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Nucleon decay in supergravity unified theories

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A general analysis of nucleon decay in supergravity unified models is given including the full set of W-gaugino, gluino, and Z-gaugino dressing diagrams (with gauge as well as Yukawa interactions at vertices) of $LLLL$ and $RRRR$ dimension-five operators generated by Higgs-triplet exchange. The analysis is carried out within the framework of a model-independent formalism of $SU(2)\times U(1)$ breaking in supergravity models. Full symmetry-breaking effects on the vertices as well as all the allowed mass splittings of the supersymmetric particles that enter the dressing diagrams (i.e., W gauginos, Z gauginos, scalar quarks, and scalar leptons) are included. L-R mixing effects on scalar-quark—scalar-lepton masses, which are quite significant for the third generation for a heavy top quark, are also taken into account. Applications of the general analysis are made and it is shown that modification of supersymmetry predictions of nucleon decay can arise due to the inclusion of the new effects. It is shown specifically that $\bar{\nu}\pi$ modes dominate nucleon decay in certain domains of the parameter space rather than $\bar{\nu}K$ modes as is conventionally the case.

I. INTRODUCTION

Nucleon decay in supersymmetric (SUSY) grand unified theories[1] (GUT's) differs from nucleon decay in nonsupersymmetric GUT's in that in the SUSY theory the decay proceeds through dressings of dimension-five operators which are generated through the exchange of heavy Higgs triplets (see Fig. 1). The dressings involve gaugino exchanges which convert scalar quarks and scalar leptons into quarks and leptons (see Figs. 2—4). The dressing procedure is a low-energy phenomena involving gaugino masses which are manifestations of $SU(2)\times U(1)$ breaking. Thus one has the remarkable feature that in the supersymmetric grand unified theory, the nucleon decay is governed by the characteristics of both the GUT sector and the low-energy sector and, in particular, the detailed manner in which $SU(2)\times U(1)$ breaks.

There exist currently in the literature a number of detailed analyses of nucleon decay in supersymmetric grand unified theories.[2-5] Surprisingly a full analysis of this phenomena is still lacking and a number of effects have been ignored: (1) Symmetry-breaking effects on the low-energy vertices, i.e., quark—scalar-quark—gaugino vertices which involve the details of $SU(2)\times U(1)$ breaking as well as mass splitting of the gauginos. [In most models there are two W gauginos and four (or five) Z gauginos, each with its characteristic mass and matter—SUSY-matter vertices.] (2) Gluino and Z-gaugino dressings. (The importance of gluino dressing was first pointed out in Ref. 6.) (3) L-R mixing effects in the scalar-quark mass matrix arising in supergravity models from soft-breaking terms which are large in the third generation due to the heaviness of the top quark ($m_t \gtrsim 30$ GeV). (4) Yukawa interactions which are similarly large in the third generation and produce additional contributions due to L-R mixing. (5) Contributions from W-gaugino dressing of $RRRR$ dimension-five Lagrangian (due to L-R mixing).

Each of the above effects can make a significant contribution to nucleon decay in different supersymmetry models. The purpose of this work is to present a full analysis of nucleon decay within the framework of $N=1$ supergravity unified models[9] in a model-independent formalism of $SU(2)\times U(1)$ breaking.[10] (For a survey of supergravity GUT models, see Ref. 11.) We take into account all the effects listed above for an SU(5) GUT theory.[12]

The superpotential for the Yukawa interactions is

$$g_Y = -\tfrac{1}{8}\epsilon_{uvwxy}H^u M_i^{vw}(f_1^\dagger)_{ij}M_j^{xy} + H'_x M'_{y_i}(f_2^\dagger)_{ij}M_j^{xy} , \tag{1.1}$$

where H^x and H'_x are $\mathbf{5}_L$ and $\overline{\mathbf{5}}_L$ of Higgs fields, and M^{xy} and M'_y are $\mathbf{10}_L$ and $\overline{\mathbf{5}}_L$ of quark-lepton fields. The indices $i,j=1,2,3$ are generation labels and f_1 and f_2 are

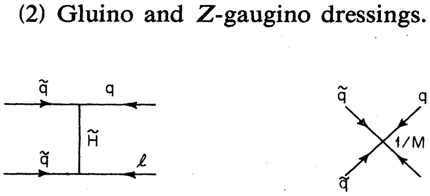

FIG. 1. Higgs-triplet-exchange diagrams which generate baryon-number-violating dimension-five operators.

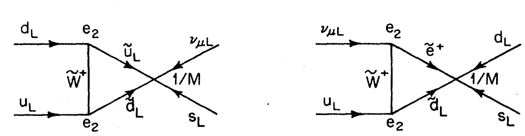

FIG. 2. Examples of W-gaugino dressing of baryon-number-violating dimension-five operators that contribute to nucleon decay.

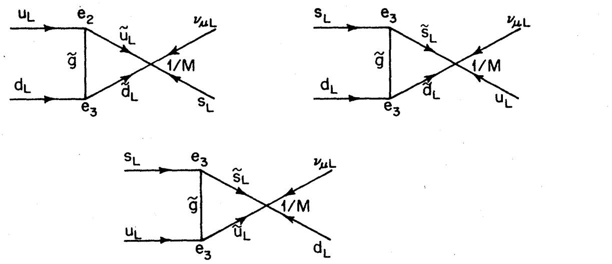

FIG. 3. Diagrams exhibiting gluino dressing of baryon-number-violating dimension-five operators that contribute to the nucleon decay.

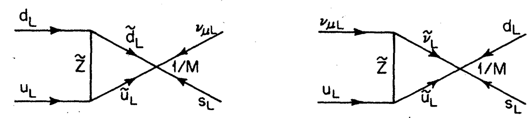

FIG. 4. Examples of Z-gaugino dressings of baryon-number-violating dimension-five operators that contribute to the nucleon decay.

Yukawa-coupling-constant matrices. After SU(5) breaks to $SU(3)\times SU(2)\times U(1)$ at the GUT mass M_{GUT}, the Higgs color-triplet fields grow a superheavy mass $M \approx M_{GUT}$. Eliminating the Higgs triplet gives the baryon-number-violating dimension-five operator which has an $LLLL$ part and an $RRRR$ part. Thus, one has

$$\mathscr{L}_5=\mathscr{L}_5^L+\mathscr{L}_5^R\,, \tag{1.2}$$

where $\mathscr{L}_5^L$ contains the left-handed matter and SUSY-matter fields which $\mathscr{L}_5^R$ contains the right-handed fields. In the representation where the quark-lepton mass matrices are diagonalized, one finds[13]

$$\begin{aligned}\mathscr{L}_5^L=\frac{\epsilon_{abc}}{M}(Pf_1^uV)_{ij}(f_2^d)_{kl}\{&\tilde{u}_{Lbi}\tilde{d}_{Lcj}[\bar{e}^c_{Lk}(Vu_L)_{al}-\bar{\nu}^c_k d_{Lal}]+\bar{u}^c_{Lbi}d_{Lcj}[\tilde{e}_{Lk}(V\tilde{u}_L)_{al}-\tilde{\nu}_k\tilde{d}_{Lal}]\\&+\bar{u}^c_{Lbi}\tilde{d}_{Lcj}[e_{Lk}(V\tilde{u}_L)_{al}-\nu_k\tilde{d}_{Lal}]+\bar{u}^c_{Lbi}\tilde{d}_{Lcj}[\tilde{e}_{Lk}(Vu_L)_{al}-\tilde{\nu}_k d_{Lal}]\\&+\tilde{u}_{Lbi}\bar{d}^c_{Lcj}[e_{Lk}(V\tilde{u}_L)_{al}-\nu_k\tilde{d}_{Lal}]+\tilde{u}_{Lbi}\bar{d}^c_{Lcj}[\tilde{e}_{Lk}(Vu_L)_{al}-\tilde{\nu}_k d_{Lal}]\}+\text{H.c.}\end{aligned} \tag{1.3}$$

In Eq. (1.3) V is the Kobayashi-Maskawa (KM) matrix, P is a diagonal phase matrix

$$P=\text{diag}(e^{i\gamma_1},e^{i\gamma_2},e^{i\gamma_3}),\quad \sum\gamma_i=0\,, \tag{1.4}$$

and $f^{u,d}$ are diagonal matrices related to the up- and down-quark masses m_i^u, m_i^d:

$$f_i^u\cos\alpha_H\left[\frac{\sin 2\theta_W}{e}\right]M_Z=m_i^u\,, \tag{1.5}$$

$$f_i^d\sin\alpha_H\left[\frac{\sin 2\theta_W}{e}\right]M_Z=m_i^d\,. \tag{1.6}$$

Here α_H is the model-dependent parameter of $SU(2)\times U(1)$ breaking

$$\begin{aligned}&\sin\alpha_H=w/(v^2+w^2)^{1/2}\,,\\&\cos\alpha_H=v/(v^2+w^2)^{1/2}\,,\\&-\pi<\alpha_H\leqq\pi\,,\\&v=\langle H^5\rangle,\quad w=\langle H_5'\rangle\,.\end{aligned} \tag{1.7}$$

Similarly $\mathscr{L}_5^R$ is given by

$$\begin{aligned}\mathscr{L}_5^R=-\frac{\epsilon_{abc}}{M}(V^\dagger f^u)_{ij}(PVf^d)_{kl}(&\bar{e}^c_{Ri}u_{Raj}\tilde{u}_{Rck}\tilde{d}_{Rbl}+\tilde{e}_{Ri}\tilde{u}_{Raj}\bar{u}^c_{Rck}d_{Rbl}+\tilde{e}_{Ri}\bar{u}^c_{Raj}\tilde{u}_{Rck}d_{Rbl}+\bar{e}^c_{Ri}\tilde{u}_{Raj}\tilde{u}_{Rck}d_{Rbl}\\&+\tilde{e}_{Ri}\bar{u}^c_{Raj}u_{Rck}\tilde{d}_{Rbl}+\bar{e}^c_{Ri}\tilde{u}_{Raj}u_{Rck}\tilde{d}_{Rbl})+\text{H.c.}\end{aligned} \tag{1.8}$$

The remaining part of this paper is organized as follows: A brief summary of the supergravity GUT models is given in Sec. II. In Sec. III we discuss dressings of dimension-five operators by W gauginos, gluinos, and Z gauginos to generate dimension-six operators which govern nucleon decay. The dressings are carried out using the model-independent low-energy interactions described in Ref. 10. The W-gaugino, gluino, and Z-gaugino dressings involve a variety of form factors arising from the loop calculations with matter—SUSY-matter—gaugino internal lines and vertices of the model-independent formalism. Size estimates of different contributions to nucleon decay are given in Sec. IV. Expressions for the dimension-six nucleon-decay amplitudes in the leading order are given in Sec. V. Results and conclusions are given in Sec. VI. A list of form factors arising from the loop integrals of W-gaugino, gluino, and Z-gaugino dressings is given in the Appendix.

A consequence of the general analysis presented here is that nucleon decay hierarchy can be modified from the conventional scheme in certain domains of the parameter space. Thus, for example, $\bar{\nu}K$ modes have conventionally been considered the most dominant in the decay pattern of the nucleon in a supersymmetric SU(5) theory.[1-4] However, we show here that this is not necessarily the case and for a class of models with a heavy top quark ($m_t \gtrsim 40$ GeV), consistent with the current experiments, $\bar{\nu}\pi$ modes may dominate the nucleon decay.

II. SUPERGRAVITY SU(5) MODELS

We briefly summarize here the properties of supergravity GUT models.[11] We make the usual assumption that supergravity is broken at the Planck scale by a super-Higgs effect and SU(5) breaks to $SU(3)\times SU(2)\times U(1)$ at $M_{\rm GUT}$. After eliminating the superheavy fields and the super-Higgs field, one is left with an effective low-energy theory involving only light fields interacting with specific supergravity soft-breaking terms.[14] We consider in this paper a "minimal" class of models by assuming that in the low-energy domain, there exists only *one* pair of color-singlet Higgs doublets H^α, H'_α which couple to the quark and lepton superfields with the usual Yukawa couplings, and possibly also a singlet field U. (Additional Higgs multiplets coupling to H^α, H'_α, but *not* the matter multiplet, may also be present. They will not effect the following discussion if they are sufficiently massive.) The low-energy dimension-three effective superpotential then has the form

$$g(Z)=\mu H^\alpha H'_\alpha+\lambda' U H^\alpha H'_\alpha-\tfrac{1}{6}\lambda'' U^3+g_Y \;, \tag{2.1}$$

where the Yukawa superpotential has the form

$$\begin{aligned} g_Y = {} & u_R^\dagger f^u u_L H^5+(d_R^\dagger f^d d_L+e_R^\dagger f^e e_L)H'_5 \\ & -u_R^\dagger f^u V d_L H^4+(d_R^\dagger f^d V^\dagger u_L+e_R^\dagger f^e \nu_L)H'_4 \;, \end{aligned} \tag{2.2}$$

where $H^{4,5}$ and $H'_{4,5}$ are the isodoublet components of the Higgs 5 and $\bar{5}$, f^u and f^d are given in Eqs. (1.5) and (1.6), and f^e is f^d with m_i^d replaced by the lepton masses $m_i^e=(m_e,m_\mu,m_\tau)$.

In the low-energy domain, the nucleon-decay amplitudes for a supergravity model depend on the following low-energy parameters: The gravitino mass $m_{3/2}$; the soft-breaking (Polonyi) constant A; the photino mass $\tilde{m}_\gamma$; the Higgs-particle coupling parameter μ; the Higgs-particle mixing angle α_H; and possibly λ' and λ''.[15] Both $m_{3/2}$ and A are related to the supergravity-breaking mechanism at the Planck mass, while $\tilde{m}_\gamma$ and μ arise from loops at the GUT sector (and possibly from tree contributions). The parameter α_H is determined from the $SU(2)\times U(1)$ breaking. Different models arise from different mechanisms used to achieve this breaking. It is convenient then to divide supergravity models into two classes: (1) Models with large D terms. Here α_H is small, e.g., $\alpha_H \approx 10°-25°$ and $SU(2)\times U(1)$ breaking requires μ to be small, i.e., $\mu \approx m_{3/2}\tan\alpha_H$. [Models of this type arise when renormalization-group (RG) corrections from the GUT mass to the W mass produce $SU(2)\times U(1)$ breaking (the RG models), e.g., Alvarez-Gaume, Polchinski, and Wise,[9] Ellis, Nanopoulos, and Tamvakis,[9] Ibañez and Lopez,[9] where also $m_{3/2} \gtrsim 60$ GeV, $|A| \lesssim 3$ and $\lambda' \equiv 0 \equiv \lambda''$.] (2) Models with small D terms. Here $\alpha_H \cong 45°$ and μ is large, i.e., $\mu \approx m_{3/2}$. [Models of this type arise when $SU(2)\times U(1)$ is broken at the tree level[16] (the "tree-breaking" or TB models), in the dimensional-transmutation models,[17] or in some "no-scale" models.[18]]

After $SU(2)\times U(1)$ breaking, the charged Higgs fermions and charged SU(2) gauginos combine to form two charged Dirac W gauginos $\tilde{W}_{(\pm)}$ with masses $\tilde{m}_\pm$. Similarly, the neutral Higgs fermions and neutral $SU(2)\times U(1)$ gauginos form four (or five) neutral Majorana Z gauginos $Z_{(k)}$, $k=0,1,2,\ldots$, with masses $\tilde{\mu}_{(k)}$, while the SU(3) gaugino, the gluino $\tilde{g}$, grows a nonzero mass $\tilde{m}_g$.[19] We label here the photino $\tilde{\gamma}$ by $\tilde{Z}_{(0)}$ (i.e., $\tilde{m}_\gamma \equiv \tilde{\mu}_{(0)}$). For most models, the photino is light, e.g., $\tilde{m}_\gamma \approx (1-15)$ GeV. Under these circumstances one generally has one low-lying W gaugino (i.e., $\tilde{m}_- < M_W$) and only two of the Z gauginos (labeled $\tilde{Z}_{(1,2)}$) couple strongly to the quarks and leptons. Hence only these two Z gauginos are important in nucleon-decay amplitudes. Finally, we note that in the simplest models where the gauginos are given, no arbitrary tree-level masses, loop corrections at the GUT level imply a fixed relation between gluino and photino masses:[20]

$$\tilde{m}_g=\frac{3}{8}\frac{\alpha_3}{\alpha}\tilde{m}_\gamma \cong 8\tilde{m}_\gamma, \quad \alpha_3 \cong 0.15 \;. \tag{2.3}$$

III. DRESSING OF DIMENSION-FIVE OPERATORS

The dimension-six nucleon-decay Lagrangian arises from the elimination of scalar quarks and scalar leptons in Eqs. (1.3) and (1.8) in terms of quarks and leptons and the $\tilde{W}_{(\pm)}$, $\tilde{Z}_{(k)}$, and $\tilde{g}$ (Figs. 2—4). In carrying out this elimination it is important to use mass-diagonal fields. Supergravity models possess a remarkable feature in having a unique set of soft-breaking terms which mix the right (R) and left (L) scalar-quark and scalar-lepton states. As we will see, this L-R mixing can play a crucial role in nucleon-decay branching ratios.

For the ith generation of up scalar quarks, the mass matrix mixing the L and R states has the form

$$\tilde{M}_{ui}=\begin{bmatrix} \tilde{m}_{Rui}{}^2 & Am_{3/2}m_i^u \\ Am_{3/2}m_i^u & \tilde{m}_{Lui}{}^2 \end{bmatrix}, \tag{3.1}$$

where m_i^u is the ith-generation u-quark mass, A is the Polonyi constant, and $\tilde{m}_{R,L}{}^2$ are the R,L diagonal mass-matrix elements. The eigenvalues of (3.1) are

$$(\tilde{m}_{ui(1,2)})^2=\tfrac{1}{2}(\tilde{m}_{Rui}{}^2+\tilde{m}_{Lui}{}^2) \pm\epsilon_{ui}[\tfrac{1}{4}(\tilde{m}_{Rui}{}^2-\tilde{m}_{Lui}{}^2)^2+A^2m_{3/2}{}^2m_i^{u\,2}]^{1/2}, \tag{3.2}$$

where

$$\epsilon_i\equiv(\tilde{m}_R{}^2-\tilde{m}_L{}^2)/|\tilde{m}_{Ri}{}^2-\tilde{m}_{Li}{}^2| .$$

The mass-diagonal scalar-quark fields $\tilde{u}_{i(1,2)}$ are related to the R and L scalar quarks by the rotation angle δ_{ui} according to

$$\tilde{u}_{Ri}=\cos\delta_{ui}\tilde{u}_{1i}+\sin\delta_{ui}\tilde{u}_{2i}, \quad \tilde{u}_{Li}=-\sin\delta_{ui}\tilde{u}_{1i}+\cos\delta_{ui}\tilde{u}_{2i}, \tag{3.3}$$

where

$$\sin 2\delta_{ui}=-2Am_{3/2}m_{ui}/(\tilde{m}_{ui1}{}^2-\tilde{m}_{ui2}{}^2) . \tag{3.4}$$

We have chosen our conventions such that in the limit $A\to 0, \tilde{u}_{1,2}\to\tilde{u}_{R,L}$. Formulas similar to Eqs. (3.1)—(3.4) hold for the down scalar quarks $\tilde{d}_i$ and the scalar electrons $\tilde{e}_i$. The up scalar quarks can now be eliminated from Eqs. (1.3) and (1.8) according to[21]

$$\tilde{u}_{iL}=2\int[\Delta_{ui}^{(L)}L_{ui}-\tfrac{1}{2}\sin 2\delta_{ui}(\Delta_{ui1}-\Delta_{ui2})R_{ui}], \tag{3.5a}$$

$$\tilde{u}_{iR}=2\int[\Delta_{ui}^{(R)}R_{ui}-\tfrac{1}{2}\sin 2\delta_{ui}(\Delta_{ui1}-\Delta_{ui2})L_{ui}], \tag{3.5b}$$

where $L_{ui}=\delta\mathscr{L}_1/\delta\tilde{u}_{iL}^\dagger$, $R_{ui}=\delta\mathscr{L}_1/\delta\tilde{u}_{iR}^\dagger$, and $\mathscr{L}_1$ is the interaction Lagrangian. In Eq. (3.5)

$$\Delta_{ui(1,2)}(k)=[k^2+(\tilde{m}_{ui(1,2)})^2]^{-1}$$

are the mass-diagonal scalar-quark propagators,

$$\Delta_{ui}^{(L)}=\sin^2\delta_{ui}\Delta_{ui1}+\cos^2\delta_{ui}\Delta_{ui2} \tag{3.6}$$

and $\Delta_{ui}^{(R)}$ is $\Delta_{ui}^{(L)}$ with $\sin\delta_{iu}\leftrightarrow\cos\delta_{ui}$. Similar formulas hold for the elimination of $\tilde{d}_{iL,R}$ scalar quarks and $\tilde{e}_{iL,R}$ scalar electrons. (The scalar neutrinos obey the simpler relation $\tilde{\nu}_{iL}=2\int\Delta_{\nu i}L_{\nu i}$.) The quantities L_{ui} and R_{ui} can be divided into their $\tilde{W}$, $\tilde{g}$, and $\tilde{Z}$ pieces, e.g.,

$$L_{ui}=L_{ui}^{\tilde{W}}+L_{ui}^{\tilde{g}}+L_{ui}^{\tilde{Z}} . \tag{3.7}$$

Thus for an arbitrary model one finds[11]

$$L_{ui}^{\tilde{W}}=\frac{ie_2}{\sqrt{2}}(\cos\gamma_-\tilde{W}_{(-)}+\sin\gamma_-\tilde{W}_{(+)})(V\gamma^0 d_L)_i-\frac{ie_2}{2\sin\alpha_H M_W}(E\cos\gamma_+\tilde{W}_{(-)}-\sin\gamma_+\tilde{W}_{(+)})(Vm^d\gamma^0 d_R)_i, \tag{3.8a}$$

$$L_{ui}^{\tilde{g}}=-ie_3\bar{\lambda}^r\tfrac{1}{2}t^r u_{iL}, \tag{3.8b}$$

$$L_{ui}^{\tilde{Z}}=-\frac{ie_2}{2}(i)^{\theta_k}O_{1k}(\cos\theta_W+\tfrac{1}{3}\sin\theta_W\tan\theta_W)\bar{\tilde{Z}}_{(k)}d_{iL}-\frac{ie_2}{2\sin\alpha_H M_W}(-i)^{\theta_k}(\sin\alpha_H O_{2k}+\cos\alpha_H O_{3k})\bar{\tilde{Z}}_{(k)}m_i^d d_{iR}$$
$$-\tfrac{2}{3}ie(i)^{\theta_k}O_{0k}\bar{\tilde{Z}}_{(k)}u_{iL}, \tag{3.8c}$$

where u_{iL}, d_{iL}, etc., are quark fields, $\lambda^r(x)$ are the gluino fields, and t^r are the SU(3) matrices, $\gamma_\pm$ are (model-dependent) angles which are functions of α_H, μ, and $\tilde{m}_2$ arising from the diagonalization of the $\tilde{W}$ mass matrix, and O_{ik} is the orthogonal rotation diagonalizing the Z-gaugino mass matrix. [$E=\pm 1$ and $(i)^{\theta_k}, \theta_k=0,1$ are phases defined by Nath, Arnowitt, and Chamseddine.[11]] For R_{ui} one has

$$R_{ui}^{\tilde{W}}=\frac{ie_2}{2\cos\alpha_H M_W}(\sin\gamma_-\tilde{W}_{(-)}-\cos\gamma_-\tilde{W}_{(+)}) \times(\gamma^0 m^u V d_L)_i, \tag{3.9a}$$

$$R_{ui}^{\tilde{g}}=-ie_3\bar{\lambda}^r\tfrac{1}{2}t^r u_{iR}, \tag{3.9b}$$

$$R_{ui}^{\tilde{Z}}=\frac{ie_2}{2\cos\alpha_H M_W}(i)^{\theta_k} \times(\cos\alpha_H O_{2k}-\sin\alpha_H O_{3k})\bar{\tilde{Z}}_{(k)}(m^u u_L)_i -\tfrac{2}{3}ie(-i)^{\theta_k}(\tan\theta_W O_{1k}+O_{0k})\bar{\tilde{Z}}_{(k)}u_{iR} . \tag{3.9c}$$

Corresponding expressions for L_{di}, R_{di}, etc., may be read off the supersymmetry interactions of Ref. 11. We note that the terms in Eqs. (3.8) and (3.9) proportional to $1/M_W$ arise from the Yukawa interactions and depend upon the quark masses m_i^u and m_i^d. The remaining terms are from the gaugino gauge interactions.

Inserting Eqs. (3.5), (3.8), (3.9), etc., into Eqs. (1.3) and (1.8) and performing the loop integrations indicated in Figs. 2—4 yields the effective dimension-six Lagrangian $\mathscr{L}_6$. Thus the W-gaugino contribution to the first term of Eq. (1.3) is

$$\mathcal{L}^{(1)}_{6\widetilde{W}}=\epsilon_{abc}\alpha_2{}^2(2MM_W\sin2\alpha_H)^{-1}[F(\widetilde{u}_i;\widetilde{d}_j;\widetilde{W})P_i m_i^u V_{ij}(Vd_{bL}\gamma^0)_i(V^\dagger u_{cL})_j - G(\widetilde{u}_i;\widetilde{d}_j;\widetilde{W})(M_W{}^2\sin2\alpha_H)^{-1}G(\widetilde{u}_i,\widetilde{d}_j)P_i m_i^u V_{ij}(Vm^d d_{bR}\gamma^0)_i(V^\dagger m^u u_{cR})_j] \times(l_L\gamma^0 m_c V^\dagger u_{aL}-\nu_L\gamma^0 m^d d_{aL}) , \tag{3.10}$$

where a,b,c are color indices. The form factors F and G come from the loop integrations and are

$$F(\widetilde{u}_i;\widetilde{d}_j;\widetilde{W})=-32\pi^2 i\int\{[\Delta_{ui}^{(L)}\cos\gamma_- -\tfrac{1}{2}\sin2\delta_{ui}(\Delta_{ui1}-\Delta_{ui2})\epsilon_i^u\sin\gamma_-] \times\widetilde{S}_{(-)}[\Delta_{di}^{(L)}E\sin\gamma_+ +\tfrac{1}{2}\sin2\delta_{di}(\Delta_{di1}-\Delta_{di2})\epsilon_i^d E\cos\gamma_+] + [\Delta_{ui}^{(L)}\sin\gamma_- +\tfrac{1}{2}\sin2\delta_{ui}(\Delta_{ui1}-\Delta_{ui2})\epsilon_i^u\cos\gamma_-]\widetilde{S}_{(+)} \times[\Delta_{di}^{(L)}\cos\gamma_+ -\tfrac{1}{2}\sin2\delta_{di}(\Delta_{di1}-\Delta_{di2})\epsilon_i^d\sin\gamma_+]\} , \tag{3.11}$$

$$G(\widetilde{u}_i;\widetilde{d}_j;\widetilde{W})=-32\pi^2 i\int\Delta_{ui}^{(L)}(E\cos\gamma_+\sin\gamma_-\widetilde{S}_{(-)}+\cos\gamma_-\sin\gamma_+\widetilde{S}_{(+)})\Delta_{dj}^L , \tag{3.12}$$

where $\widetilde{S}_{(\pm)}$ are the $\widetilde{W}_{(\pm)}$ propagators and

$$\epsilon_i^u=m_i^u/(\sqrt{2}\cos\alpha_H M_W),\quad \epsilon_i^d=m_i^d/(\sqrt{2}\sin\alpha_H M_W) . \tag{3.13}$$

One may similarly calculate the $\widetilde{g}$ and $\widetilde{Z}$ contributions to this part of Eq. (1.3) as well as the $\widetilde{W}$, $\widetilde{g}$, and $\widetilde{Z}$ contributions to $\mathcal{L}_6$ from the other 11 terms of Eqs. (1.3) and (1.8) to obtain the total dimension-six baryon-number-violating interaction.

IV. SIZE ESTIMATES

As can be seen from the previous discussion, the total baryon-number-violating amplitude is quite complicated. Thus from (3.5) and (3.10), L-R mixing produces four terms for each W gaugino, gluino, and Z gaugino for each of the 12 dimension-five terms of Eqs. (1.3) and (1.8). In addition each term has a double sum over the three generations. It is thus useful to know under what circumstances different contributions have significant size, particularly since much of the analysis of this question in the literature is incomplete. While it is generally agreed that the W-gaugino contributions are large, there exist models where each of the other gaugino contributions are also large, and we discuss here the circumstances for this.

As is well known,[22] the three gluino dressing contributions of Fig. 3 cancel among each other if the up and down scalar quarks are degenerate in the first two generations. In general, one has[23]

$$\widetilde{m}_{uL}{}^2=m_{3/2}{}^2+(-\tfrac{1}{2}+\tfrac{2}{3}\sin^2\theta_W)M_Z{}^2\cos2\alpha_H ,$$
$$\widetilde{m}_{dL}{}^2=m_{3/2}{}^2+(\tfrac{1}{2}-\tfrac{1}{3}\sin^2\theta_W)M_Z{}^2\cos2\alpha_H . \tag{4.1}$$

Thus, this condition is satisfied for theories with small D terms ($\alpha_H\cong45°$). More generally, the mass splitting becomes relatively unimportant if $m_{3/2}$ is much larger than the D term. However, in the RG models ($\alpha_H\approx10°$–$25°$) with $m_{3/2}\lesssim M_W$ one has

$$\widetilde{m}_{dL}{}^2-\widetilde{m}_{uL}{}^2=\cos^2\theta_W\cos2\alpha_H M_Z{}^2\cong\tfrac{2}{3}M_Z{}^2 \tag{4.2}$$

and there is a large mass splitting. The $\widetilde{g}$ dressing effect is enhanced for these models since the angles $\gamma_\pm$ are small[11] (e.g., $\gamma_\pm\approx10°$). Thus from (3.8a) the $\widetilde{W}_{(+)}$-u_L coupling is suppressed while examination of $\mathcal{L}^{\widetilde{W}}_{di}$ shows that the $\widetilde{W}_{(-)}$-u_L coupling is correspondingly suppressed. Hence, for both $\widetilde{W}_{(-)}$ and $\widetilde{W}_{(+)}$ at least one of the vertices in Fig. 2 is suppressed while all the vertices in Fig. 3 are relatively enhanced by a factor e_3/e_2. Detailed calculations[7] show that the gluino contribution is comparable to the W-gaugino contribution for gluino masses in the physically interesting range, as indicated in Table I.

A more complicated phenomena occurs for the Z-gaugino dressing diagrams. For very light Z-gaugino masses, the loop diagram of Fig. 4 vanishes linearly with the Z-gaugino masses, and hence a light photino ($\widetilde{m}_\gamma\cong1-15$ GeV) makes only a small contribution. In most models, the $\widetilde{Z}_{(3)}$ (and $\widetilde{Z}_{(4)}$) couple only weakly to matter. Thus, only $\widetilde{Z}_{(1)}$ and $\widetilde{Z}_{(2)}$ can make a significant contribution. However, the phase factors in the Z-gaugino interactions, e.g., Eqs. (3.8c) and (3.9c) lead to a coefficient $(-1)^{\theta_k}$ in Z-gaugino dressing diagrams, and in almost all models[11] one has $\theta_1=0,\theta_2=1$. Hence the $\widetilde{Z}_{(1)}$ and $\widetilde{Z}_{(2)}$ contributions enter with opposite sign and will cancel each other unless there is a significant mass splitting between the two states. This can indeed occur in models with small D terms ($\alpha_H\cong45°$) where the $\widetilde{Z}_{(2)}$ can lie well below the Z-boson mass. Thus one can have a sizable Z-gaugino dressing contribution

TABLE I. Ratio of gluino to W-gaugino amplitudes for $N\to\bar{\nu}_\mu K$ decay for various gluino masses $\widetilde{m}_g$. (The calculation used $m_{3/2}=80$ GeV, $m_{\widetilde{W}_{(-)}}=30$ GeV, $\alpha_3=0.15$.)

$\widetilde{m}_g$ (GeV)	$A(\widetilde{g})/A(\widetilde{W})$
30	0.5
60	0.6
90	0.8

for such models, but not generally for models with large D terms (where $\tilde{Z}_{(1)}$ and $\tilde{Z}_{(2)}$ are nearly degenerate with the Z boson). The situation for Z-gaugino contributions is thus the opposite of the gluino case considered above.

The size of L-R mixing effects depends upon several factors. From Eqs. (3.5a) and (3.9a) the W-gaugino contributions arise from the Yukawa couplings and hence are scaled by ϵ_i^u of Eq. (3.13) for $\tilde{u}_L$, and similarly by ϵ_i^d for $\tilde{d}_L$. Thus there will be significant effects only for the third generation, particularly with a heavy top quark.[24] Further, from Eqs. (3.5) one will only get significant L-R mixing if $\sin 2\delta_{ui}$ is large (e.g., $\delta_{ui}\approx 45°$) and there is significant mass splitting between $\tilde{u}_{i1}$ and $\tilde{u}_{i2}$. From Eqs. (3.2) and (3.4) one sees this generally can occur in the third generation with a heavy top quark ($m_t \gtrsim 40$ GeV) provided $Am_{3/2}$ is not anomalously small. Thus, one expects significant L-R mixing effects in nucleon-decay amplitudes in the third-generation contribution.

Due to the fact that gluino and Z-gaugino dressings are generation diagonal,[13] color antisymmetry makes their contributions from $\mathscr{L}_5^R$ of Eq. (1.8) vanish identically for nucleon decay. Because of this, it is often argued that $\mathscr{L}_5^R$ makes no contribution for nucleon decay.[25] However, the presence of L-R mixing in Eq. (3.5b) shows that nonzero W-gaugino dressing contributions from $\mathscr{L}_5^R$ exist. As discussed above for $\mathscr{L}_5^L$, these are largest for the third generation, where they make non-negligible contributions to decay amplitudes (through generally smaller than those from $\mathscr{L}_5^L$).

Finally, in examining the generational sums, Yukawa couplings (unlike gauge couplings) enhance higher generations (in spite of the smallness of the third-generation KM matrix elements) due to the experimental quark mass spectrum. For example, in the expression $m_i^u V_{i1}$ one has, for $m_t=50$ GeV,

$$m_u V_{11} : m_c V_{21} : m_t V_{31} \cong 1:60:250 \; . \tag{4.3}$$

Thus one expects large third-generation contributions to nucleon decay amplitudes (in addition to the above contribution to L-R mixing).

V. DECAY AMPLITUDES

The complete expressions for the dimension-six nucleon-decay amplitudes are quite complicated. However, making use of the discussion of Sec. IV, it is possible to pick out the leading pieces for each decay mode (i.e., those pieces which are not significantly suppressed by quark mass ratios and/or KM factors). We discuss here the major decay modes.

The $N\to\bar{\nu}_i K$ amplitudes ($i=e,\mu,\tau$) can be written in the form

$$\begin{aligned}\mathscr{L}_6(N\to\bar{\nu}_i K) = {} & [(\alpha_2)^2(2MM_W{}^2\sin 2\alpha_H)^{-1}P_2 m_c m_i^d V_{i1}^\dagger V_{21}V_{22}][F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})] \\ & \times\{[1+y_i^{tK}+(y_{\tilde{g}}+y_{\tilde{Z}})\delta_{i2}+\Delta_i^K]\alpha_i^L+[1+y_i^K-(y_{\tilde{g}}-y_{\tilde{Z}})\delta_{i2}]\beta_i^L+(y_1^{(R)}\alpha_3^R+y_2^{(R)}\beta_3^R)\delta_{i3}\}\;,\end{aligned} \tag{5.1}$$

where

$$\alpha_i^L \equiv \epsilon_{abc}(d_{aL}\gamma^0 u_{bL})(s_{cL}\gamma^0\nu_{iL})\;,$$

α_i^R is α_i^L with $(d_L,u_L\to d_R,u_R)$ and $\beta_i^{L,R}$ is $\alpha_i^{L,R}$ with $d\leftrightarrow s$. The quantity y_i^{tK} is the main third-generation contribution given by

$$y_i^{tK}=\frac{P_3}{P_2}\left[\frac{m_t V_{31}V_{32}}{m_c V_{21}V_{22}}\right]\left[\frac{F(\tilde{t};\tilde{d}_i;\tilde{W})+F(\tilde{t};\tilde{e}_i;\tilde{W})}{F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})}\right], \tag{5.2}$$

where F is given by Eq. (3.11). The gluino and Z-gaugino pieces are given by

$$y_{\tilde{g}}=\frac{4}{3}\frac{P_1}{P_2}\frac{\alpha_3}{\alpha_2}\frac{m_u V_{11}}{m_c V_{21}V_{21}^\dagger V_{22}}\frac{H(\tilde{u};\tilde{d};\tilde{g})-H(\tilde{d};\tilde{d};\tilde{g})}{F(\tilde{c};\tilde{s};\tilde{W})+F(\tilde{c};\tilde{\mu};\tilde{W})}\;, \tag{5.3}$$

$$\begin{aligned}y_{\tilde{Z}}= {} & \frac{P_1}{P_2}\frac{m_u V_{11}}{m_c V_{21}V_{21}^\dagger V_{22}}(-1)^{\theta_k}\tfrac{1}{2}O_{1k}{}^2\{[J(\tilde{u};\tilde{d};\tilde{Z}_{(k)})+J(\tilde{d};\tilde{d};\tilde{Z}_{(k)})]+[J(\tilde{\nu}_\mu;\tilde{d};\tilde{Z}_{(k)})+J(\tilde{\nu}_\mu;\tilde{u};\tilde{Z}_{(k)})]\} \\ & \times[F(\tilde{c};\tilde{s};\tilde{W})+F(\tilde{c};\tilde{\mu};\tilde{W})]^{-1}\;,\end{aligned} \tag{5.4}$$

where the form factors are given in the Appendix, Eqs. (A3)—(A7). [In Eqs. (5.3) and (5.4) we have neglected the generation splitting in the $\tilde{u}$ scalar quarks and $\tilde{d}$ scalar quarks for the first two generations. We have also neglected the photino contribution to $y_{\tilde{Z}}$, which is valid for a light photino (e.g., $\tilde{m}_\gamma\approx 1-15$ GeV).] Δ_i^K are additional (generally small) first- and third-generation $\tilde{W}$ contributions:

$$\begin{aligned}\Delta_2^K= {} & -\frac{P_1}{P_2}\frac{m_u V_{11}}{m_c V_{21}V_{21}^\dagger V_{22}}\frac{F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{u};\tilde{\mu};\tilde{W})}{F(\tilde{c};\tilde{s};\tilde{W})+F(\tilde{c};\tilde{\mu};\tilde{W})} \\ & -\frac{P_3}{P_2}\frac{m_t V_{31}V_{31}^\dagger V_{33}}{m_c V_{21}V_{21}^\dagger V_{22}}\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{c};\tilde{s};\tilde{W})+F(\tilde{c};\tilde{\mu};\tilde{W})}\;,\end{aligned} \tag{5.5}$$

$$\Delta_3^K=-\frac{P_1}{P_2}\frac{m_u V_{11}V_{32}V_{33}^\dagger}{m_c V_{21}V_{31}^\dagger V_{22}}\frac{F(\tilde{t};\tilde{\tau};\tilde{W})-F(\tilde{u};\tilde{\tau};\tilde{W})}{F(\tilde{c};\tilde{b};\tilde{W})+F(\tilde{c};\tilde{\mu};\tilde{W})}\;, \tag{5.6}$$

and $\Delta_1^K=0$. Finally, the L_5^R contributions are

$$y_1^{(R)}=\frac{P_1}{P_2}\frac{m_t m_d V_{11}V_{32}V_{33}^\dagger}{m_c m_b V_{21}V_{22}V_{31}^\dagger}\frac{Q(\tilde{\tau};\tilde{t};\tilde{W})}{F(\tilde{c};\tilde{b};\tilde{W})+F(\tilde{c};\tilde{\tau};\tilde{W})}, \tag{5.7}$$

$$y_2^{(R)}=\frac{P_1}{P_2}\frac{m_t m_s V_{31}V_{12}V_{33}^\dagger}{m_c m_b V_{21}V_{22}V_{31}^\dagger}\frac{Q(\tilde{\tau};\tilde{t};\tilde{W})}{F(\tilde{c};\tilde{b};\tilde{W})+F(\tilde{c};\tilde{\tau};\tilde{W})}, \tag{5.8}$$

where the form factor Q is given in Eq. (A8).

We note here the explicit realization of some of the qualitative comments made in Sec. III. The gluino and Z-gaugino contributions are appreciable only in the $\bar{\nu}_\mu K$ mode (since their couplings are flavor diagonal) while $\mathscr{L}_5^R$ contributes only to the $\bar{\nu}_\tau K$ mode (since L-R mixing is significant only in the third generation). The gluino contribution Eq. (5.3) indeed vanishes when the $\tilde{u}$ and $\tilde{d}$ scalar quarks are degenerate, but in models with large D terms can be quite sizable (see Table I) even though $m_u/(m_c V_{21}V_{21}^\dagger)\cong 0.06$. In contrast, the Z-gaugino contribution Eq. (5.4) does not vanish in the degenerate-scalar-quark limit, though the sign factor produces significant cancellations when the Z gauginos are degenerate.

The interactions governing $N\to\bar{\nu}_i\pi$ and $N\to\bar{\nu}_i\eta$ can be written in the form

$$\mathscr{L}_6(N\to\bar{\nu}_i\pi,\bar{\nu}_i\eta)=[(\alpha_2)^2(2MM_W{}^2\sin2\alpha_H)^{-1}P_2 m_c m_i^d(V_{21})^2V_{i1}^\dagger][F(\tilde{c},\tilde{d}_i)+F(\tilde{c},\tilde{e}_i)][(1+y_i^{t\pi}+y_i^{u\pi}+\Delta_i^\pi)\gamma_i^L+y_R\delta_{i3}\gamma_3^R], \tag{5.9}$$

where

$$\gamma_i^L=\epsilon_{abc}(d_{cL}\gamma^0 u_{aL})(\nu_{iL}\gamma^0 d_{bL})$$

and γ_i^R is γ_i^L with $(d_{cL},u_{aL}\to d_{cR},u_{aR})$. The quantity $y_i^{t\pi}$ is the main third-generation contribution, while $y_i^{u\pi}$ is a corresponding (small but non-negligible) first-generation piece:

$$y_i^{t\pi}=\frac{P_3}{P_2}\left[\frac{m_t(V_{31})^2}{m_c(V_{21})^2}\right]\left[\frac{F(\tilde{t};\tilde{d}_i;\tilde{W})+F(\tilde{t};\tilde{e}_i;\tilde{W})}{F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})}\right], \tag{5.10}$$

$$y_i^{u\pi}=\frac{P_1}{P_2}\left[\frac{m_u[(V_{11})^2-\delta_{i1}]}{m_c(V_{21})^2}\right]\times\left[\frac{F(\tilde{u};\tilde{d}_i;\tilde{W})+F(\tilde{u};\tilde{e}_i;\tilde{W})}{F(\tilde{c};\tilde{d}_i;\tilde{W})+F(\tilde{c};\tilde{e}_i;\tilde{W})}\right]. \tag{5.11}$$

Δ_i^π are additional contributions given by

$$\Delta_3^\pi=-\frac{P_1}{P_2}\frac{m_u V_{11}V_{31}V_{33}^\dagger}{m_c(V_{21})^2V_{31}^\dagger}\frac{F(\tilde{t};\tilde{\tau};\tilde{W})-F(\tilde{u},\tilde{\tau},\tilde{W})}{F(\tilde{c},\tilde{b},\tilde{W})+F(\tilde{c},\tilde{\tau},\tilde{W})}, \tag{5.12}$$

$$\Delta_1^\pi=-\frac{P_3}{P_1}\frac{m_t V_{31}V_{33}V_{31}^\dagger}{m_c(V_{21})^2V_{11}^\dagger}\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{c};\tilde{d};\tilde{W})+F(\tilde{c};\tilde{e};\tilde{W})}, \tag{5.13}$$

with $\Delta_2^\pi=0$. The $\mathscr{L}_5^R$ contribution is

$$y_R=\frac{P_1}{P_2}\left[\frac{m_t m_d V_{11}V_{31}V_{33}^\dagger}{m_c m_b(V_{21})^2V_{31}^\dagger}\right]\frac{Q(\tilde{\tau};\tilde{t};\tilde{W})}{F(\tilde{c};\tilde{b};\tilde{W})+F(\tilde{c};\tilde{\tau};\tilde{W})}. \tag{5.14}$$

Note from Eqs. (5.2) and (5.10) that

$$\frac{y_i^{t\pi}}{y_i^{tk}}=\frac{V_{31}V_{22}}{V_{32}V_{21}}\cong 1.5-2.0 \tag{5.15}$$

so that the third-generation effects are considerably larger for the pion modes than the kaon modes.

The charged-lepton-mode branching ratios are generally smaller in supersymmetry than the neutral-lepton modes. Here the gluino and Z-gaugino dressing contributions to indeed cancel identically due to color antisymmetry, and the $\mathscr{L}_5^R$ contributions are nonzero but negligibly small. For the kaon modes one finds

$$\mathscr{L}(p\to\mu^+K^0)=[(\alpha_2)^2(2MM_W{}^2\sin2\alpha_H)^{-1}P_1 m_u m_s][F(\tilde{u};\tilde{s};\tilde{W})+F(\tilde{\nu}_\mu;\tilde{s};\tilde{W})](1-V_{12}V_{21}^\dagger-y_\mu^{tK})(u_{cL}\gamma^0 s_{aL})(\mu_L\gamma^0 u_b), \tag{5.16}$$

$$\mathscr{L}_6(p\to e^+K)=[(\alpha_2)^2(2MM_W{}^2\sin2\alpha_H)^{-1}P_1 m_u m_d V_{12}V_{11}^\dagger][F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{\nu}_e;\tilde{d};\tilde{W})](1+y_e^{tK})(s_{bL}\gamma^0 u_{cL})(e_L\gamma^0 u_{aL}), \tag{5.17}$$

where $F(\tilde{\nu}_i;\tilde{d};\tilde{W})$ is given by Eq. (3.11) with $\delta_{\nu i}=0$ and

$$y_\mu^{tK}=\frac{P_3}{P_1}\left[\frac{m_t V_{21}^\dagger V_{32} V_{33} V_{31}^\dagger}{m_u}\right]\times\left[\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{\nu}_\mu;\tilde{s};\tilde{W})}\right], \tag{5.18}$$

$$y_e^{tK}=\frac{P_3}{P_1}\frac{m_t V_{32} V_{33} V_{31}^\dagger}{m_u V_{12}}\left[\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{\nu}_e;\tilde{d};\tilde{W})}\right]. \tag{5.19}$$

For the pion modes one finds

$$\begin{aligned}\mathscr{L}_6(N\rightarrow\mu\pi)=&[(\alpha_2)^2(2MM_W{}^2\sin2\alpha_H)^{-1}P_1 m_u m_s V_{11}V_{21}^\dagger]\\&\times[F(\tilde{u};\tilde{s};\tilde{W})+F(\tilde{\nu}_\mu;\tilde{s};\tilde{W})]\\&\times(1+y_\mu^{t\pi})(d_{bL}\gamma^0u_{cL})(\mu_L\gamma^0u_{aL}),\end{aligned} \tag{5.20}$$

$$\begin{aligned}\mathscr{L}_6(N\rightarrow e\pi)=&[(\alpha_2)^2(2MM_W{}^2\sin2\alpha_H)^{-1}P_1 m_u m_d]\\&\times[F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{\nu}_e;\tilde{d};\tilde{W})]\\&\times(1-V_{11}V_{11}^\dagger-y_e^{t\pi})(u_{cL}\gamma^0d_{aL})(e_L\gamma^0u_{bL}),\end{aligned} \tag{5.21}$$

where

$$y_\mu^{t\pi}=\frac{P_3}{P_1}\frac{m_t V_{31} V_{33} V_{31}^\dagger}{m_u V_{11}}\left[\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{u};\tilde{s};\tilde{W})+F(\tilde{\nu}_\mu;\tilde{s};\tilde{W})}\right], \tag{5.22}$$

$$y_e^{t\pi}=\frac{P_3}{P_1}\frac{m_t V_{31} V_{33} V_{31}^\dagger V_{11}^\dagger}{m_u}\left[\frac{F(\tilde{t};\tilde{b};\tilde{W})-F(\tilde{t};\tilde{d};\tilde{W})}{F(\tilde{u};\tilde{d};\tilde{W})+F(\tilde{\nu}_e;\tilde{d};\tilde{W})}\right]. \tag{5.23}$$

We note that the third-generation effects can be quite large here, i.e., the KM factor in $y_\mu^{t\pi}$ is $\cong 2.0$–2.5 while in y_e^{tK} it is $\cong 25$ for $m_t \cong 50$ GeV. However, these modes in general are still minor ones.

VI. RESULTS AND CONCLUSIONS

The decay amplitudes obtained in the previous section, Eqs. (5.1), (5.9), (5.16), (5.17), (5.20), and (5.21) have been written so that the quantities in the first square brackets represent the usual results arising from W-gaugino dressing of the first two generations. Thus, if one were to neglect all the corrections y_i^t, $y_{\tilde{g}}$, etc., due to the dominant nucleon-decay modes are $p\rightarrow\bar{\nu}_\mu K^+$ and $n\rightarrow\bar{\nu}_\mu K^0$ (e.g., Ellis *et al.*[22]). However, as discussed in Sec. IV, it is possible to construct models where one or more of the corrections y_i^t, $y_{\tilde{g}}$, etc., are large. If these terms are large and enter constructively with the usual W-gaugino contribution, then the dominance of the $\bar{\nu}K$ modes will generally be preserved. It is, however, possible for destructive interference to occur canceling out the dominant mode, making underlying modes experimentally accessible. The possibility that gluino dressing could cancel either the $p\rightarrow\bar{\nu}_\mu K^+$ or $n\rightarrow\bar{\nu}_\mu K^0$ amplitude has been discussed previously in Ref. 7. We consider here the more general possibility of cancellation of *all* the $p\rightarrow\bar{\nu}_i K^+$ and $n\rightarrow\bar{\nu}_i K^0$ modes from the third-generation effects[26] of y_i^{tK}. To see how this can come about we note from Eq. (5.2) that y_i^{tK} possesses a universal (generation-independent) KM factor (the first parentheses). Further, from Eq. (3.11) considerable enhancement of the form-factor ratio of Eq. (5.2) can occur when a heavy top quark exists ($m_t \gtrsim 40$ GeV) since then ϵ_3^u and $\sin2\delta_{u3}$ are large and the $\tilde{t}_1$ and $\tilde{t}_2$ scalar quarks can be highly split. As an example, we examine the class of models with large D terms (and hence $\alpha_H,\gamma_\pm$ small). Using the analysis of Buras *et al.*,[27] for $m_t=50$ GeV, one finds[28] the KM parentheses of Eq. (5.2) to be $\cong 0.25$ while if one $\tilde{t}$-scalar-quark eigenvalue of Eq. (3.2) is low lying (e.g., $\tilde{m}_t \cong 30$ GeV) it is easy to get an enhancement of the form-factor bracket of Eq. (5.2) of $\cong -(3.5$–$4.0)$. Hence if the phase factor $P_3/P_2 \cong +1$, one has $y_i^{tK} \approx -1$ and the third-generation effects due to L-R mixing then cancel the usual W-gaugino dressing part *universally* for all the $\bar{\nu}_i K$ modes. On the other hand, from Eq. (5.10) one finds the KM parentheses of $y_i^{t\pi}$ about twice as large as in y_i^{tK} and hence a similar cancellation does *not* occur for the $\bar{\nu}_i\pi$ or $\bar{\nu}_i\eta$ modes.

In general, if $-1.2 \lesssim y_i^{tK} \lesssim -0.8$, one may expect substantial cancellation of the leading $\bar{\nu}_i K$ decay modes. If the $\tilde{g}$, $\tilde{Z}$, and $\mathscr{L}_5^R$ pieces are not large, then the $\bar{\nu}_i\pi,\bar{\nu}_i\eta$ branching ratios can become comparable to or significantly larger than the $\bar{\nu}_i K$ modes. One may qualitatively state the conditions that can lead to a suppression of the $\bar{\nu}_i K$ mode in the fashion discussed above. These are (1) $m_t \gtrsim 40$ GeV, (2) $2Am_t \cong m_{3/2}$, (3) $P_2/P_3 \cong +1$, (4a) $m_{3/2} \gtrsim 150$ GeV for models with large D terms, or (4b) $\mu \lesssim \frac{1}{2}M_Z$ for models with small D terms [where μ is defined in Eq. (2.1)], and (5) the B meson lifetime obeys $\tau_B \lesssim 1.6$ ps with the "upper" choice in the KM-matrix analysis of Ref. 27 being the correct one. Condition (1) above enhances the L-R mixing in the third generation produced by Yukawa couplings, while condition (2) produces large $\tilde{t}_1$-$\tilde{t}_2$ scalar-quark mass splitting and large $\tilde{t}_L$-$\tilde{t}_R$ mixing in Eqs. (3.2) and (3.4). [These conditions are needed to make the second term of Eq. (3.5a) significant.] Condition (3) makes the third-generation effects y_i^{tK} distructively interfere with the usual W-gaugino dressing. Condition (4a) suppresses the gluino dressing since then the $\tilde{u}$-$\tilde{d}$ mass splitting in Eq. (4.2) is small relative to the masses themselves. [This mass splitting (and hence gluino dressing) is negligible in model with small D terms, see, e.g., Ref. 11.] Condition (4b) suppresses the Z-gaugino dressing by reducing the $\tilde{Z}_{(1)}$ and $\tilde{Z}_{(2)}$ mass splitting. [This mass splitting (and hence Z-gaugino dressing) is generally always small in models with large D terms.] Finally, condition (5) leads[27] to a PC-violating phase $\delta \gtrsim 160°$ and hence to a $\bar{\nu}_i K$ amplitude which is mainly real. (It is generally difficult to find reasonable constraints on the parameters of the models which will cancel *both* the real and imaginary parts of the $\bar{\nu}_i K$ amplitude simultaneously for the smaller values of δ arising in the "lower" solution of Ref. 27.)

In order to discuss quantitatively the above ideas, it is necessary to convert the quark decay amplitudes to nucleon decay rates. We use here the chiral-Lagrangian ap-

TABLE II. Branching ratios for proton and neutron decays for two models. Model 1 has $m_t=50$ GeV, $A=1.5$, $m_{3/2}=184$ GeV, $\alpha_H=15°$, and $P_3/P_2=1=P_2/P_1$. Model 2 has $m_t=66$ GeV, $A=1.2$, $m_{3/2}=184$ GeV, $\alpha_H=15°$, and $P_3/P_2=1=-P_2/P_1$.

p decay mode	Branching ratio (%) Model 1	Model 2	n decay mode	Branching ratio (%) Model 1	Model 2
$\bar\nu_\tau K^+$	18.2	10.2	$\bar\nu_\tau K^0$	18.0	26.2
$\bar\nu_\mu K^+$	30.0	5.4	$\bar\nu_\mu K^0$	42.8	11.5
$\bar\nu_e K^+$	0.4	0.2	$\bar\nu_e K^0$	1.0	0.5
$\bar\nu_\tau \pi^+$	47.4	40.0	$\bar\nu_\tau \pi^0$	28.0	20.9
$\bar\nu_\mu \pi^+$	3.1	40.1	$\bar\nu_\mu \pi^0$	1.8	20.9
$\bar\nu_e \pi^+$	0.5	4.0	$\bar\nu_e \pi^0$	0.3	2.1
$\mu^+ K$	0.4	0.02	$\bar\nu_\tau \eta$	7.3	8.8
			$\bar\nu_\mu \eta$	0.7	8.4
			$\bar\nu_e \eta$	0.1	0.7

proach[29,3] which we have extended to include the $\mathscr{L}_6^{RRLL}$ amplitudes[30] appearing in Eqs. (5.1) and (5.9) and take into account all the gluino and Z-gaugino dressings, and the supersymmetry and SU(2)$\times$U(1)-breaking effects. ($\mathscr{L}_6^{RRRR}$ make only negligible contribution to nucleon decay rates.) Branching ratios for two characteristic models with large D terms exhibiting the suppression of $\bar\nu_i K$ modes relative to the $\bar\nu_i\pi$ and $\bar\nu_i\eta$ modes is given in Table II. In model 1 the $\bar\nu\pi^+$ modes account for about 50% of the proton decay while the $\bar\nu K^+$ modes are reduced about 50%. In the neutron decay, the $\bar\nu K^0$ modes are reduced to 60% and the $\bar\nu\pi^0$ and $\bar\nu\eta$ decay account for the remaining 40%. [In addition one finds $\Gamma(n\to\bar\nu K^0)/\Gamma(p\to\bar\nu K^+)=1.1$.] For model 2, the $\bar\nu K$ suppression is even more extreme where the $\bar\nu K^+$ modes account for only 16% of the proton decay and the $\bar\nu K^0$ modes for only 38% of the neutron decay. [Here one finds $\Gamma(n\to\bar\nu K^0)/\Gamma(p\to\bar\nu K^+)=2.1$.] While by appropriate choice of parameters it is possible to reduce the $\bar\nu K$ modes even more than in these two examples, it is most likely not possible to simultaneously reduce both the proton and neutron $\bar\nu K$ modes below $\approx 10\%$ since the small corrections Δ_i^K, etc., in Eq. (5.1) are characteristically ≈ 0.1.

While it is not possible to predict the absolute rate for nucleon decay in supersymmetry (since the superheavy-color-Higgs-triplet mass m is model dependent) one may insert the experimental lower bound on the partial lifetime for one decay mode and then predict lower bounds for the partial lifetimes for other modes. These predictions may then be compared with the existing experimental lower bounds.[31] We use as our experimental input the decay $n\to\bar\nu K^0$ and compare in Table III the predictions of the two models of Table II with the experimental bounds of the Kamiokande experiment.[31] We note that aside from the μ^+K^0 mode the theoretical lower bounds are close to or slightly above the correct experimental lower bounds.

While the Higgs-triplet mass M cannot be theoretically calculated, one may estimate a lower bound for it using the experimental lower bound for τ/B of one mode, e.g., $p\to\bar\nu K^+$. From Eq. (5.1), the chiral-Lagrangian approach[3] yields for $p\to\bar\nu_\tau K^+$ the decay rate

$$\Gamma(p\to\bar\nu_\tau K^+)=\frac{\beta^2}{M^2}\frac{m_N}{32\pi f_\pi^{\,2}}\left[1-\frac{m_K^{\,2}}{m_N^{\,2}}\right]|A_{\nu_\tau K}|^2A_L^{\,2}(A_S^L)^2 \times\left|\left[1+\frac{m_N(D+3F)}{3m_B}\right]\left[1+y_3^{tK}+\Delta_3^K+\frac{A_S^R}{A_S^L}y_1^R\right]\frac{2}{3}\frac{m_N}{m_B}D\left[1+y_3^{tK}+\frac{A_S^R}{A_S^L}y_2^R\right]\right|^2, \tag{6.1}$$

where

$$A_{\nu_\tau K}=\alpha_2^{\,2}(M_W^{\,2}2\sin 2\alpha_H)^{-1}P_2 m_b m_c V_{31}^\dagger V_{21}V_{22}[F(\tilde c;\tilde b;\widetilde W)+F(\tilde c;\tilde\tau;\widetilde W)]\ . \tag{6.2}$$

A_S^L (A_S^R)=0.91 (0.48) and $A_L=0.22$ are the short-range and long-range RG suppression factors (Ellis, Nanopoulos, and Rudaz[2]) and β is the three-quark matrix element of the nucleon wave function $u_L^\gamma(\gamma=1,2)$:

$$\epsilon_{abc}\langle 0|\epsilon_{\alpha\beta}d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma|p\rangle=\beta u_L^\gamma\ . \tag{6.3}$$

The experimental bound on $p\to\bar\nu K^+$ allow a determination of a lower bound on M/β. Thus from Table III one finds for model 1, for example,

$$M/\beta>2.0\times10^{18}\ \mathrm{GeV}^{-2}\ . \tag{6.4}$$

A number of different calculations for β exist in the literature[32] ranging from $\beta=0.003$ GeV^3 to 0.03 GeV^3. Hence

$$M\gtrsim(0.6\text{–}6.0)\times10^{16}\ \mathrm{GeV} \tag{6.5}$$

which is to be compared with the SUSY GUT mass $M_{\rm GUT}\cong1\times10^{16}$ GeV. Since one expects the Higgs-triplet mass to be comparable to $M_{\rm GUT}$, Eq. (6.5) suggests that

TABLE III. Comparison between the models of Table II and experiment [Koshiba (Ref. 31)] for lower bounds on the partial lifetimes. The decay $n \to \bar{\nu} K^0$ was used as input in the theoretical calculation.

	Lower bound τ/B (10^{31} yr)		
	Theory		
Decay mode	Model 1	Model 2	Experiment (90% C.L.)
$p \to \bar{\nu} K^+$	1.7	3.7	1.5
$p \to \bar{\nu} \pi^+$	1.5	0.7	0.4
$p \to \mu^+ K^0$	176.7	3.0×10^3	1.1
$n \to \bar{\nu} K^0$	(1.6)	(1.6)	1.6
$n \to \bar{\nu} \pi^0$	3.3	1.9	2.1
$n \to \bar{\nu} \eta^0$	12.1	3.2	3.4

the theoretical lower bounds of Table III are close to being saturated. Hence, if models of this type are correct, nucleon decay should be experimentally accessible, and one might hope to see the decay events in the relatively near future as the experimental bounds improve.

Note added: A recent analysis by Enqvist, Masiero, and Nanopoulos[33] has been given of the bound on the value of M for certain dimensional-transmutation models (neglecting third-generation effects) where $\bar{\nu} K$ modes are still dominant. These authors find there that $M \gtrsim 10^{17}$ GeV for minimal models to be consistent with current limits on proton lifetime.

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APPENDIX

We list here, for ease of reference, the various form factors arising from the loop integrals of Figs. 2—4. The W-gaugino form factor is given in Eq. (3.11). We note that from the definitions Eq. (3.6), etc., this form factor is a linear combination of the basic loop integral

$$f(m_1,m_2,m_3) \equiv -16\pi^2 i \int \Delta_1 \Delta_2 \widetilde{S}_3 , \tag{A1}$$

where

$$f = \frac{m_3}{m_2{}^2 - m_3{}^2} \left[\frac{m_2{}^2}{m_1{}^2 - m_2{}^2} \ln \frac{m_1{}^2}{m_2{}^2} - \frac{m_3{}^2}{m_1{}^2 - m_3{}^2} \ln \frac{m_1{}^2}{m_3{}^2} \right] . \tag{A2}$$

For the gluino form factor one has then

$$H(\tilde{u};\tilde{d};\tilde{g}) = f(\widetilde{m}_u;\widetilde{m}_d,\widetilde{m}_g), \text{ etc.} \tag{A3}$$

while for the Z gauginos one obtains

$$J(\tilde{u};\tilde{d};\widetilde{Z}_{(k)}) = (\cos^2\theta_W - \tfrac{1}{9}\sin^2\theta_W \tan^2\theta_W) \times f(\widetilde{m}_u,\widetilde{m}_d,\widetilde{\mu}_{(k)}) , \tag{A4}$$

$$J(\tilde{d};\tilde{d};\widetilde{Z}_{(k)}) = (\cos\theta_W + \tfrac{1}{3}\sin\theta_W \tan\theta_W)^2 \times f(\widetilde{m}_d,\widetilde{m}_d,\widetilde{\mu}_{(k)}) , \tag{A5}$$

$$J(\tilde{\nu}_\mu;\tilde{d};\widetilde{Z}_{(k)}) = (1 + \tfrac{1}{3}\tan^2\theta_W) f(\widetilde{m}_{\nu_\mu},\widetilde{m}_d,\widetilde{\mu}_{(k)}) , \tag{A6}$$

$$J(\tilde{\nu}_\mu;\tilde{u};\widetilde{Z}_{(k)}) = (1 - \tfrac{1}{3}\tan^2\theta_W) f(\widetilde{m}_{\nu_\mu},\widetilde{m}_\mu,\widetilde{\mu}_{(k)}) , \tag{A7}$$

where $\widetilde{\mu}_{(k)}$ are the $\widetilde{Z}_{(k)}$ masses.

The form factor arising from $\mathscr{L}_5^R$ is

$$Q(\tilde{\tau};\tilde{t};\widetilde{W}) = -32\pi^2 i \frac{m_\tau}{\sqrt{2} M_W \sin\alpha_H} \int \{ \Delta_\tau^{(L)} E \cos\gamma_+ [\epsilon_3^u \Delta_t^{(L)} \sin\gamma_- - \tfrac{1}{2}\sin 2\delta_t (\Delta_{t1} - \Delta_{t2}) \cos\delta_-] \widetilde{S}_{(-)} + \Delta_\tau^{(L)} \sin\gamma_+ [\epsilon_3^u \Delta_t^{(L)} \cos\gamma_- + \tfrac{1}{2}\sin 2\delta_t (\Delta_{t1} - \Delta_{t2}) \sin\gamma_-] \widetilde{S}_{(+)} \} , \tag{A8}$$

where $\delta_t \equiv \delta_{u3}$, $\Delta_t^{(L)}$, and ϵ_3^u are defined in Eqs. (3.4), (3.6), and (3.13).

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