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# Muon capture in a general class of weak models

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We study muon capture by $^{12}$C in a general class of weak models. There is always a parameter characteristic of the weak model that can be extracted in a nuclear-model-independent way from the average polarization $\mathbf{P}_{av}$, the longitudinal polarization $P_L^N$ and the asymmetry $\alpha$ in the angular distribution of recoils. For a less general class of models the asymmetry $\alpha$ is unnecessary. Using the experimental values of $P_L^N$ and $\mathbf{P}_{av}$ we get a lower bound for the mass of the right-handed gauge boson of the left-right-symmetric model, $M_{W_R} \geq 2.5 M_{W_L}$, in a nuclear-model-independent way. The dependence of this bound on the experimental values is also discussed.

In a recent series of papers[1,2] the relevance of muon capture in analyzing the weak charged currents structure has been shown. In Ref. 1 it was shown how to analyze the muon capture by $^{12}$C in a model-independent way. In Ref. 2 an upper bound was extracted for the mixing of the charged gauge boson of the left-right-symmetric model using the available experimental data.

As a consequence of the large momentum transfer in the process $^{12}\mathrm{C}(\mu^-,\nu_\mu)^{12}\mathrm{B}$(g.s.) it is possible to study the magnitude of the induced pseudoscalar current—which is inaccessible in $\beta$-decay experiments. That is why this is a field of current experimental activity,[3] and this means that it is worthwhile to analyze in more detail the possibilities of the muon-capture process to provide information about the precise structure of the weak interactions.

In this paper we present a calculation of all the observables in the muon-capture process $^{12}\mathrm{C}(\mu^-,\nu_\mu)^{12}\mathrm{B}$(g.s.) in the most general class of weak models with an effective Lagrangian build up with vector and axial-vector currents. The calculation is performed in a nuclear model-independent way, that is, using covariant form factors for the nuclear matrix elements. The first interesting result we arrive at is that for this general class of models there is a combination of parameters of the weak model that can be extracted, independent of the form factors, from the experimental values of a complete experiment;[1] that is, measuring the rate $\Gamma$, the average and longitudinal recoil polarization $\mathbf{P}_{av}$ and $P_L^N$, and the angular asymmetry $\alpha$. The asymmetry $\alpha$ has not been measured, so we show that by a suitable restriction of the weak model there exists a less general class of models where the previous result also applies, but only using the well-known experimental observables $\Gamma$, $\mathbf{P}_{av}$, $P_L^N$; in fact, $\Gamma$ is not necessary. Among this last class of models there is the manifest-left-right-symmetric model[4] in the limit of vanishing mixing between left- and right-handed charged gauge bosons, so using $\mathbf{P}_{av}$ and $P_L^N$ we get a lower bound for mass of the right-handed bosons, $M_{W_R} \geq 2.5 M_{W_L}$. Finally we analyze the sensitivity of this bound to the precision of $\mathbf{P}_{av}$ and $P_L^N$.

The most general effective Lagrangian for the process $l+u \to \nu_l + d$ with the structure "current$\times$current" is, using vector and axial-vector currents,

$$\mathscr{L} = -\frac{G_F \cos\theta_C}{\sqrt{2}}[C_V(\bar{\nu}\gamma^\mu l)(\bar{d}\gamma_\mu u) + C_A(\bar{\nu}\gamma^\mu\gamma_5 l)(\bar{d}\gamma_\mu\gamma_5 u) + C_V'(\bar{\nu}\gamma^\mu\gamma_5 l)(\bar{d}\gamma_\mu u) + C_A'(\bar{\nu}\gamma^\mu l)(\bar{d}\gamma_\mu\gamma_5 u)] \ . \tag{1}$$

In order to avoid nuclear-model dependences we parametrize the hadronic matrix elements in this way:[5]

$$\begin{aligned} \langle \mathbf{P}_2, 1^+, \lambda \mid V^\mu \mid \mathbf{P}_1, 0^+ \rangle &= i\frac{F_M(q^2)}{4Mm_P}\epsilon^{\mu\nu\alpha\beta}\xi_\nu^*(\lambda)k_\alpha q_\beta \ , \\ \langle \mathbf{P}_2, 1^+, \lambda \mid A^\mu \mid \mathbf{P}_1, 0^+ \rangle &= -F_A(q^2)\xi^{\mu *}(\lambda) + F_P(q^2)\frac{(q\xi^*)}{m_\pi^2}q^\mu - F_E(q^2)\frac{(q\xi^*)}{4Mm_p}k^\mu \ . \end{aligned} \tag{2}$$

 

$V^{\mu}$ and $A^{\mu}$ are the vector and axial-vector quark currents, $\xi^{\mu}(\lambda)$ is the polarization vector of the spin-1 state, $q^{\mu}$ is the momentum transfer, and $k^{\mu}$ is the sum of both $0^+$ and $1^+$ nuclear moments. In what follows we will use the same notation as in Refs. 1 and 2, for undefined magnitudes. The relevant combinations of covariant form factors entering in the amplitudes are:[6]

$$G_V=-\frac{E^{(\nu)}}{2m_P}F_M\ ,$$
$$G_A=-F_A-\frac{E^{(\nu)}}{2m_p}F_M\ , \tag{3}$$
$$G_P=\frac{E^{(\nu)}m_\mu}{m_\pi{}^2}F_P-\frac{E^{(\nu)}}{2m_p}F_E-\frac{E^{(\nu)}}{2m_p}F_M\ ,$$

$E^{(\nu)}$ is the neutrino energy, and this expression tells us that all the form factors except $F_A$ are induced ones. In Ref. 1 all the observables are written in terms of the four independent reduced helicity amplitudes $T(\lambda_N,\lambda_\nu)$ where $\lambda_N$ and $\lambda_\nu$ refer to the helicities of the $1^+$ recoiling nucleus and the neutrino, respectively. Using the same method presented in Ref. 2 we get up to a global normalization

$$T(1,\tfrac{1}{2})=-\sqrt{2}N\beta X',\quad T(-1,-\tfrac{1}{2})=\sqrt{2}N\ ,$$
$$T(0,\tfrac{1}{2})=-N\beta X,\quad T(0,-\tfrac{1}{2})=NX\ , \tag{4}$$

where the four combinations of parameters entering in this equation are

$$N=\frac{C_A+C_A'}{2}(G_A-t_+G_V)\ ,$$
$$X=\frac{G_A-G_P}{G_A-t_+G_V}\ ,$$
$$X'=\frac{G_A-t_-G_V}{G_A-t_+G_V}\ , \tag{5}$$
$$\beta=\frac{C_A-C_A'}{C_A+C_A'}\ ,$$
$$t_\mp=1-\frac{C_V}{C_A}\left[\frac{1\mp C_V'/C_V}{1\mp C_A'/C_A}\right]\ ,$$

where the set of Eqs. (4) and (5) with the relations among observables and reduced helicity amplitudes obtained in Ref. 1 provides us with a calculation of all the observables of muon capture in the general class of models defined by Lagrangian (1). Equations (4) and (5) are independent of any nuclear model, all the nuclear dependences are under control by the set of parameters defined in Eq. (3). It must be emphasized that this set of equations is valid for any nuclear transition $0^+\rightarrow 1^+$ where we can neglect $E^{(\nu)}$ and the muon mass $m_\mu$ with respect to the nuclear average mass.[7] As a check of Eqs. (4) and (5) we note that in the parity conservation limit we must have $T(\lambda_N,\lambda_\nu)=-T(-\lambda_N,-\lambda_\nu)$, this relation implies $C_A'=C_V'=0$ or $C_A'=G_V=0$. Both eliminate the parity violating pieces coming from the Lagrangian. As long as we are involved with time-reversal-invariant interactions—we consider here only this type of model—the amplitudes $T(\lambda_N,\lambda_\nu)$ are relatively real,[1] and so only four observables are independent, for example, $\Gamma$, $\mathbf{P}_{\rm av}$, $P_L^N$, and $\alpha$. In this case the parameters entering in the right-hand side of Eq. (4) are real and looking at Eq. (5) it is evident that *with a complete experiment we can extract, independent of any nuclear assumption, the parameter* $\beta$ *which only depends on the weak model under consideration.* Having in mind that $N$ only enters in the rate of capture $\Gamma$ we see that in order to get $\beta$ we need $\mathbf{P}_{\rm av}$, $P_L^N$, and $\alpha$. It is evident that Eqs. (22) of Ref. 1 gives us a functional relation of the type[8] $f(\beta;\mathbf{P}_{\rm av},P_L^N,\alpha)=0$.

As an example of a model where $\beta$ is an interesting parameter we have for the manifest-left-right-symmetric model[9]

$$C_V=C[(1-\sin2\xi)+\Delta(1+\sin2\xi)]\ ,$$
$$C_A=C[(1+\sin2\xi)+\Delta(1-\sin2\xi)]\ , \tag{6}$$
$$C_A'=C_V'=C(1-\Delta)\cos2\xi\ ,$$

where $\xi$ is the mixing angle between the left- and right-handed gauge bosons and $\Delta$ is the ratio of their masses $\Delta=(M_{W_L}/M_{W_R})^2$. The constant $C$ gets the value $C=(1+\Delta^2)^{-1/2}$ if it is used in the muon decay rate to define the Fermi coupling constant $G_F$ (Ref. 9). So using the results of the previous analysis we can conclude that *measuring the asymmetry* $\alpha$ it would be possible to get the combination of $\Delta$ and $\xi$ defined by

$$\beta=\frac{\tan\xi(1+\tan\xi)+\Delta(1-\tan\xi)}{(1+\tan\xi)-\Delta\tan\xi(1-\tan\xi)}\ , \tag{7}$$

in a nuclear-model-independent way. This type of analysis would represent an interplay between the usual muon-decay and $\beta$-decay experiments[10] in the sense that we are using a semileptonic process involving the muon family.

As has been shown elsewhere,[11] in the standard model there are only two independent observables $\Gamma$ and $\mathbf{P}_{\rm av}$, and in the general case there are four. We have experimental values for $\Gamma$, $\mathbf{P}_{\rm av}$ and $P_L^N$ so in order to use now these observables to obtain information on possible extensions of the standard model, there are two strategies one may pursue. The first is to use some nuclear information, either theoretical or experimental, from related processes such as $\beta$ decay. This was the line pursued in Ref. 2. The second strategy is to restrict in some suitable way the general Lagrangian (1) in order to have a model with only three independent parameters so that we have the chance that $\Gamma$, $\mathbf{P}_{\rm av}$, and $P_L^N$ affords us with a complete experiment. In other words the idea is to restrict the weak model in such a way that the four parameters in the right-hand side (RHS) of Eq. (4) become three only. Looking at the set of Eq. (5) and having in mind that we can only impose conditions on the weak model parameters it is evident that the only possibility there is to impose $t_-=t_+$, so that we get $X'=1$. In turn this means we have restricted the general class of models of Lagrangian (1) to the models that verify

$$\frac{C_V'}{C_V}=\frac{C_A'}{C_A}\ . \tag{8}$$

The bonus we get from Eq. (8) is that we have eliminated one parameter $X'$ that depends on the nuclear structure. We are left with a class of models where the number of independent observables is three. By explicit calculation we will check later that $\Gamma$, $\mathbf{P}_{\rm av}$, and $P_L^N$ are in fact independent, so these three experimental observables represent a complete experiment for the class of models described by Eq. (8) and for these models we can extract the parameter $\beta$. This only contains information on the weak model.

Among the theories that verify the set of Eqs. (1) and (8) is the left-right-symmetric model in the case of vanishing mixing among the left and right charged gauge bosons, even in the case that we include an additional Cabibbo-type angle for the right-handed sector. Restricting ourselves to the so-called "manifest-left-right-symmetric model" the condition (8) can be realized by the set of Eqs. (6) provided $\xi=0$, and so we have[12]

$$\beta=\Delta=\left[\frac{M_{W_L}}{M_{W_R}}\right]^2 \quad X=\frac{G_A-G_P}{G_A} . \tag{9}$$

Using Eqs. (9) and (4) with $X'=1$ and the results of Ref. 1 we get

$$\Gamma=\Gamma^0 G_A{}^2(2+X^2)$$

$$P_L^N=-\frac{2}{(2+X^2)}\left[\frac{1-\Delta^2}{1+\Delta^2}\right]=\frac{2}{2+X^2}P_L^\nu , \tag{10}$$

$$\mathbf{P}_{\rm av}=P^N\mathbf{P}_\mu=\frac{2}{3}\left[\frac{1+2X}{2+X^2}\right]\mathbf{P}_\mu ,$$

where $\Gamma^0$ is defined in Ref. 2 and takes account of the muon wave function at the origin and $\mathbf{P}_\mu$ is the polarization of the muon in the $1S$ shell.

As we have mentioned before, $\Gamma$ fixes the global normalization of the form factors and looking at Eq. (10) it is apparent that $P^N$ and $P_L^N$ are independent and so we conclude that with these experimental values *it is possible to get in a nuclear-model-independent way a lower bound for the $W_R$ mass of the manifest-left-right-symmetric model in the limit of vanishing mixing.* It is worthwhile to point out that from nonleptonic $\Delta S=1$ weak decays and from semileptonic decays (see Ref. 18) there have been derived, respectively, the bounds $|\xi|\leq 0.004$ and $|\xi|\leq 0.005$, so that Eq. (8) is a very good approximation for the left-right model considered here.

In the set of Eq. (10) we have also included implicitly the value of the neutrino longitudinal polarization[1] $P_L^\nu$ in order to simplify the numerical analysis. In fact, the parametrization (10) is the same that has been used[13] to extract the neutrino helicity—the neutrino longitudinal polarization $P_L^\nu$ in our terminology.[14] So if we take into account, from Eq. (10) that

$$P_L^\nu=-\left[\frac{1-\Delta^2}{1+\Delta^2}\right] \tag{11}$$

and the reported "experimental"[13] value $P_L^\nu=-1.06\pm0.11$ we immediately get the lower bound $\Delta\leq 0.16$ or $M_{W_R}\geq 2.5\, M_{W_L}$. If we do not take into account the lower bound extracted from the $K_L$-$K_S$ mass difference,[15], the best experimental bound for $M_{W_R}$ comes from the muon-decay experiment of Carr *et al.*[10] and in the case of neglecting the mixing $\xi$ the bound they get is $M_{W_R}\geq 5.7 M_{W_L}$. It must be emphasized once more that the bound obtained in this work has its own interest because it comes from a different piece of the electroweak Lagrangian. In any case, keeping in mind the current experimental activity in the muon capture by $^{12}$C let us analyze the dependence of $\beta$ on the experimental values.

It is apparent from Eq. (10) that if $X$ were equal to zero it would be the upper limit of $P_L^N$ that would fix the upper limit on $\Delta$. This is not the case although $X$ is of the order of 0.26, so we have plotted in the Fig. 1, $\Delta$ as a function of $X$ for different values of $P_L^N$. The reported value of $P_L^N$ in Ref. 13 is $P_L^N=-1.02\pm0.11$ so the upper bound of $P_L^N$ is $-0.91$ and is the region under this curve ($P_L^N=-0.91$) that is allowed by the $P_L^N$ value. The corresponding $X$ value[13] is $X=0.268\pm0.062$, so the shadowed region of the figure is the one allowed by experiment and gives the correlated bounds on $\Delta$ coming from $P_L^N$ and $X$. The previously given value $\Delta\leq 0.16$ corresponds to the average of this correlated upper bound shown in the figure.

It is apparent also from the figure that the essential parameters to set an upper bound on $\Delta$ is the upper limit of $P_L^N$, although the lower bound of $X$ may also be relevant. For illustrative purposes we have plotted $\Delta(X)$ for $P_L^N=-0.96$ in this case the upper bound of $\Delta$ is more sensitive to the lower bound of $X$ because the $\Delta(X)$ function has a big dumping at the end point. From the figure we can see that by improving the upper bound of $P_L^N$ to the value $-0.96$, it would be possible to reach an average upper bound for $\Delta$ of the order of $\Delta\leq 0.06$.

We must also point out that the experimental values previously used correspond to the experimental values suitably corrected to take into account the muon capture leading to higher excited states of the $^{12}$B (Ref. 13). So we have put ourselves in the worst situation, in other words, we have taken from Ref. 13 the set of values that gives the poorest upper bound for $\Delta$. If, for example, we had used $X=0.26\pm0.06$ and $P_L^N=-1.04\pm0.11$ that means $P_L^\nu=-1.08\pm0.11$ we would have $\Delta\leq 0.12$, that is $M_{W_R}\geq 2.9 M_{W_L}$.

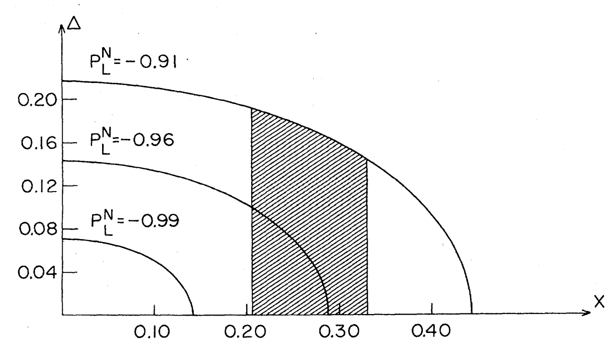


FIG. 1. The curves represent $\Delta(X)$ for different values of $P_L^N$. The shaded region is the experimentally allowed region and gives us the upper bound on $\Delta$.

Before concluding, in order to check the consistency of the set of Eq. (10) with the way $X$ is extracted from the experimental values we must make some additional comments on this last topic. In fact, the set of experimental values are two linear combinations of $\mathbf{P}_{av}$ and $P_L^N \hat{P}_\mu$ and a magnitude related to $\mathbf{P}_\mu$ of $\mathbf{P}_\mu$ itself.[13] Because the ratio of $\mathbf{P}_{av}$ and $P_L^N$ is less affected by excited states corrections, the best way to make the numerical analysis is to use this ratio and either $\mathbf{P}_{av}$ or $P_L^N$. As long as $\mathbf{P}_\mu$ is measured directly the obvious way is to use $\mathbf{P}_{av}$ to get $X$ and the ratio, with $X$, to get $\Delta$. But this is not the case in Ref. 13. If we define $\mathbf{P}_\mu = k\mathbf{P}_\mu(\pi)$, where $\mathbf{P}_\mu(\pi)$ is the muon polarization in the $\pi$ decay, what they measured is the depolarization factor $k$, so it is a straightforward calculation in the model under consideration that from pion decay[17]

$$\mathbf{P}_\mu(\pi) = \left[ \frac{1-\Delta^2}{1+\Delta^2} \right] \hat{\mathbf{P}}_\mu \,. \tag{12}$$

In this case we can see that the ratio measured in Ref. 13

$$R = \frac{\mathbf{P}_{av} \cdot \hat{\mathbf{P}}_\mu}{k P_L^N} = -\tfrac{1}{3}(1+2X) \tag{13}$$

is independent of $\Delta$ and so we must use $R$ to extract $X$ and $P_L^N$ to get $\beta$. The values of $R$ corresponding to the $X$ value used in the figure is $R = -0.512 \pm 0.041$. Note that it has been crucial to check Eq. (12) in the model used here in order to translate here, *mutatis mutandis*, the numerical analysis performed in Ref. 13.

In conclusion we can say that muon capture by $^{12}C$ has shown itself to be a powerful tool in the analysis of the structure of the weak interactions. The crucial point is that for the models described by Lagrangian (1) there always exists a parameter of the weak model that can be extracted from a complete set of experimental data independent of the way we describe microscopically the nuclear degrees of freedom. So we must stress once more that by measuring the angular asymmetry $\alpha$ this kind of experiment can be used to analyze the structure of weak interactions. To have a numerical idea of which type of bounds can be obtained we have restricted ourself to a less general class of models in order to use the actual experimental values. For the manifest-left-right-symmetric model neglecting the mixing between left- and right-handed charged bosons we get the conservative bound $M_{W_R} \geq 2.5 M_{W_L}$, of course, in a nuclear-model-independent way. We have also shown that by improving the upper bounds of the longitudinal polarization a little it will be possible to reach precisions comparable with other types of experiments that prove different combinations of the weak currents.

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[8]Equation (22) of Ref. 1 gives us a functional relation of the type $f(A_L, P_L^N, \mathbf{P}_{av}, \alpha) = 0$, where $A_L$ is the longitudinal alignment. It is a straightforward exercise to get

$$A_L = 1 + (3P_L^N - \alpha)(1+\beta^2)(1-\beta^2)^{-1}$$

so that using the last equation we get $f(\beta, \mathbf{P}_{av}, P_L^N, \alpha) = 0$.

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[12]In a nonmanifest-left-right-symmetric model with two generations and an additional right-handed Cabibbo-type angle, no *CP* violation, Eq. (9) must be replaced by

$$\beta = \Delta \frac{\cos\theta_R}{\cos\theta_L}$$

and as long as the Cabibbo angle $\cos\theta_L \sim 1$ an upper bound on $\beta$ will translate in a worst upper bound for $\Delta$.

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[14]To avoid misunderstanding we want to emphasize that the parametrization (10) used in Ref. 13 is not valid in a general model. This is shown in Ref. 5. It is valid accidentally in our particular model because essentially all of the parity-violating observables become renormalized multiplicatively by the factor

$$\left[ \frac{1-\Delta^2}{1+\Delta^2} \right] = -P_L^\nu \,.$$

[15]Of course, the best lower bound for $M_{w_R}$ comes from the mass difference $K_L - K_S$. This bound is placed in the TeV region. Independent of the gauge problems in the box graph, which appear to be correctable without drastic changes (Ref. 16), we must stress that this bound is not the result of a fit to the ex-

perimental values, but it is the result of assuming that the left-right box graph must give, by itself, a contribution smaller than the standard box graph. Although this assumption is extremely reasonable, we must say that this lower bound is in some sense model dependent. On the contrary we must also add that the bounds considered in the present paper are only valid for Dirac neutrinos, although the one obtained from the box graph is independent of the neutrino type.

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