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Spin analysis of $0+1 \rightarrow 0+1$ and its application to $\pi+d \rightarrow \pi+d$ data

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The polarization structure of the reaction $0+1 \rightarrow 0+1$ is discussed in the optimal transversity frame. First, the relationship between the observables and the bilinear products of amplitudes ("bicoms") is given when only Lorentz invariance is imposed. Then parity conservation and time-reversal invariance are also imposed, resulting in modified relationships. The measurements of spin correlations between initial- and final-state spins needed to determine the amplitudes completely are enumerated. The results are applied to the existing π-d data, and the consequences of any possible dibaryon resonances are examined.

I. INTRODUCTION

The polarization structure of the reaction $0+1 \rightarrow 0+1$ is about as simple as any incorporating a spin-1 particle, and yet, reactions of this type are largely unexplored. In the most familiar case, $\pi+d \rightarrow \pi+d$, only a few types of polarization experiments have been performed. The existing data, although covering a large energy range (60—600 MeV), are basically limited, besides the cross section, to T_{11} and T_{20}. In addition, the presently existing data by different experimental groups are not necessarily in mutual agreement regarding dynamical implications such as the dibaryon resonance structure.[1,2] Extracting complete information about the dynamical structure of a reaction is possible if all the amplitudes of the reaction are determined. The aim of this paper, therefore, is to discuss an experimental program designed to completely determine all the amplitudes and to provide, even from a yet uncompleted set of experiments, useful partial information about the amplitudes.

II. THE GENERAL POLARIZATION STRUCTURE

In the optimal formalism[3] reactions of the type

$$0+S_A \rightarrow 0+S_C \quad (1)$$

(where S_A, S_C, and 0 refer to particles with spins S_A, S_C, and 0, respectively) can be treated as a special case of a general four-particle reaction and therefore, the observables can be written as bilinear combinations of amplitudes as

$$\mathscr{L}(UVH_p;\Xi W_Q) = ZH_W[D(\Xi,U)D^*(\Omega,V)+PD(\Xi,V)D^*(\Omega,U)] \ , \quad (2)$$

where $U,V=1,2,\ldots,(2S_A+1)$ denote the spin projections of the initial-state spin-1 particle, $\Xi,\Omega=1,2,\ldots,(2S_C+1)$ denote the spin projections of the final-state spin-1 particle. $H_P=R$ ("real") or I ("imaginary") if $P=1$ or $P=-1$, respectively, and $H_Q=R$ ("real") or I ("imaginary") if $Q=1$ or $Q=-1$, respectively. We also have $Z=1+PQ-P+Q$ and

$$H_W=\begin{cases}\text{real if } W=PQ=+1 \ ,\\ \text{imaginary if } W=PQ=-1 \ .\end{cases}$$

The amplitudes are denoted by the $D(i,j)$'s, where i refers to the final-state spin-1 particle and j refers to the initial-state spin-1 particle. This notation is the same as that of Ref. 3.

Since we are dealing with spin-1 particles, the spin projection indices will be denoted by $+,0,-$. In the argument of the observables $\mathscr{L}$, the arguments for the spin-1 particles will be denoted as $++$, 00, $--$, and various R's and I's. We also use the following notations:

$$A \text{ (average)}=(++)+(00)+(--) \ ,$$

$$\Delta \text{ (difference)}=(++)-(--) \ ,$$

$$\Lambda=(++)-2(00)+(--) \ .$$

The notation for the R's and I's is

$$\mathrm{Re}(+-)=R^0,\ \mathrm{Re}(+0)=R^+,\ \mathrm{Re}(-0)=R^- \ ,$$

and three analogous equations for the imaginary parts.

Equation (2) reveals that the size of the matrix connecting the observables with the products of the amplitudes for $0+1 \rightarrow 0+1$ will be 81×81. This matrix in the optimal formalism is as diagonal as possible, namely, in the most general case, it consists of only 1×1 and 2×2 submatrices along the main diagonal, numbering 45 and 18, respectively. Under Lorentz invariance only there are nine independent amplitudes. Table I relates the observables to the bilinear products of the amplitudes for only Lorentz invariance being assumed, including observables with arguments A,Δ,Λ.

TABLE I. Relationship between observables and the bilinear products of amplitudes for the reaction $0+1\rightarrow 0+1$ in the optimal formalism for the most general case when only Lorentz invariance holds.

		+	+	+	−	−	−	0	0	0	Λ
1_M		+	−	0	+	0	−	+	0	−	L
$\frac{1}{4}\mathcal{L}(UVH_P,\Xi\Omega H_Q)$		+	+	+	−	−	−	0	0	0	Λ'
UV	$\Xi\Omega$	+	−	0	+	0	−	+	0	−	L'
$++R$	$++R$	+									
$--R$	$++R$		+								
$00R$	$++R$			+							
$++R$	$--R$				+						
$00R$	$--R$					+					
$--R$	$--R$						+				
$++R$	$00R$							+			
$00R$	$00R$								+		
$--R$	$00R$									+	
A	A	+	+	+	+	+	+	+	+	+	
Δ	Δ	+	−		−		+				
Λ	Λ	+	+	−2	+	−2	+	−2	+4	−2	
A	Δ	+	+	+	−	−	−				
Δ	A	+	−		+		−	+		−	
A	Λ	+	+	+	−	−	−	−2	−2	−2	
Λ	A	+	+	−2	+	−2	+	+	−2	+	
Δ	Λ	+	−		+		−	−2		+2	
Λ	Δ	+	+	−2	−	+2	−				
A	$++R$	+	+	+							
A	$--R$				+	+	+				
A	$00R$							+	+	+	
$++R$	A	+			+			+			
$--R$	A		+				+			+	
$00R$	A			+		+			+		
Δ	$++R$	+	−								
Δ	$--R$				+		−				
Δ	$00R$							+		−	
$++R$	Δ	+			−						
$--R$	Δ		+				−				
$00R$	Δ			+		−					
Λ	$++R$	+	+	−2							
Λ	$--R$				+	−2	+				
Λ	$00R$							+	−2	+	
$++R$	Λ	+			+			−2			
$--R$	Λ		+				+			−2	
$00R$	Λ			+		+			−2		

TABLE I. (*Continued*).

		R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	
		+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	0	0	0	Λ
1_1		+	+	−	0	−	+	+	−	0	−	0	+	+	+	−	+	+	−	L
$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		0	+	0	0	+	+	−	−	−	0	0	0	−	−	−	0	0	0	Λ′
UV	ΞΩ	+	0	−	0	0	−	+	−	0	−	0	+	−	0	0	−	0	0	L′
+ +	+0	+																		
+0	+ +		+																	
− −	+0			+																
00	+0				+															
−0	+ +					+														
+ −	+ +						+													
+ +	+ −							+												
− −	+ −								+											
00	+ −									+										
− −	−0										+									
00	−0											+								
+ +	−0												+							
+ −	− −													+						
+0	− −														+					
−0	− −															+				
+ −	00																+			
+0	00																	+		
−0	00																		+	
A	+0	+		+	+															
A	+ −							+	+	+										
A	−0										+	+	+							
+0	A														+			+		
−0	A					+										+				
+ −	A						+							+			+			
Δ	+0	+		−																
Δ	+ −							+	−											
Δ	−0										−		+							
+0	Δ		+												−					
−0	Δ					+										−				
+ −	Δ						+							−						
Λ	+0	+		+	−2															
Λ	+ −							+	−2	+										
Λ	−0										−2	+	+							
+0	Λ		+												+			−2		
−0	Λ					+										+			−2	
+ −	Λ						+							+			−2			

TABLE I. (*Continued*).

			I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
			+	+	+	+	+	+	+	+	+	−	−	−	−	−	−	0	0	0
	1_1		+	+	−	0	−	+	+	−	0	−	0	+	+	+	−	+	+	−
	$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		0	+	0	0	+	+	−	−	−	0	0	0	−	−	−	0	0	0
	UVH_P	$\Xi\Omega H_Q$	+	0	−	0	0	−	+	−	0	−	0	+	−	0	0	−	0	0
(−)	$++R$	$+0I$	+																	
	$+0I$	$++R$		+																
(−)	$--R$	$+0I$			+															
(−)	$00R$	$+0I$				+														
	$-0I$	$++R$					+													
	$+-I$	$++R$						+												
(−)	$++R$	$+-I$							+											
(−)	$--R$	$+-I$								+										
(−)	$00R$	$+-I$									+									
(−)	$--R$	$-0I$										+								
(−)	$00R$	$-0I$											+							
(−)	$++R$	$-0I$												+						
	$+-I$	$--R$													+					
	$+0I$	$--R$														+				
	$-0I$	$--R$															+			
	$+-I$	$00R$																+		
	$+0I$	$00R$																	+	
	$-0I$	$00R$																		+
	A	$+0I$	−		−	−														
	A	$-0I$										−	−	−						
	A	$+-I$							−	−	−									
	$+0I$	A		+												+			+	
	$-0I$	A					+										+			+
	$+-I$	A						+							+			+		
	Δ	$+0I$	−		+															
	Δ	$-0I$										−		−						
	Δ	$+-I$							−	+										
	$+0I$	Δ		+												−				
	$-0I$	Δ					+										−			
	$+-I$	Δ						+							−					
	Λ	$+0I$	−		−	+2														
	Λ	$-0I$										−	+2	−						
	Λ	$+-I$							−	−	+2									
	$+0I$	Λ		+												+			−2	
	$-0I$	Λ					+										+			−2
	$+-I$	Λ						+							+			−2		

TABLE I. (*Continued*).

			R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
			+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	0	−	−
	2_1		+	0	+	−	−	0	+	0	+	−	−	0	+	0	+	+	0	−
	$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		0	0	0	0	0	0	−	−	−	−	−	−	0	0	0	−	0	0
	UVH_P	$\Xi\Omega H_Q$	0	+	−	+	0	−	0	+	−	+	0	−	0	+	−	−	−	0
	$+0R$	$+0R$	+	+																
	$+0I$	$+0I$	+	−																
	$+-R$	$+0R$			+	+														
	$+-I$	$+0I$			+	−														
	$-0R$	$+0R$					+	+												
	$-0I$	$+0I$					+	−												
	$+0R$	$+-R$							+	+										
	$+0I$	$+-I$							+	−										
	$+-R$	$+-R$									+	+								
	$+-I$	$+-I$									+	−								
	$-0R$	$+-R$											+	+						
	$-0I$	$+-I$											+	−						
	$+0R$	$-0R$													+	+				
	$+0I$	$-0I$													+	−				
	$+-R$	$-0R$															+	+		
	$+-I$	$-0I$															+	−		
	$0-R$	$-0R$																	+	+
	$0-I$	$-0I$																	+	−

			I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
			+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−
	2_1		+	0	+	−	−	0	+	0	+	−	−	0	+	0	+	−	0	−
	$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		0	0	0	0	0	0	−	−	−	−	−	−	0	0	0	0	0	0
	UVH_P	$\Xi\Omega H_Q$	0	+	−	+	0	−	0	+	−	+	0	−	0	+	−	+	−	0
(−)	$+0R$	$+0I$	+	+																
	$+0I$	$+0R$	+	−																
(−)	$+-R$	$+0I$			+	+														
	$+-I$	$+0R$			+	−														
(−)	$-0R$	$+0I$					+	+												
	$-0I$	$+0R$					+	−												
(−)	$+0R$	$+-I$							+	+										
	$+0I$	$+-R$							+	−										
(−)	$+-R$	$+-I$									+	+								
	$+-I$	$+-R$									+	−								
(−)	$-0R$	$+-I$											+	+						
	$-0I$	$+-R$											+	−						

TABLE I. (*Continued*).

			I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I	I
			+	+	+	+	+	+	+	+	+	+	+	+	−	−	−	−	−	−
	2_1		+	0	+	−	−	0	+	0	+	−	−	0	+	0	+	−	0	−
$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$			0	0	0	0	0	0	−	−	−	−	−	−	0	0	0	0	0	0
	UVH_P	$\Xi\Omega H_Q$	0	+	−	+	0	−	0	+	−	+	0	−	0	+	−	+	−	0
(−)	+0R	−0I													+	+				
	+0I	−0R													+	−				
(−)	+−R	−0I															+	+		
	+−I	−0R															+	−		
(−)	0−R	−0I																	+	+
	0−I	−0R																	+	−

III. POLARIZATION STRUCTURE WITH PARITY CONSERVATION

The additional imposition of parity conservation will create restrictions on the number of independent amplitudes. These restrictions are different depending on what optimal frame is chosen. We will choose the transversity frame, since we know that for parity-conserving reactions it provides the simplest form. In such a frame the complex spin amplitudes must satisfy the relation[4]

$$D^{\tau}=(-1)^{l+L}D^{\tau}(l,L)(-1)^{S_A+S_C}\,, \tag{3}$$

where l and L are the spin components of the particles S_A and S_C, respectively. For $0+1\to 0+1$ we then have

$$D(+,0)=D(0,+)=D(0,-)=D(-,0)=0 \tag{4}$$

and the remaining nonzero amplitudes are

$$D(+,+),\ \ D(+,-),\ \ D(-,+)\,,$$
$$D(-,-),\ \ D(0,0)\,. \tag{5}$$

Table II features the relations between the observables and the amplitudes under Lorentz invariance plus parity conservation.

IV. POLARIZATION STRUCTURE WITH PARITY CONSERVATION AND TIME-REVERSAL INVARIANCE

Further reduction on the number of independent amplitudes comes from the imposition of time-reversal invariance on the reaction. In the transversity frame the time-reversal constraints read as follows:

$$D(l,L)=D(L,l)\,, \tag{6}$$

which in our case reads

$$D(+,-)=D(-,+) \tag{7}$$

so we end up with only four independent amplitudes. The relations between observables and these amplitudes are given in Table III.

V. THE DETERMINATION OF THE AMPLITUDES IN π-d ELASTIC SCATTERING

As mentioned earlier, the existing experimental measurements for $\pi+d\to\pi+d$ are limited to only measurements of T_{11} and T_{20} of the final-state deuteron and so there is not enough data for this reaction to provide the (at least) seven different polarization measurements required to determine the four amplitudes completely. There are plans to use polarized deuteron targets.[5] Methods of measuring the vector and tensor polarization have already been developed and used. Therefore, we are not far from a situation in which one could perform the complete experimental program for this reaction, one that is able to determine the reaction amplitudes phenomenologically, thus testing various theoretical models completely. Such a program consists of two parts:

TABLE II. Relationship between observables and bilinear products of amplitudes for the reaction $0+1\to 0+1$ in the transversity frame with Lorentz invariance and parity conservation.

1_n		+	+	−	−	0
$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		+	−	+	−	0
		+	+	−	−	0
UVH_P	$\Xi\Omega H_Q$	+	−	+	−	0
$++R$	$++R$	+				
$--R$	$++R$		+			
$++R$	$--R$			+		
$--R$	$--R$				+	
$00R$	$00R$					+
A	$++R$	+	+			
A	$--R$			+	+	
A	$00R$					+
$++R$	A	+		+		
$--R$	A		+		+	
Δ	$++R$	+	−			
Δ	$--R$			+	−	
$++R$	Δ	+		−		
$--R$	Δ		+		−	
Λ	$00R$					−2
A	A	+	+	+	+	+
Δ	Δ	+	−	−	+	
Λ	Λ	+	+	+	+	+4
A	Δ	+	+	−	−	
Δ	A	+	−	+	−	
A	Λ	+	+	+	+	−2
Λ	A	+	+	+	+	−2

1_1		R	R	R	R
		+	+	+	−
$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		+	+	−	+
		+	−	−	−
UV	$\Xi\Omega$	−	+	−	−
$+-$	$++$	+			
$++$	$+-$		+		
$--$	$+-$			+	
$+-$	$--$				+
A	$+-$		+	+	
$+-$	A	+			+
Δ	$+-$		+	−	
$+-$	Δ	+			−
Λ	$+-$		+	−2	
$+-$	Λ	+			+

	1_1		I	I	I	I
			+	+	+	−
	$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		+	+	−	+
			+	−	−	−
	UVH_P	$\Xi\Omega H_Q$	−	+	−	−
	$+-I$	$++R$	+			
(−)	$++R$	$+-I$		+		
(−)	$--R$	$+-I$			+	
	$+-I$	$--R$				+
	A	$+-I$		−	−	
	$+-I$	A	+			+
	Δ	$+-I$		−	+	
	$+-I$	Δ	+			−
	Λ	$+-I$		−	−	
	$+-I$	Λ	+			+

TABLE II. (*Continued*).

2_1		R	R	R	R	R	R
		+	+	+	+	−	−
$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q$		+	−	+	−	+	−
		0	0	−	−	0	0
UVH_P	$\Xi\Omega H_Q$	0	0	−	+	0	0
$+0R$	$+0R$	+					
$-0R$	$+0R$		+				
$+-R$	$+-R$			+	+		
$+-I$	$+-I$			+	−		
$+0R$	$-0R$					+	
$0-R$	$-0R$						+

	2_1		I	I	I	I	I	I
			+	+	+	+	−	−
	$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		+	−	+	−	+	−
			0	0	−	−	0	0
	UVH_P	$\Xi\Omega H_Q$	0	0	−	+	0	0
(−)	$+0R$	$+0I$	+					
(−)	$-0R$	$+0I$		+				
(−)	$+-R$	$+-I$			+	+		
	$+-I$	$+-R$			+	−		
(−)	$+0R$	$-0I$					+	
(−)	$0-R$	$-0I$						+

TABLE III. Relationship between observables and bilinear product of amplitudes for the reaction $0+1\to0+1$ in the transversity frame with Lorentz invariance and parity conservation plus time-reversal invariance. The notations for the amplitudes are $D(++)\equiv\alpha$, $D(+-)\equiv\beta$, $D(--)\equiv\gamma$, $D(00)\equiv\delta$.

1_n $\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		R	R	R	R
UVH_P	$\Xi\Omega H_Q$	$\lvert\alpha\rvert^2$	$\lvert\beta\rvert^2$	$\lvert\gamma\rvert^2$	$\lvert\delta\rvert^2$
++	++	+			
−−	++		+		
−−	−−			+	
00	00				+
A	++	+	+		
A	−−		+	+	
A	00				+
Δ	++	+	−		
Δ	−−		+	−	
Λ	00				−2
A	A	+	+2	+	+
Δ	Δ	+	−2	+	
Λ	Λ	+	+2	+	+4
A	Λ	+	+2	+	−2
A	Δ	+		−	

1_n $\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		R $\alpha\beta^*$	R $\beta\gamma^*$
++	R^0	+	
−−	R^0		+
A	R^0	+	+
Δ	R^0	+	−
Λ	R^0	+	−2

TABLE III. (*Continued*).

1_1		I	I			
$\frac{1}{4}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		$\alpha\beta^*$	$\beta\gamma^*$			
UVH_P	$\Xi\Omega H_G$					
$++$	I^0	$-$				
$--$	I^0		$-$			
A	I^0	$-$	$-$			
Δ	I^0	$-$	$+$			
Λ	I^0	$-$	$-$			
2_1		R	R	R	R	R
$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		$\alpha\delta^*$	$\beta\delta^*$	$\alpha\gamma^*$	$\gamma\delta^*$	$\lvert\beta\rvert^2$
UVH_P	$\Xi\Omega H_Q$					
R^-	R^-	$+$				
R^+	R^-		$+$			
R^0	R^0			$+$		$+$
I^0	I^0			$+$		$-$
R^+	R^+				$+$	
2_1		I	I	I	I	
$\frac{1}{2}\mathscr{L}(UVH_P,\Xi\Omega H_Q)$		$\alpha\delta^*$	$\beta\delta^*$	$\alpha\gamma^*$	$\gamma\delta^*$	
UVH_P	$\Xi\Omega H_Q$					
R^-	I^-	$-$				
R^+	I^-		$-$			
R^0	I^0			$-$		
R^+	I^+				$-$	

A. Determination of the magnitudes of the amplitudes

In the transversity frame the magnitudes of the four complex amplitudes are all in 1_M of Table III and can be determined from a set of observables consisting of A's, Δ's, and/or Λ's only. The determination of the magnitudes of the amplitudes are unambiguous, because the magnitudes are always positive and hence, the determination of the squares from the equations which are linear in them gives also unique values for the magnitudes themselves.

B. Determination of the phases of the amplitudes

There are several choices for determining the relative phases of the amplitudes. Since we measure only the relative phases, we can let one of the amplitudes be purely real or purely imaginary. The determination of the remaining three relative phases (up to some discrete ambiguities) require three more observables to determine phases. In order to remove the discrete ambiguities also, one may need to perform additional experiments. Criteria for resolving the discrete ambiguities have been developed.[6] Using these criteria, we give different choices of experiments which determine the phases.

As we see from Table IV, one possible set of experiments which determines the magnitudes of the amplitudes is related to the amplitudes as

$$\begin{aligned}
48\,|\alpha|^2 &= 2(A,A)+(A,\Lambda)+6(A,\Delta)\,,\\
48\,|\beta|^2 &= 2(A,A)+(A,\Lambda)-3(\Delta,\Delta)\,,\\
48\,|\gamma|^2 &= 2(A,A)+(A,\Lambda)+3(\Delta,\Delta)-6(A,\Delta)\,,\\
48\,|\delta|^2 &= 4(A,A)-4(A,\Lambda)\,.
\end{aligned} \tag{8}$$

As to the experiments which determine the phases also, there are more choices. Table IV shows these observables in three different notations: The notation of the optimal formalism, the Cartesian notation,[7] and spherical tensors. What the simplest set of experiments for determining the phases is depends on whether the Cartesian or spherical quantities interface easier with the particular experimental arrangements used.

One particular set of observables with none of them in the 1_M submatrix is also a set with a minimum number of observables which will determine both the magnitudes and the phases of the amplitudes uniquely, even eliminating discrete ambiguities. This set is

$$(R^0,R^0),\ (I^0,I^0);\ (R^+,R^-),\ (R^+,I^-)\,,$$

$$(R^-,R^-),\ (I^-,R^-),\ (R^0,I^0)\,.$$

TABLE IV. The listing of an experimental program to determine the amplitudes at an arbitrary energy and angle using the pure transversity frame. The notations for amplitudes are $D(++)\equiv\alpha$, $D(+-)\equiv\beta$, $D(--)\equiv\gamma$, $D(00)\equiv\delta$. The legend for the column headings is as follows: 1. Optimal-formalism notation for observables in the transversity frame as given in Table III. 2. Observables in the Cartesian-tensor notation with Z axis being the polarization direction. 3. Observables in the spherical-tensor notation with Z axis being the polarization direction. 4. Expression for the observable in terms of bilinear products of amplitudes in the transverse frame.

1	2	3	4
(A,A)	$2(I;I)$	$2(I;I)$	$4(\lvert\alpha\rvert^2+2\lvert\beta\rvert^2+\lvert\gamma\rvert^2+\lvert\delta\rvert^2)$
(Δ,Δ)	$2(P_Z;P_Z)$	$\frac{2}{3}\sqrt{2}(T_{10};T_{10})$	$4(\lvert\alpha\rvert^2-2\lvert\beta\rvert^2+\lvert\gamma\rvert^2)$
(Λ,Λ)	$2(P_{ZZ};P_{ZZ})$	$2\sqrt{2}(T_{20};T_{20})$	$4(\lvert\alpha\rvert^2+2\lvert\beta\rvert^2+\lvert\gamma\rvert^2+4\lvert\delta\rvert^2)$
(A,Δ)	$2(I;P_Z)$	$2(I;(\sqrt{6}/3)T_{10})$	$4(\lvert\alpha\rvert^2-\lvert\gamma\rvert^2)$
(A,Λ)	$2(I;P_{ZZ})$	$2(I,\sqrt{2})$	$4(\lvert\alpha\rvert^2+2\lvert\beta\rvert^2+\lvert\gamma\rvert^2-2\lvert\delta\rvert^2)$
(R^-,R^-)	$\sqrt{2}(\frac{1}{2}P_X+\frac{1}{3}P_{XZ};\frac{1}{2}P_X+\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}(-T_{21}+T_{2-1}-T_{11}+T_{1-1};-T_{21}+T_{2-1}-T_{11}+T_{1-1})$	$2\,\mathrm{Re}\alpha\delta^*$
(R^+,R^-)	$\sqrt{2}(\frac{1}{2}P_X-\frac{1}{3}P_{XZ};\frac{1}{2}P_X+\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}(T_{21}-T_{2-1}-T_{11}+T_{1-1};-T_{21}+T_{2-1}-T_{11}+T_{1-1})$	$2\,\mathrm{Re}\beta\delta^*$
(R^+,R^+)	$\sqrt{2}(\frac{1}{2}P_X-\frac{1}{3}P_{XZ};\frac{1}{2}P_X-\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}(T_{21}-T_{2-1}-T_{11}+T_{1-1};T_{21}-T_{2-1}-T_{11}+T_{1-1})$	$2\,\mathrm{Re}\gamma\delta^*$
(I^-,R^-)	$\sqrt{2}(\frac{1}{2}P_Y+\frac{1}{3}P_{YZ};\frac{1}{2}P_X+\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}[i(T_{21}+T_{2-1}+T_{11}+T_{1-1};-T_{21}+T_{2-1}-T_{11}+T_{1-1})]$	$2\,\mathrm{Im}\alpha\delta^*$
(I^+,R^+)	$\sqrt{2}(\frac{1}{2}P_Y-\frac{1}{3}P_{YZ};\frac{1}{2}P_X-\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}[-i(T_{21}+T_{2-1}-T_{11}-T_{1-1};T_{21}-T_{2-1}-T_{11}+T_{1-1})]$	$-2\,\mathrm{Im}\gamma\delta^*$
(I^+,R^-)	$\sqrt{2}(\frac{1}{2}P_Y-\frac{1}{3}P_{YZ};\frac{1}{2}P_X+\frac{1}{3}P_{XZ})$	$\frac{1}{\sqrt{6}}[-i(T_{21}+T_{2-1}-T_{11}-T_{1-1};-T_{21}+T_{2-1}-T_{11}+T_{1-1})]$	$2\,\mathrm{Im}\beta\delta^*$
(A,R^0)	$\frac{1}{3}(6I;P_{XX}-P_{YY})$	$\frac{\sqrt{3}}{3}(2\sqrt{3}I;T_{22}+T_{2-2})$	$4\,\mathrm{Re}(\alpha+\gamma)\beta^*$
(Δ,R^0)	$\frac{1}{3}(6P_Z;P_{XX}-P_{YY})$	$\frac{\sqrt{3}}{3}(2\sqrt{2}T_{20};T_{22}+T_{2-2})$	$4\,\mathrm{Re}(\alpha-\gamma)\beta^*$
(A,I^0)	$\frac{1}{3}(6I;-P_{XY})$	$\frac{\sqrt{3}}{3}(2\sqrt{3}I;iT_{22}-iT_{2-2})$	$-4\,\mathrm{Im}(\alpha-\gamma)\beta^*$
(Δ,I^0)	$\frac{1}{3}(6P_Z;-P_{XY})$	$\frac{\sqrt{3}}{3}(2\sqrt{2}T_{20};iT_{22}-iT_{2-2})$	$-4\,\mathrm{Im}(\alpha+\gamma)\beta^*$
(Λ,R^0)	$\frac{1}{3}(6P_{ZZ};P_{XX}-P_{YY})$	$\frac{\sqrt{3}}{3}(2\sqrt{6}T_{20};T_{22}+T_{2-2})$	$4\,\mathrm{Re}(\alpha-2\gamma)\beta^*$
(Λ,I^0)	$\frac{1}{3}(6P_{ZZ};-P_{XY})$	$\frac{\sqrt{3}}{3}(2\sqrt{6}T_{20};iT_{22}-iT_{2-2})$	$-4\,\mathrm{Im}(\alpha-\gamma)\beta^*$
(R^0,R^0)	$\frac{1}{3}(P_{XX}-P_{YY};P_{XX}-P_{YY})$	$\frac{\sqrt{3}}{3}(T_{22}+T_{2-2};T_{22}+T_{2-2})$	$2\,\mathrm{Re}\alpha\gamma^*+\lvert\beta\rvert^2$
(I^0,I^0)	$-\frac{1}{3}(P_{XY};P_{XY})$	$i\frac{\sqrt{3}}{3}(T_{22}-T_{2-2};T_{22}-T_{2-2})$	$2\,\mathrm{Re}\alpha\gamma^*-\lvert\beta\rvert^2$
(R^0,I^0)	$\frac{1}{3}(P_{XX}-P_{YY};-P_{XY})$	$\frac{\sqrt{3}}{3}(T_{22}+T_{2-2};iT_{22}-iT_{2-2})$	$2\,\mathrm{Im}\alpha\gamma^*$

TABLE V. Vanishing observables in the case of direct-channel resonances.

$A=C$		$A=-C$	
Optimal notation	Grein-Locher notation	Optimal notation	Grein-Locher notation
Δ,Δ	$\frac{4}{3}(10\,\vert\,10)$	Δ,Δ	$(10\,\vert\,10)$
Δ,I^0	$i(10\,\vert\,21)$	$\Delta,R^-=\Delta,R^+$	$-\frac{4}{3}(10\,\vert\,11)$
Δ,I^+		$\Delta,I^-=2I^0,R^-$	$\frac{4}{3}i(10\,\vert\,21)$
R^0,R^-	$\frac{\sqrt{6}}{6}[t_{21}-\sqrt{2}(20\,\vert\,21)]$	Δ,I^0	$\frac{\sqrt{2}}{3}i(10\,\vert\,22)$
I^0,I^0	$\frac{2}{3}(10\,\vert\,10)$	I^0,I^-	$\frac{2}{3}(10\,\vert\,11)$
R^0,I^-	$-\frac{\sqrt{6}}{6}[t_{21}-\sqrt{2}(20\,\vert\,21)]$	I^0,I^0	$\frac{2}{3}[(22\,\vert\,22)-(22\,\vert\,2-2)]$

C. Testing resonances

1. Direct-channel resonances

In a parity-conserving reaction, for a resonance state of spin J, the following relations hold among the helicity amplitudes:[8]

$$D_J(c,a;d,b)=\pm D_J(-c,a;-d,b)\ , \tag{9}$$

where the $\pm$ sign reflects the naturality of the resonance. For $\pi+d\rightarrow\pi+d$, four helicity amplitudes are

$$A\equiv(+,+),\quad B\equiv(+,0)\ ,$$
$$C\equiv(+,-),\quad D\equiv(0,0)\ , \tag{10}$$

where in (i,j), i and j are the spin indices of the final and initial deuterons, respectively.

Equation (9) implies that for a resonance at certain reaction angles (see below) we have

$$A=\pm C\ . \tag{11}$$

TABLE VI. Relation among the observables due to the direct-channel resonances.

Optimal notation	$A=C$ Grein-Locher notation (Ref. 8)
$(R^-,I^-)=(R^+,I^-)$	$\frac{2}{3}i(11\,\vert\,21)=\frac{2}{3}i(11\,\vert\,2-1)$
$(I^-,R^-)=(I^+,R^-)$	$-\frac{2}{3}i(11\,\vert\,21)=-\frac{2}{3}i(11\,\vert\,2-1)$
$(R^-,R^-)=(I^+,I^-)$	$\frac{2}{3}(21\,\vert\,21)=-\frac{3}{2}(11\,\vert\,1-1)$
$(I^-,I^-)=(R^+,R^-)$	$\frac{2}{3}(11\,\vert\,11)=\frac{2}{3}(21\,\vert\,2-1)$
$2(A,R^-)=(\Lambda,R^-)$	$\sqrt{2}t_{21}=(20\,\vert\,21)$
$(A,I^-)=-2(\Lambda,I^-)$	$\frac{4}{\sqrt{6}}it_{11}=-\frac{2}{\sqrt{3}}i(11\,\vert\,20)$

Optimal notation	$A=-C$ Grein-Locher notation (Ref. 8)
$(R^-,I^-)=(I^+,R^-)$	$\frac{2}{3}i(11\,\vert\,21)=-\frac{2}{3}i(11\,\vert\,2-1)$
$(I^-,R^-)=(R^+,I^-)$	$-\frac{2}{3}i(11\,\vert\,21)=\frac{2}{3}i(11\,\vert\,2-1)$
$-(R^-,R^-)=(R^+,R^-)$	$-\frac{2}{3}(11\,\vert\,11)=\frac{2}{3}(11\,\vert\,1-1)$
$(I^-,I^-)=-(I^+,I^-)$	$\frac{2}{3}(11\,\vert\,11)=\frac{2}{3}(11\,\vert\,1-1)$

This in turn implies certain constraints on the observables. In particular, some of the observables will vanish. Table V gives the vanishing observables both in optimal notations and the notations of Grein and Locher.[9] Furthermore, some additional relations also exist among the various observables as listed in Table VI. Tables V and VI are true, however, only for particular c.m. scattering angles. The reason for this is that if one decomposes[9] the amplitudes A,B,C,D in a partial-wave series, states with $(L=J\pm1)$ contribute to all amplitudes while states with $L=J$ contribute to only A and C. A and C however, have different $d^J_{m'm}(\theta)$. Therefore $L=J$ states do not

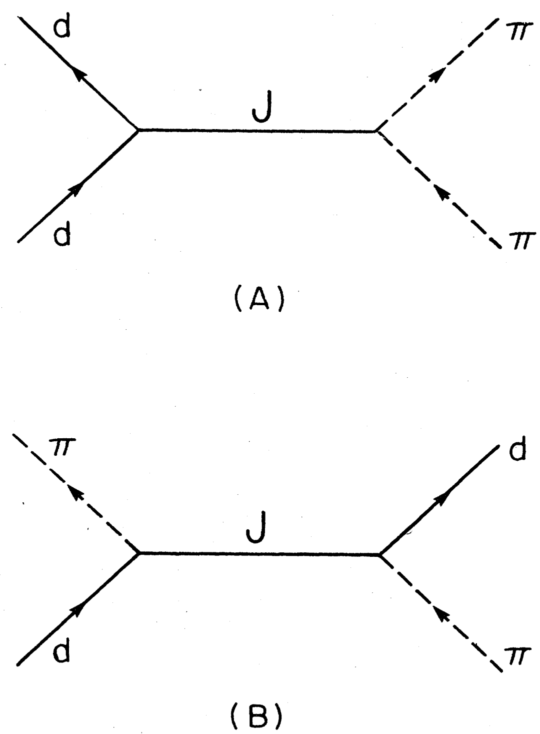

FIG. 1. (A) Exchange of a particle of spin J between two vertices, one of which has both the incoming and outgoing deuteron lines, and the other both the incoming and outgoing pion lines. (B) Crossed diagram to (A).

TABLE VII. Results of t-channel resonances.

Process \ Exchanged J	$J=0$	$J=1$	$J\geq 2$
Diag. A	$A=B=C=0$	$C=0$	
Diag. B	$A=B=C=0$	$A=-C$	

lead to an angle-independent relation between A and C. *The phase between A and C will however, be 0° or 180° for any resonance state.* An examination of their angular dependences will reveal that $A=C$ for those c.m. scattering angles for which $[d^J_{11}(\theta)-d^J_{1-1}(\theta)]$ vanishes. Therefore solutions of this will be the angles for which relations of Tables V and VI hold. For $J=1$ the equation

$$d^J_{11}(\theta)-d^J_{1-1}(\theta)=0$$

does not have a solution but for $J=2$ it does at $\theta=90°$. For $J=3$ angles will be 63.43° and 116.56°, and for higher values of J, there will be even more such angles. As a consequence, any oscillation of those observables which are listed in Tables V and VI involving zero values at the above discussed scattering angles represents an indication of a resonance.

2. *t-channel resonances*

Two different diagrams need to be considered here (Fig. 1). The first (diagram A) exchanges a particle of spin J between two vertices, one of which has both the incoming and the outgoing deuteron lines, and the other both the incoming and outgoing pion lines. The other diagram (diagram B) is the "crossed" one, in which one vertex at the end of the exchanged particle of spin J has the incoming deuteron and the outgoing pion line, while the other vertex has the incoming pion line and the outgoing deuteron line.

The results of Ref. 10 which are formulated in the "magic" optimal frame, when applied to this reaction, give the following results.

For diagram A, a $J=0$ resonance would allow only one amplitude to be nonzero, namely, $D_m\equiv(0,0)$ where the subscript m stands for magic. If the resonance is $J=1$, then three amplitudes are allowed but $C_m\equiv(+,-)$ is forbidden. For $J=2$ or higher the test imposes no constraints on the amplitudes.

For diagram B, a $J=0$ resonance would again require A_m, B_m, and C_m to vanish with only $D_m\equiv(0,0)$ contributing. If the resonance is $J=1$, where all four amplitudes contribute but two of them are related, namely, we have $A_m=-C_m$. If the resonance has $J=2$ or higher, again we have no constraints on the amplitudes.

The results are summarized in Table VII. It should be noted that these t-channel resonance tests apply at *all* angles and hence the same test also applies to a situation when the dynamics is dominated by a *sum* of several resonances, possibly with different J values, but sharing the same naturality.

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[1]Compare the measurement of it_{11} reported by J. Bolger *et al.* [Phys. Rev. Lett. **48**, 1667 (1982); **46**, 197 (1981)] with a more recent report of G. R. Smith *et al.* [Phys. Rev. C **29**, 2206 (1984)].

[2]For the case of the tensor polarization t_{20}, see R. J. Holt *et al.* [Phys. Rev. Lett. **43**, 1229 (1976); **47**, 472 (1981)] and compare it with the results of W. Gruebler *et al.* [*ibid.* **49**, 444 (1982)], and J. Ulbricht *et al.* [*ibid.* **48**, 311 (1982)].

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