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Renormalization of the axial-vector current in QCD

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Following the method of Ioffe and Smilga, the propagation of the baryon current in an external constant axial-vector field is considered. The close similarity of the operator-product expansion with and without an external field is shown to arise from the chiral invariance of gauge interactions in perturbation theory. Several sum rules corresponding to various invariants both for the nucleon and the hyperons are derived. The analysis of the sum rules is carried out by two independent methods, one called the ratio method and the other called the continuum method, paying special attention to the nondiagonal transitions induced by the external field between the ground state and excited states. Up to operators of dimension six, two new external-field-induced vacuum expectation values enter the calculations. Previous work determining these expectation values from PCAC (partial conservation of axial-vector current) are utilized. Our determination from the sum rules of the nucleon axial-vector renormalization constant G_A, as well as the Cabibbo coupling constants in the SU_3-symmetric limit ($m_s=0$), is in reasonable accord with the experimental values. Uncertainties in the analysis are pointed out. The case of broken flavor SU_3 symmetry is also considered. While in the ratio method, the results are stable for variation of the fiducial interval of the Borel mass parameter over which the left-hand side and the right-hand side of the sum rules are matched, in the continuum method the results are less stable. Another set of sum rules determines the value of the linear combination $7F-5D$ to be ≈ 0, or $D/(F+D)\approx \frac{7}{12}$.

I. INTRODUCTION

In the past few years the sum-rule method in quantum chromodynamics (QCD) has emerged as a major tool for computing the masses and coupling constants of low-lying hadron states.[1] The procedure, while approximate, has met with a considerable amount of success. In particular, in a series of papers,[2–4] Ioffe and his collaborators have successfully computed the masses of the nucleon and its octet partners as well as the isobar and the decimet members. In a later extension of this work, Ioffe and Smilga[5] and independently Balitsky and Yung[6] considered the correlation function of the baryon current in an external electromagnetic field $F_{\mu\nu}$. By computing the term linear in $F_{\mu\nu}$ in the current correlation function, they were able to calculate the magnetic moments of the proton and neutron to within 10% accuracy.

In this present work we follow the ideas of Ioffe and Smilga and consider the propagation of the baryon current, i.e., of the nucleon and the hyperons, in an external axial-vector field Z_μ, and compute the terms proportional to Z_μ. In this way we are able to determine the axial-vector coupling constants of the nucleon and hyperons. We find that the theoretical determination of the axial-vector renormalization constants are in reasonable agreement with experiment for hyperons as well as the nucleon, provided the vacuum expectation value of the chiral-symmetry-breaking parameter is about 20% smaller than the value usually assumed,

$$\langle \bar{q}q\rangle = -(0.25\ \text{GeV})^3 .$$

It is known that if the axial-vector current is conserved exactly like the vector current and the physical vacuum is invariant under chiral $SU_3\times SU_3$ symmetry, then the axial-vector coupling to the baryon octet will be of the pure D type and the F-type coupling will be zero, as follows from SU_3 symmetry and charge-conjugation invariance. Furthermore, there will be parity doubling in the baryon spectrum. The lack of conservation of the axial-vector current is attributed to the breaking of chiral invariance by the physical vacuum and is essentially a low-energy phenomenon. Therefore the sum-rule calculation should reveal that the F-type coupling should tend to zero and D-type coupling tend to unity in the chiral-symmetric limit. We shall see that QCD sum-rule approach brings out these basic features of hadron physics very clearly.

The QCD sum-rule method is basically the following. To compute the properties of a given hadron, one chooses a current which has a nonzero matrix element between the physical vacuum and the hadron in question. For the case of the proton we shall use the current[2,7]

$$\eta(x)=[u^a(x)C\gamma_\mu u^b(x)]\gamma_\mu\gamma^5 d^c(x)\epsilon^{abc} . \quad (1.1)$$

One then computes the correlation function

$$\pi(p^2)=i\int d^4x\, e^{ip\cdot x}\langle 0\,|\,T(\eta(x)\bar{\eta}(0))\,|\,0\rangle \tag{1.2}$$

which satisfies a dispersion relation of the form

$$\pi(p^2)=\frac{1}{\pi}\int \frac{\mathrm{Im}\pi(p'^2)}{p'^2-p^2-i\epsilon}dp'^2 \,. \tag{1.3}$$

For large p^2, that is in the limit $x\to 0$, the product $\eta(x)\bar{\eta}(0)$ can be computed in terms of the quark and gluon degrees of freedom via the operator-product expansion (OPE). This in turn leads to an expansion of $\pi(p^2)$ in terms of the various vacuum correlation functions, such as the chiral-symmetry-breaking parameter $\langle \bar{q}q\rangle$. On the other hand, using a dispersion relation, the correlation $\pi(p^2)$ can be computed as an integral over the absorptive part, that is to say, in terms of the nucleon and excited states which have the same quantum number as the nucleon, apart from parity. By matching the Borel transforms of these two calculations, one in terms of the operator-product expansion and the other in terms of the physical intermediate states, over a range of values of the Borel mass parameter in the region of the nucleon mass, one is able to deduce self-consistently the proton mass and the coupling strength of the current η to the one-proton state.

In computing the OPE for the baryon-current correlation function in an external field Z_μ, one encounters terms of two types: (a) Z_μ interacting with a hard quark, i.e., a quark which carries a substantial part of the current momentum p appearing in Eq. (1.2). (b) Terms which correspond to modification of the quark and gluon vacuum correlation functions by the external field.[5,6] These latter terms are analogous to the susceptibility terms one introduces in the discussion of dielectric and magnetic substances.

In our calculation, the fact that all gauge interactions in perturbation theory are chirality preserving (SU_3 color gauge interaction, as well as the usual weak interactions) leads to the fact that the terms of the type (a), i.e., the hard-quark terms, can be simply related to the propagation function in the absence of the external field Z_μ. In this context the excellence of the choice of the baryon current, Eq. (1.1), made by Ioffe manifests itself very clearly, as will be discussed in detail in Sec. II.

For deriving the sum rules, it is simpler to calculate the correlation function in configuration space first. It is easy to see that the most general form to first order in the external field is

$$\langle 0\,|\,T(\eta(x)\bar{\eta}(0))\,|\,0\rangle = f(x^2)\hat{x}+g(x^2)1+f_1(x^2)x\cdot Z\,\hat{x}\gamma_5+f_2(x^2)\hat{Z}\gamma_5+g_1(x^2)x\cdot Z\,\gamma_5+g_2(x^2)\sigma^{\alpha\beta}Z_\alpha x_\beta\gamma_5 \,. \tag{1.4}$$

Here and in the following for a four-vector A_μ, $\hat{A}=A_\mu\gamma^\mu$. The structures f_1, f_2, and f are chiral odd, while the rest are chiral even. In momentum space the correlator takes the general form

$$\pi(p^2)=F(p^2)\hat{p}+G(p^2)1+F_1(p^2)p\cdot Z\,\hat{p}\gamma_5 +F_2(p^2)\hat{Z}\gamma_5+G_1(p^2)x\cdot p\,\gamma_5+G_2(p^2)\sigma_{\alpha\beta}Z^\alpha p^\beta\gamma_5 \,. \tag{1.5}$$

It will be shown in detail in Sec. II that the leading terms in p^2 for the coefficients $F_1(p^2)$, $F_2(p^2)$ via the OPE are identical to the leading terms which are obtained for the coefficient $F(p^2)$ occurring in the mass sum rule written down by Ioffe.[2,3]

The current correlation function $\pi(p^2)$ in the absence of the external field corresponds to the creation of a state which has the quantum numbers identical to the nucleon apart from parity by the current $\bar{\eta}(0)$ and its subsequent annihilation by $\eta(x)$. In the presence of the external field, as pointed out by Ioffe and Smilga,[5] one should take into account the fact that the intermediate nucleon state can make transitions to excited states under the influence of Z_μ. These nondiagonal terms should also be kept in the computation of the current correlation functions in terms of the physical intermediate states and are *a priori* of arbitrary and unknown strength. As will be discussed in detail in Sec. III, these nondiagonal terms interfere differently in the sum rules corresponding to the coefficient $F_1(p^2)$ and $F_2(p^2)$ for the structures $p\cdot Z\,\hat{p}\gamma_5$ and $\hat{Z}\gamma_5$, respectively. In particular, states of opposite parity add destructively in the $p\cdot Z\,\hat{p}\gamma_5$ sum rule, while in the $\hat{Z}\gamma_5$ sum rule they add constructively. As a result, in the latter sum rule, the nondiagonal terms are quite significant.

In Sec. IV we extend the calculations to hyperons. Bearing in mind that if chiral symmetry were exact the Cabibbo coupling strengths would be $F=0$ and $D=1$, we have chosen the transitions $\Sigma\to\Sigma$ and $\Sigma\to\Lambda$ which involve, respectively, only pure F and D couplings. In addition the sum rule for the $\Xi\to\Xi$ vertex which involves the difference $D-F$ is also written down.

We have analyzed the sum rules by two independent methods. One of them we call the ratio method; it does not require an explicit knowledge of the coupling strength $\tilde{\beta}_N$ of the current $\eta(x)$ to the nucleon state and is described in detail in Sec. V. It depends on utilizing the ratio of the sum rules for the correlation functions with and without the external field Z_μ. This procedure is quite stable as a function of the Borel mass parameter.

In the other method we follow the procedure of Ioffe and sum over the excited contributions in the right-hand side of the sum rules using asymptotic freedom. This procedure requires an explicit determination of the coupling strength $\tilde{\beta}_N$ and is determined from the mass sum rule. Although this latter method is more sensitive to the precise value for the Borel mass variable over which the left-hand side and the right-hand side of the sum rule are matched, we find that the results are compatible with the ratio method.

We find that the value of the axial-vector coupling constant for the baryon octet depends quite sensitively on the vacuum expectation value of the chiral-symmetry-breaking correlation

$$a=-(2\pi)^2\langle 0\,|\,\bar{q}q\,|\,0\rangle \,. \tag{1.6}$$

A value of $a \approx 0.45$ GeV^3 is preferred by our sum rules rather than the value of $a \approx 0.55$ GeV^3 which is more commonly used.[8] This confirms Belyaev and Ioffe's observation[3] that the value $a=0.55$ GeV^3 is an overestimate and a reduction of about 20% in its value should bring their determination of the baryon masses into closer agreement with experiment.

As discussed in detail in Sec. V, the determination of the precise values of the renormalization constants is subject to some uncertainties. On the other hand, choosing the same parameter for the Borel mass variable, as is done in earlier calculations of the nucleon mass, the G_A value does come out close to the experimental result. The Cabibbo couplings D and F in the SU_3-symmetric limit are also not far from the experimental data. In the broken-SU_3-symmetry case, the results depend significantly on the methods of analyzing the sum rules.

We have summarized in the Appendix for the reader's convenience a collection of Fourier transforms and Borel transforms needed in the text.

During the course of this work, we became aware of the work of Koniuk and Tarrach[9] and Belyaev and Kogan,[10] who have discussed some of the problems to which this paper is addressed. We disagree with Belyaev and Kogan on several points. More importantly we believe that their method of analyzing the sum rules which involves subtracting two different sum rules may not be *a priori* reliable and moreover they used an unrealistic value of the coupling strength $\widetilde{\beta}_N$ in their calculation. On the other hand, in our analysis we use two independent methods, one of which involves no explicit knowledge of $\widetilde{\beta}_N$, and the other which uses $\widetilde{\beta}_N$ as determined by the mass sum rule. The consistency between these two methods assures us that our $\widetilde{\beta}_N$ is indeed close to its true value. Further in our analysis of the sum rules, we do not restrict the value of the external-field-induced correlation $\langle 0 | \bar{q}\widetilde{G}_{\mu\nu}\gamma^\nu q | 0\rangle$ to that given by the analysis of Novikov *et al.*[11] and regard it as a parameter.

II. DERIVATION OF THE SUM RULES: CONFIGURATION-SPACE RESULTS

We introduce an external axial-vector field Z_μ (not to be confused with the intermediate boson Z^0), whose interaction with the quark field q is written as

$$\mathscr{L}_I=\bar{q}i\hat{D}q, \quad \text{with} \quad \hat{D}=\gamma^\mu(\nabla_\mu+ig_qZ_\mu\gamma_5)\,, \qquad \nabla_\mu=\partial_\mu+ig_cA_\mu^n\frac{\lambda^n}{2}\,, \tag{2.1}$$

g_c is the QCD gauge coupling constant, and λ^n are the Gell-Mann matrices. The value of the weak coupling constant g_q depends on the quark type as well as the field Z_μ. If, for instance, Z_μ is the third component of an isovector, then $g_u=-g_d$ and $g_s=0$, with corresponding assignments for other SU_3-flavor currents.

Following Ioffe[2,7] we take the nucleon current to be

$$\eta(x)=\epsilon^{abc}[u^a_{(x)}C\gamma_\mu u^b_{(x)}]\gamma^5\gamma_\mu d^c(x)\,, \tag{2.2}$$

$$=4[(u_R^aCd_R^b)u_L^c-(u_L^aCd_L^b)u_R^c]\epsilon^{abc}\,, \tag{2.3}$$

where a,b,c denote the color indices, C is the charge-conjugation matrix and the right- and left-handed projections of the quark field $q(x)$ are

$$q_R(x)=\frac{1-\gamma_5}{2}q(x)\,,$$

$$q_L(x)=\frac{1+\gamma_5}{2}q(x)\,.$$

Equation (2.3) is obtained from Eq. (2.2) by a Fierz rearrangement. The adjoint nucleon current can be written as

$$\bar{\eta}(y)=\epsilon^{a'b'c'}[\bar{u}^{b'}(y)\gamma_\nu C\bar{u}^{a'}(y)]\bar{d}^{c'}(y)\gamma_5\gamma^\nu\,, \tag{2.4}$$

$$=4\epsilon^{a'b'c'}[(\bar{d}_R^{b'}C\bar{u}_R^{a'})\bar{u}_L^{c'}-(\bar{d}_L^{b'}C\bar{u}_L^{a'})\bar{u}_R^{c'}] \tag{2.5}$$

We are interested in the correlation function

$$\pi(p)=i\int d^4x\langle 0|T(\eta(x)\bar{\eta}(0))|0\rangle e^{ip\cdot x}\,. \tag{2.6}$$

We shall find it advantageous to use the form for the current in the helicity representation given by Eqs. (2.3) and (2.5), respectively. Since the u and d quark masses can be considered zero for our purpose, the splitting of the field into left-handed and right-handed pieces is very useful. Furthermore, both the SU_3-color gauge interactions and weak interactions in perturbation theory leave the chirality of the fermion unchanged. The product $T(\eta(x)\bar{\eta}(0))$ consists of terms of the type

$$T(\eta(x)\bar{\eta}(0))=O_R(x)+O_L(x)-E_1(x)-E_2(x)\,, \tag{2.7}$$

where

$$O_R(x)=T(u_R^a(x)Cd_R^b(x)u_L^c(x),\bar{d}_R^{b'}C\bar{u}_R^{a'}\bar{u}_L^{c'})16\epsilon^{abc}\epsilon^{a'b'c'}\,, \tag{2.8}$$

$$O_L(x)=T(u_L^a(x)Cd_L^b(x)u_R^c(x),\bar{d}_L^{b'}C\bar{u}_L^{a'}\bar{u}_R^{c'})16\epsilon^{abc}\epsilon^{a'b'c'}\,, \tag{2.9}$$

$$E_1(x)=T(u_R^a(x)Cd_R^b(x)u_L^c(x),\bar{d}_L^{b'}C\bar{u}_L^{a'}\bar{u}_L^{c'})16\epsilon^{abc}\epsilon^{a'b'c'}\,, \tag{2.10}$$

and

$$E_2(x)=T(u_L^a(x)Cd_L^b(x)u_R^c(x),\bar{d}_R^{b'}C\bar{u}_R^{a'}\bar{u}_L^{c'})16\epsilon^{abc}\epsilon^{a'b'c'}\,. \tag{2.11}$$

We shall calculate the OPE expansion for these operators using the procedure of Ioffe and Smilga.[5] In essence, it consists of using standard perturbation theory but with provision being made for the fact that normal products like $:\bar{q}q:$, which by definition in the perturbative vacuum have zero expectation value, acquire a nonzero expectation value in the physical vacuum.

Returning to Eq. (2.8) it is evident that in the product $O_R(x)$, the d quark remains right-handed in its propagation from the space-time point 0 to x, while the two u quarks either simultaneously retain their chirality or change their chirality. Thus the terms $O_R(x)$ and $O_L(x)$ contribute only to the chiral-odd invariants $f(x^2)$, $f_1(x^2)$, $f_2(x^2)$ in Eq. (1.5). Similarly the products $E_1(x)$ and $E_2(x)$contribute only to the chiral-even invariants.

Following Ref. 5, the quark propagation function in the presence of the external field Z_μ can be written as

$$\langle 0|\,T(q_i^a,\bar q_k^b)\,|0\rangle=\frac{i\delta^{ab}}{2\pi^2}\frac{(\hat x)_{ik}}{x^4}-\frac{\delta^{ab}}{2\pi^2}g_q\frac{x\cdot Z(\hat x)_{ik}}{x^4}\gamma_5+\frac{i}{32\pi^2}g_c\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\frac{(\hat x\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat x)_{ik}}{x^2}$$
$$-\tfrac{1}{12}\delta^{ab}\delta_{ik}\langle 0|\,\bar q q\,|0\rangle+\tfrac{1}{12}g_q\chi\delta^{ab}(\hat Z\gamma_5)_{ik}\langle 0|\,\bar q q\,|0\rangle+\tfrac{1}{36}g_q x^\alpha Z^\beta\sigma_{\alpha\beta}\gamma_5\langle 0|\,\bar q q\,|0\rangle$$
$$+\frac{\delta^{ab}}{192}\delta_{ik}x^2\langle 0|\,\bar q\sigma\cdot Gqg_c\,|0\rangle+\frac{\delta^{ab}g_q\kappa\langle\bar q q\rangle}{72}[(\tfrac{5}{2}x^2\hat Z-x\cdot Z\,\hat x)\gamma_5]_{ik}+\text{higher-order terms}\,. \tag{2.12}$$

The first three terms correspond, respectively, to the free propagation function for a massless quark, the propagation with interaction with the external Z_μ field and the vacuum gluon field $G^n_{\mu\nu}$, where g_q is the coupling constant defined in Eq. (2.1) and g_c is the SU(3)-color gauge coupling constant and λ^n are the SU(3) Gell-Mann matrices. The fourth term represents the breaking of chiral symmetry by the physical vacuum. The fifth term arises from the fact that in the presence of the external field Z_μ, Lorentz invariance of the vacuum is broken and $\langle\bar q\gamma_\mu\gamma_5 q\rangle\neq 0$. We have defined a susceptibility χ by

$$\langle 0|\,\bar q\gamma_\mu\gamma_5 q\,|0\rangle=g_q\chi Z_\mu\langle 0|\,\bar q q\,|0\rangle\,. \tag{2.13}$$

The sixth, seventh, and eighth terms arise on expanding the vacuum correlation $\langle 0|\,q(x)\bar q(0)\,|0\rangle$ in a Taylor series around $x=0$. Thus

$$\langle 0|\,q_i^a(x)\bar q_k^b(0)\,|0\rangle=\langle 0|\,q_i^a(0)\bar q_k^b(0)\,|0\rangle+x_\mu\langle 0|\,(\nabla^\mu q_i^a)\bar q_k^b\,|0\rangle+\frac{x_\mu x_\nu}{2!}\langle 0|\,(\nabla^\mu\nabla^\nu q_i^a)\bar q_k^b\,|0\rangle+\cdots\,, \tag{2.14}$$

where we have followed the conventional approach and used fixed-point gauge, $x^\mu A_\mu=0$, so that covariant derivatives are equivalent to ordinary derivatives.

Using the equation of motion $(\hat\nabla+ig_q\hat Z\gamma_5)q=0$, one can write

$$x_\mu\langle(\nabla^\mu q_i^a)\bar q_k^b\rangle=\delta^{ab}\frac{g_q}{36}x^\mu Z^\nu\sigma_{\mu\nu}\gamma_5\langle\bar q q\rangle\,, \tag{2.15}$$

and

$$\tfrac{1}{2}x_\mu x_\nu\langle(\nabla^\mu\nabla^\nu q_i^a)\bar q_k^b\rangle=\delta^{ab}\left[\frac{x^2\delta_{ik}}{192}\langle\bar q g_c\sigma\cdot G q\rangle+\frac{g_q\kappa\langle\bar q q\rangle}{72}[(\tfrac{5}{2}x^2\hat Z-x\cdot Z\,\hat x)\gamma_5]_{ik}\right]. \tag{2.16}$$

The external-field-induced susceptibility κ appearing in (2.16) is defined by

$$\langle\bar q\,\tilde G_{\mu\nu}\gamma_\nu q\rangle=Z_\mu\kappa\langle 0|\,\bar q q\,|0\rangle\,, \tag{2.17}$$

where

$$\tilde G_{\mu\nu}=\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}\,,$$

and

$$G^{\alpha\beta}=G^{n\alpha\beta}\frac{\lambda^n}{2}\,.$$

We note that in Eq. (2.12) the first three terms are odd in chirality and are the "hard-quark" terms mentioned earlier. The terminology "hard" reflects the fact that in momentum space these correspond to the quark carrying large momentum. The soft-quark terms, i.e., the chiral condensate and its modification thereof by the external field, are all even in chirality with the exception of $\bar q\gamma_\mu\gamma_5 q$ and $\bar q\,\tilde G_{\mu\nu}\gamma^\nu q$ which are odd.

It is now straightforward to compute the operator-product expansion in Eqs. (2.8)–(2.11). As an illustration, consider the coefficient of the identity operator which arises from the first two terms in the propagator in Eq. (2.12). Using chiral projections, these two terms can be written as

$$\frac{i\delta^{ab}}{2\pi^2}\frac{1\pm\gamma_5}{2}\hat x\frac{1\mp\gamma_5}{2}(1\pm ig_q x\cdot Z)\,, \tag{2.18}$$

which simply reflects the fact that the left-handed and the right-handed quarks have opposite couplings to the external field and, more importantly, the effect of the interaction with the external field appears simply as a multiplicative factor in the hard-quark term. To comprehend closely the nature of the similarity between the OPE with and without an external field, it is useful to keep both the terms independent of Z_μ as well as terms linear in Z_μ.

We list below the contribution in perturbation theory to the Wilson coefficients[12] in the OPE Eq. (2.7) to Eq. (2.11). The coefficients of the identity operator and of Z_μ are given by Figs. 1(a) and 1(b), respectively:

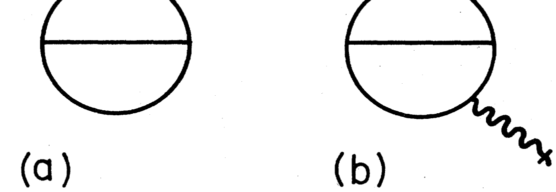


FIG. 1. Diagrams for Eq. (2.19).

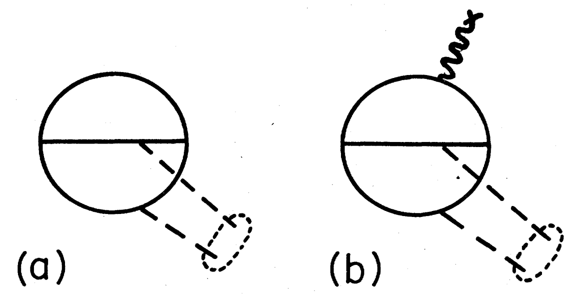


FIG. 2. Diagrams for Eq. (2.20).

$$\text{Fig. }1=+\frac{24i}{\pi^6}\frac{\hat{x}}{x^{10}}(1+ig_d x\cdot Z\gamma_5)\ . \tag{2.19}$$

Notice that there is no contribution from the interaction of the Z_μ field with the u quark.

The coefficient of $G_n^{\alpha\beta}G^n_{\alpha\beta}$ and $Z_\mu G_n^{\alpha\beta}G^n_{\alpha\beta}$ are given by Figs. 2(a) and 2(b).

$$\text{Fig. }2=+\frac{i}{32\pi^6}\frac{\hat{x}}{x^6}(1-ig_u x\cdot Z\gamma_5)\langle g_c{}^2G^2\rangle\ , \tag{2.20}$$

where

$$\langle g_c{}^2G^2\rangle=\langle g_c{}^2G_n^{\alpha\beta}G^n_{\alpha\beta}\rangle\ . \tag{2.21}$$

The contribution from the diagrams in which the vacuum gluons interact with different u lines are zero,[13] both in the sum rule for the mass as well as for the external field case. Therefore there is no dependence on g_d in Eq. (2.20). One can check this readily in the helicity representation, Eq. (2.8).

We assume factorization[12] for the four-quark correlation function, which receives contributions both from the hard-quark term in the first three terms in the propagator Eq. (2.12) as well as the soft-quark terms. The hard-quark contribution to the $(\bar{q}q)^2$ are given by Figs. 3(a) and 3(b):

$$\text{Fig. }3=-\frac{i}{3\pi^2}\frac{\hat{x}}{x^4}(1+ig_d x\cdot Z\gamma_5)\langle\bar{q}q\rangle^2\ . \tag{2.22}$$

For the odd chiral structures $x\cdot Z\,\hat{x}\gamma_5$ and $\hat{Z}\gamma_5$, these are the only diagrams containing hard quarks which appear in the coefficient of operators of dimensions $d\le 7$. Notice that in all the cases above, the proportionality factor between the hard-quark terms and the corresponding terms in the absence of the external field is evident as promised in the Introduction. Next we turn to the soft-quark terms.

The coefficient of $\bar{q}\gamma_\mu\gamma_5 q$ is given by Fig. 4:

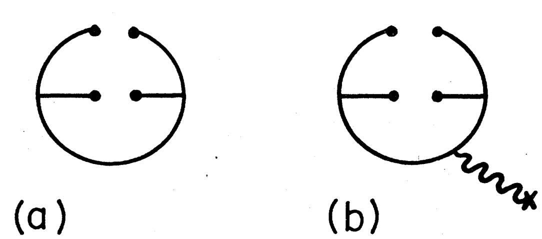


FIG. 3. Diagrams for Eq. (2.22).

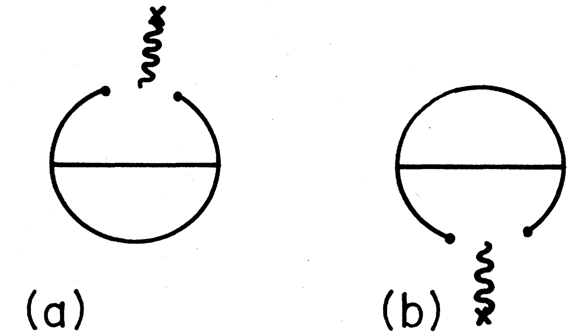


FIG. 4. Diagrams for Eq. (2.23).

$$\text{Fig. }4=+\frac{4}{\pi^4x^8}[(g_u+g_d)Z\cdot x\,\hat{x}\gamma_5-g_u x^2\hat{Z}\gamma_5]\chi\langle\bar{q}q\rangle\ . \tag{2.23}$$

Notice that for an isovector field, $g_u=-g_d$ and therefore the coefficient of $Z\cdot x\,\hat{x}\gamma_5$ vanishes.

The coefficient of $\bar{q}\,\tilde{G}_{\mu\nu}\gamma_\nu q$ is given by Figs. 5 and 6:

$$\text{Fig. }5=-\frac{1}{\pi^4}\left[(\tfrac{5}{3}g_u+g_d)\frac{x\cdot Z\,\hat{x}}{x^6}\gamma_5-\tfrac{5}{3}g_u\frac{\hat{Z}}{x^4}\gamma_5\right]\kappa\langle\bar{q}q\rangle\ , \tag{2.24}$$

$$\text{Fig. }6=+\frac{1}{\pi^4}(g_u+g_d)\left[\frac{x\cdot Z\,\hat{x}}{x^6}\gamma_5+\frac{1}{2}\frac{\hat{Z}\gamma_5}{x^4}\right]\kappa\langle\bar{q}q\rangle\ . \tag{2.25}$$

Combining the expressions of Figs. 5 and 6, we get

$$\text{Fig. }5+\text{Fig. }6=-\frac{1}{\pi^4}\left[g_u\left[\frac{2}{3}\cdot\frac{x\cdot Z\,\hat{x}\gamma_5}{x^6}-\frac{13}{6}\frac{\hat{Z}\gamma_5}{x^4}\right]+g_d\left[-\frac{\hat{Z}\gamma_5}{2x^4}\right]\right]\kappa\langle\bar{q}q\rangle\ . \tag{2.26}$$

Notice that the coefficient of $x\cdot Z\,\hat{x}\gamma_5$ is independent of g_d.

Next consider the soft-quark contributions to the coefficient of $Z_\mu\langle 0|\bar{q}q|0\rangle^2$. This arises from the seventh term in the propagator expansion given in Eq. (2.12) and

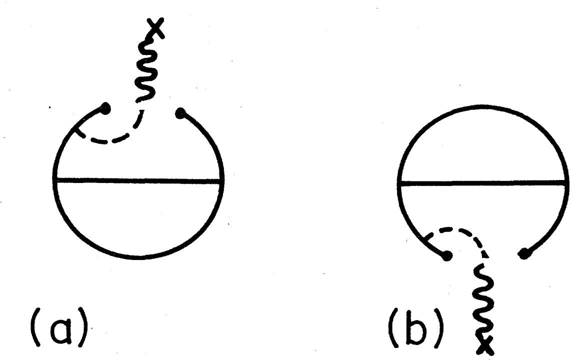


FIG. 5. Diagrams for Eq. (2.24).

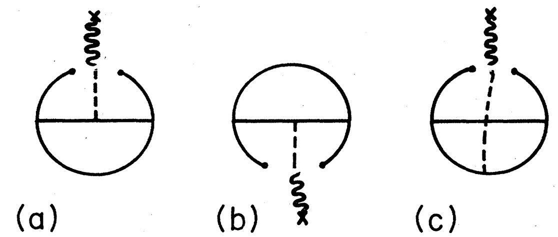


FIG. 6. Diagrams for Eq. (2.25).

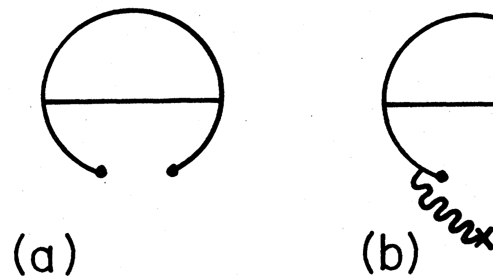


FIG. 8. Diagrams for (a) Eq. (2.28) and (b) Eq. (2.29).

is given by Fig. 7:

$$\text{Fig. } 7=-\frac{2}{9\pi^2}g_u\left[\frac{x\cdot Z\hat{x}}{x^4}-\frac{\hat{Z}}{x^2}\right]\gamma_5\langle\bar{q}q\rangle^2 . \tag{2.27}$$

So far all the terms considered have odd chirality. Next we consider even chirality terms.

The leading terms in the OPE are the operators $\bar{q}q$ and $Z_\mu\bar{q}q$ for the external-field-independent and the external-field-dependent cases, respectively. These terms are given by Figs. 8(a) and 8(b) and are computed using the fourth and the sixth terms in the propagator expansion Eq. (2.12):

$$\text{Fig. } 8(\text{a})=-\frac{2^5}{(2\pi)^4x^6}\langle\bar{q}q\rangle , \tag{2.28}$$

$$\text{Fig. } 8(\text{b})=-\frac{2^5g_d}{3(2\pi)^4}\sigma_{\alpha\beta}\frac{x^\beta Z^\alpha}{x^6}\langle\bar{q}q\rangle . \tag{2.29}$$

Note that unlike the odd chiral structures, the leading contribution to the even chiral case arises from the external field interacting with a soft quark and not a hard quark.

The coefficient of $\bar{q}q\bar{q}\gamma_\mu\gamma_5q$ is given by Fig. 9:

$$\text{Fig. } 9=+\frac{2g_u}{3\pi^2}\sigma_{\alpha\beta}\frac{Z^\alpha x^\beta}{x^4}\langle\bar{q}q\rangle^2\chi . \tag{2.30}$$

Finally the coefficient of $\bar{q}\gamma_\mu\gamma_5q\,\bar{q}\sigma\cdot Gq$ is given by Fig. 10:

$$\text{Fig. } 10=-\frac{1}{48\pi^2}g_u\chi VV_c\left[\frac{\sigma_{\alpha\beta}x^\beta}{x^2}\right]$$
$$=-\frac{1}{48\pi^2}g_u\frac{\sigma_{\alpha\beta}x^\beta}{x^2}\chi\langle\bar{q}q\rangle\langle\bar{q}\sigma\cdot Gq\rangle . \tag{2.31}$$

To compute the contribution to $\pi(p)$ from these various Wilson coefficients, we need to compute integrals like

$$[\cdots]_n\equiv\int d^4x\frac{e^{ip\cdot x}(\cdots)}{(x^2-i\epsilon)^n} ,$$

with (2.32)

$$(\cdots)=1,\ x_\alpha,\ \text{or } x_\alpha x_\beta ,$$

where ϵ is a positive infinitesimal quantity corresponding to the usual Feynman boundary condition and they are listed in the Appendix for the readers' convenience. Using these, the coefficients of the structures $p\cdot Z\hat{p}\gamma_5$, $\hat{Z}\gamma_5$, and $i\sigma_{\alpha\beta}Z^\alpha p^\beta\gamma_5$ can be written and are presented in Table I.

We next carry out the Borel transformation as follows. Remembering that the OPE coefficients are calculated for spacelike values of $P^2=-p^2$,

$$B[f(p^2)]=f_B(M^2)$$
$$=\left[\lim_{\substack{n\to\infty\\ P^2\to\infty\\ P^2/n=M^2\text{ fixed}}}\frac{(P^2)^{n+1}}{n!}\left[-\frac{d}{dP^2}\right]^nf(P^2)\right] . \tag{2.33}$$

The Borel transforms needed for our purpose are listed in the Appendix. Collecting all the results, the Borel transforms of the correlation $\pi(p)$ corresponding to the structures $p\cdot Z\hat{p}\gamma_5$, $\hat{Z}\gamma_5$, and $i\sigma_{\alpha\beta}Z^\alpha p^\beta\gamma_5$ are given by the following. For $p\cdot Z\hat{p}\gamma_5$,

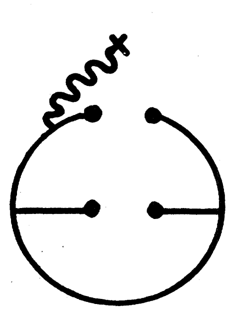


FIG. 7. Diagram for Eq. (2.27).

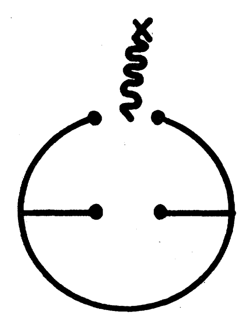


FIG. 9. Diagram for Eq. (2.30).

TABLE I. Wilson coefficients (a) with no external field and (b) with external field, in momentum space. Apart from a multiplicative factor $i/(2\pi)^4$.

(a)

	$\hat{p}$	1
Fig. 1(a)	$-\frac{1}{4}p^4\ln(-p^2)$	
Fig. 2(a)	$-\frac{1}{8}\langle g_c{}^2G^2\rangle\ln(-p^2)$	
Fig. 3(a)	$-\frac{2}{3}a^2\frac{1}{p^2}$	
Fig. 8(a)		$-ap^2\ln(-p^2)$

(b)

	$p\cdot Z\hat{p}\gamma_5$	$\hat{Z}\gamma_5$	$i\sigma_{\alpha\beta}Z^{\alpha}p^{\beta}\gamma_5$
Fig. 1(b)	$g_dp^2\ln(-p^2)$	$\frac{1}{4}g_dp^4\ln(-p^2)$	
Fig. 2(b)	$-\frac{1}{4}g_u\langle g_c{}^2G^2\rangle\frac{1}{p^2}$	$-\frac{1}{8}g_u\langle g_c{}^2G^2\rangle\ln(-p^2)$	
Fig. 3(b)	$-\frac{4}{3}g_da^2\frac{1}{p^4}$	$\frac{2}{3}g_da^2\frac{1}{p^2}$	
Fig. 4	$-\frac{2}{3}(g_d+g_u)\chi a\ln(-p^2)$	$-\frac{1}{3}(g_d-5g_u)\chi ap^2\ln(-p^2)$	
Fig. 5	$-(2g_d+\frac{10}{3}g_u)\kappa a\frac{1}{p^2}$	$-(g_d-5g_u)\kappa a\ln(-p^2)$	
Fig. 6	$2(g_u+g_d)\kappa a\frac{1}{p^2}$	$3(g_u+g_d)\kappa a\ln(-p^2)$	
Figs. 5 and 6	$-\frac{4}{3}\kappa ag_u\frac{1}{p^2}$	$2(g_d+4g_u)\kappa a\ln(-p^2)$	
Fig. 7	$\frac{8}{9}g_ua^2\frac{1}{p^4}$	$\frac{4}{9}g_ua^2\frac{1}{p^2}$	
Fig. 8(b)			$-\frac{2}{3}g_da\ln(-p^2)$
Fig. 9			$\frac{4}{3}\chi a^2g_u\frac{1}{p^2}$
Fig. 10			$-\frac{1}{6}g_u\chi a^2m_0{}^2\frac{1}{p^4}$

$$\frac{8}{(2\pi)^4}\left[(-g_d)\frac{M^4}{8}+(g_u+g_d)\frac{1}{12}\chi aM^2+(5g_u+3g_d)\frac{1}{12}\kappa a+g_u\frac{1}{32}\langle g^2G^2\rangle+(g_u+g_d)(-\frac{1}{4}\kappa a)+(g_u-\frac{3}{2}g_d)\frac{a^2}{9}\frac{1}{M^2}\right]. \tag{2.34}$$

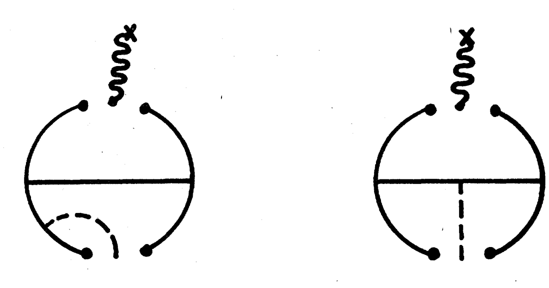
For $\hat{Z}\gamma_5$,

$$\frac{4}{(2\pi)^4}\left[(-g_d)\frac{M^6}{8}+(g_d-5g_u)\frac{1}{12}\chi aM^4+(g_d-5g_u)\frac{1}{4}\kappa aM^2+g_u\frac{1}{32}\langle g^2G^2\rangle M^2+(g_u+g_d)(-\frac{3}{4}\kappa a)M^2-(g_u+\frac{3}{2}g_d)\frac{a^2}{9}\right]. \tag{2.35}$$

For $Z_{\alpha}p_{\beta}\sigma_{\alpha\beta}\gamma_5$,

$$\frac{i}{(2\pi)^4}(g_d\tfrac{2}{3}aM^2-g_u\tfrac{4}{3}\chi a^2+g_u\tfrac{1}{6}m_0{}^2\chi a). \tag{2.36}$$

We shall refer to them as the left-hand side (LHS) of the sum rules. The right-hand side (RHS) of the sum rules are to be computed in terms of the physical hadron intermediate states and coupling strengths, which are given in the next section.



(a) (b)

FIG. 10. Diagrams for Eq. (2.31).

III. THE CORRELATION $\pi(p)$ IN TERMS OF THE PHYSICAL INTERMEDIATE STATES

The QCD sum rules are obtained as usual by computing $\pi(p)$ in terms of physical intermediate states as given by the dispersion integrals in Eq. (1.3). In the presence of the external field Z_μ, there are not only diagonal transitions in which the initial state $|j\rangle$ is the same as the final state $|k\rangle$, but there are also nondiagonal transitions $|j\rangle \neq |k\rangle$.

The diagonal nucleon contribution to $\pi(p)$ can be written as

$$i\lambda_N{}^2 \frac{i}{\not p - m_N}(-Z^\mu i G_A \gamma_\mu \gamma_5)\frac{i}{\not p - m_N}$$
$$= -G_A \lambda_N{}^2 Z^\mu \frac{1}{(p^2-m_N{}^2+i\epsilon)^2} \times \{[(p^2-m_N{}^2)+2m_N{}^2]\gamma_\mu - 2p_\mu \not p + 2m_N(i\sigma^{\mu\nu}p_\nu)\}\gamma_5 \,. \quad (3.1)$$

The axial-vector renormalization constant G_A is defined by

$$\langle N \mid J_\mu^5 \mid N\rangle = \langle N \mid (\bar u \gamma_\mu \gamma_5 u - \bar d \gamma_\mu \gamma_5 d) \mid N\rangle = g_A \bar v(p)\gamma_\mu \gamma_5 v(p) \quad (3.2)$$

and

$$\langle 0 \mid \eta \mid N\rangle = \lambda_N v(p) \,, \quad (3.3)$$

where $v(p)$ is the nucleon spinor with the normalization

$$\bar v v = 2m_N \,. \quad (3.4)$$

We must also take into account the so-called single-pole terms, in which either the initial or the final state is the nucleon, while the other is an excited state. Further since η is a fermion current these excited states can be of either parity. This is crucial in the following, since the contribution from the positive- and the negative-parity states combine differently in the different helicity structures. To see this in detail, and at the same time to keep the discussion simple, let us momentarily ignore the width of the excited states and introduce the definitions

$$\langle 0 \mid \eta \mid P_j\rangle = \lambda_j^+ v(m_j^+) \,, \quad (3.5)$$

and

$$\langle 0 \mid \eta \mid S_j\rangle = \lambda_j^- \gamma_5 v(m_j^-) \,, \quad (3.6)$$

where $|P_j\rangle$ denotes a positive-parity state of mass m_j^+, while $|S_j\rangle$ denotes a negative-parity state of mass m_j^-. $\lambda_j^\pm$ are the coupling strengths of the baryon currents to the physical states in question. The weak-field-Z_μ-induced transition matrix elements are defined by

$$\langle P_j \mid J_\mu^5 \mid P_k\rangle = -G_{jk}^{++}\bar v(m_j^+)\gamma_\mu\gamma_5 v(m_k^+) \,, \quad (3.7)$$

$$\langle S_j \mid J_\mu^5 \mid P_k\rangle = G_{jk}^{-+}\bar v(m_j^-)\gamma_\mu v(m_k^+) \,. \quad (3.8)$$

Using these definitions, Eqs. (3.5)—(3.8), we collect the coefficients of the various invariants in the expansion of $\pi(p)$:

$$\text{coefficients of } 2p\cdot Z\,\hat p\gamma_5 = \sum_{j,k}\left[-\frac{G_{jk}^{++}\lambda_j^+\lambda_k^+}{(p^2-m_j^{+2})(p^2-m_k^{+2})} - \frac{G_{jk}^{--}\lambda_j^-\lambda_k^-}{(p^2-m_j^{-2})(p^2-m_k^{-2})} - \frac{G_{jk}^{-+}\lambda_j^-\lambda_k^+}{(p^2-m_j^{-2})(p^2-m_k^{+2})} + \frac{G_{jk}^{+-}\lambda_j^+\lambda_k^-}{(p^2-m_j^{+2})(p^2-m_k^{-2})}\right], \quad (3.9)$$

$$\text{coefficients of } \hat Z\gamma_5 = \sum_{j,k}\left[\frac{G_{jk}^{++}\lambda_j^+\lambda_k^+(p^2+m_j^+m_k^+)}{(p^2-m_j^{+2})(p^2-m_k^{+2})} + \frac{G_{jk}^{--}\lambda_j^-\lambda_k^-(p^2+m_j^-m_k^-)}{(p^2-m_j^{-2})(p^2-m_k^{-2})} - \frac{G_{jk}^{-+}\lambda_j^-\lambda_k^+(p^2-m_j^-m_k^+)}{(p^2-m_j^{-2})(p^2-m_k^{+2})} - \frac{G_{jk}^{+-}\lambda_j^+\lambda_k^-(p^2-m_j^+m_k^-)}{(p^2-m_j^{+2})(p^2-m_k^{-2})}\right], \quad (3.10)$$

$$\text{coefficients of } i\sigma_{\alpha\beta}Z^\alpha p^\beta\gamma_5 = \sum_{j,k}\left[\frac{G_{jk}^{++}\lambda_j^+\lambda_k^+(m_j^++m_k^+)}{(p^2-m_j^{+2})(p^2-m_k^{+2})} - \frac{G_{jk}^{--}\lambda_j^-\lambda_k^-(m_j^-+m_k^-)}{(p^2-m_j^{-2})(p^2-m_k^{-2})} - \frac{G_{jk}^{-+}\lambda_j^-\lambda_k^+(m_j^--m_k^+)}{(p^2-m_j^{-2})(p^2-m_k^{+2})} - \frac{G_{jk}^{+-}\lambda_j^+\lambda_k^-(m_j^+-m_k^-)}{(p^2-m_j^{+2})(p^2-m_k^{-2})}\right]. \quad (3.11)$$

In particular when either $|j\rangle$ or $|k\rangle$ is the nucleon for the coefficient of $2p\cdot Z\,\hat p\gamma_5$, we have the combination

$$\sum_k\left[\frac{-G_{Nk}^{++}\lambda_N\lambda_k^+}{(p^2-m_N{}^2)(p^2-m_k^{+2})} + \frac{G_{Nk}^{+-}\lambda_N\lambda_k^-}{(p^2-m_N{}^2)(p^2-m_k^{-2})}\right] + \sum_j\left[\frac{-G_{jN}^{++}\lambda_j^+\lambda_N}{(p^2-m_j^{+2})(p^2-m_N{}^2)} + \frac{G_{jN}^{-+}\lambda_j^-\lambda_N}{(p^2-m_j^{-2})(p^2-m_N{}^2)}\right], \quad (3.12)$$

which is sharply different from the corresponding coefficient of $\hat Z\gamma_5$ given by

$$\sum_k \left[\frac{G_{Nk}^{++}\lambda_N\lambda_k^+(p^2+m_Nm_k^+)}{(p^2-m_N{}^2)(p^2-m_k^{+2})} - \frac{G_{Nk}^{+-}\lambda_N\lambda_k^-(p^2-m_Nm_k^-)}{(p^2-m_N{}^2)(p^2-m_k^{-2})} \right] + \sum_j \left[\frac{G_{jN}^{++}\lambda_j^+\lambda_N(p^2+m_j^+m_N)}{(p^2-m_j^{+2})(p^2-m_N{}^2)} - \frac{G_{jN}^{-+}\lambda_j^-\lambda_N(p^2-m_Nm_j^-)}{(p^2-m_j^{-2})(p^2-m_N^2)} \right] \quad (3.13)$$

as well as at $i\sigma_{\alpha\beta}Z^\alpha p^\beta\gamma_5$

$$\sum_k \frac{G_{Nk}^{++}\lambda_N\lambda_k^+(m_N+m_k^+)}{(p^2-m_N{}^2)(p^2-m_k^{+2})} - \frac{G_{Nk}^{+-}\lambda_N\lambda_k^-(m_N-m_k^-)}{(p^2-m_N{}^2)(p^2-m_k^{-2})} + \sum_j \frac{G_{jN}^{++}\lambda_j^+\lambda_N(m_j^++m_N)}{(p^2-m_j^{+2})(p^2-m_N{}^2)} + \frac{G_{jN}^{-+}\lambda_j^-\lambda_N(m_j^--m_N)}{(p^2-m_j^{-2})(p^2-m_N{}^2)} . \quad (3.14)$$

Imagine now for the excited states, chiral symmetry is realized in the Wigner-Weyl mode, that is, parity doubling. Then it is reasonable to expect

$$m_j^+ \approx m_j^- , \quad \lambda_j^+ \approx \lambda_j^- , \quad G_{Nk}^{++} \approx G_{Nk}^{+-} , \quad G_{jN}^{++} \approx G_{jN}^{-+} . \quad (3.15)$$

Then in the coefficient of the structure $p\cdot Z\hat{p}\gamma_5$, the contribution from the positive- and negative-parity states will cancel each other, while for the structures at $\hat{Z}\gamma_5$ and $i\sigma_{\alpha\beta}Z^\alpha p^\beta$, there is no such destructive interference.

Now one might ask whether our assumption that chiral symmetry is realized in the Wigner-Weyl mode for the excited baryon states has any justification provided by the QCD sum rules? To see that this is indeed the case, we turn to the mass sum rules derived by Belyaev and Ioffe.[3] The structure at $\hat{p}$,

$$\frac{M^6}{L^{4/9}} + \frac{bM^2}{4L^{4/9}} + \tfrac{4}{3}a^2L^{4/9} - \frac{a^2m_0{}^2}{3M^2} = 2(2\pi)^4 \left[\sum_j (\lambda_j^+)^2 e^{-m_j^{+2}/M^2} + \sum_j (\lambda_j^-)^2 e^{-m_j^{-2}/M^2} \right] . \quad (3.16)$$

The structure at 1,

$$2aM^4 - \frac{ab}{9} + \frac{8\times 17}{81}\frac{\alpha_s}{\pi}\frac{a^3}{M^2} = 2(2\pi)^4 \sum_j [m_j^+(\lambda_j^+)^2 e^{-(m_j^+)^2/M^2} - m_j^-(\lambda_j^-)^2 e^{-m_j^{-2}/M^2}] . \quad (3.17)$$

Here, and in the following a is as defined in Eq. (1.7),

$$a = -(2\pi)^2\langle 0 | \bar{q}q | 0\rangle ,$$

$$b = \langle 0 | g_c^2 G_{\mu\nu}^n G^{n\mu\nu} | 0 \rangle ,$$

$$\alpha_s = g_c{}^2/4\pi ,$$

$$L = \ln(M^2/\Lambda^2)/\ln(\mu^2/\Lambda^2) ,$$

$$am_0{}^2 = (2\pi)^2\langle 0 | \bar{q}\sigma\cdot Gq | 0\rangle ,$$

where μ is the renormalization scale taken to be 500 MeV and Λ is the QCD scale parameter taken to be 100 MeV.

It is seen that in the RHS of these sum rules, the contribution of each state is a positive-definite quantity $(\lambda_j^\pm)^2\exp(-m_j^{+2}/m^2)$. In the $\hat{p}$ sum rule, Eq. (3.16), the positive- and negative-parity state contributions add, while in the second sum rule, Eq. (3.17) they combine with opposite signs. On the other hand, the asymptotic behavior in the Borel variable M^2 is sharply different in the LHS of these two sum rules. *A forteriori,* the OPE on the left-hand side becomes more and more accurate for larger values of M^2, therefore the behavior of the sum over the physical states on the right-hand side is more exactly described for large M^2 by the leading term on the left-hand side. Hence the only way by which the two sum rules can be consistent is for the excited state contributions in the right-hand side of the second sum rule to cancel asymptotically. In particular it strongly suggests that $|\lambda_j^{+2}-\lambda_j^{-2}| \to 0$ and $|m_j^{+2}-m_j^{-2}| \to 0$ for large j.

Returning now to the case where the external field Z_μ is present, we can write the right-hand side of $\pi(p)$ as the sum of three pieces: (i) one containing the diagonal nucleon term which has a double pole at the nucleon mass in momentum space, Eq. (3.1), (ii) one involving transition between the nucleon and excited states, Eq. (3.12), which has a simple pole at the nucleon mass in momentum space, and (iii) the pure excited-state contributions. Upon Borel transforming, one can write the sum rules as follows: For $p\cdot Z\hat{p}\gamma_5$,

$$\left[\frac{M^4}{8L^{4/9}} + \frac{\kappa a}{6L^{68/81}} + \frac{\langle g_c{}^2G^2\rangle}{32M^2L^{4/9}} + \frac{5a^2L^{4/9}}{18M^2} \right] = \tilde{\beta}_N{}^2 \left[\frac{G_A}{M^2} + A \right] e^{-m_N{}^2/M^2} + \text{excited-state contribution} . \quad (3.18)$$

For $\hat{Z}\gamma_5$,

$$\left[\frac{M^6}{8L^{4/9}} - \frac{M^4\chi a}{2L^{4/9}} - \frac{3}{2}M^2\kappa aL^{-68/81} + \frac{M^2\langle g_c{}^2G^2\rangle}{32L^{4/9}} + \frac{a^2}{18}L^{4/9} \right] = \tilde{\beta}_N{}^2 \left[G_A \left[1 - \frac{2m_N{}^2}{M^2} \right] + B \right] e^{-m_N{}^2/M^2} + \text{excited-state contribution} . \quad (3.19)$$

For $i\sigma_{\alpha\beta}Z^{\alpha}p^{\beta}\gamma_5$,

$$\left[\frac{M^4a}{12}+\frac{M^2\chi a^2}{6}+\frac{\kappa a^2}{12L^{32/81}}-\frac{m_0{}^2aM^2}{48L^{4/9}}\right]=\widetilde{\beta}_N{}^2(m_NG_A+cM^2)e^{-m_N{}^2/M^2}+\text{excited-state contributions}\ , \tag{3.20}$$

where $\widetilde{\beta}_N{}^2=(2\pi)^4\lambda_N{}^2/4$. In writing down the sum rules following Ioffe and Smilga[5] we have also incorporated the effects of the anomalous dimensions of the operators η, $\bar{q}q,\ldots$ for the Wilson's coefficients in the left-hand side.[14]

For our purposes it is convenient to rewrite the mass sum rules (3.16) and (3.17) as follows:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\tfrac{1}{6}a^2L^{4/9}-\frac{a^2m_0{}^2}{24M^2}=\widetilde{\beta}_N{}^2e^{-m_N{}^2/M^2}+\text{excited states}\ , \tag{3.21}$$

$$\frac{aM^4}{4L^{4/9}}-\frac{ab}{72}+\frac{17}{81}\frac{\alpha_s}{\pi}\frac{a^3}{M^2}=\widetilde{\beta}_N{}^2M_Ne^{-m_N{}^2/M^2}+\text{excited states}\ . \tag{3.22}$$

Our discussion in the previous paragraphs, in particular Eq. (3.12) to Eq. (3.14), shows that in the sum rule (3.18) we can expect the nondiagonal single-pole term A to be small, while similar coefficients B and C appearing in Eqs. (3.19) and (3.20) are expected to be non-negligible. A comparison with the mass sum rules Eqs. (3.21) and (3.22) is also quite illuminating. Since we know the experimental value of G_A to be close to unity, we see that the $p\cdot Z\hat{p}\gamma_5$ sum rule Eq. (3.18) bears a close similarity to the mass sum rule Eq. (3.21). On the other hand, in the $\hat{Z}\gamma_5$ sum rule, Eq. (3.19), the sign of the nucleon contribution is reversed in the RHS. In the even chiral sum rule Eq. (3.20), even the leading term on the LHS is different from the leading term on the LHS of the mass sum rule Eq. (3.22). We shall not consider the even chiral structure sum rule Eq. (3.20) any further in this paper.

IV. EXTENSION TO HYPERONS

Since significant experimental data for the hyperon leptonic decays exist and the Cabibbo theory works reasonably well, it is important to extend the calculation to hyperons. To this end it is useful to consider transition amplitudes of the type $\Sigma\to\Sigma$, $\Sigma\to\Lambda$, and $\Xi\to\Xi$ induced by the external isovector field Z_μ and write for the current

$$J_\mu^5=(\bar{u}\gamma_\mu\gamma_5u-\bar{d}\gamma_\mu\gamma_5d)\ , \tag{4.1}$$

since SU_3-flavor symmetry predicts these coupling strengths to be

$$\langle p\,|J_\mu^5|\,p\rangle=G_A=D+F\ , \tag{4.2}$$

$$\langle\Sigma^+\,|J_\mu^5|\,\Sigma^+\rangle=G_\Sigma=2F\ , \tag{4.3}$$

$$\langle\Lambda\,|J_\mu^5|\,\Sigma^0\rangle=G_{\Sigma\Lambda}=\frac{2}{\sqrt{3}}D\ , \tag{4.4}$$

$$\langle\Xi^-\,|J_\mu^5|\,\Xi^-\rangle=G_\Xi=D-F\ . \tag{4.5}$$

We have suppressed the baryon spinors for simplicity. The second and third matrix elements are particularly interesting since if $SU_3\times SU_3$ chiral symmetry were exact, i.e., $m_u=m_d=m_s=0$ and the physical vacuum is invariant under this transformation, when $D=1$ and $F=0$. To derive the sum rules corresponding to these transitions we make use of the currents

$$\eta_{\Xi^-}=(s^aC\gamma_\mu s^b)\gamma_5\gamma_u d^c\epsilon^{abc}\ , \tag{4.6}$$

$$\eta_{\Sigma^+}=(u^aC\gamma_\mu u^b)\gamma_5\gamma_\mu s^c\epsilon^{abc}\ , \tag{4.7}$$

$$\eta_\Lambda=\sqrt{2/3}[(u^aC\gamma_\mu s^b)\gamma_5\gamma_\mu d^c-(d^aC\gamma_\mu s^b)\gamma_5\gamma_\mu u^c]\epsilon^{abc}\ , \tag{4.8}$$

$$\eta_{\Sigma^0}=\frac{1}{\sqrt{2}}[(u^aC\gamma_\mu d^b)\gamma_5\gamma_\mu s^c+(d^aC\gamma_\mu u^b)\gamma_5\gamma_\mu s^c]\epsilon^{abc}\ . \tag{4.9}$$

We shall first consider the sum rules in the hypothetical SU_3-flavor-symmetric limit: $m_u=m_d=m_s=0$ and $\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=\langle\bar{s}s\rangle\neq0$. The modification taking into account the strange-quark mass and the difference between $\langle\bar{u}u\rangle$ and $\langle\bar{s}s\rangle$ are considered in Sec. V.

To derive the sum rules corresponding to the $\Sigma^+\to\Sigma^+$ transition vertex, for example, all we need to do is to carry out our calculations of Sec. II using the current Eq. (4.7) instead of the proton current Eq. (2.1). This consists of a simple replacement of the d quark by the s quark. Since we assume Z_μ to be an isovector, its coupling to the s quark is zero. So the sum rules for the transition (4.3) are simply obtained by setting $g_d=0$ in the formulas (3.18) to (3.20). It is easy to see that the leading term is absent, reflecting the fact that in the exact-symmetry limit we expect that $F=0$. Writing down explicitly the sum rule for the $\Sigma^+\to\Sigma^+$ vertex, at the odd chiral structures, for $p\cdot Z\hat{p}\gamma_5$,

$$\frac{M^4}{12L^{4/9}}\chi a+\frac{M^2}{6L^{68/81}}\kappa a+\frac{M^2}{32L^{4/9}}\langle g_c{}^2G^2\rangle+\tfrac{1}{9}a^2L^{4/9}-\tfrac{1}{12}m_sa_s\chi a=\widetilde{\beta}_\Sigma{}^2e^{-m_\Sigma{}^2/M^2}(2F+BM^2)+\text{excited-state contributions}\ , \tag{4.10}$$

and for $\hat{Z}\gamma_5$,

$$-\frac{5}{12}\frac{M^4}{12L^{4/9}}\chi a-2\frac{M^2}{L^{68/81}}\kappa a+\frac{M^2}{32L^{4/9}}\langle g_c{}^2G^2\rangle-\frac{a^2}{9}L^{4/9}=\tilde{\beta}_\Sigma{}^2e^{-m_\Sigma{}^2/M^2}\left[2F\left[1-\frac{2m_\Sigma{}^2}{M^2}\right]+B'\right]$$
$$+\text{excited-state contributions}\ . \tag{4.11}$$

The sum rules for the other transitions can be written similarly. For the transition of $\Sigma\to\Lambda$, for $Z\cdot p\,\hat{p}\gamma_5$,

$$\frac{M^6}{8L^{4/9}}-\frac{M^4}{24L^{4/9}}\chi a+\frac{M^2}{12L^{68/81}}\kappa a+\frac{M^2}{64L^{4/9}}\langle g_c{}^2G^2\rangle+\tfrac{2}{9}a^2L^{4/9}=\tilde{\beta}_\Lambda\tilde{\beta}_\Sigma e^{-m_\Sigma{}^2/M^2}(D+C''M^2)+(\text{excited states})\ , \tag{4.12}$$

and for $\hat{Z}\gamma_5$,

$$\frac{M^6}{8L^{4/9}}-\frac{7M^4}{24L^{4/9}}\chi a-\frac{M^2}{2L^{68/81}}\kappa a+\frac{M^2}{64L^{4/9}}\langle g_c{}^2G^2\rangle+\frac{a^2L^{4/9}}{9}=\tilde{\beta}_\Lambda\tilde{\beta}_\Sigma\left[D\left[1-\frac{2m_\Sigma{}^2}{M^2}\right]+C'''M^2\right]e^{-m_\Sigma{}^2/M^2}$$
$$+(\text{excited states})\ . \tag{4.13}$$

For $\Xi\to\Xi$, for $p\cdot Z\,\hat{p}\gamma_5$,

$$\frac{M^6}{8L^{4/9}}-\frac{M^4}{12L^{4/9}}\chi a+\frac{a^2}{6}L^{4/9}+\tfrac{1}{6}a_sm_s\chi a-\tfrac{1}{6}m_sm_0{}^2a_s=\tilde{\beta}_\Xi{}^2e^{-m_\Xi{}^2/M^2}(D-F+CM^2)+(\text{excited states})\ , \tag{4.14}$$

and for $\hat{Z}\gamma_5$,

$$\frac{M^6}{8L^{4/9}}-\frac{M^4\chi a}{12L^{4/9}}+\frac{M^2\kappa a}{2L^{68/81}}+\frac{a^2L^{4/9}}{6}=\tilde{\beta}_\Xi{}^2e^{-m_\Xi{}^2/M^2}\left[(D-F)\left[1-\frac{2m_\Xi{}^2}{M^2}\right]+C'M^2\right]+(\text{excited states})\ . \tag{4.15}$$

It should be borne in mind that the flavor-symmetric limit defined by $m_u=m_d=m_s=0$ does not coincide with the usual SU_3-flavor-symmetric limit defined by the Gell-Mann—Okubo mass formula. Therefore the hyperon mass should be set equal to the nucleon mass in this limit, that is, $m_\Xi=m_\Sigma=m_\Lambda=m_N$, and not to the value given by the SU_3-symmetric term in the mass formula.

It is illuminating to compare these sum rules with the mass sum rule Eq. (3.21). It is seen that the leading term is absent in the $\Sigma\to\Sigma$ transition, while it is the same for the other two $\Sigma\to\Lambda$, $\Xi\to\Xi$, just as in the nucleon case. The coefficient of the $G^n_{\mu\nu}G^{n\mu\nu}$ term, that is the b term, however, varies from sum rule to sum rule. While it is identical for the nucleon and the mass sum rule, it is absent in the $\Xi\to\Xi$ transition.[15]

V. ANALYSIS OF THE SUM RULES

It is worthwhile to remind ourselves that the experimental data on hyperon semileptonic decays is consistent with the Cabibbo model. In Ref. 17, a detailed discussion of the recent experimental data is given. The constants F and D are determined to good accuracy. A typical fit, as given in column 5 of Table III of Ref. 17 gives $F=0.477\pm0.012$ and $D=0.756\pm0.011$. There are variations of a few percent, which depend on the detailed assumptions made in fitting the data. Therefore it is sensible to analyze all of our sum rules in the SU_3-symmetric limit, which in the context of QCD corresponds to setting $m_s=0$, that is to say, that the hyperon masses are degenerate with the nucleon mass.

A. Determination of χ and κ by PCAC

To proceed further and determine the value of G_A we need to know the values of the susceptibility parameters χ and κ introduced in Sec. II. As pointed out by Belyaev and Kogan[10] these parameters can be evaluated through the use of PCAC. In particular, the matrix element

$$\langle 0|\bar{u}\gamma_\mu\gamma_5u\,|0\rangle_Z=-iZ_\nu\int d^4x\,e^{iQ\cdot x}\langle 0|\,T(\bar{u}(x)\gamma_\nu\gamma_5u(x)-\bar{d}(x)\gamma_\nu\gamma_5d(x),\bar{u}\gamma_\mu\gamma_5u)\,|0\rangle\,|_{Q_\mu\to 0} \tag{5.1}$$

can be determined by the pion-pole contribution and is equal to

$$\langle 0|\bar{u}\gamma_\mu\gamma_5u\,|0\rangle_Z=-Z_\mu f_\pi{}^2\ , \tag{5.2}$$

where

$$\langle 0|\bar{u}\gamma_\mu\gamma_5d\,|\pi^+\rangle=f_\pi q_\mu \tag{5.3}$$

and

$$f_\pi=133\text{ MeV}\ .$$

Returning to the equation for the propagator expansion given by Eq. (2.12) where we had defined

$$\langle\bar{q}\gamma_\mu\gamma_5q\rangle=g_qZ_\mu\chi\langle\bar{q}q\rangle=-Z_\mu f_\pi{}^2g_q\ , \tag{5.4}$$

this leads to the identification, that $\chi\langle\bar{q}q\rangle=-f_\pi{}^2$. An analogous determination of the expectation value

$\langle 0 | \bar{q} \tilde{G}_{\mu\nu}\gamma^\nu q | 0 \rangle$ requires a knowledge of the matrix element

$$\langle 0 | \bar{q} \tilde{G}_{\mu\nu}\gamma^\nu q | \pi \rangle = i q_\mu \kappa_\pi . \tag{5.5}$$

There is an estimate of κ_π by Novikov *et al.*,[11] which is adopted by Belyaev and Kogan.[10] This value is obtained in Ref. 11 by a rather tenuous chain of argument. While it is likely that the determination of the susceptibility χ given by Eq. (5.4) is quite accurate for our purpose, *a priori* the determination of the matrix element Eq. (5.5) may not be as reliable. Therefore we have included the possibility that the matrix element (5.5) may have a different value from Ref. 11 by writing

$$\langle 0 | \bar{q} \tilde{G}_{\mu\nu}\gamma^\nu q | 0 \rangle = \kappa Z_\mu \langle 0 | \bar{q} q | 0 \rangle \tag{5.6}$$

with

$$\kappa \langle 0 | \bar{q} q | 0 \rangle = -\tfrac{1}{3}\xi f_\pi{}^2 0.2\ \text{GeV}^2 , \tag{5.7}$$

where $\xi = 1$ corresponds to the value given in Ref. 11. In our analysis, we shall consider different values of ξ.

With this all the terms in the left-hand side of the sum rules are now known and we shall analyze the sum rules using two different methods. The fundamental assumption in either of the methods is of course the principle of duality, i.e., there exists an interval in the Borel-mass variable M^2, which includes in its range the mass of the hadron whose properties we are trying to determine, over which the left-hand side and the right-hand side of the sum rules match.

B. The ratio method

Let us consider the nucleon sum rule at the structure $p \cdot Z \hat{p} \gamma_5$, Eq. (3.19), multiplying the equation by M^2 and comparing it with the mass sum rule (3.21) derived by Belyaev and Ioffe:

$$\frac{M^6}{8L^{4/9}} + \frac{M^2 b}{32L^{4/9}} + a^2 L^{4/9}\left(\tfrac{1}{6}+\tfrac{1}{9}\right) + \frac{M^2 \kappa a}{6L^{68/81}} = \tilde{\beta}_N{}^2 [G_A + AM^2] e^{-m_N{}^2/M^2} + \sum_{j\neq N} (\beta_j{}^2 G_j + A_j M^2) e^{-m_j{}^2/M^2} , \tag{3.19$'$}$$

$$\frac{M^6}{8L^{4/9}} + \frac{M^2 b}{32L^{4/9}} + \tfrac{1}{6} a^2 L^{4/9} = \tilde{\beta}_N{}^2 e^{-m_N{}^2/M^2} + \sum_{j\neq N} \beta_j{}^2 e^{-m_j{}^2/M^2} . \tag{3.21$'$}$$

The asymptotic behaviors for large M^2 for the two sum rules are identical. This again strongly suggests that the coupling of the axial-vector current to the excited states G_j tend to 1 as M_j gets larger and the single-pole coefficients A_j tend to zero. Let us introduce the ratio function,

$$R(M^2) = \frac{M^6/8L^{4/9} + M^2 b/32L^{4/9} + a^2 L^{4/9}\left(\tfrac{1}{6}+\tfrac{1}{9}\right) + M^2 \kappa a/6L^{68/81}}{M^6/8L^{4/9} + M^2 b/32L^{4/9} + a^2 L^{4/9}/6} . \tag{5.8}$$

Computing this ratio in terms of physical intermediate states we can write

$$R(M^2)|_{\text{RHS}} = \frac{G_A + AM^2 + \sum_{j\neq N} (\beta_j{}^2 G_j + A m_j{}^2) \tilde{\beta}_N^{-2} e^{-(m_j{}^2 - m_N{}^2)/M^2}}{1 + \sum_{j\neq N} \beta_j{}^2 \tilde{\beta}_N^{-2} e^{-(m_j{}^2 - m_N{}^2)/M^2}} . \tag{5.9}$$

The function $R(M^2)$ is plotted in Fig. 11.

Now for the right-hand side we make the ansatz

$$R(M^2)|_{\text{RHS}} = G_A + Am^2 + [\gamma + \delta(W^2 - m_N{}^2 + M^2)] \exp\left[-\frac{W^2 - m_N{}^2}{M^2}\right] . \tag{5.10}$$

In writing this we are assuming that the excited-state contribution in the ratio $R(M^2)$ can be effectively represented by a state with mass W^2 somewhat analogous to effective pole approximation used frequently in dispersion theory calculations. If our ansatz for the effective contribution (5.10) is good, then we expect the right-hand side and the left-hand side to match over a large region of M^2, for $M^2 \gtrsim m_N{}^2$. In fact if the two sides matched asymptotically then

$$G_A + \gamma = 1, \quad \text{coefficient of constant term} , \tag{5.11}$$

$$\delta + A = 0, \quad \text{coefficient of } M^2 \text{ term} . \tag{5.12}$$

To find the constants γ and δ we proceed as follows. We fix δ at an initial value, say, zero, and start with an arbitrary value of γ and compute

$$R(M^2) - \gamma \exp\left[-\frac{W^2 - m_N{}^2}{M^2}\right] = S(M^2) . \tag{5.13}$$

The function $S(M^2)$ is fitted by

$$S(M^2) = \rho + \sigma M^2 \tag{5.14}$$

in the fiducial region,

$$0.9 \le M^2 \le 1.2\ \text{GeV}^2 . \tag{5.15}$$

If the output value did not satisfy the condition $\rho + \gamma = 1$, a new value of

$$\gamma = (\gamma_{\text{in}} + 1 - \rho)/2 \tag{5.16}$$

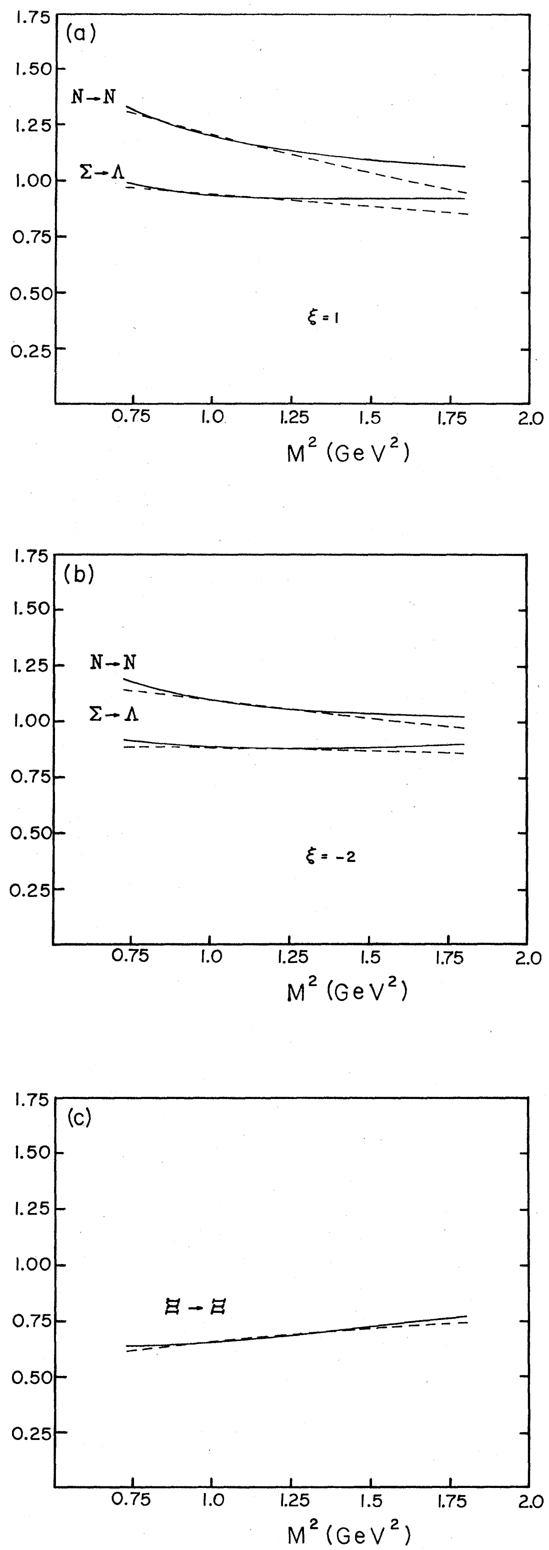


FIG. 11. The function $R(M^2)$, Eq. (5.8), is given by the solid curves. The dashed curves correspond to our ansatz fit in terms of the baryon pole and excited states, Eq. (5.10) with $\delta=0$. (a) and (b) corresponding to the value $\xi=1$ and -2, respectively, while (c) is the $\Xi\to\Xi$ amplitude and is independent of ξ.

was chosen and the processes were iterated. The convergence of this iteration is shown in Fig. 12. We note the following points. The iteration in γ coverages rapidly, and more importantly the final value of ρ is independent of the initial value δ_0.

We have also tried to satisfy Eq. (5.12) by iterating δ. However, a small nonzero value of $A+\delta$ persists. It seems proper to us to choose $\delta=0$ and let the constraint of Eq. (5.12) be mildly violated in the large M^2 region. Figure 11 shows the match between the function $R(M^2)$ and our ansatz, Eq. (5.10). It is seen that our failure to match Eq. (5.12) has little effect in the mass region of interest, Eq. (5.15). It is of course unreasonable to expect a fit over the entire M^2 region. Therefore we take our final value of G_A to be the limit to which ρ converges. Our results are displayed in Table II.

We have investigated the sensitivity of the final results

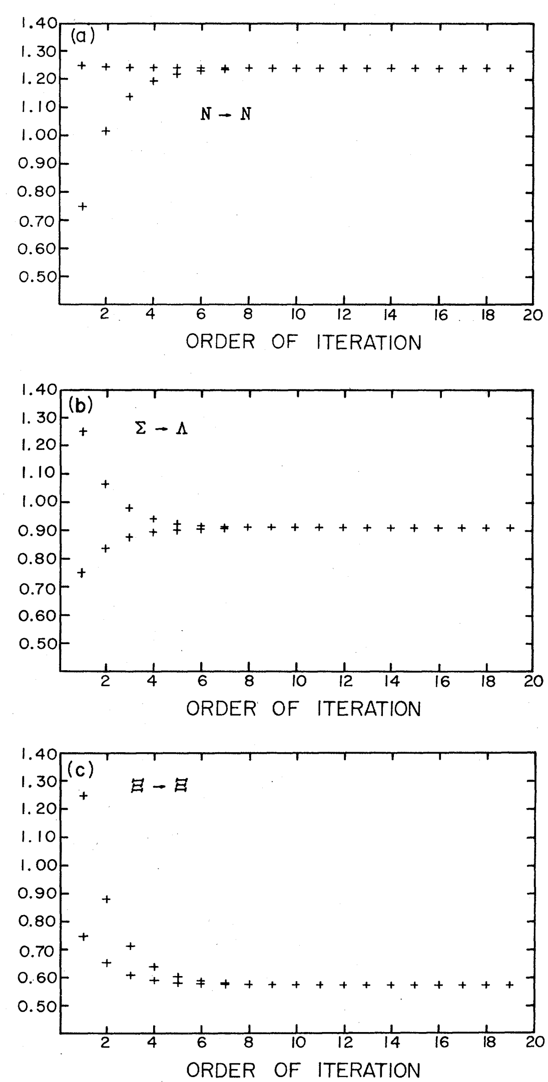


FIG. 12. Convergence of the iteration in γ to determine the renormalization constants for $N\to N$, $\Sigma\to\Lambda$, and $\Xi\to\Xi$. Note that the final value is always independent of the initial value of γ.

TABLE II. Renormalization constants determined from $p\cdot Z\hat{p}\gamma_5$ sum rules based on the ratio method. (a) In the symmetrical limit the baryon mass is 0.94 GeV; the continuum mass is taken to be $W^2=2.3$ GeV^2. Different columns correspond to different values of the susceptibility κ and the chiral-symmetry-breaking parameter a. Values in parentheses correspond to the fiducial region $0.8\le M^2\le 1.2$ GeV^2. First number is the renormalization constant and the second is the single-pole coefficient σ. Numbers immediately above are from fitting the region $0.9\le M^2\le 1.2$ GeV^2. (b) For the broken-symmetry case, the Ξ mass is 1.32 GeV, the fiducial region is $1.5\le M^2\le 1.9$ GeV^2. $a=0.45$ GeV^2, $W^2=3.6$ GeV^2. This sum rule does not depend on ξ.

(a) $SU_{(3)}$-symmetric limit: $m_u=m_d=m_s=0$				
	$a=0.45$ GeV^3		$a=0.55$ GeV^3	
	$\xi=1$	$\xi=-2$	$\xi=1$	$\xi=-2$
$N\to N$	1.52, −0.19	1.24, −0.07	1.61 −0.21	1.37, −0.11
$D+F$	(1.57, −0.22)	(1.24, −0.08)	(1.65, −0.23)	(1.38, −0.12)
$\Sigma\to\Lambda$	1.05, −0.10	0.91, −0.04	1.14, −0.13	1.02, −0.08
D	(1.08, −0.12)	(0.92, −0.05)	(1.18, −0.15)	(1.04, −0.09)
$\Xi\to\Xi$	0.58, −0.01	0.58, −0.01	0.67, −0.05	0.67, −0.05
$D-F$	(0.60, −0.02)	(0.60, −0.02)	(0.70, −0.06)	(0.70, −0.06)
(b) Broken $SU_{(3)}$ symmetry: $m_u=m_d=0$, $m_s=0.15$ GeV, $f=-0.2$.				
$\Xi\to\Xi$	0.56, 0.03			
$D-F$				

to the following two variations: (i) The fiducial range of the Borel-mass variable over which the duality is assumed to be valid. For example, we have increased the fiducial range to $0.8\le M^2\le 1.2$ GeV^2 and the results for the renormalization constants are displayed in Table I. (They appear within parentheses immediately below the values for the shorter range.) (ii) We have also varied the effective-mass parameter W^2. The results are quite stable and the variations are less than the variation due to the change of the fiducial region in (i).

It is also straightforward to carry out the calculation in the SU_3 limit for the $\Sigma\to\Lambda$ transition vertex, as well as for the $\Xi\to\Xi$ vertex (see Figs. 11 and 12). By this procedure, we determine the constants D and $D-F$. Table II displays the value of $(D+F)_N$, $D_{\Sigma\Lambda}$, $(D-F)_\Xi$ for different choice of values for the chiral-symmetry-breaking parameter: $a=0.45$ and 0.55 GeV^3. The results for variations in the susceptibility κ or equivalently the ξ parameter introduced earlier are also tabulated in the same table. It is seen from the table that the determination of the values of $D+F$, D, and $D-F$ are mutually consistent.

C. The continuum method

In their analysis of the mass sum rules, Belyaev and Ioffe[3] sum over the contribution of the excited states, i.e., the states above the nucleon occurring in the right-hand side of the sum rules, by using the asymptotic freedom expressions given by OPE, in analogy with the situation in the cross sections of e^+e^- to hadrons, where it is well known that the cross section for the excited states or the continuum is very well reproduced by the asymptotic freedom expression. Therefore in the baryon case, one approximates the absorptive part of the physical excited states by the imaginary part of the OPE expression for the creation and the absorption of the three-quark state, for Borel mass greater than some effective mass, $M^2\ge W^2$. We refer to this approximation of the excited-state contribution as the continuum method. Within this approximation, the sum rules after transferring the right-hand side excited-state contributions to the left-hand side can be written as follows. For $N\to N$,

$$\frac{M^6E_1}{8L^{4/9}}+\frac{M^2\kappa a}{6L^{68/81}}+\frac{M^2b}{32L^{4/9}}+\frac{5a^2L^{4/9}}{18}=\tilde{\beta}_N{}^2e^{-m_N{}^2/M^2}(D+F+AM^2)\ . \qquad (5.17)$$

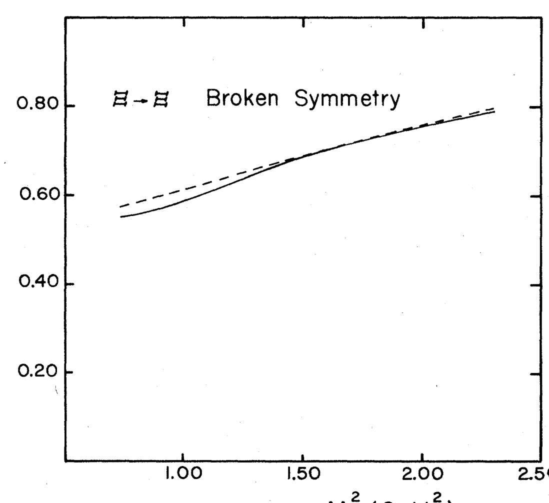


FIG. 13. The $\Xi\to\Xi$ transition in the broken-symmetry case by the ratio method, $m_s=0.15$ GeV and $f=-0.2$. (See text.)

For $\Sigma \to \Lambda$,

$$\frac{M^6 E_1}{8L^{4/9}} - \frac{M^4 E_0 \chi a}{24L^{4/9}} + \frac{M^2 \kappa a}{12L^{68/81}} + \frac{M^2 b}{64L^{4/9}} + \frac{2}{9}a^2 L^{4/9} = \tilde{\beta}_N{}^2 e^{-M_N{}^2/M^2}(D + A'M^2) . \quad (5.18)$$

For $\Sigma \to \Sigma$,

$$\frac{M^4 E_0}{12L^{4/9}}\chi a + \frac{M^2 \kappa a}{6L^{68/81}} + \frac{M^2 b}{32L^{4/9}} + \frac{a^2 L^{4/9}}{9} = \tilde{\beta}_N{}^2 e^{-m_N{}^2/M^2}(F + A''M^2) . \quad (5.19)$$

And for $\Xi \to \Xi$,

$$\frac{M^6 E_1}{8L^{4/9}} - \frac{M^4 E_0}{12L^{4/9}}\chi a + \frac{a^2}{6}L^{4/9} = \tilde{\beta}_N{}^2 e^{-m_N{}^2/M^2}(D - F + A'''M^2) . \quad (5.20)$$

We have introduced the functions

$$E_0(u) = 1 - e^{-u}, \quad E_1 = 1 - e^{-u}(1+u)$$

with (5.21)

$$u = \frac{W^2}{M^2} .$$

To analyze the sum rules, we must determine the value of the constants $\tilde{\beta}_N{}^2$ as accurately as possible. We follow Ioffe and Smilga[5] and use the mass sum rule (3.16) with the experimental nucleon mass 0.94 GeV. For $a=0.55$ GeV^3 and $W^2=2.3$ GeV^2, one obtains $\tilde{\beta}_N{}^2=0.26$ GeV^6.

Since in the ratio method we found somewhat better agreement with experiment if the value of a is decreased, we have also analyzed the mass sum rule, Eq. (3.16), with $a=0.45$ GeV^3 and find the corresponding value of $\tilde{\beta}_N{}^2$ to be $\tilde{\beta}_N{}^2=0.22$ GeV^6 if W^2 is once again taken to be $W^2=2.3$ GeV^2. Now we proceed to determine the renormalization constants and the coefficients of the single-pole term. Using the least χ^2 criterion, we match the left-hand side and the right-hand side of Eqs. (5.17)–(5.20) in the mass region $0.9 \le M^2 \le 1.2$ GeV^2. The results are displayed in Table III. Figure 14 displays the left-hand sides of Eqs. (5.17)–(5.20) and their right-hand sides corresponding to the best-fit values of the renormalization constants and the single-pole terms.

As in the ratio method, we have also investigated the variation in the value for the renormalization constants due to the change of the fiducial region and the results are again displayed in Table III. It is seen that the variations in the final results here are somewhat larger than in the ratio method. However, this should not be surprising,

TABLE III. Renormalization constants determined from $p\cdot Z\hat{p}\gamma_5$ sum rules based on the continuum method. (a) The symmetrical baryon mass is 0.94 GeV, the continuum mass is taken to be $W^2=2.3$ GeV^2. Different columns correspond to different values of the susceptibility κ and the chiral-symmetry-breaking parameter a. The double-number entries are the renormalization constant and the single-pole coefficient σ. The values not in parentheses are for $0.9 \le M^2 \le 1.2$ GeV^2, and the values in parentheses are for $0.8 \le M^2 \le 1.2$ GeV^2. (b) For the broken-symmetry case, the double-number entries in the ξ columns are the renormalization constant and the single-pole coefficient σ.

(a) SU$_3$-symmetric limit: $m_u = m_d = m_s = 0$.

	$a=0.45$ GeV^3, $\tilde{\beta}_N{}^2=0.22$ GeV^6		$a=0.55$ GeV^3, $\tilde{\beta}_N{}^2=0.26$ GeV^6	
	$\xi=1$	$\xi=-2$	$\xi=1$	$\xi=-2$
$N\to N$	1.26, 0.46	1.07, 0.47	1.58, 0.17	1.41, 0.18
$D+F$	(1.40, 0.34)	(1.19, 0.36)	(1.73, 0.03)	(1.56, 0.04)
$\Sigma\to\Lambda$	0.85, 0.38	0.75, 0.38	1.13, 0.14	1.05, 0.14
D	(0.96, 0.28)	(0.86, 0.29)	(1.25, 0.03)	(1.17, 0.04)
$\Sigma\to\Sigma$	0.41, 0.09	0.31, 0.09	0.44, 0.03	0.36, 0.04
F	(0.44, 0.06)	(0.33, 0.07)	(0.48, 0.01)	(0.39, 0.01)
$\Xi\to\Xi$	0.45, 0.28	0.45, 0.28	0.69, 0.11	0.69, 0.11
$D-F$	(0.52, 0.22)	(0.52, 0.22)	(0.77, 0.03)	(0.77, 0.03)

(b) Broken SU$_3$ symmetry: $m_u = m_d = 0$, $m_s = 0.15$ GeV, $f = -0.2$.

	$\tilde{\beta}^2$ in GeV^6 ($a=0.45$ GeV^3)	m_B in GeV	W^2 in GeV^2	Fiducial in GeV^2	$\xi=1$	$\xi=-2$
$\Sigma\to\Sigma$	0.42	1.19	3.2	$0.9 < M^2 < 1.2$	0.54, −0.10	0.41, −0.05
				$1.3 < M^2 < 1.7$	0.32, −0.08	0.24, 0.08
$\Xi\to\Xi$	0.58	1.32	3.6	$0.9 < M^2 < 1.2$	0.37, 0.32	0.37, 0.32
				$1.5 < M^2 < 1.9$	−0.02, 0.63	−0.02, 0.63

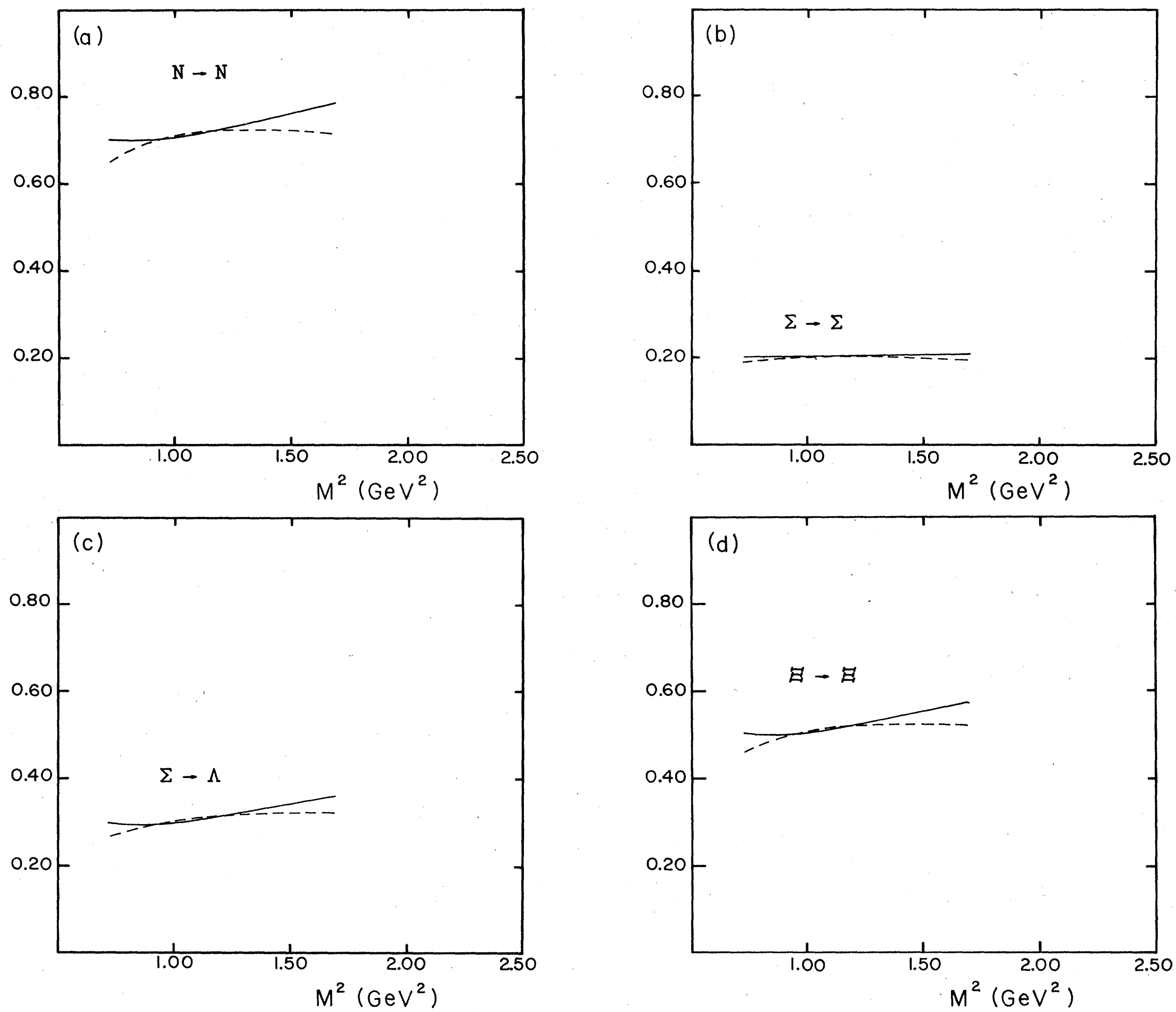


FIG. 14. Comparison of the right-hand side and the left-hand side of the nucleon and hyperon amplitudes using continuum approximation, $W^2=2.3$ GeV^2, $\tilde{\beta}_N{}^2=0.22$ GeV^6, $\xi=1$, $m_N=m_\Lambda=m_\Sigma=m_\Xi$. The solid curve is the left-hand side of Eqs. (5.17) to (5.20), divided by $\tilde{\beta}_N{}^2$. The dashed curve is the least-square fit in the fiducial region $0.9\leq M^2\leq 1.2$ GeV^2, assuming the functional form, $\exp(-m_N{}^2/M^2)\exp(-m_N{}^2/M^2)[D+F+AM^2]$, etc., as given by the right-hand side of Eqs. (5.17) to (5.20). (a) $N\to N$. (b) $\Sigma\to\Sigma$. (c) $\Sigma\to\Lambda$. (d) $\Xi\to\Xi$.

since the parameter $\tilde{\beta}_N{}^2$ itself enters explicitly in the continuum method and itself is determined not too accurately. Moreover, it is seen from Fig. 14 as we move away from the fiducial region, the difference between the LHS and the RHS is substantial except for the $\Sigma\to\Sigma$ case. It is seen from Table III that as in the ratio method in the SU_3-symmetric limit, the determination of the D and F from different sum rules is mutually consistent, i.e., the equalities

$$D_{\Sigma\Lambda}+F_\Sigma\approx(D+F)_N\ , \tag{5.22}$$

as well as

$$D_{\Sigma\Lambda}-F_\Sigma\approx(D-F)_\Xi \tag{5.23}$$

are satisfied.

D. SU_3-symmetry breaking

Now we turn to the effect of SU_3 breaking. At this point, it is sensible to keep $m_u=m_d=0$ still, but we must allow for $m_s\neq 0$. Moreover, one knows that[4] $\langle 0|\bar{s}s|0\rangle\neq\langle 0|\bar{u}u|0\rangle$. Following Belyaev and Ioffe,[4] we incorporate the effect of these corrections by modifying the strange-quark propagator function from the massless limit given by Eq. (2.12) to

$$\langle 0|s^a(x)\bar{s}^b(0)|0\rangle = \text{RHS of (2.12)} + \delta^{ab}\frac{im_s}{48}\hat{x}\left|\langle 0|\bar{s}s\,|0\rangle+\frac{x^2}{24}\langle 0|\bar{s}\sigma\cdot Gs\,|0\rangle\right| - \frac{m_s}{4\pi^2x^2}+\cdots\ . \tag{5.24}$$

Define

$$a_s \equiv -(2\pi)^2 \langle 0 | \bar{s}s | 0 \rangle = (1+f)a \ . \quad (5.25)$$

Following Ioffe and Smilga,[16] we take $f=-0.2$ and $m_s=0.15$ GeV.

Using Eq. (5.24), it is straightforward to calculate the corrections to the sum rules for the $\Sigma\to\Sigma$ and $\Xi\to\Xi$ amplitudes. However, the effect of the strange-quark mass on the $\Sigma\to\Lambda$ transition is too complicated to analyze and will not be considered here. The modified sum rules are as follows: For $\Sigma\to\Sigma$,

$$\frac{M^4}{12L^{4/9}}+\frac{M^2\kappa a}{6L^{68/81}}+\frac{M^2 b}{32L^{4/9}}+\frac{a^2L^{4/9}}{9}-\frac{m_s}{12}\chi a a_s = \tilde{\beta}_\Sigma{}^2 \exp(-m_\Sigma{}^2/M^2)(2F+\beta'' M^2)+ \text{excited states} \ , \quad (5.26)$$

and for $\Xi\to\Xi$,

$$\frac{M^6}{8L^{4/9}}-\frac{M^4\chi a}{12L^{4/9}}+\frac{a_s{}^2}{6}L^{4/9}+\frac{m_s}{6}\chi a a_s-\frac{m_s}{6}m_0{}^2 a_s = \tilde{\beta}_\Xi{}^2 \exp(-m_\Xi{}^2/M^2)(D-F+\beta''' M^2) + \text{excited states} \ . \quad (5.27)$$

The modified mass sum rules given by Belyaev and Ioffe,[4] for Σ,

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{6}-\frac{a^2m_0{}^2}{24M^2}\frac{am_sM^2}{4L^{4/9}}-\frac{am_0{}^2m_s}{24} = \tilde{\beta}_\Sigma \exp(-m_\Sigma{}^2/M^2)+\text{excited states} \ , \quad (5.28)$$

and for Ξ,

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a_s{}^2L^{4/9}}{6}-\frac{am_sm_0{}^2}{12} = \tilde{\beta}_\Xi{}^2 \exp(-m_\Xi{}^2/M^2)+\text{excited states} \ . \quad (5.29)$$

We have again analyzed the sum rules using the ratio and the continuum methods. In the former we have used for the ratio $R(M^2)$ the left-hand side as given by Eqs. (5.27) and (5.29) and used the fiducial range $1.5 \le M^2 \le 1.9$ GeV^2 to determine the new $D-F$ value. As seen from Table II, there is only a small departure from the symmetry limits. The fit for the $\Xi\to\Xi$ case is shown in Fig. 13.

For the continuum method (see Fig. 15), first the coupling strengths $\tilde{\beta}_\Sigma{}^2$ and $\tilde{\beta}_\Xi{}^2$ are redetermined using the mass sum rules (5.28) and (5.29) and the experimental values of the hyperon masses. We find for $a=0.45$ GeV^3

$$\tilde{\beta}_\Sigma{}^2=0.42 \ \text{GeV}^6, \quad W^2=3.2 \ \text{GeV}^2 \ ,$$
$$\tilde{\beta}_\Xi{}^2=0.58 \ \text{GeV}^6, \quad W^2=3.6 \ \text{GeV}^2 \ . \quad (5.30)$$

Following the least-χ^2 criterion as before we have determined the broken-symmetry values of F_Σ and $(D-F)_\Xi$ using the fiducial mass regions:

$$\text{for } \Sigma, \quad 1.2 \lesssim M^2 \lesssim 1.6 \ \text{GeV}^2 \ ,$$
$$\text{for } \Xi, \quad 1.5 \lesssim M^2 \lesssim 1.9 \ \text{GeV}^2 \ . \quad (5.31)$$

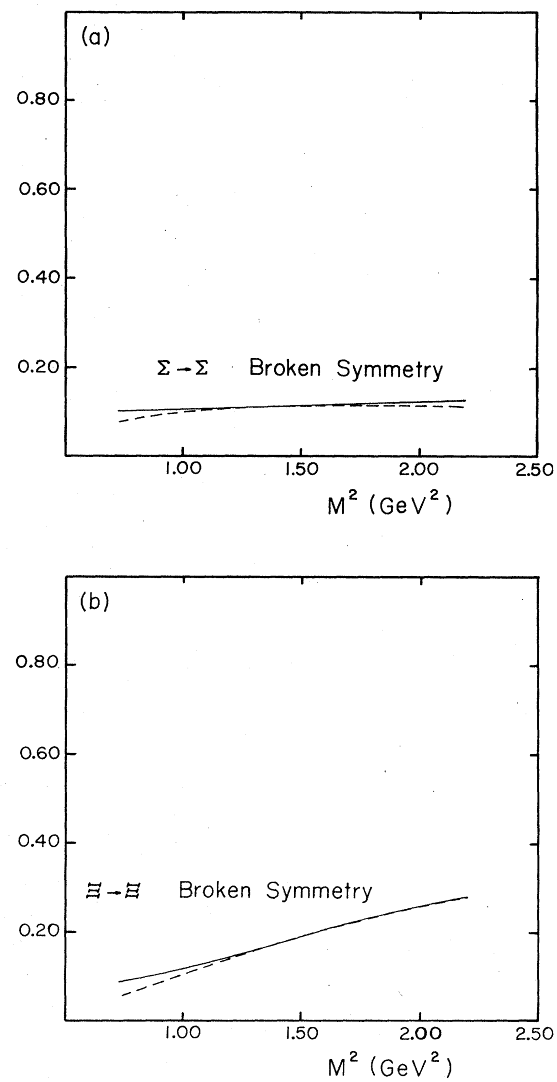


FIG. 15. Broken-symmetry case using the continuum method. Comparison of the RHS and the LHS of the sum rules Eqs. (5.26) and (5.27). As in Eqs. (5.17)–(5.20), the continuum-states contributions are transferred to the LHS in these figures. (a) $\Sigma\to\Sigma$. The fiducial region $1.2 \le M^2 \le 1.6$ GeV^2 is used to match the left-hand side (solid curve) and the right-hand side (dashed curve) after dividing through by $\tilde{\beta}_\Sigma{}^2$. (b) $\Xi\to\Xi$. The fiducial region $1.5 \le M^2 \le 1.9$ GeV^2 is used to match the left-hand side (solid curve) and right-hand side (dashed curve) after dividing through by $\tilde{\beta}_\Xi{}^2$.

The results are again displayed in Table III. It is seen that there is substantial change from the SU_3-symmetric case. A large part of this change of course comes from the change in the definition of the fiducial region. As remarked earlier, the results are sensitive to the fiducial region and the departure from the symmetry limit can be reduced by moving the fiducial region to lower M^2 value. On the other hand, it is not clear to us within the spirit of duality whether such a choice is proper.

E. The $\hat{Z}\gamma_5$ sum rules

We now turn to the $\hat{Z}\gamma_5$ sum rules (3.19) and (4.15). In the SU_3-symmetric limit for the proton and the cascade, they are given below as Eqs. (A) and (B):

$$\frac{M^6}{8L^{4/9}}-\frac{\chi a}{2L^{4/9}}M^4-\frac{3}{2}\frac{\kappa a}{L^{68/81}}M^2+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{18}$$
$$=\tilde{\beta}_N{}^2e^{-m_N{}^2/M^2}\left[\left[1-\frac{2m_N{}^2}{M^2}\right](D+F)+B'\right]$$
$$+\text{continuum}\ , \qquad \text{(A)}$$

$$\frac{M^6}{8L^{4/9}}-\frac{\chi a}{12L^{4/9}}M^4+\frac{1}{2}\frac{\kappa a}{L^{68/81}}M^2+\tfrac{1}{6}a^2L^{4/9}$$
$$=\tilde{\beta}_N{}^2e^{-m_N{}^2/M^2}\left[\left[1-\frac{2m_N{}^2}{M^2}\right](D-F)+B''\right]$$
$$+\text{continuum}\ , \qquad \text{(B)}$$

respectively. As usual, when $m_s=0$, we must set $m_\Xi=m_N$ and $\tilde{\beta}_N{}^2=\tilde{\beta}_\Xi{}^2$.

As noted earlier sum rule (3.19) has a structure which is very different from either the mass rule Eq. (3.16) or the G_A sum rule at the structure $p\cdot Z\hat{p}\gamma_5$, Eq. (3.18). For one thing, the diagonal-nucleon double-pole term enters the right-hand side with a negative sign. Further, as pointed out in Sec. III, we expect the nondiagonal single-pole term at the nucleon mass to be significant, unlike in the sum rule, (3.18). Therefore it is not possible to extract directly the renormalization constants $D+F$ and $D-F$ from these sum rules. However, it is interesting to consider a linear combination along with the mass sum rule (3.16), given below as Eq. (C)

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2}{6}L^{4/9}=\tilde{\beta}_N{}^2\exp(-m_N{}^2/M^2)$$
$$+\text{continuum}\ . \qquad \text{(C)}$$

It is easy to see that the linear combination $\frac{1}{6}[6(B)-5(C)-(A)]$ of these three equations eliminates both the leading M^6 term arising from the unit operator the M^4 term arising from the coefficient of $\langle\bar{q}\gamma_\mu\gamma_5 q\rangle$. After multiplying both sides by

$$e^{m_N{}^2/M^2}/2m_N{}^2\tilde{\beta}_N{}^2\ ,$$

we obtain

$$\frac{e^{m_N{}^2/M^2}}{12m_N{}^2\tilde{\beta}_N{}^2}\left[\frac{-3}{16}\cdot\frac{bM^4}{L^{4/9}}+\frac{9\kappa aM^4}{2L^{68/81}}\frac{+a^2}{9}L^{4/9}M^2\right]$$
$$=\tfrac{1}{6}(7F-5D)-\frac{1}{12m_N{}^2}[5+(7F-5D)+B'-6B'']M^2$$
$$+\text{excited states}\ . \qquad (5.32)$$

The left-hand side of this equation is plotted in Fig. 16. We have fitted this curve with the form $\lambda+\sigma M^2$. Notice that the fit works well over a large M^2 region, suggesting that for this particular linear combination the continuum is probably not too significant. Identifying λ with $(7F-5D)/6$, we arrive at the values listed in Table IV.

We have also determined the coefficient $(7F-5D)/6$ using the continuum method. As in the earlier discussion the effective mass for the continuum is taken to be $W^2=2.3$ GeV^2. Transferring the excited-state contribution to the left-hand side, one arrives at

$$\frac{e^{-m_N{}^2/M^2}}{12m_N{}^2{}_N{}^2}\left[\frac{-3}{16}\frac{6M^4}{L^{4/9}}E_0+\frac{9\kappa aM^4}{2L^{68/81}}+\frac{a^2}{9}L^{4/9}M^2\right]$$
$$=\tfrac{1}{6}[7F-5D]-\frac{1}{12m_N{}^2}[5+(7F-5D)+B'$$
$$-6B'']M^2\ . \qquad (5.33)$$

The left-hand side of this sum rule for the case $a=0.45$ is illustrated in Fig. 16. It is seen that a straight line fit, $\lambda+\sigma M^2$, works very well. The best-fit values of $\lambda=\frac{1}{6}(7F-5D)$ and σ are displayed in Table IV. Notice that the λ value is very small numerically although F and D by themselves are not small compared to unity. Table IV therefore suggests the relation

$$7F\approx 5D\ . \qquad (5.34)$$

This in turn gives for the ratio

$$\alpha\equiv\frac{D}{F+D}=\frac{7}{12}\approx 0.58$$

which is quite close to the experimental number[17]

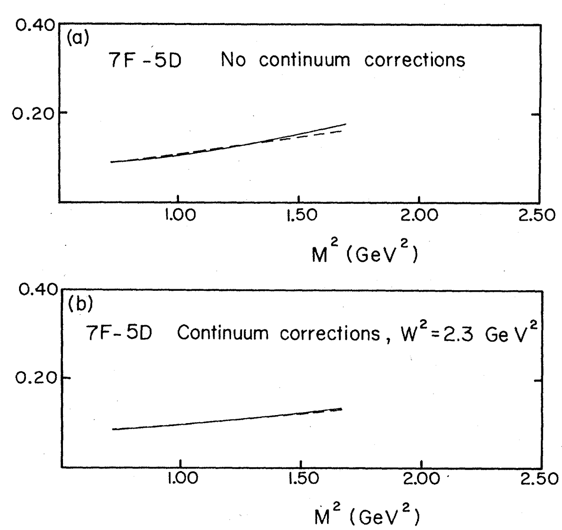


FIG. 16. The $\hat{Z}\gamma_5$ sum rule, Eq. (5.32) for the linear combination $\frac{1}{6}(7F-5D)$ in the SU_3-symmetric limit, with $\xi=1$. (a) The solid curve is the left-hand side of Eq. (5.32). The dashed curve is the right-hand side without including the excited-state contributions. (b) The left-hand side (solid curve) and the right-hand side (dashed curve) of Eq. (5.33) with $W^2=2.3$ GeV^2.

TABLE IV. Results from $\hat{Z}\gamma_5$ sum rule for $\frac{1}{6}(7F-5D)$ in SU(3) limit ($m_u=m_d=m_s=0$). The parameters $a=0.45$ GeV^3, $\tilde{\beta}_N{}^2=0.22$ GeV^6. In the symmetrical limit the baryon mass is 0.94 GeV and the fiducial region $0.9\leq M^2\leq 1.2$ GeV^2.

	No continuum corrections		Continuum approximation with $W^2=2.3$ GeV^2	
	$\xi=1$	$\xi=-2$	$\xi=1$	$\xi=-2$
$\frac{1}{6}(7F-5D)$	0.036	-0.003	0.053	-0.092
σ	0.075	-0.373	0.049	-0.242
$\frac{1}{6}(6B''-B')$	1.00	0.17	1.02	0.32

$\alpha=0.613\pm0.009$.

On the other hand, from Eq. (5.33)

$$\sigma=-\frac{1}{12m_N{}^2}[5+(7F-5D)+B'-6B''] .$$

The values of $\frac{1}{6}[B'-6B'']$ are given in Table IV for $a=0.45$ GeV^3 and different values of ξ. It suggests that the single-pole coefficients, B' and B'', should be of the order unity and therefore quite comparable to the double-pole terms; this is in sharp contrast to the $p\cdot Z\hat{p}\gamma_5$ sum rule. This confirms our remarks in Sec. III concerning the nature of the interference between the odd- and the even-parity states.

VI. DISCUSSION

In our analysis of the sum rules, we find both the strength and the weakness of the sum-rule approach for the determination of hadron properties. On the one hand, the sum rules illustrate some basic features of hadron dynamics clearly. For example, the fact that in the $SU_3\times SU_3$-chiral-symmetric limit, the coupling constant F tends to zero and D tends to unity is brought out by the structures of the hard-quark terms. In the limit where chiral $SU_3\times SU_3$ is broken, but flavor SU_3 remains intact, we find that the coupling D decreases to a value below unity, while the value of $F+D$ moves up from unity. These very general features emerge as a consequence of the structure of the OPE with and without the external field. The sum rules also suggest that chiral symmetry perhaps is realized in the Wigner-Weyl mode for the high-mass states.

On the other hand, to extract the precise values of hadron couplings from the sum rules is a difficult task. We are faced with several problems. First, we must make some approximation for the excited-state contributions in the right-hand side of the sum rules. Second, the duality principle in itself is only an approximate statement. We have taken this principle to mean, for example, in the case of the nucleon, the left-hand side and the right-hand side of the sum rule should match over the Borel mass variable $0.9\lesssim M^2\lesssim 1.2$ GeV^2. While it is obvious that the same fiducial region should also be applicable for the hyperon amplitudes in the SU_3-symmetric case, it is less clear in the experimentally relevant broken-symmetry situation what is the correct Borel-mass region over which this matching should be done. We have found that the results of the calculation change if we change this fiducial range, especially in the continuum method.

Nevertheless the following conclusions emerge from our calculations. It is worth stressing that the key parameter that enters in all calculations of baryon properties is the quark chiral condensate $\langle 0|\bar{q}q|0\rangle$. To a first approximation[2] the nucleon and isobar masses are proportional to $|\langle 0|\bar{q}q|0\rangle|^{1/3}$. Belyaev and Ioffe[3] in their analysis of the mass sum rule for the nucleon and isobar including a large number of terms in the OPE found that the computed masses came out uniformly higher than the experimental number and suggested that perhaps the value of a is overestimated by about 20%. The G_A sum rules confirm their observation and thus reducing a would help to bring both the mass calculation and the G_A calculation closer to experiment.

We have seen that in the presence of the external field Z_μ, counting operators to dimension 6, we have introduced two new vacuum expectation values, whose values are phenomenologically characterized by the susceptibilities χ and κ. Using PCAC the susceptibility χ is related[10] to the pion decay constant f_π. On the other hand, the determination of the susceptibility κ requires a knowledge of the matrix element $\langle 0|\bar{q}\tilde{G}_{\mu\nu}\gamma_\nu q|\pi\rangle$ which is not directly related to experiment. *A priori* we do not know even its order of magnitude. It is therefore impressive that the value obtained for this matrix element by Novikov *et al.*,[11] when employed in the G_A sum rules leads to sensible results. In fact, if we use the continuum method to analyze the sum rules and use the best fiducial region recommended in Ref. 3 with $0.9\leq M^2\leq 1.2$ GeV^2, the axial-vector renormalization constants D and F come out very close to the experimental values. Perhaps this is not altogether a coincidence.

Comparing the ratio method, which does not involve an explicit knowledge of $\tilde{\beta}_N{}^2$, and the continuum method, it is gratifying that the two methods are compatible if we recognize that the sum-rule procedure itself is only approximate. It is difficult to pinpoint the precise reason for the difference in the values of the renormalization constants determined by the two methods. It could arise from either of the following: (i) Errors in the value of $\tilde{\beta}_N$. (ii) The different methods of accounting for the excited-state contributions in the right-hand side.

We have seen that in the ratio method, the constant $D+F$ determined from the nucleon sum rules comes out somewhat higher than the experimental number. The susceptibility χ does not enter the nucleon sum rule. Therefore keeping it at its PCAC value, Eq. (5.2), if we decrease the susceptibility κ from the value given in Ref. 11, we find that the agreement with experiment improves. On the other hand, the discrepancy could also be due to higher-dimensional operators in the OPE in the left-hand side.

We have seen that the sum rules at the $\hat{Z}\gamma_5$ structure cannot be individually used to extract experimental information, because of the reversal of the sign of the G_A term on the right-hand side, as well as the relative importance of the single-pole terms. However, we have been able to combine several sum rules and determine the ratio $F/(F+D)$ to be approximately $\frac{7}{12}$, which is close to the experimental value of 0.6. Moreover, it also confirms the relative importance of the single-pole term as analyzed in Sec. III.

The analysis of the broken-SU_3 situation is considerably more complicated. Since the Cabibbo theory is empirically successful, it makes sense to investigate vertices like $\Sigma \to \Sigma$ and $\Xi \to \Xi$, which are of course not accessible for experimental measurement. On the other hand, the choice of these amplitudes simplifies theoretical analysis very considerably. If the initial and final baryons are very different in mass, then the distinction between the double pole whose residue contains the physical coupling of interest and the single-pole term is blurred. We have found that in the ratio method, the $D-F$ value obtained from the $\Xi \to \Xi$ amplitude is quite stable. In the continuum method, however, the $\Xi \to \Xi$ amplitude is less stable than the $\Sigma \to \Sigma$ amplitude.

As for future work, the following questions are worth pursuing. A global analysis of all baryon sum rules, for the masses, magnetic moments, and G_A value should help to determine quite accurately the value of the quark chiral condensate. Alternative calculation of the QCD vacuum susceptibilities especially κ would help to narrow down the uncertainties in the present calculation. Finally the amplitude $\Sigma \to \Lambda$ which is experimentally known is a realistic candidate for investigating the broken-SU_3-symmetry case, since the mass difference $m_\Sigma - m_\Lambda$ is only about 80 MeV, and moreover, the corresponding axial-vector coupling constant D is experimentally close to unity.

Note added. After the submission of the present manuscript for publication, our attention was called to a work by V. M. Belyaev, B. L. Ioffe, and Ya. I. Kogan [Phys. Lett. **151B**, 290 (1985)]. In this paper, taking into account those terms proportional to $f_\pi^2 m_1^2$, these authors stated that they had reevaluated the sum rules of Ref. 10 and obtained $F=0.45$ and $D=0.95$. This solution is essentially the same as our solution given in column one of Table III.

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APPENDIX

In the text we need to calculate the Fourier transforms of expressions like

$$\frac{f(x)}{(x^2-i\epsilon)^n}\,, \tag{A1}$$

where ϵ is a positive infinitesimal supplying the appropriate boundary conditions and $f(x)=1$, x_α, $x_\alpha x_\beta$. We define the following bracket symbol

$$[f]_{2n} = \int d^4x\, e^{ip\cdot x} \frac{f(x)}{(x^2-i\epsilon)^n}\,. \tag{A2}$$

Equation (A2) is most easily computed by introducing a parameter a^2 and evaluating

$$\int d^4x\, e^{ip\cdot x} \frac{1}{(x^2-a^2-i\epsilon)}\,. \tag{A3}$$

The integral in Eq. (A3) is well known. To obtain the value of Eq. (A2) one merely differentiates (A3) with respect to $a^2 n$ times and sets $a^2=0$.

One obtains

$$[1]_{2n} = C_n (p^2)^{n-2} \ln(-p^2)\,, \tag{A4}$$

where

$$C_n = \frac{i\pi^2(-1)^n}{(n-2)!(n-1)!4^{n-2}}\,. \tag{A5}$$

Similar formulas for $[x_\alpha]_{2n}$, $[x_\alpha x_\beta]_{2n}$, etc., may be obtained from Eq. (A4) by differentiating with $-i\,\partial/\partial p_\alpha$, $(-i\,\partial/\partial p_\alpha)(-i\,\partial/\partial p_\beta)$, etc. For convenience we write out a few:

$$\begin{aligned}
&[1]_2 = -4i\pi^2 \frac{1}{p^2}\,,\\
&[x_\alpha]_2 = 8\pi^2 \frac{p_\alpha}{p^4}\,,\\
&[x_\alpha x_\beta]_2 = -8i\pi^2 \left[\frac{g_{\alpha\beta}}{p^4} - 4\frac{p_\alpha p_\beta}{p^6}\right],
\end{aligned} \tag{A6}$$

$$\begin{aligned}
&[1]_4 = i\pi^2 \ln(-p^2)\,,\\
&[x_\alpha]_4 = 2\pi^2 \frac{p_\alpha}{p^2}\,,\\
&[x_\alpha x_\beta]_4 = -2i\pi^2 \left[\frac{g_{\alpha\beta}}{p^2} - 2\frac{p_\alpha p_\beta}{p^4}\right],
\end{aligned} \tag{A7}$$

$$\begin{aligned}
&[1]_6 = \frac{-i\pi^2}{8} p^2 \ln(-p^2)\,,\\
&[x_\alpha]_6 = -\frac{\pi^2}{4} p_\alpha \ln(-p^2)\,,\\
&[x_\alpha x_\beta]_6 = \frac{i\pi^2}{4} \left[g_{\alpha\beta} \ln(-p^2) + 2\frac{p_\alpha p_\beta}{p^2}\right],
\end{aligned} \tag{A8}$$

$$\begin{aligned}
&[1]_8 = \frac{i\pi^2}{3\times 2^6} p^4 \ln(-p^2)\,,\\
&[x_\alpha]_8 = \frac{\pi^2}{3\times 2^4} p_\alpha p^2 \ln(-p^2)\,,\\
&[x_\alpha x_\beta]_8 = \frac{-i^2}{3\times 2^4} [g_{\alpha\beta} p^2 \ln(-p^2) + 2p_\alpha p_\beta \ln(-p^2)]\,,
\end{aligned} \tag{A9}$$

$$[1]_{10}=\frac{-i\pi^2}{3^2\times 2^{10}}p^6\ln(-p^2)\ ,$$

$$[x_\alpha]_{10}=-\frac{\pi^2}{3\times 2^9}p_\alpha p^4\ln(-p^2)\ , \qquad \text{(A10)}$$

$$[x_\alpha x_\beta]_{10}=\frac{i\pi^2}{3\times 2^9}(g_{\alpha\beta}p^4+4p_\alpha p_\beta p^2)\ln(-p^2)\ .$$

The Borel transform is defined by the following limit,

$$B[f(p^2)] = \lim_{\substack{n\to\infty \\ -p^2\to\infty \\ -p^2/n=M^2 \text{ fixed}}} \frac{1}{n!}(-p^2)^{n+1}\left[\frac{d}{dp^2}\right]^n f(p^2)\ . \qquad \text{(A11)}$$

Note that p^2 is assumed to be spacelike. For the right-hand side of the sum rules which are written in the form of a dispersion integral, the Borel transform is given simply by

$$B[f(p^2)]=\frac{1}{\pi}\int_0^\infty ds\,\mathrm{Im}f(s)e^{-s/M^2}\ . \qquad \text{(A12)}$$

On the left-hand side there are two types of terms that occur. Their Borel transforms are

$$B\left[\frac{1}{(p^2+i\epsilon)^m}\right]=\frac{(-1)^m}{(m-1)!}\frac{1}{(M^2)^{m-1}}\ , \qquad \text{(A13)}$$

$$B\left[(p^2)^m\ln\left[-\frac{1}{p^2}\right]\right]=m!(M^2)^{m+1}\ . \qquad \text{(A14)}$$

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