Current-definition freedom in a derivative-coupling model

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The problem of the construction of a covariant current operator in a two-dimensional field theory is reviewed and subsequently applied to a recently proposed derivative-interaction model. It is found both by solving the equations of motion as well as by path-integral techniques that there exists a family of admissible solutions labeled by a parameter ξ, only one choice of which corresponds to that previously derived. This is shown to be in agreement with results for the other known soluble two-dimensional field theories in which a gauge principle is absent.

I. INTRODUCTION

It is well known that for the case of a massless spinor field the current operator j^μ in two dimensions has the property that (at least in the absence of interactions) it has vanishing divergence and curl, i.e.,

$$\partial_\mu j^\mu = \partial_\mu \epsilon^{\mu\nu} j_\nu = 0 \ .$$

This implies that the spectrum of j^μ consists solely of a zero-mass particle and is presumably the principal reason for the solubility of the various models in which this current is coupled either to itself or to a vector field. In the former case one has the Thirring model[1] while the latter is either the Schwinger model[2] or what will be referred to here as the current-coupled vector meson depending upon whether the vector field is massless or massive. Of these, the Schwinger model has probably enjoyed the greatest popularity since it possesses an exact local gauge invariance, a concept which is widely held to be the appropriate starting point for a complete four-dimensional theory. Because of this, the current operator is uniquely prescribed and the solution of the model is fixed up to a gauge transformation.

Recently, however, there has been work reported[3] on a coupling of the current operator j^μ to the derivative of a scalar field ϕ. This model does not possess a local gauge invariance and consequently cannot be expected to resemble the Schwinger model with its necessarily conserved current operator. Thus one expects that at least qualitatively it should resemble the soluble theories which lack a gauge principle. Since, however, it is known that both the Thirring model[4] and the current-coupled vector meson[5] are dependent on a parameter which does not appear in the Lagrangian (but is necessary to specify a particular current construction) there immediately arises the question as to whether the solution obtained in Ref. 3 is the most general result. It will be shown in this paper using the approach of Refs. 4 and 5 as well as the path-integral techniques due to Fujikawa[6–8] that the derivative-coupling model in fact possesses a one-parameter family of solutions as suggested here.

In Sec. II the issue of current construction in two dimensions is briefly reviewed and the origin of the so-called ξ parameter displayed. In Sec. III application of this approach is made to the derivative-coupling model and the existence of the more general solution demonstrated. Similar results are shown to emerge from the path-integral approach in Sec. IV with some concluding remarks offered in Sec. V.

II. CURRENT DEFINITION WITH AN EXTERNAL SOURCE

Since the solutions of the known two-dimensional field theories can all be obtained from the more manageable case of a spinor field coupled to a classical vector field $A^\mu(x)$, one conveniently starts from the Lagrangian

$$L = \frac{i}{2}\psi\alpha^\mu\partial_\mu\psi + j^\mu A_\mu \ . \tag{2.1}$$

In writing (2.1) the field $\psi(x)$ is taken to be Hermitian with the current given by

$$j^\mu(x) = \tfrac{1}{2}\psi(x)\alpha^\mu q\psi(x) \ , \tag{2.2}$$

where

$$\alpha^0 = -\alpha_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \alpha^1 = \alpha_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

and q is a matrix of the form

$$q = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

in an internal charge space.

Since the definition (2.2) is a formal one, it becomes necessary to prescribe a technique for the circumventing of divergences which appear in the calculation of its matrix elements. One such approach is that of regularization which seems to be quite adequate to the case in which current conservation is required for the model. Since, however, one is particularly concerned here with those instances in which the current need not satisfy such a condition, it will be convenient to begin with the definition proposed by Schwinger for the gauge-invariant case, i.e.,

$$j^\mu(x) = \lim_{x\to x'} \tfrac{1}{2}\psi(x)\alpha^\mu q \exp\left[iq\int_{x'}^{x} dx''_\mu A^\mu(x'')\right]\psi(x') , \tag{2.3}$$

where the limit $\mathbf{x}=\mathbf{x}'$ is to be taken after setting $x^0=x'^0$. This specification of the current operator is clearly appropriate to the case of a conserved current inasmuch as it preserves the invariance of $j^\mu(x)$ under the combined local gauge transformations

$$\psi(x) \to (1+iq\delta\lambda)\psi(x) ,$$
$$A^\mu(x) \to A^\mu(x) + \partial^\mu\delta\lambda ,$$

during the process of taking the limit. Specifically, one obtains[4]

$$\langle j^\mu(x)\rangle \equiv \frac{\langle 0|j^\mu(x)|0\rangle}{\langle 0|0\rangle} = \int D_V^{\mu\nu}(x-x')A_\nu(x')dx' , \tag{2.4}$$

where

$$D_V^{\mu\nu}(x) = -\frac{1}{\pi}\epsilon^{\mu\sigma}\epsilon^{\nu\tau}\partial_\sigma\partial_\tau D(x)$$

and $D(x)$ satisfies

$$-\partial^2 D(x) = \delta(x) \tag{2.5}$$

subject to the usual causal boundary conditions.

Although Eq. (2.4) implies $\partial_\mu j^\mu = 0$, it does not yield $\partial_\mu \epsilon^{\mu\nu} j_\nu = 0$ despite the *formal* invariance of the Lagrangian under

$$\psi(x) \to (1+iq\gamma_5\delta\lambda')\psi(x) , \tag{2.6}$$

where $\gamma_5 = \alpha_1$. This symmetry, however, can be restored by replacing the exponential in the definition of $j^\mu(x)$ by

$$\exp\left[-iq\int_{x'}^{x} dx''_\mu \gamma_5 \epsilon^{\mu\nu} A_\nu(x'')\right]$$

which preserves the invariance under (2.6) together with

$$A_\mu(x) \to A_\mu(x) + \epsilon_{\mu\nu}\partial^\nu\delta\lambda .$$

Calculation now yields the result[4]

$$\langle j^\mu(x)\rangle = \int D_A^{\mu\nu}(x-x')A_\nu(x')dx' ,$$

where

$$D_A^{\mu\nu}(x) = -\frac{1}{\pi}\partial^\mu\partial^\nu D(x) . \tag{2.7}$$

Although Eq. (2.7) clearly implies the conservation of the dual of $j^\mu(x)$, it requires that one dispense with the usual idea of a conserved current. At the same time it raises the suspicion that there may be a definition of $j^\mu(x)$ intermediate between the case of current conservation and axial current conservation. This is indeed accomplished by taking the exponential in $j^\mu(x)$ to be

$$\exp\left[iq\int_{x'}^{x} dx''_\mu [\xi A^\mu(x'') - \eta\gamma_5\epsilon^{\mu\nu}A_\nu(x'')]\right] , \tag{2.8}$$

where ξ and η are required for reasons of covariance[5] to satisfy

$$\xi + \eta = 1 . \tag{2.9}$$

The current correlation functions $D_V^{\mu\nu}(x)$ and $D_A^{\mu\nu}(x)$ are subsequently found to be replaced by the more general result

$$D^{\mu\nu}(x) = -\frac{1}{\pi}(\xi\epsilon^{\mu\sigma}\epsilon^{\nu\tau} + \eta g^{\mu\sigma}g^{\nu\tau})\partial_\sigma\partial_\tau D(x) \tag{2.10}$$

with $\xi=1$ and $\eta=1$ corresponding, respectively, to the cases of vector and axial-vector current conservation.

The case of a quantized interaction is now easily handled by use of functional derivatives. In the case of the Thirring model with a term

$$\frac{\lambda}{2} j^\mu j_\mu$$

in the Lagrangian one finds that the two parts of $D^{\mu\nu}(x)$ undergo different renormalizations as a consequence of the interaction. The result is

$$D^{\mu\nu}(x) = -\frac{1}{\pi}\left[\epsilon^{\mu\sigma}\epsilon^{\nu\tau}\frac{1}{1+\lambda\xi/\pi} + g^{\mu\sigma}g^{\nu\tau}\frac{1}{1-\lambda\eta/\pi}\right]\partial_\sigma\partial_\tau D(x) . \tag{2.11}$$

The corresponding calculations for the case of the current-coupled vector meson are also readily carried out[5] and display a similar dependence upon the ξ and η parameters. In point of fact, the appearance of ξ in the expression for the renormalized mass makes the dependence on the current definition even more striking than in the case of the Thirring model.

In sum then one finds that all the known soluble models in two dimensions which involve the coupling of a current operator depend upon a parameter which can be prescribed at will. (The sole exception to this is the Schwinger model whose equations of motion require current conservation for consistency.) That such an ambiguity must also characterize the model of Ref. 3 would appear logical, the explicit demonstration of which is given in the next section.

III. GENERAL SOLUTION OF THE DERIVATIVE INTERACTION MODEL

The Lagrangian of the model of Ref. 3 can be written as

$$L = \frac{i}{2}\psi\alpha^\mu\partial_\mu\psi + \phi^\mu\partial_\mu\phi + \tfrac{1}{2}\phi^\mu\phi_\mu - \tfrac{1}{2}m^2\phi^2 + g\partial^\nu\phi j^\mu\epsilon_{\mu\nu} + K\phi + A^\mu j_\mu , \tag{3.1}$$

where K and A^μ are classical sources coupled, respectively, to the spin-zero field ϕ and the current j^μ. The action principle implies that the variation with g of the vacuum-to-vacuum transition amplitude is given by

$$\frac{\delta}{\delta g}\langle 0|0\rangle^g = i\langle 0|\partial^\nu\phi j^\mu|0\rangle\epsilon_{\mu\nu} = -i\frac{\delta}{\delta A_\mu}\epsilon_{\mu\nu}\partial^\nu\frac{\delta}{\delta K}\langle 0|0\rangle^g$$

or

$$\langle 0|0\rangle^g = \exp\left[-ig\int\frac{\delta}{\delta A_\mu(x)}\epsilon_{\mu\nu}\partial^\nu\frac{\delta}{\delta K(x)}dx\right]\langle 0|0\rangle_{K,A}^{g=0} . \tag{3.2}$$

Using the fact that for $g=0$ one has the factorization

$$\langle 0|0\rangle_{K,A}^{g=0} = \langle 0|0\rangle_K\langle 0|0\rangle_A ,$$

one readily obtains that

$$\langle 0|0\rangle_{K,A}^{g=0} = \exp\left[\frac{i}{2}\int A_\mu(x)D^{\mu\nu}(x-x')A_\nu(x')dx\,dx'\right]\exp\left[\frac{i}{2}\int K(x)\Delta(x-x')K(x')dx\,dx'\right] , \tag{3.3}$$

where $D_{\mu\nu}(x)$ is given by the ξ-dependent form of Eq. (2.10) and $\Delta(x)$ is the usual mass-m propagator

$$(-\partial^2+m^2)\Delta(x)=\delta(x) . \tag{3.4}$$

The insertion of Eq. (3.3) into Eq. (3.2) yields a precise prescription for the calculation of the vacuum-to-vacuum transition amplitude. Following somewhat tedious calculations analogous to those of Refs. 4 and 5 it follows that

$$\langle 0|0\rangle^g = C\exp\left[\frac{i}{2}\int A_\mu(x)D_g^{\mu\nu}(x-x')A_\nu(x')dx\,dx'\right]\exp\left[\frac{i}{2}\int K(x)\Delta_g(x-x')K(x')dx\,dx'\right]$$
$$\times\exp\left[i\int K(x)M^\mu(x-x')A_\mu(x')dx\,dx'\right] , \tag{3.5}$$

where C is a constant and

$$D_g^{\mu\nu}(x)=D^{\mu\nu}(x)-\frac{g^2\xi^2}{\pi^2}\epsilon^{\mu\sigma}\partial_\sigma\epsilon^{\nu\tau}\partial_\tau\Delta_g(x) ,\quad \Delta_g(x)=\left[1-\frac{g^2\xi}{\pi}\right]^{-1}\Delta'(x) ,\quad M^\mu(x)=-\frac{\xi g}{\pi}\epsilon^{\mu\sigma}\partial_\sigma\Delta_g(x) , \tag{3.6}$$

with $\Delta'(x)$ being the propagator for a field of mass squared $m^2(1-g^2\xi/\pi)^{-1}$, i.e.,

$$\left[-\partial^2+\frac{m^2}{1-g^2\xi/\pi}\right]\Delta'(x)=\delta(x) . \tag{3.7}$$

The fermionic matrix elements are also computed in straightforward fashion using the source method. In particular one finds that for vanishing external sources the $2n$-point function has the form[9]

$$G(x_1,\ldots,x_{2n})=G_{g=0}(x_1,\ldots,x_{2n})\exp\left[\frac{ig^2}{2}\sum_{i,j}q_iq_j\gamma_{5i}\gamma_{5j}\Delta_g'(x_i-x_j')\right] .$$

This implies for the two-point function the result

$$G(x-x')=G_{g=0}(x-x')\exp\left[\frac{-ig^2}{1-g^2\xi/\pi}[\Delta'(x-x')-\Delta'(0)]\right] . \tag{3.8}$$

One finds upon comparison with Ref. 3 that in all cases there is agreement for the gauge-invariant (i.e., $\xi=1$) choice while at the same time one has accomplished an explicit demonstration of the fact that as in the previously known gauge-noninvariant models there exists a one-parameter family of solutions.

Before leaving this brief discussion of the Green's functions of the model it is of some interest to remark that operator calculations of the divergence of the current and its dual give

$$\partial_\mu j^\mu=0,\quad \partial_\mu\epsilon^{\mu\nu}j_\nu=-\frac{g^2\xi}{\pi}\partial^2\phi . \tag{3.9}$$

It is the latter equation which readily yields the previously calculated mass renormalization of the model. Perhaps even more interesting, however, is the fact that when all the external sources are removed, the current is conserved despite its gauge noninvariant form. This, however, is also identical to what happens in the Thirring and current-coupled vector-meson models. In fact in each of these cases it is an accidental conservation law which could be broken by the inclusion of additional (but soluble) coupling terms.

IV. PATH-INTEGRAL SOLUTION

The previous two sections have shown that in the absence of a dynamical gauge-symmetry principle the definition of the current and hence the most general solution of the model with a derivative coupling involves an arbitrary parameter. Attention will now be directed to a study of the same model following methods of Fujikawa[6–8] with the avowed goal of deriving the one-parameter class of solutions within the context of path integrals. The Lagrangian in Euclidean space has the form

$$L=\frac{1}{2}\partial_\mu\phi\partial_\mu\phi+\frac{m^2}{2}\phi^2-i\bar{\psi}\gamma_\mu\partial_\mu\psi+g\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\phi \tag{4.1}$$

and the generating functional is given by

$$Z=\int D\bar{\psi}D\psi D\phi\, e^{-S} . \tag{4.2}$$

In order to proceed it is convenient to imagine the interaction of the fermion fields as a gauge interaction, i.e., one writes the fermion interaction Lagrangian as

$$g\bar{\psi}\gamma_5\gamma_\mu\psi A_\mu^5 .$$

This can always be done in the path-integral formalism by introducing a delta functional of the form

$$\delta(A_\mu^5(x)-\partial_\mu\phi(x))$$

and functionally integrating over the new field $A_\mu^5(x)$. Furthermore, using the two-dimensional identity

$$\gamma_5\gamma_\mu A_\mu^5=\gamma_\mu A_\mu ,$$

where $A_\mu=-i\epsilon_{\mu\nu}A_\nu^5$ in Euclidean space, the complete Euclidean Lagrangian can be written as

$$L=\frac{1}{2}\partial_\mu\phi\partial_\mu\phi+\frac{m^2}{2}\phi^2-i\bar{\psi}\gamma_\mu(\partial_\mu+igA_\mu)\psi$$

with

$$A_\mu=-i\epsilon_{\mu\nu}A_\nu^5=-i\epsilon_{\mu\nu}\partial_\nu\phi . \tag{4.3}$$

Note that this allows one to mimic the form of a gauge interaction even though there is no dynamical gauge principle operating in this model. There are many models (at least in two space-time dimensions) with this property and it is useful, therefore, to study the most general behavior of the fermion functional in the absence of a gauge-invariance principle. To do that it is sufficient to look at only the Lagrangian of the fermion interacting with a background field. That is,

$$L_f=-i\bar{\psi}\gamma_\mu(\partial_\mu+igA_\mu)\psi$$

with

$$Z_f=\int D\bar{\psi}D\psi\, e^{-S_f} . \tag{4.4}$$

If the fermion fields inside the functional integral are redefined by

$$\delta\psi=-ig\epsilon(x)\psi(x)\quad(\text{gauge } tf^n) \tag{4.5a}$$

and

$$\delta\psi=ig\bar{\epsilon}(x)\gamma_5\psi(x)\quad(\text{chiral gauge } tf^n) \tag{4.5b}$$

the invariance of the generating functional leads to the naive conservation laws

$$\partial_\mu j_\mu=\partial_\mu(\bar{\psi}\gamma_\mu\psi)=0 ,$$
$$\partial_\mu j_\mu^5=\partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi)=0 . \tag{4.6}$$

However, those conservation laws are known to be anomalous because of quantum corrections.[10] As is known from the analysis of Fujikawa,[6–8] the measure is not necessarily invariant under such transformations and the noninvariance leads to anomalous conservation laws. As an illustration of this point it is useful to calculate the most general change in the fermionic measure under an infinitesimal chiral transformation.

Let $\phi_n(x)$ satisfy the eigenvalue equation

$$\gamma_\mu D_\mu(\xi)\phi_m(x)=\gamma_\mu(\partial_\mu+i\xi gA_\mu)\phi_m(x)$$
$$=\lambda_m\phi_m(x) . \tag{4.7}$$

Here ξ is an arbitrary parameter and hence $\phi_n(x)$ are the eigenfunctions of a gauge-noninvariant operator. Note, however, that for $\xi=1$, the operator is gauge invariant. If the ϕ_m's form a complete set, then they must satisfy

$$\int d^2x\, \phi_m^\dagger(x)\phi_l(x)=\delta_{ml} ,$$
$$\sum_m\phi_m(x)\phi_m^\dagger(y)=\delta^2(x-y) . \tag{4.8}$$

This allows expansion of the fermion fields as

$$\psi(x)=\sum_m a_m\phi_m(x),\quad \bar{\psi}(x)=\sum_m\phi_m^\dagger(x)b_m , \tag{4.9}$$

so that

$$D\bar{\psi}D\psi=\prod_m db_m da_m . \tag{4.10}$$

It is helpful to emphasize here the reason for selecting a basis that is not gauge invariant. Normally, if a theory has a gauge invariance one expands in a basis which respects the symmetry of the theory. Since, however, in the absence of any such symmetry principles, one basis is as good as any other, it is clearly desirable to choose a very general basis. Note here that the deviation of the parameter ξ from unity measures the gauge noninvariance.

To find the change in the measure under an infinitesimal chiral transformation, recall that the fields transform as

$$\psi(x)\to\psi'(x)=\psi(x)+\delta\psi(x)=[1+ig\gamma_5\bar{\epsilon}(x)]\psi(x) ,$$
$$\bar{\psi}(x)\to\bar{\psi}'(x)=\bar{\psi}(x)+\delta\bar{\psi}(x)=\bar{\psi}(x)(1+ig\gamma_5\bar{\epsilon}) . \tag{4.11}$$

Further, by expanding

$$\psi'(x)=\sum_m a_m'\phi_m(x)=\sum_{m,l}C_{ml}a_l\phi_m(x) ,$$

where

$$C_{ml}=\delta_{ml}+ig\int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5\phi_l(x) \tag{4.12}$$

it follows that

$$\det C_{ml} = \exp\left[\operatorname{Tr}\ln\left[\delta_{ml} + ig\int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5(x)\phi_l(x)\right]\right]$$
$$= \exp\left[ig\sum_m \int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5\phi_m(x)\right]$$
$$= 1 + ig\sum_m \int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5\phi_m(x)$$
$$= 1 + \lim_{M^2\to\infty} ig\sum_m \int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5\phi_m(x)e^{-\lambda_m{}^2/M^2} = 1 + \lim_{M^2\to\infty} ig\sum_m \int d^2x\,\bar{\epsilon}(x)\phi_m^\dagger(x)\gamma_5 e^{-\not{D}^2(\xi)/M^2}\phi_m(x) .$$

Using the completeness relation this can be written as

$$\det C_{ml} = 1 + \lim_{M^2\to\infty} ig\int d^2x\frac{d^2k}{(2\pi)^2}\bar{\epsilon}(x)\operatorname{Tr} e^{-ik\cdot x}\gamma_5 e^{-\not{D}^2(\xi)/M^2}e^{ik\cdot x}$$
$$= 1 + \lim_{M^2\to\infty} ig\int d^2x\frac{d^2k}{(2\pi)^2}\bar{\epsilon}(x)\xi g\frac{\epsilon_{\mu\nu}F_{\mu\nu}}{M^2}e^{-k^2/M^2}$$
$$= 1 + \lim_{M^2\to\infty} i\xi g^2\int d^2x\,\bar{\epsilon}(x)\frac{\epsilon_{\mu\nu}F_{\mu\nu}}{M^2}\frac{\pi M^2}{(2\pi)^2} = 1 + \frac{i\xi g^2}{4\pi}\int d^2x\,\bar{\epsilon}(x)\epsilon_{\mu\nu}F_{\mu\nu} , \tag{4.13}$$

where use has been made of the two-dimensional identity

$$\gamma_\mu\gamma_\nu D_\mu(\xi)D_\nu(\xi) = (-\delta_{\mu\nu} - i\epsilon_{\mu\nu}\gamma_5)D_\mu(\xi)D_\nu(\xi) = -D_\mu(\xi)D_\mu(\xi) + \frac{\xi g}{2}\gamma_5\epsilon_{\mu\nu}F_{\mu\nu} . \tag{4.14}$$

It is clear now that under the infinitesimal chiral transformation

$$\prod_m da_m \to \prod_m da_m' = \det C_{ml}\prod_m da_m = \left[1 + \frac{i\xi g^2}{4\pi}\int d^2x\,\bar{\epsilon}(x)\epsilon_{\mu\nu}F_{\mu\nu}\right]\prod_m da_m . \tag{4.15}$$

Similarly it is easy to see that under the same transformation

$$\prod_m db_m \to \prod_m db_m' = \left[1 + \frac{i\xi g^2}{4\pi}\int d^2x\,\bar{\epsilon}(x)\epsilon_{\mu\nu}F_{\mu\nu}\right]\prod_m db_m \tag{4.16}$$

so that the change in the measure is given by

$$\delta(D\bar{\psi}D\psi) = \frac{i\xi g^2}{2\pi}\int d^2x\,\bar{\epsilon}(x)\epsilon_{\mu\nu}F_{\mu\nu}D\bar{\psi}D\psi .$$

Consequently the naive conservation law for the axial-vector current becomes

$$\partial_\mu j_\mu^5 = \partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) = \frac{i\xi g^2}{2\pi}\epsilon_{\mu\nu}F_{\mu\nu} . \tag{4.17}$$

Similar calculations show that the measure is invariant under a gauge transformation so that the conservation laws take the form

$$\partial_\mu j_\mu = \partial_\mu(\bar{\psi}\gamma_\mu\psi) = 0 ,$$
$$\partial_\mu j_\mu^5 = \partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) = \frac{i\xi g^2}{2\pi}\epsilon_{\mu\nu}F_{\mu\nu} = -\frac{\xi g^2}{\pi}\partial_\mu A_\mu^5 = -\frac{\xi g^2}{\pi}\partial_\mu\partial_\mu\phi . \tag{4.18}$$

Use has been made here of Eq. (4.3), namely, $A_\mu = -i\epsilon_{\mu\nu}\partial_\nu\phi$. It is informative to compare this with the point-splitting results of the previous section given in Eq. (3.9). This shows that in the absence of a gauge principle the axial anomaly is ambiguous.

To solve for the derivative-coupling model, let us note that if we make a finite chiral transformation

$$\psi(x) \to \psi'(x) = e^{ig\gamma_5\alpha(x)}\psi(x), \quad \bar{\psi}(x) \to \bar{\psi}'(x) = \bar{\psi}(x)e^{ig\gamma_5\alpha(x)} , \tag{4.19}$$

the fermion Lagrangian changes to

$$L_f = -i\bar{\psi}\gamma_\mu[\partial_\mu + ig\gamma_5\partial_\mu\alpha(x) - ig\gamma_5\partial_\mu\phi(x)]\psi(x) . \tag{4.20}$$

It is clear, therefore, that if one chooses

$$\alpha(x)=\phi(x)$$

the fermions would decouple.

Following Roskies and Schaposnik[11] one can now calculate how the functional measure changes under a finite chiral transformation. The finite transformation, of course, can be thought of as N infinitesimal transformations with parameter $\bar{\epsilon}(x)$ such that

$$\lim_{\bar{\epsilon}\to 0, N\to\infty} N\bar{\epsilon}(x)=\alpha(x)=\phi(x) \ . \tag{4.21}$$

Supposing that n such infinitesimal transformations have already been done, the fermion Lagrangian would have the form

$$L_f=-i\bar{\psi}\gamma_\mu(\partial_\mu+ig\gamma_5 n\partial_\mu\bar{\epsilon}-ig\gamma_5\partial_\mu\phi)\psi=-i\bar{\psi}\gamma_\mu[\partial_\mu-ig\gamma_5 A_\mu^5(n)]\psi=-i\bar{\psi}\gamma_\mu[\partial_\mu+igA_\mu(n)]\psi \ , \tag{4.22a}$$

where as before

$$A_\mu(n)=-i\epsilon_{\mu\nu}A_\nu^5(n)=-i\epsilon_{\mu\nu}\partial_\nu(\phi-n\bar{\epsilon}) \ . \tag{4.22b}$$

It is clear now that upon expansion in the basis

$$\gamma_\mu D_\mu(\xi,n)\phi_m=\gamma_\mu[\partial_\mu+i\xi gA_\mu(n)]\phi_m=\lambda_m\phi_m(x) \ , \tag{4.23}$$

the functional measure changes under an infinitesimal transformation as [see Eq. (4.16)]

$$\begin{aligned} D\bar{\psi}D\psi\to D\bar{\psi}D\psi\left[1+\frac{i\xi g^2}{2\pi}\int d^2x\,\bar{\epsilon}(x)\epsilon_{\mu\nu}F_{\mu\nu}(n)\right]&=D\bar{\psi}D\psi\left[1-\frac{\xi g^2}{\pi}\int d^2x\,\bar{\epsilon}(x)\partial_\mu A_\mu^5(n)\right]\\ &=D\bar{\psi}D\psi\left[1-\frac{\xi g^2}{\pi}\int d^2x\,\bar{\epsilon}(x)\partial_\mu\partial_\mu(\phi-n\bar{\epsilon})\right] \ . \end{aligned} \tag{4.24}$$

Thus under a finite chiral transformation, the measure changes as

$$\begin{aligned} D\bar{\psi}D\psi\to &\lim_{\bar{\epsilon}\to 0, N\to\infty, N\bar{\epsilon}=\alpha=\phi}\ \prod_{n=0}^{N}\left[1-\frac{\xi g^2}{\pi}\int d^2x\,\bar{\epsilon}(x)\partial_\mu\partial_\mu(\phi-n\bar{\epsilon})\right]D\bar{\psi}D\psi\\ &=D\bar{\psi}D\psi\exp\left[-\frac{\xi g^2}{\pi}\int d^2x\,\alpha(x)\partial_\mu\partial_\mu[\phi-\tfrac{1}{2}\alpha(x)]\right]\\ &=D\bar{\psi}D\psi\exp\left[-\frac{\xi g^2}{2\pi}\int d^2x\,\phi(x)\partial_\mu\partial_\mu\phi(x)\right]=D\bar{\psi}D\psi\exp\left[\frac{\xi g^2}{2\pi}\int d^2x\,\partial_\mu\phi(x)\partial_\mu\phi(x)\right] \ . \end{aligned} \tag{4.25}$$

Clearly, therefore, the finite chiral transformation which decouples the fermions leads to an effective action of the form

$$Z=\int D\bar{\psi}D\psi D\phi\, e^{-S_{\text{eff}}} \ ,$$

where

$$S_{\text{eff}}=\int d^2x\left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi+\frac{m^2}{2}\phi^2-i\bar{\psi}\gamma_\mu\partial_\mu\psi-\frac{\xi g^2}{2\pi}\partial_\mu\phi\partial_\mu\phi\right]=\int d^2x\left[\left[1-\frac{\xi g^2}{\pi}\right]\partial_\mu\phi\partial_\mu\phi+\frac{m^2}{2}\phi^2-i\bar{\psi}\gamma_\mu\partial_\mu\psi\right] \ . \tag{4.26}$$

This is the one-parameter solution of the model. For $\xi=1$ there follows, of course, the results of Ref. 3 corresponding to a special case of the most general solution. The Green's functions can be calculated in a straightforward manner by introducing sources and give results already quoted in Sec. III.

V. CONCLUSION

In this paper the most general solution of the derivative-coupling model has been derived both in the point-splitting method as well as in the path-integral formalism. The results obtained forcefully indicate that even though the gauge principle is a beautiful physical idea, an insistence on a gauge-invariant regularization in the absence of a gauge symmetry can be too restrictive. Gauge invariance may, of course, be necessary for renormalizability when true gauge interactions are present. On the other hand, when not required by such considerations, insisting on gauge invariance does not allow for the richer spectrum of solutions as obtained in detail here for the derivative-coupling model. In general, therefore, one should retain the flexibility of not specifying a value for the parameter ξ even though it may be determined from other considerations in a particular problem.

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