Scalar field theories in curved space

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We investigate the behavior of scalar fields (ϕ) in curved space which have a potential $V[\phi]=V_0+(1/2)m^2\phi^2+(1/3!)\eta\phi^3+(1/4!)\lambda\phi^4$ and a general coupling to gravity $\frac{1}{2}\xi R\phi^2$. The back-reaction of gravity strongly affects the stability of scalar fields. By examining the scalar field equations and the Einstein equations, we clarify conditions for the system to have an absolutely stable ground state in which ϕ is constant and a metric is either Minkowski, or de Sitter, or anti—de Sitter. We find that (i) cubic interactions cause instability, unless $\xi=0$, (ii) Higgs scalar fields in the standard model must have $\xi\leqq 0$ or $\xi\geqq\frac{1}{6}$, (iii) negative quartic interaction couplings ($\lambda<0$) can make sense, and (iv) a free scalar field with a tiny mass can reduce the bare large vacuum energy density V_0 to an extremely small value ($\sim m^2G^{-1}\xi^{-1}$). Based on the last observation, the vanishing-cosmological-constant problem is viewed not as a problem of how to reduce the bare vacuum energy density, but as that of how to get a large gravitational constant ($G\gg|m^2V_0|$).

I. INTRODUCTION

It is well known that scalar fields (ϕ) in curved space have an additional coupling to scalar curvature R of the form $\frac{1}{2}\xi R\phi^2$. The conformal invariance[1] dictates $\xi=\frac{1}{6}$ and Nambu-Goldstone bosons[2] have a minimal coupling $\xi=0$. However, there is no preferential value of ξ for such scalar fields as physical Higgs scalar fields in unified gauge theories of electroweak and strong interactions. The $\frac{1}{2}\xi R\phi^2$ interaction becomes important under those circumstances where spacetime curvature gets very large as in the very early universe.

Is the parameter ξ completely arbitrary? We approach this question from the viewpoint of classical stability. Nonvanishing ξ generally leads to nonvanishing R, which in turn affects the evolution of ϕ through the $\frac{1}{2}\xi R\phi^2$ interaction. We evaluate the back-reaction of gravity to the classical stability of scalar fields by solving scalar field equations and the Einstein equations, simultaneously. We clarify conditions for a scalar field system with a potential

$$V[\phi]=V_0+\kappa\phi+\tfrac{1}{2}m^2\phi^2+\frac{\eta}{3!}\phi^3+\frac{\lambda}{4!}\phi^4$$

to have an absolutely stable classical ground state. As corollaries we find that (i) the cubic term must vanish ($\eta=0$), unless $\xi=0$; (ii) Higgs scalar fields in unified theories must have $\xi\leqq 0$ or $\xi\geqq\frac{1}{6}$; (iii) a negative quartic interaction coupling ($\lambda<0$) can make sense in curved space; and (iv) an almost massless, free scalar field can relax a bare cosmological constant to an extremely small value. Many of our results are obtained where quantum gravity effects are negligible.

II. CRITERION FOR STABILITY

To be precise, we consider a real scalar field ϕ in Einstein's general relativity:

$$I=\int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R+\tfrac{1}{2}g^{jk}\partial_j\phi\partial_k\phi-V[\phi]-\tfrac{1}{2}\xi R\phi^2\right]. \tag{1}$$

Equations of motion are

$$\phi^{;k}{}_{k}+V'[\phi]+\xi R\phi=0, \tag{2}$$

$$R_{jk}-\tfrac{1}{2}g_{jk}R=-8\pi G T_{jk}, \tag{3}$$

$$T_{jk}=\phi_{;j}\phi_{;k}+g_{jk}(-\tfrac{1}{2}\phi_{;m}\phi^{;m}+V)+\xi[g_{jk}(\phi^2)_{;m}{}^{m}-(\phi^2)_{;jk}-(R_{jk}-\tfrac{1}{2}g_{jk}R)\phi^2]. \tag{4}$$

We address the question of whether or not the system (1) admits an absolutely stable ground state at the classical level in which ϕ is constant and R_{jk} is proportional to g_{jk}, i.e., spacetime is either Minkowski, de Sitter, or anti—de Sitter spacetime. In curved space the energy density is not a good criterion for the stability. The classical stability must be examined by analyzing the combined Eqs. (2) and (3).

By taking a trace of Eq. (3) and using Eq. (2), one finds

$$R=\frac{8\pi G\phi_c{}^2}{\phi^2-\phi_c{}^2}[(1-6\xi)\phi_{;k}\phi^{;k}-4V+6\xi\phi V'], \tag{5}$$

where $\phi_c{}^2=[8\pi G\xi(1-6\xi)]^{-1}$. Substitution of (5) into Eq. (2) yields[3].

$$\phi^{;k}{}_{k}+U'[\phi]+\frac{\phi}{\phi^2-\phi_c{}^2}\phi_{;k}\phi^{;k}=0, \tag{6}$$

$$U'[\phi]=-\frac{\phi_c{}^2}{\phi^2-\phi_c^2}[(1-8\pi G\xi\phi^2)V'+32\pi G\xi\phi V]. \tag{7}$$

Note that Eq. (6) is exact. $U[\phi]$, which plays the role of a potential in flat-space field theory, may be called a pseudopotential. Though not having any relation to energy density, $U[\phi]$ determines the stability of the scalar field through Eq. (6). The last term in Eq. (6) might be interpreted as a "velocity-dependent" potential.

It is known that a local, but not global minimum of $V[\phi]$ (or of $U[\phi]$ in general cases) can be stable because of the vanishing tunneling probability. Coleman and de Luccia showed,[4] for instance, that the transition probability from a Minkowski spacetime to an anti—de Sitter space vanishes under certain conditions. Furthermore, Breitenlohner and Freedmann, and Gibbons *et al.*, showed that anti—de Sitter spaces with a scalar field at the top of a potential can be stable in gauged supergravity theories, provided that a boundary condition is imposed at spatial infinity such that a total "energy" in anti—de Sitter spaces is finite.[5] The positive-energy theorem in asymptotically anti—de Sitter spaces has been proved. Since anti—de Sitter spaces have zero energies and all other configurations have positive energies, anti—de Sitter spaces are stable. The boundary condition at spatial infinity is crucial in this argument. It has been argued that it is natural to impose such conditions to define a quantum field theory, or in other words a Hilbert space, in asymptotically anti—de Sitter spaces.

In this paper we examine the stability problem without imposing such boundary conditions for various reasons. In the cosmological context the real issue is not the stability of "exact" anti—de Sitter spaces. In the standard big-bang theory the universe is very hot at the very early stage of its evolution, energy-momentum tensors being dominated by finite-temperature ($T\neq 0$) effects. Suppose that the Lagrangian of particle-physics theory contains constant negative energy density V_0. Then, the question is what happens when the universe, as it expands and T drops ($T^4 \ll |V_0|$), enters an approximately anti—de Sitter space? Boundary conditions at spatial infinity must be imposed such that they can be applied at all times without any inconsistency. When energy-momentum tensors are dominated by finite-temperature effects, it seems quite unnatural to impose boundary conditions at spatial infinity which are special to anti—de Sitter spaces. In a spatially open universe, finite temperature, and so finite energy density, would mean an infinite total energy because of infinite total spatial volume. In such circumstances it is perfectly legitimate to ask about the stability of scalar fields ϕ without requiring ϕ to approach some ϕ_0 (one of the minima of $U[\phi]$) at spatial infinity.

We encounter a similar problem when the universe enters an anti—de Sitter phase through tunneling from a de Sitter or Minkowski space. As was shown in Ref. 4, just after materialization of a "bubble" of the anti—de Sitter space a scalar field is not at the bottom (ϕ_0) of a potential, but at some ϕ, close to the bottom. In this context boundary conditions are not at our disposal, but are chosen by dynamics.

Furthermore, it has been recently argued that quantum field theory in anti—de Sitter space, constructed by imposing the boundary conditions at spatial infinity as in Ref. 5, would suffer serious diseases at the one-loop level. Sakai and Tanii showed[6] in a two-dimensional model that (i) nonrenormalizable divergences appear, (ii) the flat-spacetime limit does not reproduce quantum field theory in Minkowski spacetime, and (iii) the one-loop effective action for constant scalar field configurations is not proportional to the spatial volume of anti—de Sitter space. All these diseases are expected to persist in four dimensions.[7] General interacting quantum field theories in anti—de Sitter space have not been satisfactorily defined at the one-loop level.

We investigate the behavior of ϕ without imposing any boundary conditions at spatial infinity. In this paper we are concerned with the stability of constant scalar field configurations against arbitrary fluctuations, and require that $U[\phi]$ defined in (7) have an absolute minimum. In a sense our condition for the stability is sufficient, but not necessary. There is no contradiction between our results and those in Ref 5. The difference lies in the boundary conditions at spatial infinity. We believe that our way of raising the question of stability is more appropriate in the cosmological context.

Let us for the moment neglect the last term in Eq. (6) and investigate the classical stability from the pseudopotential $U[\phi]$. Consider first a positive G. For $\xi\leqq 0$ or $\xi\geqq\frac{1}{6}$, $U[\phi]$, which is regular everywhere, must be such that (A) $\phi U'[\phi]>0$ as $\phi^2\to\infty$. For $0<\xi<\frac{1}{6}$, ${\phi_c}^2>0$ and $U'[\phi]$ has poles at $\phi=\pm\phi_c$. The stability requires, in addition to (A), that (B) the residue of $U'[\phi]$ at $\phi=\pm\phi_c$ be negative or zero. Though $U[\phi]$ has an infinitely high barrier at $\phi=\pm\phi_c$ under the condition (B), the condition (A) also must be imposed to have an absolutely stable ground state. This is because if $\dot\phi(t)$ is sufficiently large, the last term in Eq. (6), which has a positive residue at $\phi=\pm\phi_c$, dominates over $U'[\phi]$ near $\phi=\pm\phi_c$ so that the total effective potential becomes attractive. Indeed, we will see below that there are classical configurations which connect the $\phi^2<{\phi_c}^2$ region with the $\phi^2>{\phi_c}^2$ region.

More explicitly, we consider a potential

$$V[\phi]=V_0+\tfrac{1}{2}m^2\phi^2+\frac{\eta}{3!}\phi^3+\frac{\lambda}{4!}\phi^4\ . \tag{8}$$

Inclusion of a linear term does not change the results very much. Curiously, the most dominant term in U' at $\phi^2\to\infty$ comes from the cubic interaction term:

$$U'[\phi]\sim-\frac{8\pi}{3}G\xi\eta{\phi_c}^2\phi^2 \tag{9}$$

as $\phi^2\to\infty$. The stability condition (A) can be satisfied, only if $\xi\eta=0$. That is, a cubic interaction term induces instability in curved space, unless $\xi=0$.

Now we restrict ourselves to $\eta=0$. Applying the criterion, we find the following conditions for the classical stability:

(a) $G>0$, $\xi\leqq 0$, or $\xi\geqq\frac{1}{6}$,

$$\frac{\lambda}{4\pi}+12\xi Gm^2>0$$

or

$$\frac{\lambda}{4\pi}=-12\xi Gm^2,\ \ m^2+32\pi G\xi V_0>0; \tag{10}$$

(b) $G>0,\ 0<\xi<\frac{1}{6}$,

$$-24\xi[(1-3\xi)Gm^2+16\pi\xi(1-6\xi)G^2V_0] \leqq \frac{\lambda}{4\pi} < -12\xi Gm^2 . \qquad (11)$$

The condition (11) can be met only if $m^2+32\pi G\xi V_0>0$. If $G<0$, we have the condition (10) for $0\leqq\xi\leqq\frac{1}{6}$ and (11) for $\xi<0$ or $\xi>\frac{1}{6}$.

To find the behavior of ϕ near the poles of $U'[\phi]$, we consider a spatially homogeneous but time-dependent configuration $\phi=\phi_c+\chi(t)$ ($|\chi/\phi_c|\ll1$). To the leading order Eq. (6) reads

$$\ddot{\chi}-\frac{K}{\chi}+\frac{\dot{\chi}^2}{2\chi}=0, \qquad (12)$$

where K is a positive constant under the condition (B). Solutions are $\chi(t)=\pm\sqrt{2K}t$ and $\sim\text{const}\times\epsilon(t)\,|t|^{2/3}$, which establishes the statement that there can be classical transitions from the $\phi^2<\phi_c^2$ region to the $\phi^2>\phi_c^2$ region. If $|\dot{\phi}|$ at $\phi=\pm\phi_c$ is larger than $\sqrt{2K}$, the velocity-dependent potential in Eq. (6) overwhelms $U'[\phi]$ so that the effective potential becomes attractive.

The last term in Eq. (6) is harmless for spatially homogeneous large fluctuations $[\phi(t)^2\to\infty]$, too. To the leading order Eq. (6) may be written in terms of $\psi\equiv\phi(t)^2$ as

$$\ddot{\psi}+a\psi-b\frac{\dot{\psi}^2}{\psi^2}=0, \qquad (13)$$

where $2\phi U'[\phi]\sim a\phi^2$ ($a>0$) as $\phi^2\to\infty$ and $b=-\frac{1}{2}\phi_c^2$. The last term in Eq. (13) with $b>0$ can destabilize the theory, if it dominates over $a\psi$ as $\psi\to\infty$. However, it is impossible, since $\ddot{\psi}\sim b\dot{\psi}^2/\psi^2$ means that $\psi=c\exp(-b/\psi)$ so that $b\dot{\psi}^2/\psi^2<a\psi$ as $\psi\to\infty$. For spatially inhomogeneous fluctuations the issue is more subtle and further investigation is necessary.

III. APPLICATIONS

Let us apply our results to special cases.

(i) Higgs scalar fields: $V=(\lambda/4!)(\phi^2-v^2)^2$. We require $V[\phi=v]=0$ to have a Minkowski spacetime solution. In the standard unified theory of electroweak and strong interactions $v^2\ll G^{-1}$. (G^{-1} is renormalized to $G_{\text{obs}}{}^{-1}=G^{-1}-8\pi\xi v^2$, whose correction is negligible for moderate ξ.) The conditions (10) and (11) lead to

(a) $\xi\leqq0$ or $\xi\geqq\frac{1}{6}$,

$$\frac{\lambda}{4\pi}(1-8\pi G\xi v^2)>0, \qquad (14a)$$

(b) $0<\xi<\frac{1}{6}$,

$$\frac{\lambda}{4\pi}(1-8\pi G\xi v^2)<0,$$

$$v^2\geqq\phi_c^2=\frac{1}{8\pi G\xi(1-6\xi)} . \qquad (14b)$$

Equation (14b) is equivalent to

$$\frac{\lambda}{4\pi}>0,\ \ v^2\geqq\phi_c^2. \qquad (14b')$$

For $v^2\ll G_{\text{obs}}{}^{-1}$, (14b) or (14b′) cannot be satisfied. We conclude that $\xi\leqq0$ or $\xi\geqq\frac{1}{6}$ for standard Higgs scalar fields.[8]

(ii) Semistable de Sitter space: $V=\frac{1}{2}m^2\phi^2+(\lambda/4!)\phi^4(m^2>0)$. ϕ is an additional scalar field which might be relevant to cause primodial inflation in the very early universe.[9] $\phi=0$ corresponds to a stable Minkowski space with $G_{\text{obs}}=G>0$.

The conditions (10) and (11) read

(a) $\xi\leqq0$ or $\xi\geqq\frac{1}{6}$,

$$y\equiv\frac{\lambda}{4\pi}\frac{1}{Gm^2}\geqq-12\xi, \qquad (15a)$$

(b) $0<\xi<\frac{1}{6}$,

$$-24\xi(1-3\xi)\leqq y<-12\xi. \qquad (15b)$$

Note that (15b) is satisfied only for negative λ.

The behavior of the pseudopotential $U[\phi]$ for $0<\xi<\frac{1}{6}$ is depicted in Fig. 1(a). There are two types of local minima, $\phi^2=0$ and $\phi^2=\phi_0^2$ where

$$\phi_0^2=-\frac{3}{2\pi G(y+12\xi)}>0. \qquad (16)$$

$\phi^2=\phi_0^2$ corresponds to a de Sitter space with

$$R=-\frac{12m^2}{y+12\xi}>0. \qquad (17)$$

One also finds that

$$U''[\pm\phi_0]=-\frac{2m^2(y+12\xi)}{y+24\xi(1-3\xi)}>0. \qquad (18)$$

Though $\phi_0^2=O(G^{-1})$, both of two important energy scales, R and $U''[\pm\phi_0]$, are of order m^2. The de Sitter space ($\phi^2=\phi_0^2$) is expected to be only semistable, decaying into the Minkowski spacetime ($\phi=0$). The reasons are (a) $U[\pm\phi_0]>U[\phi=0]$, (b) there is a bounce solution to Eqs. (2) and (3) in the Euclidean signature with a finite Euclidean action which describes semiclassical tunneling from the de sitter space to the Minkowski spacetime, and (c) the effective gravitational constant in the de Sitter space,

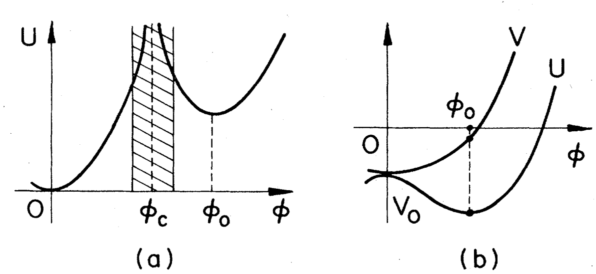

FIG. 1. (a) $U[\phi]$ in the semistable de Sitter-space model ($0<\xi<\frac{1}{6}$). (b) $V[\phi]$ and $U[\phi]$ in the vanishing-cosmological-constant model (case I). In both cases $U[\phi]$ is normalized by $U[0]=V_0$. In the shaded region in (a), quantum gravity effects are expected to be important.

$$G_{\rm DS}{}^{-1}=G^{-1}-8\pi\xi\phi_0{}^2=G^{-1}(y+24\xi)(y+12\xi)^{-1},$$

is negative so that the space is unstable at the quantum level. The model with $0<\xi<\frac{1}{6}$ is interesting in that it admits a semi-stable de Sitter space and a negative λ assures the asymptotic freedom in the Minkowski spacetime.

At the very early stage of the universe (at a Planck time scale) large quantum fluctuations might put the universe in the $\phi^2=\phi_0{}^2$ state.[10] If it so happens, the universe undergoes an exponential expansion for a while, eventually making a transition to the Minkowski spacetime to settle down.

(iii) Vanishing cosmological constant: $V=V_0+\frac{1}{2}m^2\phi^2$. Consider a scalar field which interacts with only gravity. Bare vacuum energy density V_0, which arises from all other particle interactions (electroweak and strong interactions), could give rise to an unacceptably large cosmological constant ($\Lambda_0=8\pi GV_0$), if $G=M_P{}^{-2}$. We propose to investigate the problem by starting with an extremely tiny bare gravitational constant G. Indeed, one will find that under certain conditions nonvanishing expectation value of the scalar field ϕ ($\equiv\phi_0$) reduces the vacuum energy density to an extremely tiny value and increases G to the observed value $G_{\rm obs}=M_P{}^{-2}$, simultaneously. The key observation is that Eq. (2)

$$\phi^{;k}{}_k+(m^2+\xi R)\phi=0 \tag{19}$$

guarantees that in a nontrivial solution ($\phi=\phi_0$ $=$const$\neq 0$), if it exists, $R=-m^2/\xi$ regardless of V_0.

To see how it works, we require that (a) the stability conditions (10) or (11) be satisfied, (b) there be a nontrivial stable solution $\phi=\phi_o\neq 0$, and (c) $G_{\rm obs}{}^{-1}=G^{-1}-8\pi\xi\phi_0{}^2=M_P{}^2>0$. All the conditions are satisfied, if either case I: $G>0$, $\xi\geq\frac{1}{6}$, $m^2>0$

$$V_0<-\frac{m^2}{32\pi G_{\rm obs}\xi}<0 \tag{20a}$$

or case II: $G<0$, $\xi<\xi_c=-0.60$, $m^2<0$,

$$V_0>-\frac{m^2}{32\pi G_{\rm obs}\xi}>0. \tag{20b}$$

ξ_c is given by $\ln 6\,|\,\xi_c\,|=1-(6\xi_c)^{-1}$. In both cases

$$R=-\frac{m^2}{\xi}, \tag{21a}$$

$$\phi_0{}^2=-\frac{1}{16\pi G_{\rm obs}\xi}-\frac{2V_0}{m^2}, \tag{21b}$$

$$\frac{1}{G_{\rm obs}}=\frac{2}{G}-\frac{32\pi V_0}{R}, \tag{21c}$$

$$V[\pm\phi_0]=\frac{R}{32\pi G_{\rm obs}}, \tag{21d}$$

and $U[\pm\phi_0]<U[\phi=0]$.

What is happening in case I is depicted in Fig. 1(b). In spite of positive m^2 the $\phi=0$ state is unstable, since $U''[\phi=0]<0$. The true minimum of U is at $\phi=\pm\phi_0$, where Eq. (17) guarantees (21a).[11] In case II $\phi=0$ is a local minimum, but is not absolutely stable. To have tiny R [$|R|<10^{-64}$ (eV)^2], $|m^2\xi|$ must be tiny. The required inequality for V_0 is, in practice, equivalent to $V_0<0$ ($V_0>0$) in case I (II).

Recently many models have been proposed to dynamically relax the cosmological constant to an extremly small value regardless of its initial value. Banks[12] has proposed a model containing third-rank antisymmetric tensor gauge fields with a nonlocal mass term. Abbott[13] has proposed, instead, a model which involves both a scalar field and non-Abelian gauge fields. Our approach is similar to that of Dolgov,[3,14] in which the ϕ^2R interaction plays a crucial role. In our scenario the effective cosmological constant is automatically reduced, regardless of the initial, large V_0, to an extremely tiny value ($\sim m^2/\xi$). Unfortunately, in view of the naturalness stressed by Abbott, our scenario does not provide a solution to the problem. Equation (21c) implies that G^{-1} must be fine-tuned to cancel $16\pi V_0/R$. In our view the cosmological constant problem is not a problem of how to reduce the effective vacuum energy density, but rather that of how to get a large effective gravitational constant ($G_{\rm obs}\gg|R/V_0|$). Also note that though the resultant theory at $\phi=\phi_0$ contains an almost massless scalar field [$U''(\phi_0)=O(|m^2|)$], it is quite different from the Bran-Dicke scalar field theory. Its observational implications have yet to be investigated.

So far our discussions have been mostly at the classical level. Quantum gravity effects are expected to get important in those regions where Riemann tensors R_{ijkm} are comparable with or larger than G^{-1} (or $G_{\rm obs}{}^{-1}$). A rough estimate may be obtained from R in Eq. (5), or its approximation

$$R\cong\frac{8\pi G(4V-6\xi\phi V')}{1-8\pi G\xi(1-6\xi)\phi^2}. \tag{22}$$

There are potentially two regions where R gets very large: $\phi^2\to\infty$ and $\phi^2\sim\phi_c{}^2$ if $\phi_c{}^2>0$. Quantum gravity effects affect transitions from the de Sitter space to the Minkowski spacetime in the semistable de Sitter model (ii), which involve the passage through the $\phi^2\sim\phi_c{}^2$ region. But many of our results are obtained in those regions where quantum gravity effects are expected to be negligible. In particular, in the vanishing cosmological constant model (iii), case I, the quantity (22) is always small for any ϕ, provided that $|V_0|\ll G_{\rm obs}{}^{-2}$ and $\xi>\frac{1}{6}$.

If there are more than one-scalar fields, general analysis of classical stability becomes extremely difficult. In general pseudopotential $U[\phi_k]$ cannot be defined for such systems. However, it is not difficult to construct a model which contains several scalar fields and has a stable ground state. For instance, one may consider a system consisting of a Higgs scalar field ψ and an almost massless free scalar field ϕ with a potential

$$V[\psi,\phi]=V_0+\frac{\lambda}{4!}(\psi^2-v^2)^2+\tfrac{1}{2}m^2\phi^2.$$

It can be shown by analyzing equations of motion that if $G>0$, $\xi_\phi>\frac{1}{6}$, $\xi_\psi<0$, $\lambda>0$, $m^2>0$, and $8\pi G\xi_\psi v<1$, the model has a stable ground state. Furthermore, if $-V_0/m^2>(32\pi G_{\rm obs})^{-1}\gg v^2\gg m^2$, the model has a nontrivial minimum at $\psi^2\sim v^2$ and $\phi^2\sim\phi_0{}^2$ in Eq. (18b), where $R=-m^2/\xi_\phi$.

IV. SUMMARY AND DISCUSSIONS

In this paper we have examined the classical stability of constant scalar field configurations in curved space against arbitrary (time-dependent) fluctuations, to find that the back-reaction of gravity greatly modifies the behavior of scalar fields. As by-products we found that the parameter ξ is not arbitrary, that cubic interactions cause instability, that negative quartic interactions can make sense, and that the cosmological constant can be reduced to an extremely small value regardless of an initial value, provided that there exists a free scalar field with a tiny mass. Our analysis is mostly at the classical level. If quantum corrections are included, the parameter ξ, for instance, is not constant, but becomes a running coupling constant, which might give rise to new kinds of phase transitions by failing to satisfy the stability conditions above or below a critical energy scale. Furthermore, the criterion for the stability of quantum field theories in curved space, particularly in the time-dependent cosmological context, is far from obvious. It depends on boundary conditions at spatial infinity. In the cosmological context boundary conditions at spatial infinity cannot be arbitrarily imposed, but must be consistent with an initial state which the universe starts with. Also it might be mentioned that a "flat" spacetime at finite temperature is not absolutely stable against formation of black holes, though its rate is totally negligible in realistic situations.[15]

We have mainly discussed the behavior of a scalar field ϕ by restricting ϕ to be spatially homogeneous, but allowing it to be time-dependent. The spatially homogeneous mode $\phi_{\rm cl}(t)$ may be viewed as a classical part of $\phi(x,t)$ in a spatially homogeneous universe. In quantum theory, only deviations of $\phi(x,t)$ from $\phi_{\rm cl}(t)$ are to be quantized, though $\phi_{\rm cl}(t)$ itself is subject to quantum corrections. A Hilbert space for $\phi(x,t)$ depends on $\phi_{\rm cl}(t)$.

We have addressed the question if there is an absolutely stable ground state in which $R_{jk} \propto g_{jk}$ and $\phi = \text{const}$. In the cosmological context R_{jk} need not be exactly proportional to g_{jk} and ϕ need not be time independent. There can be other kinds of cosmologically stable configurations.

Certainly, we need better understanding of field theories in curved space.

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