Mass splittings within composite Goldstone supermultiplets from broken supersymmetry

T. E. Clark and S. T. Love
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 28 May 1985)

The supersymmetric (SUSY) Dashen formulas are modified to include effects of softly broken supersymmetry and are used to compute the mass splittings and differences in decay constants among the various components of a Goldstone supermultiplet. The general results are applied to chiral-symmetry breaking in two-flavor SUSY QCD.

I. INTRODUCTION

Effective actions describing the low-energy behavior of strongly interacting supersymmetric (SUSY) theories have been shown to exhibit some interesting properties.[1,2] In particular, a SUSY extension of the Dashen formula was found which allowed the masses and decay constants of composite Goldstone bosons and their bosonic and fermionic (SUSY) partners to be expressed in terms of the constituent masses and condensates.[3–5] In SUSY QCD-like theories these Dashen formulas implied a singular behavior in the chiral limit[3,4] and led to speculations concerning the nonperturbative structure of the vacuum state.[6] Similarly, these formulas were employed to prove the absence of radiative mass shifts between composite Goldstone supermultiplets to arbitrary order in the perturbative gauge interactions.[7] In all of these considerations, the SUSY was treated as an unbroken symmetry. (See Ref. 8 for a short review of the superspace effective-action approach.)

However, if supersymmetry is to have any phenomenological applicability, it must be as a broken symmetry. The purpose of this paper is to calculate the effects of SUSY breaking on the mass spectrum of the (quasi-) Goldstone particles by simultaneously investigating both the broken-SUSY and internal-global-symmetry-group Ward identities. An alternate approach, which uses an explicit form for the effective Lagrangian, can be found in Ref. 9. More specifically, we consider the consequences on the SUSY Dashen formulas of soft SUSY breaking arising at the constituent level which would result, for instance, from a hidden supergravity sector.[10] Towards this end, the results of this breaking on superfield time-ordered functions is determined in Sec. II by solving the broken-SUSY Ward-identity differential equations to lowest order in the mass parameters characterizing the SUSY breaking. Since the Noether currents associated with the spontaneous breakdown of a global internal-symmetry group G to an invariant subgroup H act as interpolating fields for the Goldstone supermultiplets, their two-point functions will carry information about the (quasi-)Goldstone-boson masses (poles) and decay constant (residues).[3] When SUSY is broken these masses and decay constants will no longer be degenerate. Furthermore, the relationship between the auxiliary field on the mass shell and the first component of the composite supermultiplet will acquire an additional relative wave-function normalization factor. These results are summarized in the broken-SUSY current-field identity [partially conserved axial-vector current (PCAC) relation] which is given by

$$-\tfrac{1}{4}\bar{D}\bar{D}J_i=e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}\frac{1}{2\sqrt{2}}[(Z_{S_i}m_{S_i}f_{S_i}S_i+iZ_{P_i}m_{P_i}f_{P_i}P_i)+2m_{f_i}f_{f_i}\theta^\alpha\psi_{\alpha_i}+\theta\theta(m_{S_i}{}^2f_{S_i}S_i-im_{P_i}{}^2f_{P_i}P_i)] \tag{1.1}$$

and relates the superfield of Noether currents $J_i(x,\theta,\bar{\theta})$ to the Goldstone-boson fields P_i and their bosonic and fermionic SUSY partners S_i and ψ_i, respectively. Here m_{P_i},m_{S_i},m_{f_i} are the masses, f_{P_i},f_{S_i},f_{f_i} the decay constants, and Z_{P_i},Z_{S_i} the above-mentioned normalization factors for the fields denoted by the subscripts. In Sec. III, the current two-point functions are analyzed by using the superspace Noether theorem along with the broken-SUSY Ward identities of Sec. II. This allows these functions to be related to the parameters characterizing the internal-symmetry group and SUSY breaking as well as various condensates and (inserted) two-point functions of the underlying theory. Then, using the current-field identities, these relations are converted into the desired Dashen formulas. Finally, in Sec. IV, these general results are applied to the specific example of two-flavor SUSY QCD with an $SU(2)_L\times SU(2)_R$ internal symmetry spontaneously broken to an $SU(2)_V$ diagonal subgroup.

II. BROKEN SUPERSYMMETRY

Consider an underlying SUSY gauge theory with (anti)chiral matter fields ($\bar{\phi}$) ϕ in some representation of a strongly interacting gauge group with Yang-Mills vector superfields V. The dynamics is given by the action

$$I=I_{\rm inv}+I_b\ , \tag{2.1}$$

where

$$I_{\rm inv}=\int dS\,L_{\rm inv}+\int d\bar{S}\,\bar{L}_{\rm inv} \tag{2.2}$$

is SUSY invariant and $(\bar{L}_{\rm inv})$ $L_{\rm inv}$ is the corresponding (antichiral) chiral Lagrangian (see Refs. 3 and 11 for superspace conventions). In addition to being SUSY invariant, $I_{\rm inv}$ is also globally G invariant except for matter-field mass terms which are soft-G breaking but H invariant. Moreover, we envision that SUSY has been broken via some unspecified means such as a supergravity Higgs mechanism.[10] This breaking manifests itself as soft explicit SUSY-breaking masses in the underlying action I_b. We include in this paper only breaking terms arising from matter fields and exclude possible gaugino mass SUSY-breaking effects. The breaking action can then be written as

$$I_b=\int dV\,\theta\theta\bar{\theta}\,\bar{\theta}\Delta(x,\theta,\bar{\theta})\ , \tag{2.3}$$

where the superfield Δ is given by

$$\Delta=\Delta_V+\Delta_S+\bar{\Delta}_S\ , \tag{2.4}$$

with the (antichiral) chiral $(\bar{\Delta}_S)$ Δ_S superfield of the form

$$\Delta_S=\mu_S{}^2\phi^2\ ,\ \ \bar{\Delta}_S=\mu_S{}^2\bar{\phi}^2\ . \tag{2.5}$$

Typically these terms will also explicitly, but softly, break the global internal-symmetry group G, while respecting the symmetries of H. The vector superfield soft SUSY-breaking term Δ_V is of the form

$$\Delta_V=\mu_V{}^2\bar{\phi}e^{2V}\phi \tag{2.6}$$

and is typically G invariant.

The SUSY Ward identity will now be broken by the presence of I_b. According to the action principle, the breaking has the form

$$i\langle 0\,|\,T\delta X\,|\,0\rangle=\langle 0\,|\,T(\delta I_b)X\,|\,0\rangle\ , \tag{2.7}$$

where $\langle 0\,|\,TX\,|\,0\rangle$ is the time-ordered function for an arbitrary product, X, of superfields, fundamental or composite and δX is their supersymmetry variation. Thus the right-hand side of Eq. (2.7) is just the SUSY variation of the action inserted into the X Green's function. In our case this is simply given by

$$\delta I_b=\int dV\,\theta\theta\bar{\theta}\,\bar{\theta}\delta\Delta(x,\theta,\bar{\theta})$$
$$=\int dV\,\theta\theta\bar{\theta}\,\bar{\theta}\left[\xi^\alpha\frac{\partial}{\partial\theta^\alpha}+\bar{\xi}^{\dot{\alpha}}\left[-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\right]\right]\Delta(x,\theta,\bar{\theta})\ , \tag{2.8}$$

with $\xi^\alpha,\bar{\xi}^{\dot{\alpha}}$ the Weyl spinor, Grassman parameters for SUSY transformations. Equation (2.7) is viewed as a differential equation for $\langle 0\,|\,TX\,|\,0\rangle$ which we will solve to lowest nontrivial order. In particular, taking M,N as vector superfields, the Ward identity for their two-point function becomes

$$i\left[\xi^\alpha\left[\frac{\partial}{\partial\theta_1^\alpha}+\frac{\partial}{\partial\theta_2^\alpha}-i(\sigma^\mu\bar{\theta}_1)_\alpha\partial_{1\mu}-i(\sigma^\mu\bar{\theta}_2)_\alpha\partial_{2\mu}\right]\right.$$
$$\left.+\bar{\xi}^{\dot{\alpha}}\left[-\frac{\partial}{\partial\bar{\theta}_1^{\dot{\alpha}}}-\frac{\partial}{\partial\bar{\theta}_2^{\dot{\alpha}}}+i(\theta_1\sigma^\mu)_{\dot{\alpha}}\partial_{1\mu}+i(\theta_2\sigma^\mu)_{\dot{\alpha}}\partial_{2\mu}\right]\right]\langle 0\,|\,TM(1)N(2)\,|\,0\rangle$$
$$=\int dV_3\theta_3\theta_3\bar{\theta}_3\,\bar{\theta}_3\left[\xi^\alpha\frac{\partial}{\partial\theta_3^\alpha}+\bar{\xi}^{\dot{\alpha}}\left[-\frac{\partial}{\partial\bar{\theta}_3^{\dot{\alpha}}}\right]\right]\langle 0\,|\,T\Delta(3)M(1)N(2)\,|\,0\rangle\ . \tag{2.9}$$

The solution to this differential equation through first order in Δ is

$$\langle 0\,|\,TM(1)N(2)\,|\,0\rangle=\exp[i(\theta_1\sigma^\mu\bar{\theta}_2-\theta_2\sigma^\mu\bar{\theta}_1)\partial_{2\mu}][\langle 0\,|\,TM(0)N(2-1)\,|\,0\rangle$$
$$+i\int d^4x\,\langle 0\,|\,T\Delta(x_3,-\theta_1,-\bar{\theta}_1)M(0)N(2-1)\,|\,0\rangle]\ . \tag{2.10}$$

In obtaining this result, we have employed the unbroken-SUSY result

$$\langle 0\,|\,T\Delta(3)M(1)N(2)\,|\,0\rangle=\exp[i(\theta_1\sigma^\mu\bar{\theta}_2-\theta_2\sigma^\mu\bar{\theta}_1)\partial_{2\mu}+i(\theta_1\sigma^\mu\bar{\theta}_3-\theta_3\sigma^\mu\bar{\theta}_1)\partial_{3\mu}]\langle 0\,|\,T\Delta(3-1)M(0)N(2-1)\,|\,0\rangle\ . \tag{2.11}$$

This is consistent with our approximation scheme since this superfield expression already contains a factor of Δ.

In a similar fashion we can find the form of the Green's functions when M,N are chiral and/or antichiral. Letting S be a chiral superfield satisfying $\bar{D}_{\dot{\alpha}}S=0$ so that

$$S(1)=\exp(i\theta_1\sigma^\mu\bar{\theta}_1\partial_{1\mu})S(x_1,\theta_1,0)\ , \tag{2.12}$$

and taking M as a vector superfield we have

$$\langle 0\,|\,TM(1)S(2)\,|\,0\rangle=\exp[i(\theta_1\sigma^\mu\bar{\theta}_1+\theta_2\sigma^\mu\bar{\theta}_2-2\theta_2\sigma^\mu\bar{\theta}_1)\partial_{2\mu}]$$
$$\times[\langle 0\,|\,TM(0)S(x_2-x_1,\theta_2-\theta_1,0)\,|\,0\rangle$$
$$+i\int d^4x\,\langle 0\,|\,T\Delta(x,-\theta_1,-\bar{\theta}_1)M(0)S(x_2-x_1,\theta_2-\theta_1,0)\,|\,0\rangle]\ . \tag{2.13}$$

Finally if $\bar{R}$ is an antichiral superfield constrained by $D_\alpha\bar{R}=0$ so that

$$\bar{R}(2)=\exp(-i\theta_2\sigma^\mu\bar{\theta}_2\partial_{2\mu})\bar{R}(x,0,\bar{\theta}_2)\ , \tag{2.14}$$

we secure, for S chiral, the form

$$\langle 0|T\bar{R}(1)S(2)|0\rangle=\exp[i(\theta_1\sigma^\mu\bar{\theta}_1+\theta_2\sigma^\mu\bar{\theta}_2-2\theta_2\sigma^\mu\bar{\theta}_1)\partial_{2\mu}]\times\left[\langle 0|T\bar{R}(0)S(x_2-x_1,0,0)|0\rangle+i\int d^4x\langle 0|T\Delta(x,-\theta_2,\bar{\theta}_1)\bar{R}(0)S(x_2-x_1,0,0)|0\rangle\right]. \quad (2.15)$$

In addition, the vacuum value of a single vector or chiral superfield obeys the simple SUSY differential equation

$$i\left[\xi^\alpha\frac{\partial}{\partial\theta_1^\alpha}-\bar{\xi}^{\dot\alpha}\frac{\partial}{\partial\bar{\theta}_1^{\dot\alpha}}\right]\langle 0|M(1)|0\rangle=\int dV\,\theta\theta\bar{\theta}\bar{\theta}\langle 0|T\left[\xi^\alpha\frac{\partial}{\partial\theta^\alpha}-\bar{\xi}^{\dot\alpha}\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}}\right]\Delta(x,\theta,\bar{\theta})M(1)|0\rangle, \quad (2.16)$$

where space-time translation invariance has been used. Solving this equation to first order in Δ for a vector superfield M then yields

$$\langle 0|M(1)|0\rangle=\langle 0|M(0)|0\rangle+i\int d^4x\langle 0|T\Delta(x,-\theta_1,-\bar{\theta}_1)M(0)|0\rangle. \quad (2.17)$$

Focusing on the $\theta_1,\bar{\theta}$, independent component leads to the constraint

$$\int d^4x\langle 0|T\Delta(x,0,0)M(0)|0\rangle=0. \quad (2.18)$$

Similarly for S, a chiral superfield, we find

$$\langle 0|S(1)|0\rangle=\langle 0|S(0)|0\rangle+i\int d^4x\langle 0|T\Delta(x,-\theta_1,\bar{\theta}_1)S(0)|0\rangle, \quad (2.19)$$

with

$$\int d^4x\langle 0|T\Delta(x,0,0)S(0)|0\rangle=0.$$

III. THE DASHEN FORMULAS

In order to obtain the component Dashen formulas we analyze the component structure, implied by the broken-SUSY Ward identity, of the Noether current two-point functions. Writing the underlying action as

$$I=\int dV(L_V+\theta\theta\bar{\theta}\bar{\theta}\Delta_V)+\int dS(L_S+\theta\theta\Delta_S)+\int d\bar{S}\,\bar{\theta}\bar{\theta}(\bar{L}_S+\bar{\theta}\bar{\theta}\bar{\Delta}_S), \quad (3.1)$$

the Noether currents associated with the global symmetry group G are given by[3]

$$J_A=J_A^{\text{inv}}+\theta\theta\bar{\theta}\bar{\theta}J_A^\Delta, \quad (3.2)$$

where the index A represents the generators of G and the superfield currents are

$$J_A^{\text{inv}}=\frac{i}{2}\left[\delta_A\bar{\phi}\frac{\partial L_V}{\partial\bar{\phi}}-\delta_A\phi\frac{\partial L_V}{\partial\phi}\right],$$
$$J_A^\Delta=\frac{i}{2}\left[\delta_A\bar{\phi}\frac{\partial\Delta_V}{\partial\bar{\phi}}-\delta_A\phi\frac{\partial\Delta_V}{\partial\phi}\right]. \quad (3.3)$$

Note that, although J_A^{inv} and J_A^Δ are superfields, J_A is not a superfield due to the presence of the explicit $\theta\theta\bar{\theta}\bar{\theta}$. We will call such quantities superspace functions as opposed to superfields.

Defining the chiral and antichiral Lagrangians as

$$L=-\tfrac{1}{8}\bar{D}\bar{D}(L_V+\theta\theta\bar{\theta}\bar{\theta}\Delta_V)+L_S+\theta\theta\Delta_S,$$
$$\bar{L}=-\tfrac{1}{8}DD(L_V+\theta\theta\bar{\theta}\bar{\theta}\Delta_V)+\bar{L}_S+\bar{\theta}\bar{\theta}\bar{\Delta}_S, \quad (3.4)$$

so that

$$I=\int dS\,L+\int d\bar{S}\,\bar{L},$$

we find that the internal-symmetry variations of L and $\bar{L}$ are given via Noether's theorem as

$$\delta_A L=\frac{i}{4}\bar{D}\bar{D}J_A+\delta_A\phi\frac{\delta I}{\delta\phi},$$
$$\delta_A\bar{L}=-\frac{i}{4}DDJ_A+\delta_A\bar{\phi}\frac{\delta I}{\delta\bar{\phi}}. \quad (3.5)$$

It follows that the current two-point functions can be written as

$$\langle 0|T\delta_A L(1)\delta_B L(2)|0\rangle=\left\langle 0\left|T\left[\frac{i}{4}\bar{D}\bar{D}J_A(1)\right]\left[\frac{i}{4}\bar{D}\bar{D}J_B(2)\right]\right|0\right\rangle+\left\langle 0\left|T\delta_A L(1)\left[\delta_B\phi\frac{\delta I}{\delta\phi}\right](2)\right|0\right\rangle+\left\langle 0\left|T\left[\delta_A\phi\frac{\delta I}{\delta\phi}\right](1)\left[\frac{i}{4}\bar{D}\bar{D}J_B(2)\right]\right|0\right\rangle, \quad (3.6a)$$

$$\langle 0|T\delta_A L(1)\delta_B\bar{L}(2)|0\rangle=\left\langle 0\left|T\left[\frac{i}{4}\bar{D}\bar{D}J_A(1)\right]\left[-\frac{i}{4}DDJ_B(2)\right]\right|0\right\rangle+\left\langle 0\left|T\delta_A L(1)\left[\delta_B\bar{\phi}\frac{\delta I}{\delta\bar{\phi}}\right](2)\right|0\right\rangle+\left\langle 0\left|T\left[\delta_A\phi\frac{\delta I}{\delta\phi}\right](1)\left[-\frac{i}{4}DDJ_B(2)\right]\right|0\right\rangle. \quad (3.6b)$$

The component structure of the above identities can be obtained by tedious but straightforward application of the broken-SUSY Ward identities to each superfield Green's function. First consider the current two-point function itself. It can be written in terms of its superfield parts as

$$\langle 0 \mid TJ_A(1)J_B(2) \mid 0\rangle = \langle 0 \mid TJ_A^{\rm inv}(1)J_B^{\rm inv}(2) \mid 0\rangle + \theta_1\theta_1\bar\theta_1\bar\theta_1 \langle 0 \mid TJ_A^{\Delta}(1)J_B^{\rm inv}(2) \mid 0\rangle + \theta_2\theta_2\bar\theta_2\bar\theta_2\langle 0 \mid TJ_A^{\rm inv}(1)J_B^{\Delta}(2) \mid 0\rangle + \theta_1\theta_1\bar\theta_1\bar\theta_1\theta_2\theta_2\bar\theta_2\bar\theta_2 \langle 0 \mid TJ_A^{\Delta}(1)J_B^{\Delta}(2) \mid 0\rangle \ . \tag{3.7}$$

Now each superfield two-point function can be expanded in terms of its independent components to first order in the SUSY-breaking parameters $\mu_V{}^2$, $\mu_S{}^2$. Consistent with this approximation, we set

$$\langle 0 \mid TJ_A^{\Delta}(1)J_B^{\Delta}(2) \mid 0\rangle = 0 \tag{3.8}$$

since it is second order in SUSY breaking and

$$\langle 0 \mid TJ_A^{\Delta}(1)J_B^{\rm inv}(2) \mid 0\rangle = \exp[i(\theta_1\sigma^\mu\bar\theta_2 - \theta_2\sigma^\mu\bar\theta_1)\partial_{2\mu}]\langle 0 \mid TJ_A^{\Delta}(0)J_B^{\rm inv}(2-1) \mid 0\rangle \tag{3.9}$$

which is the unbroken SUSY relation and follows since J_A^{Δ} is already first order in the SUSY breaking. For the $J_A^{\rm inv}$ Green's function we find upon using Eq. (2.10) that

$$\int d^4x_1 \langle 0 \mid T[-\tfrac{1}{4}\bar D\,\bar D J_A^{\rm inv}(1)][-\tfrac{1}{4}\bar D\,\bar D J_B^{\rm inv}(2)] \mid 0\rangle$$
$$= i\int d^4x d^4y \langle 0 \mid T[\Delta^{(0,2)}(y) - \theta_1^{\alpha}\Delta_\alpha^{(1,2)}(y) + \theta_1{}^2\Delta^{(2,2)}(y)] \times J_A^{{\rm inv}(0,0)}(x)[J_B^{{\rm inv}(0,2)}(0) + (\theta_2-\theta_1)^\alpha J_{B\alpha}^{{\rm inv}(1,2)}(0) + (\theta_2-\theta_1)^2 J_B^{{\rm inv}(2,2)}(0)] \mid 0\rangle \ . \tag{3.10}$$

Here the superscripts (m,n) denote the respective powers of $(\theta,\bar\theta)$ that the component field is multiplied by in its corresponding superfield decomposition. For example, the superfield Δ has the component decomposition

$$\Delta(1) = \Delta^{(0,0)}(x_1) + \theta_1^\alpha\Delta_\alpha^{(1,0)}(x_1) + \bar\theta_{1\dot\alpha}\Delta^{\dot\alpha(0,1)}(x_1) + \theta_1{}^2\Delta^{(2,0)}(x_1) + \bar\theta_1{}^2\Delta^{(0,2)}(x_1) + \theta_1\sigma^\mu\bar\theta_1\Delta_\mu^{(1,1)}(x_1) + \theta_1{}^2\bar\theta_{1\dot\alpha}\Delta^{\dot\alpha(2,1)}(x_1) + \bar\theta_1{}^2\theta_1^\alpha\Delta_\alpha^{(1,2)}(x_1) + \theta_1{}^2\bar\theta_1{}^2\Delta^{(2,2)}(x_1) \ . \tag{3.11}$$

By recombining these various equations we obtain the component structure of

$$\langle 0 \mid T\bar D\,\bar D J_A(1)\bar D\,\bar D J_B(2) \mid 0\rangle \ .$$

The component structure of the remaining terms on the right-hand side (RHS) of Eqs. (3.6a) and (3.6b) can also be analyzed. That is, by applying the action principle

$$\left\langle 0 \left| T\left[\delta\phi\frac{\delta I}{\delta\phi}\right](1)X\right| 0\right\rangle = i\left\langle 0\left| T\delta\phi(1)\frac{\delta}{\delta\phi(1)}X\right| 0\right\rangle \tag{3.12}$$

to the term where X is $\delta_A L$ we have

$$\left\langle 0\left| T\delta_A L(1)\left[\delta_B\phi\frac{\delta I}{\delta\phi}\right](2)\right| 0\right\rangle = i\left\langle 0\left| T\delta_B\phi(2)\frac{\delta}{\delta\phi(2)}\delta_A L(1)\right| 0\right\rangle \ . \tag{3.13}$$

Then if L_V and Δ_V are invariant under G (as is our case), we can factor a chiral δ function from this to simply write it as the double variation of the chiral Lagrangian to obtain

$$\delta_B\phi(2)\frac{\delta}{\delta\phi(2)}\delta_A L(1) = -\tfrac{1}{4}\bar D\,\bar D\delta(1-2)\delta_B(1)\frac{\partial\delta_A L(1)}{\partial\phi(1)} = -\tfrac{1}{4}\bar D\bar D\delta(1-2)\delta_B\delta_A L(1) \ . \tag{3.14}$$

Combining this with Eq. (3.13), then yields

$$\left\langle 0\left| T\delta_A L(1)\left[\delta_B\phi\frac{\delta I}{\delta\phi}\right](2)\right| 0\right\rangle = -\frac{i}{4}\bar D\,\bar D\delta(1-2)\langle 0 \mid \delta_B\delta_A L(1) \mid 0\rangle \ . \tag{3.15}$$

Finally, using the chirality of L and the broken-SUSY Ward-identity result of Eq. (2.19) we secure

$$\left\langle 0\left| T\delta_A L(1)\left[\delta_B\phi\frac{\delta I}{\delta\phi}\right](2)\right| 0\right\rangle = -\frac{i}{4}\bar D\,\bar D\langle 0 \mid \delta_B\delta_A L(0) \mid 0\rangle + \tfrac{1}{4}\bar D\,\bar D\delta(1-2)\int d^4y \langle 0 \mid T\Delta(y,-\theta_1,0)\delta_B\delta_A L(0) \mid 0\rangle \ . \tag{3.16}$$

In a similar way the action principle can be applied to the term where X is $\bar D\bar D J_B$. We find that this term can be related to a different condensate and broken-SUSY insertion corrections and takes the form

$$\int d^4x_1 \left\langle 0 \left| T\left[\delta_A\phi \frac{\delta I}{\delta\phi}\right](1)\left[\frac{i}{4}\bar{D}\bar{D}J_B(2)\right]\right| 0\right\rangle$$
$$=\theta_1{}^2\theta_2{}^2\left\langle 0\left|\left[\delta_A\phi\frac{\partial J_B^\Delta}{\partial\phi}\right](0)\right|0\right\rangle$$
$$+i(\theta_1-\theta_2)^2\int d^4x\left\langle 0\left|T[\Delta^{(0,2)}(x)-\theta_2^\alpha\Delta_\alpha^{(1,2)}(x)+\theta_2{}^2\Delta^{(2,2)}(x)]\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right](0)\right|0\right\rangle . \quad (3.17)$$

Reassembling all of the above terms along with the corresponding expressions in the $\langle 0|T\delta_A L\delta_B\bar{L}|0\rangle$ function we find 7 equations relating the components of the $\delta_A L$ and $\delta_A\bar{L}$ propagators to double variations of the Lagrangian plus SUSY-breaking corrections. (An additional 2 relations are obtained, but these are merely consistency relations among the breaking terms and contain no new information about the $\delta_A L, \delta_A\bar{L}$ propagators.) The component zero-momentum two-point functions are given by

$$\int d^4x_1\langle 0|T\delta_A L^{(0,0)}(x_1)\delta_B L^{(0,0)}(x_2)|0\rangle=-i\int d^4x\,d^4y\langle 0|T\Delta^{(0,2)}(x)J_A^{(0,0)}(0)J_B^{(0,2)}(y)|0\rangle , \quad (3.18a)$$

$$\int d^4x_1\langle 0|T\delta_A L^{(0,0)}(x_1)\delta_B L^{(2,0)}(x_2)|0\rangle$$
$$=i\langle 0|\delta_B\delta_A L^{(0,0)}(0)|0\rangle-\int d^4x\langle 0|TJ_B^{\Delta(0,0)}(0)J_A^{(0,2)}(x)|0\rangle$$
$$+i\int d^4x\left\langle 0\left|T\Delta^{(0,2)}(x)\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right](0)\right|0\right\rangle-i\int d^4x\,d^4y\langle 0|T\Delta^{(0,2)}(x)J_A^{(0,0)}(0)J_B^{(2,2)}(y)|0\rangle , \quad (3.18b)$$

$$\int d^4x_1\langle 0|T\delta_A L^{(2,0)}(x_1)\delta_B L^{(2,0)}(x_2)|0\rangle$$
$$=i\langle 0|\delta_B\delta_A\Delta_S^{(0,0)}(0)|0\rangle+\left\langle 0\left|\left[\delta_A\phi\frac{\partial J_B^\Delta}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle-\int d^4x\langle 0|T\Delta^{(2,0)}(x)\delta_B\delta_A L^{(0,0)}(0)|0\rangle$$
$$+i\int d^4x\left\langle 0\left|T\Delta^{(2,2)}(x)\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle-\int d^4x\langle 0|TJ_A^{\Delta(0,0)}(0)J_B^{(2,2)}(x)|0\rangle$$
$$-\int d^4x\langle 0|TJ_A^{(2,2)}(x)J_B^{\Delta(0,0)}(0)|0\rangle-i\int d^4x\,d^4y\langle 0|T\Delta^{(2,2)}(x)J_A^{(0,0)}(0)J_B^{(2,2)}(y)|0\rangle , \quad (3.18c)$$

$$\int d^4x_1\langle 0|T\delta_A L^{\alpha(1,0)}(x_1)\delta_B L_\alpha^{(1,0)}(x_2)|0\rangle$$
$$=4i\langle 0|\delta_B\delta_A L^{(0,0)}(0)|0\rangle+4i\int d^4x\left\langle 0\left|T\Delta^{(0,2)}(x)\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle$$
$$-4i\int d^4x\,d^4y\langle 0|T\Delta^{(0,2)}(x)J_A^{(0,0)}(0)J_B^{(2,2)}(y)|0\rangle+2i\int d^4x\,d^4y\langle 0|T\Delta^{\alpha(1,2)}(x)J_A^{(0,0)}(0)J_{B\alpha}^{(1,2)}(y)|0\rangle , \quad (3.18d)$$

$$\int d^4x_1\langle 0|T\delta_A L^{(0,0)}(x_1)\delta_B\bar{L}^{(0,0)}(x_2)|0\rangle$$
$$=-\left\langle 0\left|\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle+\int d^4x\langle 0|TJ_A^{(0,0)}(0)J_B^{\mathrm{inv}(2,2)}(x)|0\rangle$$
$$+i\int d^4x\,d^4y\langle 0|T\Delta^{(0,2)}(x)J_A^{(0,0)}(0)J_B^{(2,0)}(y)|0\rangle$$
$$+i\int d^4x\,d^4y\langle 0|T\Delta^{(0,0)}(x)J_A^{(0,0)}(0)J_B^{(2,2)}(y)|0\rangle-\frac{i}{2}\int d^4x\,d^4y\langle 0|T\Delta_{\dot\alpha}^{(0,1)}(x)J_A^{(0,0)}(0)J_B^{\dot\alpha(2,1)}(y)|0\rangle , \quad (3.18e)$$

$$\int d^4x_1\langle 0|T\delta_A L^{(0,0)}(x_1)\delta_B\bar{L}^{(0,2)}(x_2)|0\rangle$$
$$=\int d^4x\langle 0|TJ_A^{(0,2)}(x)J_B^{\Delta(0,0)}(0)|0\rangle-i\int d^4x\left\langle 0\left|T\Delta^{(0,2)}(x)\left[\delta_A\phi\frac{\partial J_B}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle$$
$$+i\int d^4x\,d^4y\langle 0|T\Delta^{(0,2)}(x)J_A^{(0,0)}(0)J_B^{(2,2)}(y)|0\rangle , \quad (3.18f)$$

$$\int d^4x_1 \langle 0 | T\delta_A L^{(2,0)}(x_1)\delta_B \bar{L}^{(0,2)}(x_2) | 0 \rangle$$
$$= -\left\langle 0 \left| \left[\delta_A \phi \frac{\partial J_B^{\Delta(0,0)}}{\partial \phi} \right](0) \right| 0 \right\rangle - i \int d^4x \left\langle 0 \left| T\Delta^{(2,2)}(x) \left[\delta_A \phi \frac{\partial J_B}{\partial \phi} \right]^{(0,0)}(0) \right| 0 \right\rangle + \int d^4x \langle 0 | TJ_A^{(2,2)}(x) J_B^{\Delta(0,0)}(0) | 0 \rangle$$
$$+ \int d^4x \langle 0 | TJ_A^{\Delta(0,0)}(0) J_B^{(2,2)}(x) | 0 \rangle + i \int d^4x\, d^4y \langle 0 | T\Delta^{(2,2)}(x) J_A^{(0,0)}(0) J_B^{(2,2)}(y) | 0 \rangle \ . \quad (3.18g)$$

When SUSY is unbroken, the PCAC current-field identity relating the Goldstone-boson fields P_i and their SUSY partners, the quasi-Goldstone-boson fields S_i, and quasi-Goldstone-fermion fields ψ_{α_i}, to their interpolating fields, the components of the superfield of currents, J_i, is simple. (The indices from the middle of the alphabet $i,j,\ldots$ denumerate the broken generators of G.) SUSY dictates a common mass m_{π_i} and a common decay constant f_{π_i}. Furthermore, the relative wave-function normalization between the bosonic fields and their auxiliary fields is unity. That is, close to mass shell the current-field identity takes the form

$$-\tfrac{1}{4}\bar{D}\,\bar{D}J_i = \tfrac{1}{2} m_{\pi_i} f_{\pi_i} \pi_i \ , \quad (3.19)$$

with the chiral Goldstone superfield π_i given by

$$\pi_i = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu} \frac{1}{\sqrt{2}} \{ (S_i + iP_i) + 2\theta^\alpha \psi_{\alpha_i} + \theta\theta[m_{\pi_i}(S_i - iP_i) + E_i] \} \quad (3.20)$$

and the E_i are Euler-Lagrange terms which arise from the elimination of constituent auxiliary fields from the interpolating fields. On mass shell these vanish but at zero momentum where the current, or equivalently the $\delta_A L$, two-point functions are being evaluated they contribute additional condensate terms necessary to derive the ordinary Dashen formula.[3–5] The explicit form of these Euler terms depends on the model considered.

On the other hand, when SUSY is broken each field will receive separate contributions to its mass, $m_{P_i}, m_{S_i} m_{f_i}$, and decay constant, $f_{P_i}, f_{S_i}, f_{f_i}$. In addition, there will now be nontrivial normalization factors, Z_{P_i}, Z_{S_i}, for the different composite fields that interpolate for P_i and S_i in the first and last components of the superfields. That is, the composite auxiliary field now has a relative wave-function normalization in its equation of motion. In a somewhat cryptic notation, the field equation for the composite auxiliary field reads $\bar{F}_{\text{composite}} = Z^{-1} m A_{\text{composite}}$. Hence, the PCAC current-field identity close to the mass shell becomes

$$-\tfrac{1}{4}\bar{D}\,\bar{D}J_i = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu} \frac{1}{2\sqrt{2}} [(Z_{S_i} m_{S_i} f_{S_i} S_i + iZ_{P_i} m_{P_i} f_{P_i} P_i) + 2m_{f_i} f_{f_i} \theta^\alpha \psi_{\alpha_i} + \theta\theta(m_{S_i}{}^2 f_{S_i} S_i - i m_{P_i}{}^2 f_{P_i} P_i + E_i)] = i\delta_i L \ , \quad (3.21)$$

where the second equality follows from Noether's theorem applied close to mass shell. Consequently $\delta_i L$ also carries the (quasi-)Goldstone particle poles and hence the PCAC relation can also be written using $\delta_i L$ as the interpolating field. Similarly for the antichiral Lagrangian we have the corresponding current-field identities

$$-\tfrac{1}{4}DDJ_i = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu} \frac{1}{2\sqrt{2}} [(Z_{S_i} m_{S_i} f_{S_i} S_i - iZ_{P_i} m_{P_i} f_{P_i} P_i) + 2m_{f_i} f_{f_i} \bar{\theta}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}_i + \bar{\theta}\bar{\theta}(m_{S_i}{}^2 f_{S_i} S_i + i m_{P_i}{}^2 f_{P_i} P_i + \bar{E}_i)] = -i\delta_i \bar{L} \ . \quad (3.22)$$

Employing these identities in our $\delta_A L, \delta_A \bar{L}$ two-point function relations and using the propagators

$$\begin{aligned}
&\int d^4x \langle 0 | TS_i(0) S_j(x) | 0 \rangle = \frac{1}{i} \frac{1}{m_{S_i}{}^2} \delta_{ij} \ , \\
&\int d^4x \langle 0 | TP_i(0) P_j(x) | 0 \rangle = \frac{1}{i} \frac{1}{m_{P_i}{}^2} \delta_{ij} \ , \\
&\int d^4x \langle 0 | TS_i(0) P_j(x) | 0 \rangle = 0 \ , \\
&\int d^4x \langle 0 | T\psi_{i\alpha}(0) \psi_{j\beta}(x) | 0 \rangle = -\frac{1}{i} \frac{\epsilon_{\alpha\beta}}{m_{f_i}} \delta_{ij} \ ,
\end{aligned} \quad (3.23)$$

we obtain 5 independent component Dashen formulas. [Of the original 7 expressions Eqs. (3.18a)–(3.18g), 2 of them are redundant and simply reduce to consistency relations among the various (inserted) two-point functions.] The independent Dashen formulas are given by (no sum on i)

$$m_{P_i}{}^2 f_{P_i}{}^2 = -i \int d^4x \langle 0 | TE_i(x) \bar{E}_i(0) | 0 \rangle - 8 \langle 0 | \delta_i \delta_i \Delta_S^{(0,0)}(0) | 0 \rangle - 8i \int d^4x \langle 0 | T\Delta^{(2,0)}(x) \delta_i \delta_i L^{(0,0)}(0) | 0 \rangle \ , \quad (3.24a)$$

$$m_{S_i}{}^2 f_{S_i}{}^2 = -i\int d^4x\,\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle + 8\langle 0\,|\,\delta_i\delta_i\Delta_S^{(0,0)}(0)\,|\,0\rangle - 8i\left\langle 0\,\middle|\,\left[\delta_i\phi\frac{\partial J_i^{\Delta}}{\partial\phi}\right]^{(0,0)}(0)\,\middle|\,0\right\rangle$$
$$+8i\int d^4x\,\langle 0\,|\,T\Delta^{(2,0)}(x)\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle + 8\int d^4x\left\langle 0\,\middle|\,T\Delta^{(2,2)}(x)\left[\delta_i\phi\frac{\partial J_i}{\partial\phi}\right]^{(0,0)}(0)\,\middle|\,0\right\rangle$$
$$+16i\int d^4x\,\langle 0\,|\,TJ_i^{(2,2)}(x)J_i^{\Delta(0,0)}(0)\,|\,0\rangle - 8\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(2,2)}(x)J_i^{(0,0)}(0)J_i^{(2,2)}(y)\,|\,0\rangle\,, \tag{3.24b}$$

$$m_{f_i}f_{f_i}{}^2 = 4\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle + 4\int d^4x\left\langle 0\,\middle|\,T\Delta^{(0,2)}(x)\left[\delta_i\phi\frac{\partial J_i}{\partial\phi}\right]^{(0,0)}(0)\,\middle|\,0\right\rangle$$
$$-4\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(0,2)}(x)J_i^{(0,0)}(0)J_i^{(2,2)}(y)\,|\,0\rangle + 2\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(2,2)}(x)J_i^{(0,0)}(0)J_i^{(0,2)}(y)\,|\,0\rangle\,, \tag{3.24c}$$

$$Z_{P_i}m_{P_i}f_{P_i}{}^2 = 8\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle\,, \tag{3.24d}$$

$$Z_{S_i}m_{S_i}f_{S_i}{}^2 = 8\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle + 8i\int d^4x\,\langle 0\,|\,TJ_i^{(0,2)}(x)J_i^{\Delta(0,0)}(0)\,|\,0\rangle$$
$$+8\int d^4x\left\langle 0\,\middle|\,T\Delta^{(0,2)}(x)\left[\delta_i\phi\frac{\partial J_i}{\partial\phi}\right]^{(0,0)}(0)\,\middle|\,0\right\rangle - 8\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(0,2)}(x)J_i^{(0,0)}(0)J_i^{(2,2)}(y)\,|\,0\rangle\,. \tag{3.24e}$$

In evaluating the breaking terms on the right-hand side (RHS) of Eqs. (3.24), one first pulls out the explicit breaking mass parameters leaving a soft operator inserted Green's function. This is then evaluated in the unbroken symmetry limit and carries a power, according to its dimension, of the underlying strong dynamics scale. Note that equations (3.24c)–(3.24e) above can be added to obtain a generalized SUSY supertrace mass formula involving masses, decay constants, normalization factors, and the SUSY-breaking vector superfield (i.e., ΔV):

$$Z_{P_i}m_{P_i}f_{P_i}{}^2 + Z_{S_i}m_{S_i}f_{S_i}{}^2 - 2m_{f_i}f_{f_i}{}^2 = 8i\int d^4x\,\langle 0\,|\,TJ_i^{\Delta(0,0)}(0)J_i^{(0,2)}(x)\,|\,0\rangle - 4\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(2,2)}(x)J_i^{(0,0)}(0)J_i^{(0,2)}(x)\,|\,0\rangle\,. \tag{3.25}$$

Note also that the only nonvanishing two-point function involving the Euler terms is $\langle 0\,|\,TE_i\bar{E}_i\,|\,0\rangle$. This follows since E_i contains derivatives with respect to the constituent auxiliary fields $\bar{F}$ which are contained only in $\bar{E}_i$.

The above Dashen formulas relate parameters characterizing the composite objects to condensates and (inserted) two-point functions containing the constituent fields. When SUSY is unbroken, the common mass and decay constant could be separately related to the underlying internal-symmetry-group-breaking masses and constituent scalar and fermion field condensates.[3] However, once SUSY is broken, this degeneracy is lifted and only the products Zmf^2 and m^2f^2 for the bosons and mf^2 for the fermions are determined from the Dashen formulas.

If the internal symmetry group's spontaneous breaking occurs at a scale comparable to the SUSY breaking scale we can simplify the above formulas by only retaining terms to first order in all broken quantities. In such a case, not only are terms such as $\langle 0\,|\,T\Delta^{(2,0)}\delta_i\delta_i L^{(0,0)}\,|\,0\rangle$ directly negligible but also, by the use of Noether's theorem and the action principle, we have indirectly that terms such as

$$\int d^4x\,d^4y\,\langle 0\,|\,T\Delta^{(0,2)}(x)J_i^{(0,0)}(0)J_i^{(0,2)}(y)\,|\,0\rangle$$

are higher order in breaking and hence can be neglected. Simplifying the Dashen formulas Eqs. (3.25), we thus find to lowest order in SUSY and group breaking the Dashen formulas (no sum on i):

$$m_{P_i}{}^2 f_{P_i}{}^2 = -\frac{i}{2}\int d^4x\,\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle - 4\langle 0\,|\,\delta_i\delta_i\Delta_S^{(0,0)}(0)\,|\,0\rangle\,, \tag{3.26a}$$

$$m_{S_i}{}^2 f_{S_i}{}^2 = -\frac{i}{2}\int d^4x\,\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle + 4\langle 0\,|\,\delta_i\delta_i\Delta_S^{(0,0)}(0)\,|\,0\rangle$$
$$+8i\left\langle 0\,\middle|\,\left[\delta_i\phi\frac{\partial J_i^{\Delta}}{\partial\phi}\right]^{(0,0)}(0)\,\middle|\,0\right\rangle - 8\int d^4x\left\langle 0\,\middle|\,T\left[\delta_i\phi\frac{\partial\Delta}{\partial\phi}\right]^{(2,2)}(x)J_i^{(0,0)}(0)\,\middle|\,0\right\rangle\,, \tag{3.26b}$$

$$m_{f_i}f_{f_i}{}^2 = 4\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle\,, \tag{3.26c}$$

$$Z_{P_i}m_{P_i}f_{P_i}{}^2 = 4\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle\,, \tag{3.26d}$$

$$Z_{S_i}m_{S_i}f_{S_i}{}^2 = 4\langle 0\,|\,\delta_i\delta_i L^{(0,0)}(0)\,|\,0\rangle - 8\int d^4x\,\langle 0\,|\,T\left[\delta_i\phi\frac{\partial\Delta}{\partial\phi}\right]^{(0,2)}(x)J_i^{(0,0)}(0)\,|\,0\rangle\,. \tag{3.26e}$$

In fact, to the order of approximation under present consideration, these relations can be even further simplified. This follows from the observation that the normalization factors Z_{S_i} and Z_{P_i} are equal to 1 plus terms proportional to the SUSY breaking parameters and that the right-hand sides of Eqs. (3.26d) and (3.26e) are already first order in the breaking (SUSY or group). Thus, in the first-order breaking approximation, we can simply take the normalization factors equal to unity.

IV. MODEL: SUSY QCD

In order to illustrate the utility of the Dashen formulas, we consider an underlying two-flavor SUSY QCD-like model possessing an $SU(2)_L \times SU(2)_R$ global chiral symmetry which is presumed to be spontaneously broken to a diagonal $SU(2)_V$ vector subgroup by vacuum condensate formation. (The question of which chiral-symmetry-breaking condensates form in SUSY theories is still not completely settled.[1,6,12]) We define constituent chiral superfields η^a and χ^a which transform under $[SU(2)_L, SU(2)_R]$ as $(2,0)$ and $(0,\bar{2})$, respectively, so that

$$\begin{aligned}
&\delta_{L_i}\eta^a = i\left[\frac{\tau^i}{2}\right]^a{}_b\eta^b\ ,\quad \delta_{L_i}\bar{\eta}_a = -i\bar{\eta}_b\left[\frac{\tau^i}{2}\right]^b{}_a\ ,\\
&\delta_{L_i}\chi_a = 0\ ,\quad \delta_{L_i}\bar{\chi}^a = 0\ ,\\
&\delta_{R_i}\eta^a = 0\ ,\quad \delta_{R_i}\bar{\eta}_a = 0\ ,\\
&\delta_{R_i}\chi_a = -i\chi_b\left[\frac{\tau^i}{2}\right]^b{}_a\ ,\quad \delta_{R_i}\bar{\chi}^a = i\left[\frac{\tau^i}{2}\right]^a{}_b\bar{\chi}^b\ ,
\end{aligned} \tag{4.1}$$

where the τ^i, $i=1,2,3$ are the usual Pauli matrices. The vector and axial-vector variations are simply defined as the sum and difference of the right and left transformations and are given by

$$\begin{aligned}
&\delta_{V_i} = \delta_{R_i} + \delta_{L_i}\ ,\\
&\delta_{5_i} = \delta_{R_i} - \delta_{L_i}\ .
\end{aligned} \tag{4.2}$$

The SUSY-invariant chiral Lagrangian has the form

$$L_{\rm inv} = -\tfrac{1}{8}\bar{D}\,\bar{D}L_V + L_S\ , \tag{4.3}$$

with

$$L_V = \bar{\eta}e^{2V}\eta + \chi(e^{2V})^T\bar{\chi}\ , \tag{4.4}$$

while the Yang-Mills vector superfield V is matrix valued in the color space. (The superscript T denotes transposition in color space.) The mass term is

$$L_S = -\mu_G\chi\eta\ , \tag{4.5}$$

which explicitly breaks $SU(2)_L \times SU(2)_R$ to $SU(2)_V$. Furthermore, the soft SUSY-breaking action is given by

$$I_b = \int dV\,\theta\theta\bar{\theta}\bar{\theta}\Delta\ , \tag{4.6}$$

with

$$\Delta = \Delta_V + \Delta_S + \bar{\Delta}_S\ . \tag{4.7}$$

Here

$$\Delta_V = \mu_V{}^2 L_V \tag{4.8}$$

respects the left-right symmetry while

$$\Delta_S = \mu_S{}^2\chi\eta = -\frac{\mu_S{}^2}{\mu_G}L_S \tag{4.9}$$

explicitly, but softly, breaks it. The complete (anti-)chiral Lagrangian is then given by Eq. (3.4) and takes the form

$$\begin{aligned}
&L = -\tfrac{1}{8}\bar{D}\,\bar{D}[(1+\mu_V{}^2\theta^2\bar{\theta}^2)L_V] + \left[1-\frac{\mu_S{}^2}{\mu_G}\theta^2\right]L_S\ ,\\
&\bar{L} = -\tfrac{1}{8}DD[(1+\mu_V{}^2\theta^2\bar{\theta}^2)L_V] + \left[1-\frac{\mu_S{}^2}{\mu_G}\bar{\theta}^2\right]\bar{L}_S\ .
\end{aligned} \tag{4.10}$$

The axial variation of the (anti-)chiral Lagrangian is then simply given as

$$\begin{aligned}
&\delta_{5_i}L = 2i(\mu_G - \theta^2\mu_S{}^2)\chi\frac{\tau^i}{2}\eta\ ,\\
&\delta_{5_i}\bar{L} = -2i(\mu_G - \bar{\theta}^2\mu_S{}^2)\bar{\eta}\frac{\tau^i}{2}\bar{\chi}\ .
\end{aligned} \tag{4.11}$$

Expanding the chiral and antichiral superfields in terms of their component decompositions as

$$\begin{aligned}
&\chi = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}(A_\chi + \sqrt{2}\theta^\alpha\psi_{\chi_\alpha} + \theta\theta F_\chi)\ ,\\
&\bar{\chi} = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}[\bar{A}_\chi + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}_\chi^{\dot{\alpha}} + \bar{\theta}\bar{\theta}\bar{F}_\chi]\ ,
\end{aligned} \tag{4.12}$$

with similar expressions for η and $\bar{\eta}$ and using the auxiliary field equations of motion

$$\begin{aligned}
&\frac{\delta I}{\delta\bar{F}_\chi} = e^{2C}F_\chi - \mu_G\bar{A}_\eta\ ,\\
&\frac{\delta I}{\delta\bar{F}_\eta} = e^{2C}F_\eta - \mu_G\bar{A}_\chi\ ,\\
&\frac{\delta I}{\delta F_\chi} = \bar{F}_\chi e^{2C} - \mu_G A_\eta\ ,\\
&\frac{\delta I}{\delta F_\eta} = \bar{F}_\eta e^{2C} - \mu_G A_\chi\ ,
\end{aligned} \tag{4.13}$$

the variation of the chiral Lagrangian takes the form

$$\begin{aligned}
\delta_{5_i}L = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}2i\Bigg[&\mu_G A_\chi\frac{\tau^i}{2}A_\eta + \sqrt{2}\theta^\alpha\mu_G\left[A_\chi\frac{\tau^i}{2}\psi_{\eta_\alpha} + \psi_{\chi_\alpha}\frac{\tau^i}{2}A_\eta\right]\\
&+\theta\theta\Bigg[\mu_G A_\chi\frac{\tau^i}{2}e^{-2C}\bar{A}_\chi + \mu_G\bar{A}_\eta(e^{-2C})^T\frac{\tau^i}{2}A_\eta - \mu_G\psi_\chi\frac{\tau^i}{2}\psi_\eta - \mu_S{}^2A_\chi\frac{\tau^i}{2}A_\eta\\
&+\mu_G A_\chi\frac{\tau^i}{2}e^{-2C}\frac{\delta I}{\delta\bar{F}_\eta} + \mu_G\frac{\delta I}{\delta\bar{F}_\chi}(e^{-2C})^T\frac{\tau^i}{2}A_\eta\Bigg]\Bigg]\ .
\end{aligned} \tag{4.14}$$

Here $\delta I/\delta\bar{F}$ is the Euler-Lagrange equation for $\bar{F}$ with

$$I=\int dS\,L+\int d\bar{S}\,\bar{L}$$

being the total action and C is the $\theta,\bar{\theta}$ independent component of the Yang-Mills field V. (In Wess-Zumino gauge, $C=0$ and the above exponentials can be ignored.)

Applying the current-field identity of Eq. (3.21), we can explicitly relate the component fields of the Goldstone multiplet to composite combinations of the constituent fields as

$$Z_{P_i}m_{P_i}f_{P_i}P_i=2\sqrt{2}i\mu_G\left[A_\chi\frac{\tau^i}{2}A_\eta-\bar{A}_\eta\frac{\tau^i}{2}\bar{A}_\chi\right], \tag{4.15a}$$

$$Z_{S_i}m_{S_i}f_{S_i}S_i=-2\sqrt{2}\mu_G\left[A_\chi\frac{\tau^i}{2}A_\eta+\bar{A}_\eta\frac{\tau^i}{2}\bar{A}_\chi\right], \tag{4.15b}$$

$$m_{f_i}f_{f_i}\psi_{i_\alpha}=-4\mu_G\left[A_\chi\frac{\tau^i}{2}\psi_{\eta_\alpha}+\psi_{\chi_\alpha}\frac{\tau^i}{2}A_\eta\right], \tag{4.15c}$$

$$m_{P_i}{}^2f_{P_i}P_i=2\sqrt{2}i\mu_G\left[\psi_\chi\frac{\tau^i}{2}\psi_\eta-\bar{\psi}_\eta\frac{\tau^i}{2}\bar{\psi}_\chi\right]+2\sqrt{2}i\mu_S{}^2\left[A_\chi\frac{\tau^i}{2}A_\eta-\bar{A}_\eta\frac{\tau^i}{2}\bar{A}_\chi\right], \tag{4.15d}$$

$$m_{S_i}{}^2f_{S_i}S_i=2\sqrt{2}\mu_G\left[\psi_\chi\frac{\tau^i}{2}\psi_\eta+\bar{\psi}_\eta\frac{\tau^i}{2}\bar{\psi}_\chi\right]-4\sqrt{2}\mu_G\left[A_\chi\frac{\tau^i}{2}e^{-2C}\bar{A}_\chi+\bar{A}_\eta(e^{-2C})^T\frac{\tau^i}{2}A_\eta\right]+2\sqrt{2}\mu_S{}^2\left[A_\chi\frac{\tau^i}{2}A_\eta+\bar{A}_\eta\frac{\tau^i}{2}\bar{A}_\chi\right], \tag{4.15e}$$

while the Euler-Lagrange derivatives are given by

$$E_i=-4\sqrt{2}\mu_G\left[A_\chi\frac{\tau^i}{2}e^{-2C}\frac{\delta I}{\delta\bar{F}_\eta}+\frac{\delta I}{\delta\bar{F}_\chi}(e^{-2C})^T\frac{\tau^i}{2}A_\eta\right],$$
$$\bar{E}_i=-4\sqrt{2}\mu_G\left[\bar{A}_\eta\frac{\tau^i}{2}(e^{-2C})^T\frac{\delta I}{\delta F_\chi}+\frac{\delta I}{\delta F_\eta}e^{-2C}\frac{\tau^i}{2}\bar{A}_\chi\right]. \tag{4.16}$$

The two-point Euler term can now be evaluated yielding

$$i\int d^4x\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle=-8\mu_G{}^2\langle 0\,|\,[A_\chi^{-2C}\bar{A}_\chi+\bar{A}_\eta(e^{-2C})^TA_\eta]\,|\,0\rangle\,, \tag{4.17}$$

and by once again using the auxiliary field equations of motion, it can be rewritten as

$$i\int d^4x\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle=-4\mu_G\langle 0\,|\,(A_\chi F_\eta+F_\chi A_\eta)(0)\,|\,0\rangle-4\mu_G\langle 0\,|\,(\bar{F}_\eta\bar{A}_\chi+\bar{A}_\eta\bar{F}_\chi)(0)\,|\,0\rangle\,. \tag{4.18}$$

Application of the broken-SUSY Ward identity then relates this scalar field condensate to a fermion field condensate plus correction terms due to the SUSY breaking. So doing, we secure the result

$$i\int d^4x\langle 0\,|\,TE_i(x)\bar{E}_i(0)\,|\,0\rangle=-4\mu_G\langle 0\,|\,(\psi_\chi\psi_\eta+\bar{\psi}_\eta\bar{\psi}_\chi)(0)\,|\,0\rangle-4i\mu_G\int d^4x\langle 0\,|\,T\Delta^{(2,0)}(x)(A_\chi A_\eta)(0)\,|\,0\rangle+4i\mu_G\int d^4x\langle 0\,|\,T\Delta^{(0,2)}(x)(\bar{A}_\eta\bar{A}_\chi)(0)\,|\,0\rangle\,. \tag{4.19}$$

From Eqs. (4.15) we see explicitly the origin of the normalization factors Z_{P_i},Z_{S_i}. By convention the decay constants are defined by the vacuum to one-particle state matrix elements of the component currents in J_{5_i}. From Noether's theorem, this translates into the matrix elements of the components of $\delta_{5_i}L$. It is the $\theta\theta$ component of $\bar{D}\bar{D}J_{5_i}$ which contains the axial-vector current $j^\mu_{5_i}$ which is conventionally used to interpolate for the Goldstone-boson field P_i. As such, we define the Goldstone-boson decay constant f_{P_i} via

$$\langle 0\,|\,j^\mu_{5_i}(0)\,|\,P_i(k)\rangle=-if_{P_i}k^\mu\,, \tag{4.20}$$

with $k^2=-m_{P_i}{}^2$. Using the Noether theorem, it follows that it is the real part of the $\theta\theta$ component of $\delta_{5_i}L$ whose current-field identity is normalized with the factor as in Eq. (4.15d). We also adopt the convention that the imaginary part of the $\theta\theta$ component of $\delta_{t_i}L$ is normalized with the f_{S_i} factor only as in Eq. (4.15e). However, the supersymmetry dictates that the $\theta,\bar{\theta}$-independent components of $\bar{D}\bar{D}J_{5_i}$ and likewise $\delta_{5_i}L$ also carry the S_i and P_i poles. When SUSY is broken the residue of these poles need no longer be the same as the residue of the poles in the $\theta\theta$ components. More explicitly the (quasi-)Goldstone-boson pole for the θ-independent term of $\delta_{5_i}L$ is carried by the composite field

$$\mu_GA_\chi\frac{\tau^i}{2}A_\eta\,.$$

If SUSY is unbroken, this term transforms into the $\theta\theta$ component of $\delta_{5_i}L$ given by

$$\mu_G\left[A_\chi\frac{\tau^i}{2}e^{-2C}\bar{A}_\chi+\bar{A}_\eta(e^{-2C})^T\frac{\tau^i}{2}A_\eta-\psi_\chi\frac{\tau^i}{2}\psi_\eta\right]$$

and consequently the poles in the two terms have the same residue.

When SUSY is broken, $A_\chi(\tau^i/2)A_\eta$ Green's functions do not simply transform into

$$A_\chi\frac{\tau^i}{2}e^{-2C}\bar{A}_\chi+\bar{A}_\eta(e^{-2C})^T\frac{\tau^i}{2}A_\eta-\psi_\chi\frac{\tau^i}{2}\psi_\eta$$

Green's functions. Furthermore, L is no longer even a superfield. This leads to different residues between the first and last components of $\delta_{5_i}L$. These differences are reflected in the relative wave-function normalization factors Z_{P_i},Z_{S_i}. Since the fermion appears in only one component of $\delta_{5_i}L$, the residue of the pole is, by convention, defined as the PCAC decay constant.

The axial-vector current superspace function has the definition

$$J_{5_i}=J_{5_i}^{\text{inv}}+\theta^2\bar{\theta}^2 J_{5_i}^{\Delta} , \tag{4.21}$$

with

$$J_{5_i}^{\text{inv}}=\frac{i}{2}\left[\delta_{5_i}\bar{\phi}\frac{\partial L_V}{\partial\bar{\phi}}-\delta_{5_i}\phi\frac{\partial L_V}{\partial\phi}\right] ,$$
$$J_{5_i}^{\Delta}=\frac{i}{2}\left[\delta_{5_i}\bar{\phi}\frac{\partial \Delta_V}{\partial\bar{\phi}}-\delta_{5_i}\phi\frac{\partial \Delta_V}{\partial\phi}\right] , \tag{4.22}$$

where ϕ stands for both η and χ. Explicitly the current takes the form

$$J_{5_i}=i(1+\mu_V{}^2\theta^2\bar{\theta}^2)\left[\bar{\eta}e^{2V}\frac{\tau^i}{2}\eta+\chi\frac{\tau^i}{2}(e^{2V})^T\bar{\chi}\right] , \tag{4.23}$$

where we recall that V acts in color space only and so commutes with τ^i. In order to apply the Dashen formulas [Eqs. (3.26)] to our model we need the double variation of the chiral Lagrangian which is given by

$$\delta_{5_i}\delta_{5_j}L=2(\mu_G-\mu_S{}^2\theta^2)\chi\left\{\frac{\tau^i}{2},\frac{\tau^j}{2}\right\}\eta$$
$$=(\mu_G-\mu_S{}^2\theta^2)\chi\eta$$
$$=(\mu_G-\mu_S{}^2\theta^2)e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}[A_\chi A_\eta+\sqrt{2}\theta^\alpha(A_\chi\psi_{\eta_\alpha}+\psi_{\chi_\alpha}A_\eta)+\theta\theta(A_\chi F_\eta+F_\chi A_\eta-\psi_\chi\psi_\eta)] . \tag{4.24}$$

However, upon evaluating the necessary two-point functions we find they vanish in this model. For instance, since

$$\delta_{5_j}\phi\frac{\partial L_V}{\partial\phi}=-\delta_{5_j}\bar{\phi}\frac{\partial L_V}{\partial\bar{\phi}} , \tag{4.25}$$

we have that

$$J_{5_i}^{\Delta}=-i\delta_{5_i}\phi\frac{\partial\Delta_V}{\partial\phi} , \tag{4.26}$$

and

$$\left[\delta_{5_i}\phi\frac{\partial\Delta}{\partial\phi}\right]^{(0,2)}=\left[\delta_{5_i}\phi\frac{\partial\Delta_V}{\partial\phi}\right]^{(0,2)}$$
$$=iJ_{5_i}^{\Delta(0,2)} . \tag{4.27}$$

Hence, it follows that

$$\int d^4x\left\langle 0\left|T\left[\delta_{5_j}\phi\frac{\partial\Delta}{\partial\phi}\right]^{(0,2)}(x)J_{5_i}^{(0,0)}(0)\right|0\right\rangle$$
$$=i\mu_V{}^2\int d^4x\langle 0|TJ_{5_j}^{(0,2)}(x)J_{5_i}^{(0,0)}(0)|0\rangle . \tag{4.28}$$

But by using the broken-SUSY Ward identity and the action principle, we find zero for the right-hand side. Similarly we obtain

$$i\left\langle 0\left|\left[\delta_{5_j}\phi\frac{\partial J_{5_i}^{\Delta}}{\partial\phi}\right]^{(0,0)}(0)\right|0\right\rangle$$
$$=\int d^4x\langle 0|T\left[\delta_{5_j}\phi\frac{\partial\Delta}{\partial\phi}\right]^{(2,2)}(x)J_{5_i}^{(0,0)}(0)|0\rangle \tag{4.29a}$$

and

$$\int d^4x\left\langle 0\left|T\left[\delta_{5_i}\phi\frac{\partial\Delta}{\partial\phi}\right]^{(0,2)}(x)J_{5_i}^{(0,0)}(0)\right|0\right\rangle=0 . \tag{4.29b}$$

Substituting these results into the Dashen formulas, Eqs. (3.26) and recalling that in the approximation of keeping terms only to first order in all explicit breaking parameters, we can set Z_{S_i} and Z_{P_i} to unity, we secure (no sum on i),

$$m_{P_i}{}^2f_{P_i}{}^2=-\frac{i}{2}\int d^4x\langle 0|TE_i(x)\bar{E}_i(0)|0\rangle$$
$$-4\langle 0|\delta_i\delta_i\Delta_S^{(0,0)}(0)|0\rangle$$
$$=2\mu_G\langle 0|(\psi_\chi\psi_\eta+\bar{\psi}_\eta\bar{\psi}_\chi)(0)|0\rangle$$
$$+2\mu_S{}^2\langle 0|(A_\chi A_\eta+\bar{A}_\eta\bar{A}_\chi)(0)|0\rangle , \tag{4.30a}$$

$$m_{S_i}{}^2f_{S_i}{}^2=-\frac{i}{2}\int d^4x\langle 0|TE_i(x)\bar{E}_i(0)|0\rangle$$
$$+4\langle 0|\delta_i\delta_i\Delta_S^{(0,0)}(0)|0\rangle$$
$$=2\mu_G\langle 0|(\psi_\chi\psi_\eta+\bar{\psi}_\eta\bar{\psi}_\chi)(0)|0\rangle$$
$$-2\mu_S{}^2\langle 0|(A_\chi A_\eta+\bar{A}_\eta\bar{A}_\chi)(0)|0\rangle , \tag{4.30b}$$

$$m_{f_i}f_{f_i}{}^2=4\langle 0|\delta_{5_i}\delta_{5_i}L^{(0,0)}(0)|0\rangle$$
$$=2\mu_G\langle 0|(A_\chi A_\eta+\bar{A}_\eta\bar{A}_\chi)(0)|0\rangle , \tag{4.30c}$$

$$m_{P_i}f_{P_i}{}^2=4\langle 0|\delta_{5_i}\delta_{5_i}L^{(0,0)}(0)|0\rangle$$
$$=2\mu_G\langle 0|(A_\chi A_\eta+\bar{A}_\eta\bar{A}_\chi)(0)|0\rangle , \tag{4.30d}$$

$$m_{S_i}f_{S_i}{}^2=4\langle 0|\delta_{5_i}\delta_{5_i}L^{(0,0)}(0)|0\rangle$$
$$=2\mu_G\langle 0|(A_\chi A_\eta+\bar{A}_\eta\bar{A}_\chi)(0)|0\rangle . \tag{4.30e}$$

From these formulas, separate expressions for m_{S_i}, m_{P_i}, f_{S_i}, and f_{P_i} may be extracted, while, for the fermion, only the combination $m_{f_i} f_{f_i}^2$ is obtained.

Since we are looking at the broken $SU(2)_L \times SU(2)_R$ Ward identities to lowest order in the comparable SUSY-breaking and internal-symmetry breaking scales we find only effects from the internal-symmetry breaking masses μ_G, μ_S^2. Of course, this first-order approximation scheme radically limits the possible Fermi-Bose mass splittings. A more complete calculation of the spectrum can be obtained by analyzing the Dashen formula (3.24) when the SUSY and internal-symmetry breaking scales differ. So doing, effects of the G-invariant but SUSY-breaking mass terms (with parameter μ_V^2) will appear. Furthermore, higher-order SUSY-breaking corrections can also be included by finding additional corrections to the broken-SUSY Ward identities. That is, Eq. (2.9) can be solved to higher order in Δ. Finally, in addition to these effects, the effect of SUSY-breaking gaugino mass terms can be included.

ACKNOWLEDGMENT

This work was supported in part by the U.S. Department of Energy.

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