Heterotic string in an arbitrary background field

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An expression for the light-cone gauge action for the first-quantized heterotic string in the presence of arbitrary background gauge, gravitational, and antisymmetric tensor fields is derived. The result is a two-dimensional local field theory with $N=\frac{1}{2}$ supersymmetry. The constraints imposed on the background fields in order to make this theory one-loop finite are derived. These constraints are identical to the equations of motion for the massless fields at the linearized level. Finally, it is shown that if there is no background antisymmetric tensor field, and if the gauge connection is set equal to the spin connection, the effective action is that of an $N=1$ supersymmetric nonlinear σ model.

I. INTRODUCTION

The discovery of anomaly cancellation[1] in type-I superstring theories[2] by Green and Schwarz has given us hope that superstrings may provide us with a unified theory of nature. Since then, two new string theories have been discovered,[3,4] one of which has SO(32) as its gauge group, while the other is based on the gauge group $E_8 \times E_8$. Both of these theories are expected to be anomaly free, since the limiting field theories obtained from them in the zero-slope limit may be shown to be free from anomaly at the one-loop level. Of these the $E_8 \times E_8$ theory seems to have a good phenomenological prospect.[5-8]

Since these theories are defined in ten dimensions, we must compactify the six extra dimensions in order to get a realistic theory of nature.[9] Two different approaches have been taken to study this problem. In the first approach one studies the compactification of the ten-dimensional field theory which is the zero-slope limit of the string theory.[5-8,10] The compactification is then achieved by giving the various massless fields (e.g., the graviton field, the antisymmetric tensor field, and the gauge field) associated with the massless excitations of the string vacuum expectation values. In the other approach,[11,12] one tries to formulate and study first-quantized string theory in an arbitrary background metric, which gives us a nonlinear σ model in $1+1$ dimensions. Attempts have also been made to formulate new kinds of string theories by adding Wess-Zumino-type terms to this nonlinear σ model.[12] The requirement of being able to formulate a consistent, reparametrization-invariant string theory gives strong constraints on the two-dimensional field theory describing the first-quantized string action. In particular, it requires that the theory should be conformally invariant, and hence all the β functions must vanish to all orders in the perturbation theory.

In a previous paper[13] we studied the connection between these two approaches by investigating the dynamics of a string in a weak background graviton and antisymmetric tensor field associated with massless closed-string excited states. In particular, we showed that the presence of a background antisymmetric tensor field is equivalent to adding a Wess-Zumino term to this string action. For the fermionic strings the effective action for the first-quantized string reduces to the supersymmetric extension of a nonlinear σ model with a Wess-Zumino term. A similar action for the heterotic string was also written down in the presence of arbitrary background gauge, gravitational, and antisymmetric tensor fields.

In this paper we further pursue this approach and study the ultraviolet behavior of the two-dimensional field theory that describes the heterotic string in arbitrary background fields. In Sec. II we derive the action for the heterotic string in the presence of arbitrary background fields. Section III is devoted to the study of the one-loop ultraviolet divergences in the theory, and deriving the requirement for finiteness of this model. In Sec. IV we discuss the various implications of our results. In particular, we show that the criterion for finiteness of the nonlinear σ model is equivalent to the field equations in the weak-field limit. From this we conjecture that there may be a deep connection between the condition for finiteness of the σ model describing the string in a given background, and the equations of motion of the string field theory. (It was argued in the first of Refs. 6 that the conformal invariance of the σ model automatically guarantees that the background is a solution of the classical equations of motion. We, however, believe that the correspondence goes both ways, namely, any background field configuration satisfying the classical equations of motion automatically describes a conformally invariant σ model.) It is also shown that in the absence of any background antisymmetric tensor field, the action derived in Sec. II reduces to that of an $N=1$ supersymmetric nonlinear σ model if the gauge connection is set equal to the spin connection. This, in turn, implies that the effective σ model is finite to all orders in the perturbation theory if the background is Ricci flat. In Appendix A we show that the effective nonlinear σ model derived in Sec. II is invariant under an $N=\frac{1}{2}$ supersymmetry transformation. Appendix B contains some details of the one-loop calculation performed in Sec. III of the text.

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II. HETEROTIC STRING IN ARBITRARY BACKGROUND FIELD

In this section we shall derive an expression for the action of the heterotic string in arbitrary weak background gauge, gravitational, and antisymmetric tensor fields. If, however, we want the background to satisfy the classical equations of motion, we should not only consider background fields corresponding to the massless states of the string, but also background fields corresponding to the massive states of the string, since the massive fields couple to the massless fields through three-point vertices. The problem may be avoided by taking all the background massless fields to be small ($\sim\epsilon$). Then a consistent solution of the field equations may be obtained where the massive fields are of order ϵ^2 (since the massive and the massless fields couple through cubic coupling). For this reason we restrict ourselves to the case of weak background fields only.

We start our analysis by writing down the light-cone gauge action for the free heterotic string,

$$S=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\left[\sum_{i=1}^{8}(\partial_\alpha X^i\partial^\alpha X^i+i\bar{\lambda}^i\rho^\alpha\partial_\alpha\lambda^i)+\sum_{I=10}^{25}\partial_\alpha X^I\partial^\alpha X^I\right], \qquad (2.1)$$

where $\alpha=0,1$ denotes the world-sheet parameters τ and σ, respectively. X^i and X^I are bosonic coordinates, and λ^i are the fermionic coordinates. (We have chosen the Ramond-Neveu-Schwarz representation[14] for the fermionic coordinates.) The λ^i's satisfy the Majorana-Weyl condition; for definiteness we take them to be left-handed. The X^I's satisfy the constraint that they are always right moving. We choose to work in the Majorana representation for the two-dimensional γ matrices:

$$\rho^0=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\quad \rho^1=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix} \qquad (2.2)$$

and define

$$\gamma_P=\rho^0\rho^1=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \qquad (2.3)$$

The constraints on X^I and λ^i may then be written as

$$(\partial_\tau-\partial_\sigma)X^I=0\,, \qquad (2.4)$$

$$(\lambda^i)^\dagger=\lambda^i\,, \qquad (2.5)$$

$$(1-\gamma_P)\lambda^i=0\,. \qquad (2.6)$$

Equations (2.5) and (2.6) tell us that λ^i is real, and its lower component must vanish. Hence we may treat the λ^i's as one-component real spinors, which we shall again denote by λ^i. The action (2.1) may then be written as

$$S=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\left[\sum_{i=1}^{8}[(\partial_\tau-\partial_\sigma)X^i(\partial_\tau+\partial_\sigma)X^i+i\lambda^i(\partial_\tau+\partial_\sigma)\lambda^i]+\sum_{I=10}^{25}(\partial_\tau+\partial_\sigma)X^I(\partial_\tau-\partial_\sigma)X^I\right]. \qquad (2.7)$$

There are massless states of the heterotic string belonging to the symmetric and antisymmetric tensor representation of SO(8), giving rise to the graviton, a massless scalar, and an antisymmetric tensor field.[2] The vertex operator for the emission of such a state with momentum k in the $k^+\to 0$ limit is given by

$$\frac{i\kappa}{\pi}\zeta_{ij}\int_0^\pi d\sigma[(\partial_\tau-\partial_\sigma)X^i+k_l\lambda^l\lambda^i](\partial_\tau+\partial_\sigma)X^j e^{ik_lX^l}, \qquad (2.8)$$

where κ is the gravitational coupling constant. The polarization tensor ζ_{ij} for the external tensor field is assumed to be transverse. It is symmetric for the graviton, and antisymmetric for the antisymmetric tensor field. We may now use Eq. (2.8) to derive the effective action for the first-quantized string in the presence of a background graviton field $h_{ij}(X)$ and antisymmetric tensor field $b_{ij}(X)$. If $\tilde{h}_{ij}(k)$ and $\tilde{b}_{ij}(k)$ are the Fourier transforms of these fields with respect to the transverse coordinates X^i, the operator whose matrix element between two states gives the correct transition amplitude from one state to another is given by

$$\frac{i\kappa}{\pi}\int d\tau\int_0^\pi d\sigma\int d^{D-2}k\,[\tilde{h}_{ij}(k)+\tilde{b}_{ij}(k)](\partial_\tau+\partial_\sigma)X^j[(\partial_\tau-\partial_\sigma)X^i+k_l\lambda^l\lambda^i]e^{ik_mX^m}$$
$$=\frac{i\kappa}{\pi}\int d\tau\int_0^\pi d\sigma(\partial_\tau+\partial_\sigma)X^j\{(\partial_\tau-\partial_\sigma)X^i[h_{ij}(X)+b_{ij}(X)]-i\lambda^l\lambda^i[h_{ij,l}(X)+b_{ij,l}(X)]\}\,, \qquad (2.9)$$

where $f_{,l}$ denotes $\partial f/\partial X_l$ for any function f. Let us define

$$g_{ij}(X)=\delta_{ij}+2\kappa h_{ij}(X)\,,$$
$$B_{ij}(X)=2\kappa b_{ij}(X)\,. \qquad (2.10)$$

Finally, note that the addition of a term

$$\frac{i\kappa}{\pi}\int d\tau\int_0^\pi d\sigma[h_{ij}(X)+b_{ij}(X)]\lambda^i(\partial_\tau+\partial_\sigma)\lambda^i \qquad (2.11)$$

to (2.9) does not affect the transition amplitudes from one state of the string to another in first order in h and b, since in the zeroth-order approximation equations of motion for λ^j give $(\partial_\tau+\partial_\sigma)\lambda^j=0$. Thus to first order in the external fields, the effective string action may be obtained by adding to (2.1) the expressions (2.9) and (2.11) without the factor of i. We may then write down the full effective string action as

$$S=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\left\{g_{ij}(X)[\partial_\alpha X^i\partial^\alpha X^j+i\bar{\lambda}^i\rho^\alpha(D_\alpha\lambda)^j]+B_{ij}(X)\epsilon^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j-iS_{ijk}(X)\bar{\lambda}^i\rho^\alpha\lambda^j\epsilon_{\alpha\beta}\partial^\beta X^k+\frac{i}{2}\partial_\alpha[\bar{\lambda}^i\rho^\alpha\lambda^j B_{ij}(X)]\right\}, \quad (2.12)$$

where

$$(D_\alpha\lambda)^i=\partial_\alpha\lambda^i+\Gamma^i_{jk}\partial_\alpha X^j\lambda^k , \quad (2.13)$$

$$S_{ijk}=(\partial_i B_{jk}+\partial_j B_{ki}+\partial_k B_{ij})/2 , \quad (2.14)$$

$$\Gamma^i{}_{jk}=\tfrac{1}{2}g^{il}(g_{lj,k}+g_{lk,j}-g_{jk,l}) . \quad (2.15)$$

Equation (2.12) may also be obtained directly from the expressions derived in Ref. 13 by using the constraint (2.6). This Lagrangian is that of a supersymmetric nonlinear σ model[15] with a Wess-Zumino term.[16–18] Equation (2.12) is invariant under the $N=\frac{1}{2}$ supersymmetry transformation laws,

$$\delta X^i=i\epsilon\lambda^i,\quad \delta\lambda^i=-(\partial_\tau-\partial_\sigma)X^i\epsilon , \quad (2.16)$$

where ϵ is a real one-component spinor. Moreover, the proof of invariance of (2.12) under the above supersymmetry transformation does not require the use of equations of motion. [This is related to the fact that the auxiliary fields vanish identically when (2.6) is satisfied.] As a result, any extra term added to (2.12) will keep the full action supersymmetric if the extra term is supersymmetric by itself. Equation (2.12) also shows that B_{ij} couples to the string as a Kalb-Ramond field.[19]

Let us now turn to the effect of introducing background gauge fields. They may be divided into two classes, one corresponding to the diagonal generators of the $E_8\times E_8$ or the SO(32) groups, and the other corresponding to the off-diagonal generators. First we shall consider the ones corresponding to the diagonal generators of the group. These states are labeled by two indices i and I, where i is the polarization index ($i=1,\ldots,8$), and I is the internal index ($10\le I\le 25$) labeling the 16 generators of the gauge group. The vertex for emission of such states with momentum k is given by

$$\frac{i\kappa}{\pi}\int_0^\pi d\sigma[(\partial_\tau-\partial_\sigma)X^i+k_m\lambda^m\lambda^i](\partial_\tau+\partial_\sigma)X^I\times e^{ik_mX^m}\zeta_{iI} . \quad (2.17)$$

Hence in the presence of a background gauge field $A_{iI}(X^i)$ the effective action acquires a term

$$S_2=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma[A_{iI}(X)(\partial_\tau-\partial_\sigma)X^i-iA_{iI,l}(X^i)\lambda^l\lambda^i]\times\left[-\frac{1}{\sqrt{2}}(\partial_\tau+\partial_\sigma)X^I\right] \quad (2.18)$$

absorbing a factor of $(-2\sqrt{2}\kappa)$ in A_{iI}. The reason for choosing this particular normalization will become clear later. It is easy to verify that (2.18) is invariant under the supersymmetry transformation (2.16).

Finally, let us turn to the off-diagonal gauge fields. These states are given by the direct product of the massless states of the fermionic strings in the left-handed sector carrying vector index i, and the ground state of the bosonic string in the right-handed sector carrying momentum p^I+k^i, where k^i is the physical momentum of the state, and p^I is a vector in the root space of the gauge group, labeling a particular off-diagonal generator of the group. The emission vertex for such a state is proportional to[3]

$$\frac{i\kappa}{\pi}\int_0^\pi d\sigma[(\partial_\tau-\partial_\sigma)X^i+k_m\lambda^m\lambda^i]e^{i(k_mX^m+2p_IX^I)}C(p_I) . \quad (2.19)$$

where $C(p_I)$ is the cocycle factor.

Hence, in the presence of a background gauge field $A_i(X^i,p^I)$, the action receives an extra term,

$$\frac{1}{2\pi}\left[-\frac{1}{\sqrt{2}}\right]\int d\tau\int_0^\pi d\sigma\sum_{p_I}[(\partial_\tau-\partial_\sigma)X^iA_i(X^j,p_I)-iA_{i,m}(X^j,p_I)\lambda^m\lambda^i]\times e^{2ip_IX^I}C(p_I) \quad (2.20)$$

with properly normalized $A_i(X,p)$. Here the sum over p^I runs over all the roots of the group. If we define

$$A_i(X^i,X^I)=\sum_{p_I}A_i(X^j,p_I)e^{2ip_IX^I}C(p_I) \quad (2.21)$$

then (2.17) may be written as

$$S_3=\frac{1}{2\pi}\left[-\frac{1}{\sqrt{2}}\right]\int d\tau\int_0^\pi d\sigma[(\partial_\tau-\partial_\sigma)X^iA_i(X^j,X^I)-iA_{i,m}(X^j,X^I)\lambda^m\lambda^i] , \quad (2.22)$$

which may again be shown to be invariant under the supersymmetry transformation law given in (2.16). Equation (2.22) may also be derived from (2.18) by applying the gauge transformation operator constructed in Ref. 20 on this expression.

Thus, in the presence of arbitrary background gravitational field g_{ij}, antisymmetric tensor field B_{ij}, and gauge field A_{iI} and $A_i(X^I)$, the full effective action for the first-quantized heterotic string is given by the sum of S_1, S_2, and S_3. This action is invariant under the $N=\frac{1}{2}$ supersymmetry transformation given in (2.16).

For doing quantum calculation, however, it is more convenient to replace the 16 right-moving bosonic coordinates X^I by 32 right-handed Majorana-Weyl fermions ψ^s ($s=1,\ldots,32$). For the heterotic string with SO(32) gauge group, the 32 fermions belong to the fundamental representation of the group, whereas for the heterotic string with $E_8\times E_8$ as its gauge group, the fermions belong to the $(16,1)+(1,16)$ representation of the $SO(16)\times SO(16)$ subgroup of the $E_8\times E_8$ group. [In the rest of the paper we shall assume that the background gauge fields always belong to this $SO(16)\times SO(16)$ subgroup.] The current $(\partial_\tau+\partial_\sigma)X^I$ may then be expressed in terms of the fermionic coordinates as

$$(\partial_\tau+\partial_\sigma)X^I=\frac{(-1)}{\sqrt{2}}\bar{\psi}T^I(\rho_0+\rho_1)\psi\ , \tag{2.23}$$

where T^I is a diagonal generator of the gauge group, normalized to $\mathrm{tr}(T^I)^2=1$. The action S_2 may then be expressed in terms of the fermionic coordinates as

$$\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\,\bar{\psi}T^I\rho^\alpha\psi[A_{iI}\partial_\alpha X^i-\frac{i}{2}\bar{\lambda}^l\rho_\alpha\lambda^i A_{iI,l}(X)]\ . \tag{2.24}$$

Since the ψ^s's transform linearly under the SO(32) or the $SO(16)\times SO(16)$ groups, the generalization of (2.24) to the case of arbitrary background gauge field $A_i^M(X)$ [T^M denotes an arbitrary generator of the SO(32) or the $SO(16)\times SO(16)$ group] may be obtained from (2.24) by a global gauge transformation, and is given by

$$\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\,\bar{\psi}T^M\rho^\alpha\psi[A_i^M(X)\partial_\alpha X^i-\frac{i}{2}\bar{\lambda}^l\rho_\alpha\lambda^i A_{i,l}^M(X)]\ . \tag{2.25}$$

Since $\bar{\lambda}^l\rho^\alpha\lambda^i$ is antisymmetric in i and l, we may replace $A_{i,l}^M$ by $F_{il}^M/2$ in the weak-field approximation, where,

$$F_{il}^M=A_{i,l}^M-A_{l,i}^M+f^{MNP}A_i^N A_l^P\ . \tag{2.26}$$

Here f^{MNP} are the structure constants of the group defined through the relation

$$[T^M,T^N]=-if^{MNP}T^P\ . \tag{2.27}$$

Thus the full action of the heterotic string in the presence of an arbitrary background field is given by

$$S=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma\left[g_{ij}(X)[\partial_\alpha X^i\partial^\alpha X^j+i\bar{\lambda}^i\rho^\alpha(D_\alpha\lambda)^j]+B_{ij}\epsilon^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j-iS_{ijk}(X)\bar{\lambda}^i\rho^\alpha\lambda^j\epsilon_{\alpha\beta}\partial^\beta X^k\right.$$
$$\left.+\frac{i}{2}\partial_\alpha[\bar{\lambda}^i\rho^\alpha\lambda^j B_{ij}(X)]+i\bar{\psi}^s\rho^\alpha\partial_\alpha\psi^s+\bar{\psi}^s(T^M)_{st}\rho^\alpha\psi^t\left[A_i^M\partial_\alpha X^i-\frac{i}{4}F_{il}^M\bar{\lambda}^l\rho_\alpha\lambda^i\right]\right]\ . \tag{2.28}$$

This action is invariant under the $N=\frac{1}{2}$ supersymmetry transformation, as shown in Appendix A.

III. ONE-LOOP ULTRAVIOLET DIVERGENCES IN THE THEORY

In the last section we derived an expression for the action for the heterotic string in the presence of arbitrary background fields. Although these results are valid only in the weak-field approximation, the resulting two-dimensional field theory has an exact $N=\frac{1}{2}$ supersymmetry, and may be of interest in its own right. With this in mind, we shall study all the one-loop divergences in this theory, and derive the constraints on the background fields required by the vanishing of all the one-loop divergences. In interpreting this result, however, we must keep in mind the fact that we should take these constraints seriously only to first order in the background fields.

We use the background field method for our analysis, which has been widely used by many authors.[17,21,22] In this method, each bosonic coordinate X^i in the action is replaced by $X^i+\pi^i$, where X^i is the background field satisfying the classical equations of motion, and π^i is the quantum field. We may then expand the action in a power series in π, and derive the Feynman rules for calculating graphs involving the π^i lines. In this method, the external lines are always made of the background fields X^i, whereas the internal lines are always made of the quantum fields π^i. The fermion fields ψ^s and λ^i are treated in the same way as in ordinary perturbation expansion, and hence they appear both in the internal as well as the external lines of a graph.

Although this method can be used directly to calculate all the counterterms, we lose explicit general coordinate invariance in this method, and hence the calculation becomes very complicated. This problem may be avoided by expanding the action in terms of the normal coordinates $\xi^i(X,\pi)$ instead of the fields π^i themselves. $\xi^i(X,\pi)$ is defined as

$$\xi^i(X,\pi)=\left.\frac{d\chi^i}{dt}\right|_{t=0}\ , \tag{3.1}$$

where χ^i is defined through the equations

$$\frac{d^2\chi^i}{dt^2}+\Gamma^i_{jk}\frac{d\chi^j}{dt}\frac{d\chi^k}{dt}=0\ , \tag{3.2}$$

$$\chi^i(0)=X^i,\ \ \chi^i(1)=X^i+\pi^i\ . \tag{3.3}$$

Physically, ξ^i may be interpreted as a vector whose direction is along the tangent vector at X to the geodesic passing through the points X and $X+\pi$, and whose length is equal to the length of the geodesic between the points X and $X+\pi$. The fields ξ^i transform covariantly under a general coordinate transformation on the manifold spanned by the coordinates X^i. Let us also define

$$\xi^a=e^a{}_i\xi^i,\ \ \lambda^a=e^a{}_i\lambda^i\ , \tag{3.4}$$

where e_i^a are the vielbein fields, satisfying

$$e^a{}_ie^a{}_j=g_{ij}\ . \tag{3.5}$$

The action for the heterotic string after the replacement of X by $X+\pi$, and using the X^i equations of motion takes a simple form when expanded in terms of the fields ξ^a. The part of the action (2.28), that does not involve the field ψ^s, represents the standard action for a supersymmetric nonlinear σ model with a Wess-Zumino term, with the restriction that the spinors λ^i are Majorana-Weyl. This theory has been studied by previous authors, and the extra term in the action from this term, obtained due to the replacement of X^i by $X^i+\pi^i$ is given by[17]

$$\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma[\partial_\alpha\xi^a\partial^\alpha\xi^a-B^{ab\alpha}(\xi^a\partial_\alpha\xi^b-\xi^b\partial_\alpha\xi^a)+B^{ca}_\alpha B^{cb\alpha}\xi^a\xi^b$$
$$+\widetilde{R}_{ibaj}(\partial_\alpha X^i\partial^\alpha X^j-\epsilon^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j)\xi^a\xi^b+O(\xi^3)+O(\lambda^2\xi)]\ , \tag{3.6}$$

where

$$B^{ab}_\alpha=(\omega^{ab}{}_k\delta_\alpha{}^\beta-\epsilon_\alpha{}^\beta e^a{}_ie^b{}_jS^{ij}{}_k)\partial_\beta X^k\ , \tag{3.7}$$

$$\omega^{ab}{}_k=\Gamma^{ij}{}_ke^a{}_ie^b{}_j-(\partial_ke^a{}_i)e^b{}_jg^{ij} \tag{3.8}$$

is the spin connection, and

$$\widetilde{R}_{iklj}\equiv e^b{}_ke^a{}_l\widetilde{R}_{ibaj}=g_{im}(\partial_l\widehat{F}^{\,m}_{jk}-\partial_j\widehat{F}^{\,m}_{lk}-\widehat{F}^{\,n}_{lk}\widehat{F}^{\,m}_{jn}+\widehat{F}^{\,n}_{jk}\widehat{F}^{\,m}_{ln})\ , \tag{3.9}$$

$$\widehat{F}_{ijk}=(\Gamma_{ijk}-S_{ijk})\equiv g_{in}\widehat{F}^{\,n}_{jk}\ . \tag{3.10}$$

$\widetilde{R}$ is the generalized curvature.[17] S_{ijk} has been defined in Eq. (2.14). In writing down (3.6), we have ignored the $O(\xi^3)$ contribution, since it does not contribute to the one-loop amplitudes. Terms of order $\xi\lambda^2$ may contribute to one-loop amplitudes involving external λ lines. The graphs involving external λ lines, however, are related to the graphs involving external X lines due to the $N=\frac{1}{2}$ supersymmetry, and hence, we shall not consider any graph involving external λ lines in our analysis. Owing to this reason we may ignore all terms of order $\lambda^2\xi$.

Next we must turn to the terms involving the ψ^s's. Again, in the term involving $\psi^s\psi^t\lambda^i\lambda^j$ we may replace $X+\pi$ by X, the extra terms being of order $\lambda^2\xi$. The analysis of the term independent of λ may be carried out in the following way. It has been shown[21] that in a suitable frame,

$$\partial_\alpha(X^i+\pi^i)=\partial_\alpha X^i+(D_\alpha\xi)^i+\tfrac{1}{3}R^i{}_{abj}(\partial_\alpha X^j)\xi^a\xi^b+O(\xi^3)\ , \tag{3.11}$$

$$A_i^M(X+\pi)=A_i^M(X)+(D_jA^M)_i\xi^j+\tfrac{1}{2}[(D_jD_kA^M)_i-\tfrac{1}{3}R^l{}_{jik}A_l^M]\xi^j\xi^k\ , \tag{3.12}$$

where

$$\begin{aligned}&(D_jA^M)_i=\partial_jA_i^M-\Gamma^l_{ji}A_l^M\ ,\\&(D_\alpha\xi)^a=\partial_\alpha\xi^a+\omega^{ab}{}_i\xi^b\partial_\alpha X^i=e^a{}_j(\partial_\alpha\xi^j+\Gamma^j{}_{ki}\xi^k\partial_\alpha X^i)\ ,\end{aligned} \tag{3.13}$$

and R is the usual Riemann tensor, given by the right-hand side of Eq. (3.9) with $\widehat{F}$ replaced by Γ.

The total action for the heterotic string, evaluated at $X+\pi$ is then given by

$$
\begin{aligned}
I_B(X^i)+\frac{1}{2\pi}\int d\tau\int_0^{\pi}d\sigma\Big[&\partial_\alpha\xi^a\partial^\alpha\xi^a-B_\alpha^{ab}(\xi^a\partial^\alpha\xi^b-\xi^b\partial^\alpha\xi^a)+B_\alpha^{ca}B^{cb\alpha}\xi^a\xi^b\\
&+\widetilde{R}_{mban}(\partial_\alpha X^m\partial^\alpha X^n-\epsilon^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n)\xi^a\xi^b+i\bar{\lambda}^a\rho^\alpha(\delta_{ab}\partial_\alpha+B_\alpha^{ab})\lambda^b\\
&+i\bar{\psi}^s\rho^\alpha[\delta_{st}\partial_\alpha-iA_i^M\partial_\alpha X^i(T^M)_{st}]\psi^t+\bar{\psi}\rho^\alpha T^M\psi[A_a^M(X)(D_\alpha\xi)^a+(D_aA^M)_l\xi^a\partial_\alpha X^l]\\
&+\bar{\psi}\rho^\alpha T^M\psi(D_bA^M)_a\xi^b(D_\alpha\xi)^a+\tfrac{1}{2}\bar{\psi}\rho^\alpha T^M\psi[(D_aD_bA^M)_l+R^i{}_{abl}A_i^M]\partial_\alpha X^l\xi^a\xi^b\\
&-\frac{i}{4}\bar{\psi}\rho^\alpha T^M\psi F_{il}^M(X)\bar{\lambda}^l\rho_\alpha\lambda^i\Big]\ ,
\end{aligned}
\tag{3.14}
$$

where $I_B(X^i)$ denotes the part of the action involving only the background fields X^i. This part is irrelevant for any quantum calculation.

With this effective action, we may now proceed to calculate all the one-loop counterterms involving the external ψ^s and X^i fields. During this calculation we always have the X^i's as external lines, ξ^a and λ^i's as internal lines, and the ψ^s's as both, internal and external lines. The details of the Feynman rules, and the calculation of various graphs have been presented in Appendix B. After using the equations of motion for the X^i and the ψ^s fields, the ultraviolet divergent part of the one-loop effective action may be shown to be proportional to

$$
i\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\left(\tfrac{1}{2}\widetilde{R}_{iaaj}(\partial_\alpha X^i\partial^\alpha X^j-\epsilon^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^j)-\tfrac{1}{4}\bar{\psi}\rho^\alpha T^M\psi\{-\partial_\alpha X^l S^{ab}{}_l F_{ba}^M+[(D_aF^M)_{al}+f^{MNP}A_a^N F_{al}^P]\partial_\alpha X^l\}\right)\ .
\tag{3.15}
$$

Hence the finiteness of the model to one-loop order gives the following constraints on the background fields:

$$\widetilde{R}_{iaaj}=0\ , \tag{3.16}$$

$$(\hat{D}_aF^M)_{al}-S^{ab}{}_lF_{ba}^M=0\ , \tag{3.17}$$

where $\hat{D}$ denotes the full covariant derivative, including the gauge, as well as the spin connection. Also, note that Eq. (3.17) may be written as $(\widetilde{D}_aF^M)_{al}=0$, where $\widetilde{D}$ is the generalized covariant derivative,[17] including torsion on the manifold.

IV. DISCUSSION

In this paper, we have derived an expression for the heterotic string in the presence of arbitrary weak background gauge, gravitational and antisymmetric tensor field. The result is a two-dimensional local field theory with $N=\frac{1}{2}$ supersymmetry. A consistent formulation of the string theory requires that the two-dimensional field theory, representing the first-quantized string action must be conformally invariant, and have vanishing β function. This requirement gives strong restrictions on the background fields. In this paper we have studied the possible ultraviolet divergences in the two-dimensional theory at one-loop order, and have obtained constraints on the background fields by demanding that all such ultraviolet divergences must vanish. These constraints have been summarized in Eqs. (3.16) and (3.17).

Although these conditions have been written down in a nice covariant form, they should be taken seriously only to order linear in the background fields, since the original action was derived by ignoring all terms containing more than one power of the external fields. To this order, the symmetric and the antisymmetric parts of Eq. (3.16) are equivalent to the equations,

$$R_{iaaj}=0\ , \tag{4.1}$$

$$\partial_iS_{ijk}=0\ , \tag{4.2}$$

whereas Eq. (3.17) gives

$$\partial_iF_{ij}^M=0\ . \tag{4.3}$$

Note, however, that these equations are precisely the equations of motion of the graviton, antisymmetric tensor and gauge fields, respectively, to first order in these fields. This leads us to believe that there is a deep connection between the condition for finiteness of the two-dimensional field theory describing the action for a string in a given background, and the full equations of motion involving the background fields.

Finally, we shall analyze the action (2.28) in the special case where $b_{ij}=0$, and the gauge field is equal to the spin connection. Most of the attempts to compactify string theories have been based on such manifolds.[6,8] In this case, the holonomy group of the underlying manifold, which is a subgroup of SO(6) is identical to the subgroup of $E_8\times E_8$ or SO(32) in which the gauge field takes its value. Components of the field ψ which are singlets of this group decouple from the theory as a set of free fields. Similarly, components of λ, which are singlets of the holonomy group, decouple from the theory. Components of λ which are not singlets of the holonomy group, on the other hand, transform in the same way as the components of ψ which couple to the background gauge field. Hence there is a one-to-one correspondence between the ψ^s's and the λ^i's, and we may combine the fields ψ and λ to get a Majorana spinor χ with both right- and left-handed components. Since the gauge field strength F is equal to the curvature tensor R in this case, we may write the full action as

$$S=\frac{1}{2\pi}\int d\tau\int_0^\pi d\sigma[g_{ij}\partial_\alpha X^i\partial^\alpha X^j +i\bar{\chi}_i\rho^\alpha(\partial_\alpha\delta^i{}_j+\Gamma^i{}_{jl}\partial_\alpha X^l)\chi^j +\tfrac{1}{8}R_{ijkl}\bar{\chi}^i(1+\gamma_P)\chi^k\bar{\chi}^j(1+\gamma_P)\chi^l] \tag{4.4}$$

which is the action for an $N=1$ supersymmetric σ model with background metric g_{ij}. This guarantees that the model is finite to all orders in the perturbation theory if the background metric is Ricci flat.[21]

APPENDIX A

In this appendix we shall show that the full effective action for the heterotic string in arbitrary background field, as given in Eq. (2.28), has an exact $N=\frac{1}{2}$ supersymmetry.[23] For this purpose, we shall express the action explicitly in terms of g_{ij}, B_{ij}, and A_i^M as follows:

$$S=\frac{1}{\pi}\int d\tau\int_0^\pi d\sigma\,\mathscr{L}\;, \tag{A1}$$

$$\mathscr{L}=\frac{1}{2}\Bigg[g_{ij}(X)(\partial_\tau-\partial_\sigma)X^i(\partial_\tau+\partial_\sigma)X^j+ig_{ij}\lambda^i(\partial_\tau+\partial_\sigma)\lambda^j-ig_{ij,l}(\partial_\tau+\partial_\sigma)X^j\lambda^l\lambda^i$$
$$+i\psi^s(\partial_\tau-\partial_\sigma)\psi^s+\psi^s(T^M)_{st}\psi^t\left[A_i^M(\partial_\tau-\partial_\sigma)X^i-iA^M_{i,l}\lambda^l\lambda^i-\frac{i}{2}f^{MNP}A_i^NA_l^P\lambda^l\lambda^i\right]\Bigg]\;, \tag{A2}$$

where we have used the one-component notation for the spinor fields λ^i and ψ^s. Under supersymmetry transformation with one-component spinor ϵ, the various fields transform as

$$\delta\lambda^i=-(\partial_\tau-\partial_\sigma)X^i\epsilon\;,$$
$$\delta X^i=i\epsilon\lambda^i\;, \tag{A3}$$
$$\delta\psi^s=0\;.$$

Thus,

$$\delta f(x)=f_{,l}i\epsilon\lambda^l\;, \tag{A4}$$

where f is any function of X. Using Eqs. (A2)—(A4), we get

$$\delta\mathscr{L}=\tfrac{1}{2}i\epsilon(\partial_\tau-\partial_\sigma)[g_{ij}\lambda^i(\partial_\tau+\partial_\sigma)\lambda^j+B_{ij}\lambda^i(\partial_\tau+\partial_\sigma)\lambda^j]$$
$$+\tfrac{1}{2}i\epsilon\psi^s(T^M)_{st}\psi^t\{(\partial_\tau-\partial_\sigma)(A_i^M\lambda^i)-if^{MNP}A_l^P\lambda^i[A^N_{i,k}\lambda^k\lambda^l+iA_i^N(\partial_\tau-\partial_\sigma)X^l]\}\;. \tag{A5}$$

Contribution to δS from the first term in (A5) vanishes except for the boundary terms.[23] Contribution from the second term may be written as, after doing an integration by parts,

$$\delta S=\frac{i\epsilon}{2\pi}\int d\tau\int_0^\pi d\sigma\{-(\partial_\tau-\partial_\sigma)(\psi^sT^M_{st}\psi^t)A_i^M\lambda^i-i\psi^sT^M_{st}\psi^tf^{MNP}A_l^P\lambda^i[A^N_{i,k}\lambda^k\lambda^l+A_i^N(\partial_\tau-\partial_\sigma)X^l]\}\;. \tag{A6}$$

From Eq. (A2) we may write the ψ field equations of motion as

$$i(\partial_\tau-\partial_\sigma)\psi^s+T^M_{st}\psi^t\left[A_i^M(\partial_\tau-\partial_\sigma)X^i-iA^M_{i,l}\lambda^l\lambda^i-\frac{i}{2}f^{MNP}A_i^NA_l^P\lambda^l\lambda^i\right]=0\;, \tag{A7}$$

which gives

$$(\partial_\tau-\partial_\sigma)(\psi^s T^M_{st}\psi^t)=\psi^s[T^M,T^L]_{st}\psi^t i\left[A^L_l(\partial_\tau-\partial_\sigma)X^l-iA^L_{i,l}\lambda^l\lambda^i-\frac{i}{2}f^{LNP}A^N_iA^P_l\lambda^l\lambda^i\right]. \tag{A8}$$

Substituting this in (A6), and using the relation

$$[T^M,T^N]=-if^{MNP}T^P, \tag{A9}$$

we get

$$\delta S=-\frac{\epsilon}{4\pi}\int d\tau\int_0^\pi d\sigma\,\psi^s T^K_{st}\psi^t f^{MLK}f^{LNP}A^N_iA^P_lA^M_j\lambda^j\lambda^l\lambda^i\,. \tag{A10}$$

Using the antisymmetry property of the product of λ's, we may replace $f^{MLK}f^{LNP}$ by

$$\tfrac{1}{3}(f^{MLK}f^{LNP}-f^{PLK}f^{LNM}-f^{NLK}f^{LMP}) \tag{A11}$$

which vanishes by the Jacobi identity. This shows that S is indeed invariant under the supersymmetry transformation (A3).

APPENDIX B

In this appendix we shall give some details of the Feynman rules, and evaluation of various one-loop ultraviolet divergent graphs in the effective two-dimensional field theory. The relevant action is given by Eq. (3.14). Absorbing an overall factor of $(1/\pi)$ in the loop counting parameter, we may write down the propagator for various fields as

$$\Delta_\xi(p)=\frac{i}{p^2+i\epsilon}\delta_{ab}, \tag{B1}$$

$$S_\lambda(p)=\frac{1+\gamma_P}{2}\frac{i\not p}{p^2+i\epsilon}\delta_{ab}, \tag{B2}$$

$$S_\psi(p)=\frac{1-\gamma_P}{2}\frac{i\not p}{p^2+i\epsilon}\delta_{st}\,. \tag{B3}$$

The various propagators are diagrammatically represented as in Fig. 1. The various vertices of the theory that are relevant for one-loop calculation are shown in Fig. 2. They are given by, respectively,

$$\begin{aligned}
&(\mathrm{a}):\tfrac{1}{4}[B^{ab}_\alpha(2p-k)^\alpha+iB^{ca}_\alpha B^{cb\alpha}+i\tilde R_{mban}(\partial_\alpha X^m\partial^\alpha X^n-\epsilon^{\alpha\beta}\partial_\alpha X^m\partial_\beta X^n)]+\begin{bmatrix}a\leftrightarrow b\\ p\leftrightarrow k-p\end{bmatrix},\\
&(\mathrm{b}):-\tfrac{1}{2}B^{ba}_\alpha\rho^\alpha,\\
&(\mathrm{c}):\frac{i}{2}A^M_i\partial_\alpha X^i\rho^\alpha T^M_{ts},\\
&(\mathrm{d}):\frac{i}{2}[(D_aA^M)_l\partial_\alpha X^l+A^M_b\omega^{ba}{}_l\partial_\alpha X^l+ik_{1\alpha}A^M_a]\rho^\alpha T^M_{ts},\\
&(\mathrm{e}):\frac{i}{4}\rho^\alpha T^M_{ts}\{(D_bA^M)_a ik_{1\alpha}+(D_bA^M)_c\omega^{ca}{}_l\partial_\alpha X^l+\tfrac{1}{2}[(D_aD_bA^M)_l+R^i{}_{abl}A^M_i]\partial_\alpha X^l\}+\begin{bmatrix}a\leftrightarrow b\\ k_1\leftrightarrow k_2\end{bmatrix},\\
&(\mathrm{f}):\tfrac{1}{8}F^M_{ab}T^M_{ts}\rho^\alpha\otimes\rho_\alpha\,.
\end{aligned} \tag{B4}$$

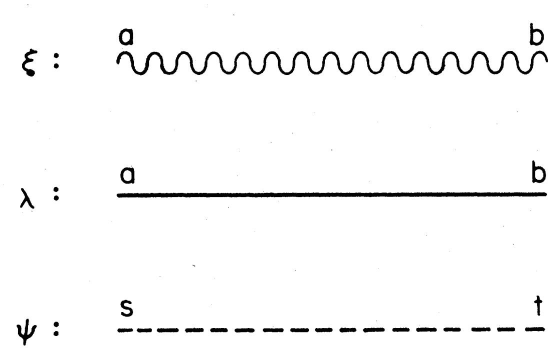

FIG. 1. Propagators for various fields.

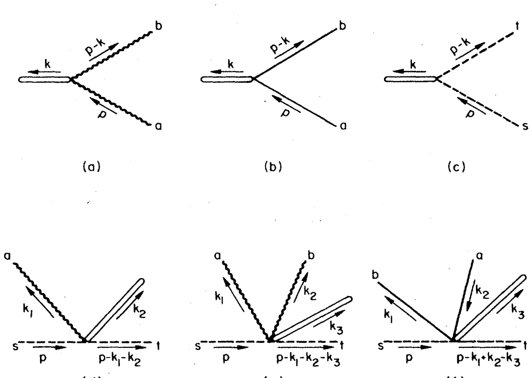

FIG. 2. Various vertices relevant for calculating one-loop counterterms.

In Fig. 2, the double lines always represent some function of the background fields, which are stated explicitly in Eq. (B4). Notice that in writing down the Feynman rules, all the quantum fields have been represented in the momentum space, while all the background fields are still represented in the position space. If the effective one-loop action calculated with these Feynman rules has any dependence on the momenta k_α carried by the background fields, we must replace it by $i\partial_\alpha$ if the momentum is incoming, and by $(-i\partial_\alpha)$ if it is outgoing. Finally, note that in calculating the combinatoric factor associated with a given graph, we must take into account the fact that the fields λ and ψ are Majorana, so that any of the two lines coming out of a vertex quadratic in ψ (or λ) may be contracted with a given external ψ (or λ) field.

In order to study the ultraviolet-divergence structure of the theory, we must list all possible operators of dimension 2. As mentioned in the text, we shall consider only those operators which do not carry any λ field, since operators with λ fields are related to those without by supersymmetry transformation. There are four such possible operators,[24] given by

$$S_{ij}(X)\partial_\alpha X^i \partial^\alpha X^j \,, \tag{B5}$$

$$T_{ij}(X)\epsilon^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^j \,, \tag{B6}$$

$$Q_i^M(X)\bar{\psi}T^M\rho^\alpha\psi\partial_\alpha X^i \,, \tag{B7}$$

$$P_i^M(X)\bar{\psi}T^M\rho^\alpha\psi\epsilon_{\alpha\beta}\partial^\beta X^i \,. \tag{B8}$$

Moreover, (B7) and (B8) are not independent since ψ is Weyl. Here, S, T, P, and Q are arbitrary functions of the fields X^i. Graphs contributing to (B5) and (B6) have been shown in Figs. 3(a)—3(d), while those contributing to (B7) and (B8) are shown in Figs. 3(e)—3(i).

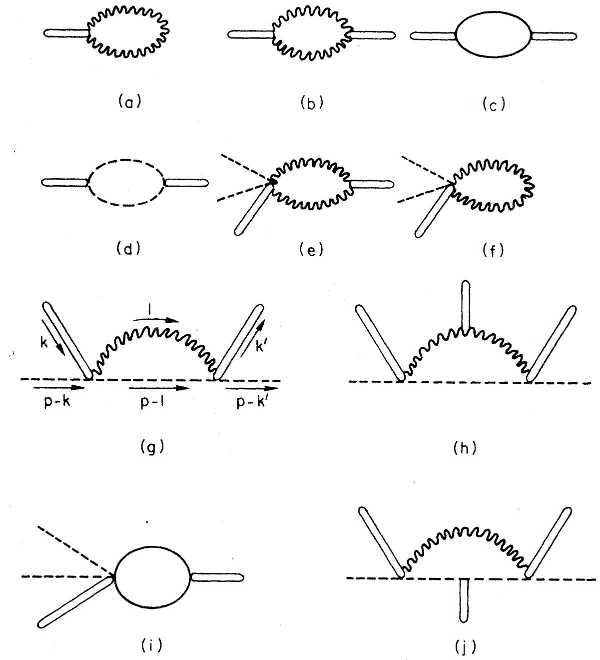

FIG. 3. One-loop ultraviolet-divergent contributions to the effective action involving external X^i and ψ^s lines.

The evaluation of most of these graphs is straightforward. The contribution from Figs. 3(c), 3(d), and 3(i) may be shown to be ultraviolet finite. The total contribution to the effective action from Figs. 3(a) and 3(b) is proportional to[17]

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\frac{1}{2}\tilde{R}_{maan}(\partial_\alpha X^m\partial^\alpha X^n - \epsilon^{\alpha\beta}\partial_\alpha X^m \partial_\beta X^n) \,. \tag{B9}$$

Total contribution to the effective action from Figs. 3(e) and 3(f) is given by, after doing an integration by parts and using the equations of motion (A7) for the ψ field,

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\bar{\psi}T^M\rho^\alpha\psi\{ -\tfrac{1}{2}\epsilon_{\alpha\beta}\partial^\beta X^l S^{ca}{}_l(D_cA^M)_a + \tfrac{1}{4}D_a[(D_aA^M)_l - (D_lA^M)_a]\partial_\alpha X^l$$
$$- \tfrac{1}{4}f^{MNP}A_l^P(D_aA^N)_a\partial_\alpha X^l\} + O(\psi^2\lambda^2) \,, \tag{B10}$$

where we have used Eq. (3.7) to express $B^{ab}{}_\alpha$ in terms of the spin connection and the torsion. Contributions from Figs. 3(h) and 3(j) are given by, respectively,

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\left[-\frac{1}{4}\right]\left[\omega^{ab}{}_i\partial_\alpha X^i f^{MNP}A_a^N A_b^P - \epsilon_{\alpha\beta}\partial^\beta X^l S^{ab}{}_l f^{MNP}A_a^N A_b^P\right]\bar{\psi}T^M\rho^\alpha\psi \tag{B11}$$

and

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\frac{1}{2}\bar{\psi}T^MT^NT^P\rho^\alpha\psi A_a^M A_l^N A_a^P\partial_\alpha X^l \,. \tag{B12}$$

Evaluation of the graph shown in Fig. 3(g) is somewhat more tricky. A direct calculation of the graph gives a contribution of the form $a+b\not{p}$, where a and b are functions of the background fields. Since p is the total incoming momentum carried by the fermion field ψ and the background fields attached to the left vertex of the graph, in writing down the effective action, we may replace $\not p$ by $i\not\partial$, where the ∂_α operator acts both on the incoming fermion as well as the background field. The contribution to the effective action from Fig. 3(g) may then be written as

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\left[-\frac{1}{2}\right](\bar\psi T^M T^N \rho^\alpha A_a^M i\partial_\alpha(A_a^N\psi)+i\bar\psi T^M T^N\rho^\alpha\psi\{A_a^M[(D_a A^N)_l\partial_\alpha X^l+A_b^N\omega^{ba}{}_l\partial_\alpha X^l] - A_a^N[(D_a A^M)_l\partial_\alpha X^l+A_b^M\omega^{ba}{}_l\partial_\alpha X^l]\}) \,. \tag{B13}$$

This may be simplified by using the equations of motion for the ψ fields given in Eq. (A7). The final result is

$$\left[i\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2+i\epsilon}\right]\left[-\frac{1}{4}\right](-\bar\psi\rho^\alpha T^M\psi f^{MNP}A_a^N\partial_l A_a^P\partial_\alpha X^l+2\bar\psi\rho^\alpha T^N T^M T^P\psi A_a^N A_l^P\partial_\alpha X^l A_a^M + f^{MNP}\bar\psi T^M\rho^\alpha\psi\{A_a^N[(D_a A^P)_l\partial_\alpha X^l+A_b^P\omega^{ba}{}_l\partial_\alpha X^l] - A_a^P[(D_a A^N)_l\partial_\alpha X^l+A_b^N\omega^{ba}{}_l\partial_\alpha X^l]\}) \,. \tag{B14}$$

Adding (B9), (B10), (B11), (B12), and (B14), we may reproduce Eq. (3.15) of the text.

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[24]There is no dimension-2 Lorentz-invariant operator involving four ψ fields, due to the Majorana-Weyl nature of these fields.