

Bound heavy- and light-quark systems in a non-Coulombic logarithmic-potential model

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A previous fit of the charmonium and Υ spectra by a simple non-Coulombic logarithmic potential is modified to include systems containing up or down and strange quarks. It is found that the spectra of both light and heavy mesons, particularly, the $1P$ and $2P$ state splittings of the Υ system can be satisfactorily explained without taking into account the short-distance Coulombic part of the potential as suggested by QCD.

I. INTRODUCTION

The nonrelativistic-potential-model approach has been notably successful in describing meson spectroscopy, even if one does not yet fully understand why this is so. Various theoretical potentials have been used to describe the $c\bar{c}$ and $b\bar{b}$ systems.¹ Purely phenomenological potentials such as

$$V(r) = V_0 + ar^\nu, \quad \nu = 0.1, \quad (1.1)$$

having nonsingular behavior for $r \rightarrow 0$, have also been extensively used by Martin and others² to reproduce the most up-to-date data on $c\bar{c}$, $b\bar{b}$, and even $s\bar{s}$ systems. A similar type of phenomenological potential of the form

$$V(r) = V_0 + a \ln(1 + r/r_0), \quad (1.2)$$

with $a > 0$ and $r_0 = 1$ fm, has been used in a recent paper³ to obtain a simultaneous fit of the spectra of charmonium and Υ systems. It has been found that the spin-averaged masses, fine-hyperfine splittings, and leptonic-decay-width ratios of these spectra can be well understood by the model (1.2) without taking into account the short-distance part of the potential as suggested by QCD [i.e., r^{-1} or $r^{-1} |\ln(r/r_0)|^{-1}$]. The potentials (1.1) and (1.2) essentially agree for distances $0.1 < r < 1$ fm, corresponding to the mean radii of the well known quarkonium levels and this is why both the potentials give a very good fit of all the levels of the $c\bar{c}$ and $b\bar{b}$ systems.

The potential (1.2) has already been investigated for the charmonium spectrum by Machacek and Tomozawa.⁴ Although work in this direction using an empirically similar potential of type (1.1) has also been done in Ref. 2, the present approach is yet another way of parameterizing the light- and heavy-meson spectra. In view of the remarkable fits obtained with this model, it is now tempting to test the applicability of this potential in the study of lighter-quark systems and see if indeed the spectra of these systems can be explained without considering the short-range Coulomb-type behavior of the quark-antiquark potential in accordance with QCD predictions. Moreover, in light of some recent experiments giving new information on the $1P$ and $2P$ state splittings of the Υ system,⁵ it is worthwhile to have a fresh look at this potential.

The present work aims to modify the previous nonrelativistic fit of the ψ and Υ levels of Ref. 3 to include systems containing light quarks. We are mainly interested in obtaining a nonrelativistic fit of the spectra of bound light- and heavy-quark systems in a unified manner with the potential given by Eq. (1.2). In fact, the usual nonrelativistic Schrödinger-type approach for heavy quarkonia is justified on the basis of the large quark masses involved. But the same approach may be unsuitable for the ordinary light-quark systems due to relativistic effects, expected to be significant in these cases. On the other hand, the relativistic generalizations attempted in some limited senses by some authors⁶ are by no means simple and straightforward. Therefore, nonrelativistic potential-model studies are often extended⁷ to include light hadrons, which gives, if not a quantitative, at least a qualitative understanding of their spectra. With this contention in mind, we feel that a unified nonrelativistic approach to the study of the light and heavy mesons taken together is possible, which would provide a comprehensive picture of the applicability of the simple phenomenological non-Coulombic logarithmic potential model. The light mesons which we select to study in the present work are ρ^0 and ϕ systems corresponding to the like-flavored quark-antiquark configurations $(1/\sqrt{2})(u\bar{u} - d\bar{d})$ and $s\bar{s}$, respectively. Here we also concentrate on a group of non-self-conjugate quark-antiquark configurations such as D ($c\bar{u}$ or $c\bar{d}$), F ($c\bar{s}$), B ($b\bar{u}$ or $b\bar{d}$), G ($b\bar{s}$), and H ($b\bar{c}$) mesons and study their ground-state hyperfine mass splittings.

II. THEORY

A solution to the Schrödinger equation with the potential (1.2) would lead to the spin-averaged masses $M_{nL}(q_1\bar{q}_2)$ for different mesonic bound states. But the quantitative explanation of the fine-hyperfine levels corresponding to the mesonic states depends on the spin structure of the static quark-antiquark potential. If we regard the potential (1.2) to be an admixture of scalar and vector components with vector fraction f , then with a nonzero quark anomalous magnetic moment g the spin-dependent correction terms generated by this potential can be written in the usual manner as⁸

$$\delta V(\mathbf{r}) = A_1(r)\mathbf{L}\cdot\mathbf{S} + A_2(r)\mathbf{S}_1\cdot\mathbf{S}_2 + A_3(r)S_{12}, \quad (2.1)$$

where \mathbf{L} and \mathbf{S} are total orbital and spin angular momenta, S_{12} is the tensor interaction, and $A_1(r)$, $A_2(r)$, and $A_3(r)$ are the radially dependent potential functions which also depend on the way the spin forces are generated. These functions are obtained easily through standard reduction formulas as

$$\begin{aligned} A_1(r) &= \frac{a}{2m_{q_1}m_{q_2}} [4f(1+g) - 1] B_1(r), \\ A_2(r) &= \frac{2a}{3m_{q_1}m_{q_2}} [(1+g)^2 f] [2B_1(r) - B_2(r)], \\ A_3(r) &= \frac{a}{3m_{q_1}m_{q_2}} [(1+g)^2 f] [B_1(r) + B_2(r)], \end{aligned} \quad (2.2)$$

where

$$B_1(r) = \frac{1}{r(1+r)} \quad \text{and} \quad B_2(r) = \frac{1}{(1+r)^2}. \quad (2.3)$$

Now using a first-order perturbation approach for the spin-dependent correction term $\delta V(\mathbf{r})$, the fine-hyperfine level splittings can be calculated to give the following mass formulas for different orbital states:

(i) For S states:

$$\begin{aligned} M(n^3S_1) &= M_{nS} + \frac{1}{4} \langle A_2(r) \rangle_{nS}, \\ M(n^1S_0) &= M_{nS} - \frac{3}{4} \langle A_2(r) \rangle_{nS}. \end{aligned} \quad (2.4)$$

(ii) For P states:

$$\begin{aligned} M(n^3P_0) &= M_{nP} + \langle A_1(r) \rangle_{nP} \\ &\quad + \frac{1}{4} \langle A_2(r) \rangle_{nP} - \frac{1}{10} \langle A_3(r) \rangle_{nP}, \\ M(n^3P_1) &= M_{nP} - \langle A_1(r) \rangle_{nP} \\ &\quad + \frac{1}{4} \langle A_2(r) \rangle_{nP} + \frac{1}{2} \langle A_3(r) \rangle_{nP}, \\ M(n^3P_2) &= M_{nP} - 2 \langle A_1(r) \rangle_{nP} \\ &\quad + \frac{1}{4} \langle A_2(r) \rangle_{nP} - \langle A_3(r) \rangle_{nP}, \\ M(n^1P_1) &= M_{nP} - \frac{3}{4} \langle A_2(r) \rangle_{nP}. \end{aligned} \quad (2.5)$$

Here M_{nL} are the spin-averaged masses for the L -orbital states of the quarkonium obtained from the exact numerical solution to the Schrödinger equation with the static potential given in Eq. (1.2) and $\langle A_1(r) \rangle_{nL}$, $\langle A_2(r) \rangle_{nL}$, and $\langle A_3(r) \rangle_{nL}$ are the corresponding expectation values of the potential functions. Hence, the knowledge of these

expectation values would enable one to determine the fine-hyperfine splittings of the mesons for their S and P states. From (2.2) it is clear that these values depend on the expectation value $\langle B_1(r) \rangle_{nL}$ and $\langle B_2(r) \rangle_{nL}$ and also on the parameters g and f .

III. RESULTS AND DISCUSSION

In our phenomenological analysis we take

$$V(r) = -2.03 + 1.067 \ln(1+r) \quad (\text{in GeV}) \quad (3.1)$$

and $m_c = 1.863$ GeV, $m_b = 5.196$ GeV, $m_s = 0.649$ GeV, $m_u = m_d = 0.293$ GeV to compute the Schrödinger spin-averaged mass spectra for both light and heavy mesons. However, for the light meson $\rho^0(770)$ considered to be a $q\bar{q}$ configuration such as $(u\bar{u} - d\bar{d})/\sqrt{2}$ we take the effective quark mass $m_q = 0.3$ GeV.

The results for the spin-averaged Schrödinger mass spectra for ρ^0 , ϕ , ψ , and Υ systems are presented in Table I. A definite quantitative comparison of these results with the experimental values cannot be made unless the fine-hyperfine splittings are computed. However, looking at the mean mass values obtained here we can notice a very good qualitative agreement with the experimental mass spectra. There is some new experimental material on the ϕ system. The data from e^+e^- annihilation (DMI at DCI), and from 20–70-GeV photon-beam-energy photoproduction (WA4 and WA57 at CERN),⁹ suggest the first radial excitation of ϕ called ϕ' at 1.68 GeV. The second radial excitation ϕ'' is experimentally obscure, with a possible candidate¹⁰ at ~ 1.9 GeV. New evidence for the old E meson,¹¹ which may be interpreted as $1P$ state of ϕ has been obtained both in hadron collisions (Ref. 12) and in radiative decay of the J/ψ (Ref. 13) with a mass 1.44 GeV. Our theoretical calculation gives the $2S$, $3S$, and $1P$ levels for ϕ at mass values 1.67, 2.08, and 1.47 GeV, respectively. For the radially excited states of the light meson ρ^0 , the experimental situation is not clear yet. There have been experiments⁹ in favor of ρ' (1570), whereas some other experiments¹⁴ at the same time claim good evidence for a ρ' (1250) as the $2S$ excited states of ρ^0 . But from our calculation we find the $2S$ and $3S$ levels for ρ^0 at mass value 1321 MeV and 1741 MeV, respectively. However, we would not like to attach too much quantitative significance to these results, since for the excited states of these light mesons, convincing experimental data are not yet available.

Now within the same framework as mentioned in (3.1) we compute the spin-averaged ground-state mass values of $D(c\bar{u})$, $F(c\bar{s})$, and $B(b\bar{u})$ mesons as

TABLE I. Spin-averaged mass spectra for ρ^0 , ϕ , ψ , and Υ systems (in GeV units).

n	L	$M_{nL}(\rho^0)$	$M_{nL}(\phi)$	$M_{nL}(\psi)$	$M_{nL}(\Upsilon)$
1	S	0.6125	1.004	3.067	9.453
2	S	1.321	1.673	3.669	9.970
3	S	1.741	2.075	4.039	10.304
4	S	2.041	2.365	4.309	10.554
1	P	1.111	1.469	3.469	9.795
2	P	1.597	1.933	3.899	10.174
1	D	1.453	1.793	3.761	10.049

TABLE II. The values $\langle B_1(r) \rangle_{nL}$ and $\langle B_2(r) \rangle_{nL}$ and the corresponding spin-averaged masses M_{nL} for $c\bar{c}$ and $b\bar{b}$ systems.

Bound state		$c\bar{c}$ systems			$b\bar{b}$ systems		
n	L	M_{nL} (GeV)	$\langle B_1(r) \rangle_S$ (GeV ²)	$\langle B_2(r) \rangle_S$ (GeV ²)	M_{nL} (GeV)	$\langle B_1(r) \rangle_S$ (GeV ²)	$\langle B_2(r) \rangle_S$ (GeV ²)
1	S	3.067	0.360	0.171	9.453	0.659	0.259
2	S	3.669	0.183	0.088	9.970	0.339	0.140
3	S	4.039	0.120	0.057	10.304	0.229	0.095
4	S	4.309	0.088	0.042	10.554	0.171	0.071
1	P	3.469	0.134	0.087	9.795	0.273	0.152
2	P	3.899	0.084	0.054	10.174	0.177	0.099

$$\begin{aligned}
 M_{1S}(c\bar{u})_{\text{calc}} &= 1.970 \text{ GeV}, \\
 M_{1S}(c\bar{u})_{\text{expt}} &= 1.970 \text{ GeV}, \\
 M_{1S}(c\bar{s})_{\text{calc}} &= 2.102 \text{ GeV}, \\
 M_{1S}(c\bar{s})_{\text{expt}} &= 2.113 \text{ GeV}, \\
 M_{1S}(b\bar{u})_{\text{calc}} &= 5.269 \text{ GeV}, \\
 M_{1S}(b\bar{u})_{\text{expt}} &= 5.272 \text{ GeV}.
 \end{aligned} \tag{3.2}$$

Our calculated results show an excellent agreement with the corresponding experimental spin-averaged mass values,¹⁵ written below each result in (3.2).

We thus find that the spin-averaged mass spectra of both the light and heavy mesons can be well understood by the potential given by Eq. (3.1), without taking into account the short-distance Coulombic part of the potential as given by QCD. The relativistic effects considered to be significant in the case of light mesons do not spoil the quantitative results. We predict the spin-averaged ground-state levels for $G(b\bar{s})$ and $H(b\bar{c})$ mesons at mass values 5.372 and 6.331 GeV, respectively.

We have presented in Table II the relevant values of $\langle B_1(r) \rangle_{nL}$ and $\langle B_2(r) \rangle_{nL}$ obtained for the S and P bound states of $c\bar{c}$ and $b\bar{b}$ systems along with the corresponding spin-averaged mass M_{nL} . Now it is a question of choosing and fitting the parameters f and g . Within the Fermi-Breit approach, it can easily be shown that for the potential given by Eq. (1.2) the spin forces can be generated only by an admixture of scalar- and vector-gluon ex-

changes with a nonzero quark anomalous magnetic moment (i.e., $g \neq 0, f < 1$). In the present calculation we find that the vector fraction $f=0.5$ with $g=0.44$ gives a very good fit to the fine-hyperfine splittings of $c\bar{c}$ and $b\bar{b}$ systems. Particularly for $b\bar{b}$ system the 1^3P_J wave splittings which come out to be

$$\begin{aligned}
 M(1^3P_2) - M(1^3P_1) &= 16.8 \text{ (19.9) MeV}, \\
 M(1^3P_1) - M(1^3P_0) &= 18.9 \text{ (21.3) MeV}, \\
 M(1^3P_1) - M(1^1P_1) &= 3.5 \text{ MeV},
 \end{aligned} \tag{3.3}$$

and the 2^3P_J wave splittings which come out to be

$$\begin{aligned}
 M(2^3P_2) - M(2^3P_1) &= 10.9 \text{ (16.5) MeV}, \\
 M(2^3P_1) - M(2^3P_0) &= 12.3 \text{ (21.5) MeV}, \\
 M(2^3P_1) - M(2^1P_1) &= 3.0 \text{ MeV},
 \end{aligned} \tag{3.4}$$

show a good agreement with the recently reported experimental values,⁵ given inside parentheses. Our calculated results for the hyperfine structures of $c\bar{c}$ and $b\bar{b}$ systems are displayed in Table III. We do not calculate the fine-structure masses of P states of $c\bar{c}$ and $b\bar{b}$ systems, as the fit to the spin-averaged masses in these cases is not good. However, taking the experimental spin-averaged masses as inputs for these states, these results may be improved.

The pseudoscalar partners of ψ and ψ' are found to be

$$M(\eta_c) = 2980 \text{ MeV} \tag{3.5}$$

TABLE III. Hyperfine structures of $c\bar{c}$ and $b\bar{b}$ systems (in GeV units).

n	State L	$c\bar{c}$ system		$b\bar{b}$ system	
		Predicted mass	Experimental mass	Predicted mass	Experimental mass
1	1S_0	2.980	2.978	9.431	
1	3S_1	3.097	3.097	9.460	9.460
2	1S_0	3.624	3.592	9.959	
2	3S_1	3.684	3.686	9.973	10.021
3	1S_0	4.010		10.297	
3	3S_1	4.049	4.029	10.307	10.350
4	1S_0	4.287		10.548	
4	3S_1	4.316	4.415	10.559	10.580

$$M(\eta'_c) = 3624 \text{ MeV} ,$$

respectively, which are in good agreement with recent experiments.¹⁶ The 1^3P_J state splitting which we obtain for $c\bar{c}$ system as

$$\begin{aligned} M(1^3P_2) - M(1^3P_1) &= 63.5 (45.8) \text{ MeV} , \\ M(1^3P_1) - M(1^3P_0) &= 74.2 (95.0) \text{ MeV} , \\ M(1^3P_1) - M(1^1P_1) &= 11.5 \text{ MeV} , \end{aligned} \quad (3.6)$$

show a reasonable agreement with experimental values, given inside parentheses. The hyperfine splittings of Υ and Υ' levels are predicted to be $M(\Upsilon) - M(\eta_b) = 29 \text{ MeV}$ and $M(\Upsilon') - M(\eta'_b) = 15 \text{ MeV}$, which are quite small.

We now turn to the calculation of the hyperfine structures for light-quark systems with the present model. For lighter mesons such as ρ^0 and ϕ the expectation values

$$\begin{aligned} \langle B_1(r) \rangle_{\rho^0} &= 0.106, \quad \langle B_2(r) \rangle_{\rho^0} = 0.066 , \\ \langle B_1(r) \rangle_{\phi} &= 0.181, \quad \langle B_2(r) \rangle_{\phi} = 0.102 \end{aligned} \quad (3.7)$$

(in GeV^2 units) along with $f=0.5$ and $g=0$ lead to their hyperfine masses, listed below with the corresponding experimental values inside the parentheses:

$$\begin{aligned} M(\rho^0) &= 756 (770 \pm 5) \text{ MeV} , \\ M(\pi^0) &= 180 (134.96 \pm 0.01) \text{ MeV} , \\ M(\phi) &= 1059 (1019.7 \pm 0.3) \text{ MeV} , \\ M(\eta') &= 840 (957.6 \pm 0.3) \text{ MeV} . \end{aligned} \quad (3.8)$$

We observe that the agreement with the experimental mass values is better than we would expect in a nonrelativistic model.

Taking the values

$$\begin{aligned} \langle B_1(r) \rangle_D &= 0.156 \quad \langle B_2(r) \rangle_D = 0.091 , \\ \langle B_1(r) \rangle_F &= 0.249, \quad \langle B_2(r) \rangle_F = 0.131 , \end{aligned} \quad (3.9)$$

in GeV^2 , along with $f=0.5$ and $g=0$ we also obtain the hyperfine structures of D and F mesons as

$$\begin{aligned} M(D^*) &= 2009.9 (2006 \pm 1.5) \text{ MeV} , \\ M(D) &= 1851.6 (1863.3 \pm 0.9) \text{ MeV} , \\ M(F^*) &= 2131.6 (2140 \pm 60) \text{ MeV} , \\ M(F) &= 2013.3 (2010 \pm 10) \text{ MeV} , \end{aligned} \quad (3.10)$$

which show a remarkable agreement with their corresponding experimental values inside the parentheses. We find that, in spite of the smallness of the constituent quark mass involved, the nonrelativistic fit for the light mesons is astonishingly good.

IV. CONCLUSION

Within the Fermi-Breit approach we thus find that the ground-state hyperfine levels of light mesons can be well explained in the framework of a nonsingular logarithmic potential given by Eq. (1.2), if the Lorentz structure of the potential is generated by an equal admixture of vector- and scalar-gluon exchanges (i.e., with $f=0.5$). The same spin structure of the potential incorporating with it only a small quark anomalous moment ($g=0.44$) is also found to give a good description of the hyperfine structures of heavy mesons, as well as the $1P$ - and $2P$ -wave splittings of the Υ system. The observation that the spin forces are generated by an equally mixed vector and scalar potential is in line with the phenomenological findings of some other authors,¹⁷ in the context of different potential models, which is further supported by the gauge-invariant formalism of Eichten and Feinberg.¹⁸ Such an observation was also made by one of the present authors in the nonrelativistic,¹⁹ as well as relativistic,²⁰ fits of meson spectra with an effective non-Coulombic power-law potential.

We also find that the short-distance singular part of the potential, which is believed to play an important role in the hyperfine splittings of light- and heavy-meson spectra, is absolutely not required to understand these spectra. Therefore, one would have serious doubts in saying that the light- and heavy-meson spectra suggest a short-range Coulomb-type behavior of the quark-antiquark potential well in accordance with the predictions of QCD.

We thus conclude that the spectra of bound heavy- and light-quark system can be well explained by a simple nonsingular-logarithmic-potential model without taking into account the short-distance part of the potential suggested by QCD. The amount of experimental data explained by this phenomenological model is quite good. The relativistic effects already known to be non-negligible, even in the case of the ψ system, do not spoil the results of the ρ^0 , ϕ , D , and F systems.

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