Application of chiral perturbation theory to $K \to 2\pi$ decays

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(Received 3 December 1984)

Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

I. INTRODUCTION

Lattice Monte Carlo techniques offer the possibility of calculating, from the fundamental theory, hadronic matrix elements of the operators which govern weak decays. In particular, it seems likely that one may solve the long-standing puzzle of the $\Delta I = \frac{1}{2}$ enhancement in hadronic weak decays with these techniques. Partial efforts in this direction have already been made;[1] an attempt to evaluate all the relevant diagrams is now in progress.[2,3]

In such lattice calculations—and indeed also in studies using other methods[4,5]—a direct evaluation of the physical matrix elements of interest is rather difficult. For example, in the mesonic sector on which we focus here, one would like to calculate the matrix element $\langle \pi\pi | \Theta | K \rangle$ where Θ is a generic weak operator. However, the lattice evaluation of this four-point function presents severe technical difficulties (even three-point functions are awkward to deal with using present methods), and it would be a lot simpler if one could look at reduced matrix elements such as $\langle \pi | \Theta | K \rangle$ or even $\langle 0 | \Theta | K \rangle$.

A systematic method of performing such reductions (i.e., of finding relations between various matrix elements of a given operator) is called chiral perturbation theory;[6] it involves the use of an effective Lagrangian for the pseudo-Goldstone-boson sector of the theory. In Sec. II, we apply the machinery of chiral perturbation theory to the decay $K \to 2\pi$. We find that[7] to lowest nontrivial order in meson masses, the value of the matrix element $\langle \pi | \Theta | K \rangle$ does not by itself determine the value of the physical matrix element $\langle \pi\pi | \Theta | K \rangle$; rather the amplitude $\langle 0 | \Theta | K \rangle$ is required in addition to $\langle \pi | \Theta | K \rangle$.

Following this calculation we explain, in Sec. III, the underlying reasons for the relations we have found between the amplitudes. In fact, the amplitude $\langle 0 | \Theta | K \rangle$ is needed in order to subtract from $\langle \pi | \Theta | K \rangle$ an unphysical contribution (off-diagonal wave-function renormalization) which does not effect the physical amplitude $\langle \pi\pi | \Theta | K \rangle$. Finally, in Sec. IV, we report the results of a lattice calculation designed as a rough test of the practicality of these ideas.

II. CHIRAL-PERTURBATION-THEORY RESULTS

Define the unitary chiral matrix field Σ by

$$\Sigma \equiv \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{1}$$

where ϕ^a ($a = 1, \ldots, 8$) are the (real) pseudoscalar-meson fields; λ^a are proportional to the Gell-Mann matrices, with $\mathrm{tr}(\lambda_a\lambda_b) = \delta_{ab}$; and f is the meson decay constant, which is the same for all mesons in this approximation and is equal to 135 MeV with our conventions. Then the chiral Lagrangian, correct to quadratic order in meson masses and momenta, is

$$\mathscr{L} = \frac{f^2}{8}\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger) + v\,\mathrm{tr}[M\Sigma + (M\Sigma)^\dagger], \tag{2}$$

where M is the quark mass matrix

$$M = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} \tag{3}$$

and v is a constant related to the meson masses by

$$v = \frac{f^2 m_{\pi^+}{}^2}{4(m_u + m_d)} = \frac{f^2 m_{K^+}{}^2}{4(m_u + m_s)} = \frac{f^2 m_{K^0}{}^2}{4(m_d + m_s)}. \tag{4}$$

Under $SU(3)_L \times SU(3)_R$, Σ transforms by

$$\Sigma \to U\Sigma V^\dagger, \tag{5}$$

where $U \in SU(3)_L$ and $V \in SU(3)_R$. This symmetry is softly broken by the mass term in (2); however it is often convenient, in order to keep track of the form of symmetry breaking, to imagine that M also transforms:

$$M \to VMU^{\dagger} . \tag{6}$$

With this fiction, $\mathscr{L}$ is "invariant" under $SU(3)_L \times SU(3)_R$.

One can now make a correspondence between an operator of interest in the underlying quark-gluon theory and the meson operators in the effective theory which have the same chiral transformation properties. This correspondence will involve a set of initially undetermined coefficients, one for each possible meson operator. The relevant matrix elements can then be calculated in the effective theory. If there are more matrix elements than there are unknown coefficients, there will in general be relations among them; it is these relations which we seek.

In the case at hand, the weak operators are all (8,1) or (27,1) under $SU(3)_L \times SU(3)_R$. This is true for both of the operators $O_{\pm}$, which appear after an operator-product expansion for large M_W,[8] and for the operators $O_1, O_2, \ldots, O_6$, which appear after a further expansion for large m_c.[4] In the leading nontrivial order of chiral perturbation theory there is only one (27,1) operator:

$$\tilde{\Theta}^{(27,1)} \equiv T^{ij}_{kl}(\Sigma\partial_\mu\Sigma^{\dagger})^k{}_i(\Sigma\partial^\mu\Sigma^{\dagger})^l{}_j , \tag{7}$$

where T^{ij}_{kl} is symmetric in i,j and in k,l and traceless on any upper and lower index. The $\Delta s = 1$, $\Delta d = -1$, $\Delta I = \frac{1}{2}$ member of this multiplet (corresponding to O_3) has the following nonzero elements of T^{ij}_{kl}:

$$\begin{aligned} &T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = T^{31}_{21} = \tfrac{1}{2} , \\ &T^{23}_{22} = T^{32}_{22} = 1 , \\ &T^{33}_{32} = T^{33}_{23} = -\tfrac{3}{2} ; \end{aligned} \tag{8}$$

whereas the $\Delta s = 1$, $\Delta d = -1$, $\Delta I = \frac{3}{2}$ member of the multiplet (corresponding to O_4) has nonzero elements:

$$\begin{aligned} &T^{13}_{12} = T^{31}_{12} = T^{13}_{21} = T^{31}_{21} = \tfrac{1}{2} , \\ &T^{23}_{22} = T^{32}_{22} = -\tfrac{1}{2} . \end{aligned} \tag{9}$$

There are, to this order, four independent (8,1) operators:

$$\begin{aligned} &\mathrm{tr}[\Lambda(\partial_\mu\Sigma)(\partial^\mu\Sigma^{\dagger})] , \\ &\mathrm{tr}(\Lambda\Sigma M) , \\ &\mathrm{tr}[\Lambda(\Sigma M)^{\dagger}] , \\ &\mathrm{tr}\partial_\mu[\Lambda(\partial^\mu\Sigma)\Sigma^{\dagger}] , \end{aligned} \tag{10}$$

where M is taken to transform according to (6), and Λ is a traceless 3×3 matrix. The $\Delta s = 1$, $\Delta d = -1$ members of these multiplets (corresponding to O_-, O_1, O_2, O_5, or O_6) have

$$\Lambda_{ij} = \delta_{i3}\delta_{j2} . \tag{11}$$

Note that total derivatives are not automatically excluded in (10) since we have not integrated the operators over all space-time; we want to retain the freedom to examine processes in which the weak operator injects energy and/or momentum. Of course, in the physical process which is of ultimate interest ($K \to 2\pi$), energy-momentum is conserved, and total derivatives do not contribute.

The number of operators in (10) may be reduced by noting that all the relevant (8,1) quark operators are invariant under an additional discrete symmetry, *CPS*, which is the product of ordinary *CP* with a "switching" symmetry, *S*, which simply switches the s and d quarks. [S is actually an element of $U(3)_{\text{vector}}$.] As in (6), one can make S an "invariance" of the entire chiral Lagrangian by switching m_s and m_d at the same time. Demanding *CPS* symmetry then reduces the number of $\Delta s = 1$, $\Delta d = -1$, (8,1) chiral operators to two:

$$\begin{aligned} &\tilde{\Theta}_1^{(8,1)} \equiv \mathrm{tr}[\Lambda(\partial_\mu\Sigma)(\partial^\mu\Sigma^{\dagger})] , \\ &\tilde{\Theta}_2^{(8,1)} \equiv \frac{8v}{f^2}\mathrm{tr}[\Lambda\Sigma M + \Lambda(\Sigma M)^{\dagger}] , \end{aligned} \tag{12}$$

where constant factors have been inserted into $\tilde{\Theta}_2^{(8,1)}$ for later convenience.

One now has the correspondence

$$\begin{aligned} &\Theta^{(8,1)} \to \alpha_1^{(8,1)}\tilde{\Theta}_1^{(8,1)} + \alpha_2^{(8,1)}\tilde{\Theta}_2^{(8,1)} , \\ &\Theta^{(27,1)} \to \alpha^{(27,1)}\tilde{\Theta}^{(27,1)} , \end{aligned} \tag{13}$$

where the operators with (without) tildes are in the effective (underlying) theory, and where the unknown α coefficients are independent of meson (or quark) masses to this order. It is then straightforward to calculate matrix elements of $\Theta^{(8,1)}$ and $\Theta^{(27,1)}$ in terms of the α's by expanding (1), (2), (7), and (12) in terms of ϕ^a and computing tree diagrams. Note first that all matrix elements will vanish quadratically with meson mass in the chiral limit; this is because the operators (7) and (12) are either manifestly proportional to meson mass squared or contain two powers of derivatives. For states at rest we find

$$\begin{aligned} \langle 0 | \Theta^{(8,1)} | K^0 \rangle &= \frac{16iv}{f^3}(m_s' - m_d')\alpha_2^{(8,1)} \\ &= \frac{4i}{f}[(m_{K^+}')^2 - (m_{\pi^+}')^2]\alpha_2^{(8,1)} , \\ \langle 0 | \Theta^{(27,1)} | K^0 \rangle &= 0 , \\ \langle \pi^+ | \Theta^{(8,1)} | K^+ \rangle &= \frac{4m_M{}^2}{f^2}(\alpha_1^{(8,1)} - \alpha_2^{(8,1)}) , \\ \langle \pi^+ | \Theta^{(27,1)} | K^+ \rangle &= -\frac{4m_M{}^2}{f^2}\alpha^{(27,1)} , \\ \langle \pi^+\pi^- | \Theta^{(8,1)} | K^0 \rangle &= \frac{4i}{f^3}(m_{K^0}{}^2 - m_{\pi^+}{}^2)\alpha_1^{(8,1)} , \\ \langle \pi^+\pi^- | \Theta^{(27,1)} | K^0 \rangle &= -\frac{4i}{f^3}(m_{K^0}{}^2 - m_{\pi^+}{}^2)\alpha^{(27,1)} , \end{aligned} \tag{14}$$

where the mass independence of the α's allows us to calculate the unphysical processes for arbitrary masses: m_s' and m_d' are the quark masses used in $K \to 0$ (m_{K^+}' and m_{π^+}' are the corresponding meson masses); m_M is a common π-K mass for $K \to \pi$ (for convenience, we chose to have $K \to \pi$ take place with no energy-momentum insertion by the operator); m_{K^0} and m_{π^+} are the physical masses. Since both elements of the (27,1) [i.e., Eqs. (8) and (9)] turn out to contribute the same to $K^+ \to \pi^+$ and to $K^0 \to \pi^+\pi^-$, we make no distinction between them in (13). The fact that $\langle 0 | \tilde{\Theta}^{(27,1)} | K^0 \rangle$ and $\langle 0 | \tilde{\Theta}_1^{(8,1)} | K^0 \rangle$ vanish is a trivial consequence of group theory and/or the form of the operators. However, the fact that $\langle \pi^+\pi^- | \tilde{\Theta}_2^{(8,1)} | K^0 \rangle$ vanishes while $\langle \pi^+ | \tilde{\Theta}_2^{(8,1)} | K^+ \rangle$ does not is more subtle and rather illuminating; the explanation is the subject of the next section.

The results in (14) can now be combined to obtain the desired relation among processes. We have, for the complete amplitude, to quadratic order,

$$[K^0 \to \pi^+\pi^-] = \frac{i(m_{K^0}{}^2 - m_{\pi^+}{}^2)}{m_M{}^2 f} \times ([K^+ \to \pi^+] - b[K^0 \to 0]) , \quad (15)$$

and for the ration of isospin amplitudes,

$$\frac{[K^0 \to \pi^+\pi^-]_{1/2}}{[K^0 \to \pi^+\pi^-]_{3/2}} = \frac{[K^+ \to \pi^+]_{1/2} - b[K^0 \to 0]_{1/2}}{[K^+ \to \pi^+]_{3/2}} , \quad (16)$$

where

$$b = i m_M{}^2 / f(m'^2_{K^+} - m'^2_{\pi^+}) ,$$

square brackets denote the amplitude, and the subscripts $\frac{1}{2}$ and $\frac{3}{2}$ indicate $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$, respectively. We have used the fact that (8,1) operators are pure $\Delta I = \frac{1}{2}$. The $K \to 0$ terms in (15) and (16) were omitted (incorrectly) in Ref. 2.

Note that (16) indicates that a $\Delta I = \frac{1}{2}$ rule for $K \to \pi$ does not in principle imply a $\Delta I = \frac{1}{2}$ rule for $K \to 2\pi$. In practice however, it appears[3] that $[K \to 0]_{1/2}$ does not change the qualitative picture: $\Delta I = \frac{1}{2}$ is strongly enhanced both in $K \to \pi$ and $K \to 2\pi$.

III. TWO QUARK OPERATORS, RENORMALIZATION, AND TOTAL DERIVATIVES

To elucidate the "subtleties" we mentioned above, let us first consider a vexing, but seemingly unrelated, problem. The matrix elements of the four-quark operators considered here have contributions like those in Fig. 1 which

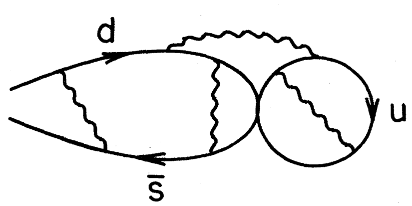

FIG. 1. A contribution to the matrix elements of a four-quark operator which generates two-quark operators such as $\bar{s}d$ and therefore produces unwanted off-diagonal renormalizations. In a complete physical process there must of course also be additional spectator quarks; this graph is only a subgraph for such a process.

mix s and d quarks. (The spectator quark is not shown.) The mass and wave-function renormalization parts of Fig. 1 are unphysical, and a way must be found to subtract them off.

To consider this problem in more detail, first note that the relevant part of the QCD Lagrangian is

$$\mathcal{L} = \bar{s}(i\not{D} - m_s)s + \bar{d}(i\not{D} - m_d)d , \quad (17)$$

where $\not{D} = \gamma^\mu D_\mu$ is the color-covariant derivative. Under the infinitesimal vector transformation

$$\delta_V s = -d, \quad \delta_V \bar{d} = \bar{s} , \quad (18)$$

the Lagrangian changes by

$$\delta_V \mathcal{L} = (m_s - m_d)\bar{s}d . \quad (19)$$

This field redefinition is what is therefore needed to "renormalize away" the two-quark operator $\bar{s}d$ which is generated by Fig. 1. (Other two-quark operators can be treated similarly: the operator $\bar{s}i\not{D}d$ is proportional to $\bar{s}d$ for on-shell hadronic matrix elements; the operator $\bar{s}\gamma_5 i\not{D}d$, which is equal to $m_d\bar{s}\gamma_5 d$ on shell, can be removed by an axial transformation.) If the matrix elements were being calculated perturbatively, renormalization would present no problem: the fields could be redefined order by order to cancel the $\bar{s}d$ terms as they appeared. However, here we might imagine evaluating the matrix elements with a numerical, lattice computation, and it is not immediately obvious how to enforce such a definition. Renormalization effects would appear to be inextricably mixed with legitimate contributions in which the quarks in the loop in Fig. 1 exchange gluons with a spectator quark. (The "penguin diagrams" are of this type.)

If we are dealing with the process $K^0 \to \pi^+\pi^-$ directly, this problem would not be present. The reason is that $\bar{s}d$ is a total divergence[9]—an obvious consequence of its proportionality to $\delta\mathcal{L}$ in (19). Explicitly

$$\bar{s}d = \frac{-i}{m_s - m_d} \partial_\mu(\bar{s}\gamma_\mu d) . \quad (20)$$

Since the weak operator carries zero momentum in the physical process $K \to 2\pi$, such total derivative terms do not contribute.

However, we wish instead to treat the process $K^+ \to \pi^+$. Here, in order for the weak operator to carry zero four-momentum, we must force K and π to be degenerate, which in turn implies $m_s = m_d$. But $\bar{s}d$ is then no longer a total divergence, as (20) shows explicitly. [Alternatively, note that the Lagrangian (17) is now invariant under the transformation (18), so there is no corresponding total divergence.] There is no advantage to keeping $m_s \neq m_d$: in that case $\bar{s}d$ remains a total divergence, but its $K \to \pi$ matrix element must carry nonzero four-momentum and therefore does not vanish. (In Ref. 2 it was overlooked that $\bar{s}d$ cannot be treated as a total divergence for $K^+ \to \pi^+$.)

The conclusion is that the amplitude for $K \to \pi$ necessarily contains unphysical renormalization contributions which must be subtracted off. In fact, this is precisely what is accomplished by the $K \to 0$ term in (15) and (16). Operator $\tilde{\Theta}_2^{(8,1)}$, which contributes to $K \to 0$ and $K \to \pi$

but not to $K\to 2\pi$, is, like $\bar{s}d$, a total divergence for $m_d \neq m_s$. Indeed, under the vector SU(3) transformation corresponding to (18),

$$\delta_V \Sigma = i[\Lambda,\Sigma] ,\quad \delta_V \Sigma^\dagger = i[\Lambda,\Sigma^\dagger] , \tag{21}$$

with Λ given by (11), the change in the chiral Lagrangian (2) is

$$\delta_V \tilde{\mathscr{L}} = iv(m_s - m_d)\mathrm{tr}(\Lambda\Sigma + \Lambda\Sigma^\dagger) . \tag{22}$$

Similarly, under the corresponding axial (right minus left) transformation,

$$\delta_A \Sigma = -i\{\Lambda,\Sigma\},\quad \delta_A \Sigma^\dagger = i\{\Lambda,\Sigma^\dagger\} ,\quad \delta_A \tilde{\mathscr{L}} = -iv(m_s + m_d)\mathrm{tr}(\Lambda\Sigma - \Lambda\Sigma^\dagger) . \tag{23}$$

Combining (12), (22), and (23), one can write

$$\tilde{\Theta}_2^{(8,1)} = -\frac{4i}{f^2}\left[\frac{m_s+m_d}{m_s-m_d}\delta_V\tilde{\mathscr{L}} - \frac{m_s-m_d}{m_s+m_s}\delta_A\tilde{\mathscr{L}}\right] . \tag{24}$$

This clearly shows that $\tilde{\Theta}_2^{(8,1)}$, like $\bar{s}d$, is a total divergence for $m_s \neq m_d$, but not for $m_s = m_d$. The correspondence can be made even more precise. Using the information that the quark operator $\Theta^{(8,1)}$ appearing in Fig. 1 is an (8,1) and is invariant under *CPS*, one finds that the two-quark operator generated must actually be proportional to

$$i\bar{s}(\overrightarrow{\not{D}} - \overleftarrow{\not{D}})(1-\gamma_5)d = (m_s + m_d)\bar{s}d - (m_s - m_d)\bar{s}\gamma_5 d . \tag{25}$$

The $\bar{s}d$ term here is proportional to $\delta_V\mathscr{L}$ (for $m_s \neq m_d$) as we saw in (19); the $\bar{s}\gamma_5 d$ term can be seen to be proportional to $\delta_A\mathscr{L}$ and corresponds to the second term in (24).

The conclusion is that the $\tilde{\Theta}_2^{(8,1)}$ operator is the chiral perturbation theory representative of the renormalization terms coming from diagrams like Fig. 1. Since $\tilde{\Theta}_2^{(8,1)}$ is a total divergence for $m_s \neq m_d$, it contributes to $K\to 0$ and $K\to\pi$ but not $K\to 2\pi$; whereas the physical operator $\tilde{\Theta}_1^{(8,1)}$ contributes to $K\to\pi$ and $K\to 2\pi$. The physical amplitude $K\to 2\pi$ can therefore be obtained from $K\to\pi$ by subtracting away a piece proportional to $K\to 0$; this is just the content of (15).

An intuitive understanding of this point can be gained by examining Fig. 2, which is a typical diagram for $K\to 0$. If the $\bar{s}$-d line is straightened out, Fig. 2 becomes Fig. 1; therefore the two have the same renormalization parts. Thus it is reasonable that the amplitude $K\to 0$ can be used to subtract off the unphysical off-diagonal renormalizations that we discuss above.

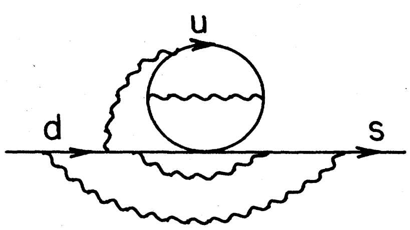

FIG. 2. A typical graph contributing to the process $K\to 0$. This graph is fundamentally the same as Fig. 1, as can be seen by straightening the $\bar{s}d$ line.

IV. MONTE CARLO CALCULATION

We have performed a rough test of the practicability, in a Monte Carlo calculation, of removing the unphysical term in $K\to\pi$ by subtracting the amplitude $K\to 0$. In the ultimate case of interest, two-quark operators are generated by four-quark operators through diagrams like Fig. 1. In our test, however, we start directly with a two-quark operator,

$$\Theta^{(3,\bar{3})} \equiv \bar{s}(1-\gamma_5)d . \tag{26}$$

In chiral perturbation theory, $\Theta^{(3,\bar{3})}$ corresponds to a unique operator to lowest (here zeroth) order in meson masses and momenta:

$$\Theta^{(3,\bar{3})} \to \alpha^{(3,\bar{3})}\tilde{\Theta}^{(3,\bar{3})} , \tag{27}$$

where

$$\tilde{\Theta}^{(3,\bar{3})} = \mathrm{tr}(\Lambda\Sigma) , \tag{28}$$

with Λ again given by (11). We have used the invariance of $\Theta^{(3,\bar{3})}$ under *CS*, with *C* charge conjugation and *S* defined above, to arrive at the unique chiral operator in (27).

Because $\Theta^{(3,\bar{3})}$ is a total divergence for $m_s \neq m_d$, it does not contribute to $K\to 2\pi$. Its contributions to the other processes are, from chiral perturbation theory,

$$\langle 0 | \Theta^{(3,\bar{3})} | K^0 \rangle = \frac{2i}{f}\alpha^{(3,\bar{3})} ,\quad \langle \pi^+ | \Theta^{(3,\bar{3})} | K^+ \rangle = -\frac{2}{f^2}\alpha^{(3,\bar{3})} . \tag{29}$$

In the presence of this operator alone, we thus have

$$\frac{[K\to 0]}{[K\to\pi]} = -if . \tag{30}$$

Our ability to "subtract away" two-quark contributions from $K\to\pi$ is therefore tested by our ability to measure f using (30). We have computed f from (30) with a lattice Monte Carlo calculation, using the methods for evaluating matrix elements which are outlined in Ref. 2. With eight independent $6^3\times 10$ quenched SU(3) configurations at $\beta = 5.7$, Wilson $r = 1$, and $k = 0.155$ and 0.162, we find

$$af = 0.55\pm 0.08,\quad k = 0.155 ,\quad af = 0.56\pm 0.17,\quad k = 0.162 , \tag{31}$$

where a is the lattice spacing, and the quoted errors are purely statistical.

The conventional method for determining f requires a knowledge of the quark masses since it makes use of the relation

$$af = (m_u + m_d)\frac{2k}{(am_\pi)^2} | \langle 0 | \bar{d}\gamma_5 u | \pi^+ \rangle | , \tag{32}$$

where the u and d fields are the dimensionless lattice fields. The value of af one gets from (32) depends on the definition of the quark masses used. With the straightforward definition (as used for example in Ref. 10),

$$m_q=\frac{1}{2k}-\frac{1}{2k_c} \tag{33}$$

and our estimate of $k_c=0.173\pm0.002$, we find

$$\begin{aligned} af&=0.47\pm0.05,\quad k=0.155\\ af&=0.49\pm0.08,\quad k=0.162\,. \end{aligned} \tag{34}$$

With the definitions used by Hamber and Parisi[11] the values of af would be about 30% higher; with the one-loop corrections as computed by Arroyo, Yndurain, and Martinelli,[12] the values would instead be about 60% higher.

The point we wish to make here is that our chiral-perturbation-theory methods produce results roughly comparable with other techniques, giving us some confidence that we will in practice be able to remove the effects of the two-quark operators to a reasonable degree of accuracy. A cautionary note should be injected, however. If one puts in the value $a^{-1}\simeq1.0$ GeV which is determined by potential and/or string-tension measurements[13] (assuming scaling, but *not* asymptotic scaling), one finds very high values for f in either (31) or (34). This is not completely unexpected: Ref. 10 shows a rather strong decrease of f with quark mass, and we are here working in a range of mass in which $m_\pi\sim700$–900 MeV (with the above value of a). It does suggest, however, that until one works at considerably smaller quark mass and, most likely, weaker coupling, Monte Carlo results will be only qualitatively and not quantitatively accurate. Luckily, the $\Delta I=\frac{1}{2}$ enhancement is such a large effect that a qualitative evaluation may be very interesting.

ACKNOWLEDGMENTS

We are grateful to Steve Otto for discussions, especially for extracting for us the value of a at $\beta=5.7$ from his potential measurement. The work of C.B. and T.D. was partially supported by the National Science Foundation. The work of A.S., H.P., and M.W. was supported by the Department of Energy. M.W. also received support from the Alfred P. Sloan Foundation. The computing was done on the MFE computing network and was supported by the Department of Energy. Two of us (C.B. and T.D.) also thank the UCLA Academic Senate Committee on Research for partial support.

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[1] N. Cabibbo, G. Martinelli, and R. Petronzio, Nucl. Phys. **B244**, 381 (1984); R. C. Brower, M. B. Gavela, R. Gupta, and G. Maturana, Phys. Rev. Lett. **53**, 1318 (1984).

[2] C. Bernard, in *Gauge Theory on a Lattice: 1984,* proceedings of the Argonne National Laboratory Workshop, edited by C. Zachos, W. Celmaster, E. Kovacs, and D. Sivers (National Technical Information Service, Springfield, Virginia, Report No. CONF-8404119, 1984).

[3] C. Bernard, T. Draper, G. Hockney, A. Soni, and M. Rushton (in preparation).

[4] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 123 (1975) [JETP Lett. **22**, 55 (1975)] Nucl. Phys. **B120**, 316 (1977).

[5] J. F. Donoghue, E. Golowich, B. R. Holstein, and W. Ponce, Phys. Rev. D **21**, 186 (1980).

[6] See, for example, S. Weinberg, Physica **96A**, 327 (1979).

[7] Some of these results have already been reported in M. Wise, Caltech Report No. CALT-68-1179, 1984 (unpublished), and in T. Draper, Ph.D. thesis, University of California at Los Angeles, 1984.

[8] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).

[9] G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. **3**, 527 (1959).

[10] K. C. Bowler *et al.,* Nucl. Phys. **B220** [FS8], 137 (1983).

[11] H. Hamber and G. Parisi, Phys. Rev. D **27**, 208 (1983).

[12] A. Gonzalez Arroyo, F. J. Yndurain, and G. Martinelli, Phys. Lett. **117B**, 437 (1982).

[13] S. Otto (private communication); D. Barkai, K. J. M. Moriarty, and C. Rebbi, Phys. Rev. D **30**, 1293 (1984).