_1-\cdots-z_{n-1})}$$

$$\sim n\times\int_{n\lambda^2}^\infty\frac{dT}{T^2}\int_{\lambda^2/T}^1\frac{dz_1\cdots dz_{n-1}}{z_1\cdots z_{n-1}}\ , \tag{A12}$$

since only the small-z_i regions are of interest. Therefore we get the final result

$$I\sim\int_{n\lambda^2}^\infty\frac{dT}{T^2}\ln^{n-1}\left[\frac{T}{\lambda^2}\right], \tag{A13}$$

which is convergent at infinity due to the denominator.

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