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Comments on rotational perturbations of Friedmann models

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In addition to the solutions we have discussed before, we show that the field equations corresponding to the slowly rotating Friedmann models admit solutions for a special class of nonseparable rotation functions of the matter distribution. We also present two analytic solutions and discuss their possible use.

It is well known that almost everything in the Universe has some form of rotation[1,2] (differential or uniform), and over the past years the possibility of the entire Universe being endowed with some rotation has intrigued many physicists.[3-6] The current observations indicate that the Universe may be rotating at the rate of $\lesssim 10^{-13}$ rad/sec.[3-7] The existence of such a small rotation when extrapolated to the early stages of the Universe would play a major role in the dynamics of the Universe as well as in the processes that involve galaxy formation.

In a previous article[3] examining some of the effects of slow rotation we studied rotational perturbations of Friedmann models, where the metric is given as

$$ds^2 = dt^2 - e^{g(t)}\left[\frac{dr^2}{1-r^2/R^2} + r^2\,d\Omega^2\right] + 2r^2\sin^2\theta e^{g}\Omega(r,t)\,d\phi\,dt \quad . \tag{1}$$

$\Omega(r,t)$ is the metric rotation function which is related to the local dragging of inertial frames.[3] Using the perfect-fluid energy-momentum tensor

$$T^{ik} = (P+\rho)u^iu^k - Pg^{ik} \tag{2}$$

with $u^1 = u^2 = 0$, $u^3 = \omega$, we obtained the following field equations:

$$8\pi P = -\frac{1}{R^2}e^{-g} - \ddot{g} - \tfrac{3}{4}\dot{g}^2 + \Lambda \quad , \tag{3}$$

$$8\pi\rho = \frac{3}{R^2}e^{-g} + \tfrac{3}{4}\dot{g}^2 - \Lambda \quad , \tag{4}$$

$$R_{03} = -8\pi(T_{03} - \tfrac{1}{2}g_{03}T) + \Lambda g_{03} \quad , \tag{5}$$

and

$$R_{13} = -8\pi T_{13} \quad . \tag{6}$$

We kept only the first-order terms in $\Omega(r,t)$.[3]

As seen from above, the first two equations are not perturbed to the order we are considering, and can be used to determine e^g for a given equation of state. The remaining equations determine $\Omega(r,t)$ for a given rotation function $\omega(r,t)$. Equation (6) can be readily integrated to yield

$$\Omega(r,t) = A(r)e^{-(3/2)g(t)} + K(t) \quad , \tag{7}$$

where $K(t)$ can be set to zero without altering the physical structure. Finally, using (7) in (5) one obtains the field equation that determines $A(r)$ as

$$\left[1-\frac{r^2}{R^2}\right]\frac{A''}{A} + \left[\frac{4}{r} - \frac{5r}{R^2}\right]\frac{A'}{A} = \left[\frac{4}{R^2} - 2\ddot{g}e^{g}\right]\left[1 - \frac{e^{(3/2)g}\omega(r,t)}{A}\right] \quad . \tag{8}$$

Since the left-hand side of this equation is only a function of r, it imposes limitations on the allowed functional form of $\omega(r,t)$.

In our paper, we considered solutions of this equation for separable $\omega(r,t)$. However, even though Eq. (8) is incompatible for an arbitrary nonseparable $\omega(r,t)$, there is a special class of nonseparable $\omega(r,t)$ which is compatible with (8), and is given as

$$\omega(r,t) = [1 - a(r)b(t)]A(r)e^{-(3/2)g} \quad , \tag{9}$$

where $a(r)$ is an arbitrary function to be supplied to the field equations and $b(t)$ is given by

$$b(t) = a_0\left[\frac{4}{R^2} - 2\ddot{g}e^{g}\right]^{-1} \quad , \tag{10}$$

where a_0 is an arbitrary constant. For a given equation of state, Eqs. (3) and (4) determine e^g, which, through the use of Eq. (10), determines $b(t)$. On the other hand, for a given $a(r)$ Eq. (8) determines $A(r)$, which completes the solution of the problem.[3] As an example we present two solutions, the first of which corresponds to critically open Friedmann models ($1/R^2 = 0$), where $a(r)$ is given as

$$a(r) = \left[-\frac{b}{a_0}\right]r^{k-1} \quad , \tag{11}$$

where b and k are constants. For this choice of $a(r)$ the general solution of (8) could be given in terms of Bessel functions. A solution is

$$A(r) = r^{-3/2}Z_p(z) \quad , \tag{12}$$

where $p = -3/(1+k)$ is the order of the Bessel function and

$$z = \pm\frac{2\sqrt{b}}{(1+k)}r^{(1+k)/2} \quad . \tag{13}$$

Our second solution corresponds to closed models ($1/R^2 > 0$) and is given as[8]

$$A(y) = \ln C_2(1-y)^{-C_1} \quad , \tag{14}$$

and

$$\omega(y,t)=\left[\ln C_2(1-y)^{-C_1}-\frac{4}{a_0R^2}\left(\frac{\frac{5}{2}C_1-2C_1y}{(1-y)}\right)b(t)\right]e^{-(3/2)g} \;; \tag{15}$$

C_1 and C_2 are constants. Also, $y=r^2/R^2$ and $y\in[0,1]$. This solution is physically meaningful near the origin. However, for $y\rightarrow 1$ both $A(r,t)$ and $\omega(r,t)$ diverge. Hence, the small rotation approximation we have made is no longer true and the solution is not valid in this limit. To determine the arbitrary constant C_2 we require $A(r)$ to be zero at the center (which could be taken as the center of the local group of galaxies). This determines C_2 as $+1$. One of the remaining arbitrary constants could be determined by using the observations of Smoot, Gorenstein, and Muller,[9] which indicate that our galaxy is moving with a velocity of 520 km/sec with respect to the background radiation. This velocity is rather large from the standpoint that the peculiar velocities of all the nearby galaxies are at the level of or below 200 km/sec. Interpreting the residual velocity ($\sim$ 320 km/sec) as the value of $\omega(r,t)$ at our present location, one could in principle determine a_0 or C_1 from (15). The arbitrary constants that will appear in $g(t)$ and its explicit time dependence will come from the specific Friedmann model that one starts out with. The second solution could also be used for open models ($1/R^2<0$) by replacing y with $-y$.

In our paper[3] in order to clarify the physical nature of $\Omega(r,t)$ we have considered the behavior of test particles that have precisely the same initial conditions as the matter of the cosmological model, viz., $u^1=u^2=0$ and $u^3=\omega$ at $t=0$. From the geodesic equations we have found out that

$$\frac{du^1}{dt}=\frac{du^2}{dt}=0 \;; \tag{16}$$

hence u^1 and u^2 remain zero for $t>0$. However, u^3 was found to be

$$u^3=\Omega+(\omega_0-\Omega_0)e^{g(t)-g(0)} \;. \tag{17}$$

ω_0 and Ω_0 are the values of ω and Ω at $t=0$. As seen from (17), even though Ω is related to the dragging of test particles, it is not precisely equal to it, unless $\omega_0=\Omega_0$. In the second solution the remaining arbitrary constant could be used to set $\omega_0=\Omega_0$.

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