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($V+A$) components from the measured observables in muon capture

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We use the data on the average polarization and the longitudinal polarizaton of recoil in the process $^{12}C(\mu^-,\nu_\mu)^{12}B$ to extract independent values for leptonic ($V+A$) currents and induced pseudoscalar hadronic coupling. Present results give $|\xi|<0.25$ for the mixing parameter of the left-right-symmetric model, whereas f_P is consistent with PCAC (partial conservation of axial-vector current). The interest of better precision in the longitudinal polarization, as well as the measurement of the asymmetry in the angular distribution, is discussed.

I. INTRODUCTION

The standard electroweak theory has met considerable success, the last great triumph being the discovery of $W^\pm$ and Z^0 at the predicted masses. The small deviations from the standard predictions allowed experimentally should be contemplated because of their possible deep implications. Alternative theoretical frameworks, such as the one provided by the manifest-left-right-symmetric model,[1] have received attention and its parameters have been bounded through muon decay and nuclear β decay. It is worthwhile to analyze the informational content of other observables in semileptonic processes, such as muon capture, in a theory such as $SU(2)_L\times SU(2)_R\times U(1)$ going beyond the standard description.

Recently,[2] the relevance of muon capture to study the space-time structure of the weak charged currents has been discussed independent of any prejudices. Four independent observables, such as the rate Γ, the average polarization of recoil P^N, the longitudinal polarization P_L^N, and the asymmetry α of the angular distribution provide a convenient set to describe a complete experiment. The dynamical information contained in these observables for the parameters of the left-right-symmetric model is the subject of this paper.

The nuclear observables in the process $^{12}C(\mu,\nu_\mu)^{12}B$ fix the neutrino polarizations, so deviations from the standard longitudinal polarization $P_L^\nu=-1$ and transverse polarization $P_{x'}^\nu=0$ give information about electroweak models. At present one has experimental values only for Γ, P^N, P_L^N, so we need to use additional information from other sources.

As we will see later, using data from β decay, we will get a bound for the mixing angle ξ of the left-right-symmetric model in the limiting case where only one intermediate boson operates, that is, $M_{W_1}/M_{W_2}\to 0$.

In Sec. II we calculate, in a nuclear-model-independent way, the reduced helicity amplitudes for the left-right-symmetric model. In Sec. III we use muon-capture observables and β-decay data to extract the ξ mixing and the pseudoscalar induced coupling.

II. DYNAMICAL ANALYSIS

If $W_L^\pm, W_R^\pm$ are fields associated with the intermediate bosons linked to $SU(2)_L$ and $SU(2)_R$, when the symmetry is spontaneously broken, the bosons with a definite mass are $W_1^\pm$, $W_2^\pm$ in such a way that

$$\begin{pmatrix} W_L^\pm \\ W_R^\pm \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix} . \qquad (1)$$

In the limit $M_{W_1}/M_{W_2}\to 0$ the effective Lagrangian at low energies for the muon-capture process is[3]

$$L_{CC}=-\frac{G\cos\theta_C}{\sqrt{2}}[\bar\nu\gamma^\mu(1+\gamma_5)\mu+\epsilon\bar\nu\gamma^\mu(1-\gamma_5)\mu] \times[\lambda_V V_\mu+\lambda_A A_\mu] , \qquad (2)$$

where

$$\epsilon=-\tan\xi ,$$
$$\lambda_V=\tfrac{1}{2}[1+\cos2\xi-\sin2\xi] , \qquad (3)$$
$$\lambda_A=\tfrac{1}{2}[1+\cos2\xi+\sin2\xi] .$$

In this Lagrangian the deviations from a pure $V-A$ structure are due to the mixing (1). As a consequence, Eq. (2) contains the dependence on the mixing angle ξ. The parametrization (2) is also relevant for the discussion of the mirror-fermion mixing model.[4] Notice that the structure of the Lagrangian is still of the current-current form.

In the scattering approach, taking the muon at rest and factorizing the muon wave function at the origin, the amplitude for our process will be proportional to

$$f=\frac{G\cos\theta_C}{\sqrt{2}}\bar u_\nu(\lambda_\nu)\gamma_\mu[(1+\gamma_5)+\epsilon(1-\gamma_5)]u_\mu(m) \times\langle 1^+,\lambda_N\,|\,\lambda_V V^\mu+\lambda_A A^\mu\,|\,0^+\rangle , \qquad (4)$$

where the hadronic vertex is parametrized, in a nuclear-model-independent way, using the elementary-particle approach[5]

$$\langle \mathbf{p}_2,1^+,\lambda_N\,|\,V^\mu(0)\,|\,\mathbf{p}_1,0^+\rangle=i\frac{F_M(q^2)}{4Mm_p}\epsilon^{\mu\nu\alpha\beta}\xi_\nu^*(\lambda_N)k_\alpha q_\beta ,$$
$$\langle \mathbf{p}_2,1^+,\lambda_N\,|\,A^\mu(0)\,|\,\mathbf{p}_1,0^+\rangle =-F_A(q^2)\xi^{\mu*}(\lambda_N) +F_p(q^2)\frac{(q\cdot\xi^*)}{m_\pi{}^2}q^\mu-F_E(q^2)\frac{(q\cdot\xi^*)}{4Mm_p}k^\mu , \qquad (5)$$

where k^μ and q^μ are the sum and the difference of the four-momenta of the initial nucleus ($J^P=O^+$) and the final one ($J^P=1^+$). $\xi_\mu(\lambda_N)$ is the polarization vector of the nucleus of spin 1. The $F_i(q^2)$ are the so-called covariant form factors. Our conventions are such that our form factors differ by a $(-2M)$ factor from the Hwang ones,[6] where M is the mass average of the initial and final nuclei.

Owing to the large neutrino energy in this process, we neglect a possible muon-neutrino mass. This implies that all deviations from the definite-neutrino-helicity situation come from (2) and not from mass effects. So the muon and neutrino spinors, up to a global normalization, are

$$u_\mu(m)=\begin{bmatrix} v^{(\mu)}(m) \\ 0 \end{bmatrix}, \quad u_\nu(\lambda_\nu)=\begin{bmatrix} v^{(\nu)}(\lambda_\nu) \\ \boldsymbol{\sigma}\cdot\hat{\boldsymbol{\nu}}v^{(\nu)}(\lambda_\nu) \end{bmatrix} \tag{6}$$

where v are Pauli bispinors and the momentum of the neutrino is $\boldsymbol{\nu}\equiv E^{(\nu)}\hat{\boldsymbol{\nu}}$. If we write

$$f=\frac{G\cos\theta_C}{\sqrt{2}}(f_{V-A}+\epsilon f_{V+A}) \tag{7}$$

neglecting $E^{(\nu)}/M$ and m_μ/M it is straightforward to obtain

$$\begin{aligned} f_{V-A}&=v^{(\nu)+}(\lambda_\nu)(1-\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\nu}})\xi^*(\lambda_N) \\ &\quad\times(G_A\boldsymbol{\sigma}+G_p\hat{\boldsymbol{\nu}})v^{(\mu)}(m)\ , \\ f_{V+A}&=v^{(\nu)+}(\lambda_\nu)[1+\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\nu}}]\xi^*(\lambda_N) \\ &\quad\times[(-G_A+2G_V)\boldsymbol{\sigma}+(G_P-2G_V)\hat{\boldsymbol{\nu}}]v^{(\mu)}(m)\ , \end{aligned} \tag{8}$$

where G_V, G_A, and G_p are the combinations of covariant form factors used by Hwang,[6] except for the redefinition of the hadronic currents in Eq. (4), i.e.,

$$\begin{aligned} G_V&=-\lambda_V\frac{E^{(\nu)}}{2m_p}F_M(q^2)\ , \\ G_A&=-\lambda_A(q^2)-\lambda_V\frac{E^{(\nu)}}{2m_p}F_M(q^2)\ , \\ G_P&=\lambda_A\left[\frac{m_\mu E^{(\nu)}}{m_\pi^2}F_P(q^2)-\frac{E^{(\nu)}}{2m_p}F_E(q^2)\right] \\ &\quad-\lambda_V\frac{E^{(\nu)}}{2m_p}F_M(q^2)\ . \end{aligned} \tag{9}$$

In Ref. 2, the nuclear observables were calculated in terms of the reduced helicity amplitudes $T_\lambda=T(\lambda_N,\lambda_\nu)$ defined by

$$f\equiv f_{\lambda;m}(\theta,\phi)\equiv\left[\frac{1}{2\pi}\right]^{1/2}\mathscr{D}^{(1/2)*}_{m,\lambda}(\phi,\theta,0)T_\lambda \tag{10}$$

from which one easily shows

$$T(\lambda_N,\lambda_\nu)=(2\pi)^{1/2}f_{(\lambda_N,\lambda_\nu);m=\lambda_N-\lambda_\nu}(\theta=0,\phi=0)\ . \tag{11}$$

Now using Eq. (8) we get the central result of this calculation, giving the helicity amplitudes in terms of the ϵ parameter and three combinations of form factors

$$\begin{aligned} &T(-1,-\tfrac{1}{2})=\sqrt{2}G_A\ , \quad T(0,-\tfrac{1}{2})=(G_A-G_P)\ , \\ &T(1,\tfrac{1}{2})=\epsilon\sqrt{2}(G_A-2G_V), \quad T(0,\tfrac{1}{2})=\epsilon(G_A-G_P)\ . \end{aligned} \tag{12}$$

The connection (12) has to be understood up to a global normalization factor. This missing factor is only relevant for the capture rate, and it is given by[7]

$$\begin{aligned} \Gamma^{\text{cap}}&=\Gamma^0\Gamma\ , \\ \Gamma^0&=\frac{(G\cos\theta_C)^2}{16\pi^2}\left[Z\alpha\frac{m_\mu M_1}{m_\mu+M_1}\right]^3\frac{[(M_1+m_\mu)^2-M_2{}^2]^2}{M(M_2+m_\mu)^3} \\ &\quad\times\left[\frac{Z_{\text{eff}}}{Z}\right]^4\ , \end{aligned} \tag{13}$$

$$\left[\frac{Z_{\text{eff}}}{Z}\right]^4=0.856\ .$$

M_1 and M_2 are the masses of the initial and final nuclei and Γ^0 takes into account the initial-state interaction. Γ is the sum of the squares of the amplitudes of Eq. (12).

As expected, the amplitudes coupled to the $\lambda_\nu=+\frac{1}{2}$ helicity state of the neutrino are proportional to ϵ, so that they get a vanishing value in the limit where we have no mixing ($\xi=0$), that is, in the $(V-A)$ limit. It is interesting to point out that, with relation (12), we have transformed the four amplitudes into three combinations of form factors G_V, G_A, G_P, and ϵ, so that if ϵ is different from zero, it would be possible to extract from muon-capture data $F_M(q^2)$, $F_A(q^2)$, and

$$\frac{m_\mu E^{(\nu)}}{m_\pi^2}F_P(q^2)-\frac{E^{(\nu)}}{2m_p}F_E(q^2)$$

at the q^2 value of the muon-capture process, together with ϵ. For this extraction one requires values for Γ, P_L^N, P^N, and α.

Before going into the numerical analysis, we express the observables in terms of the helicity amplitudes. One obtains[2]

$$\begin{aligned} \Gamma&=\sum_\lambda|T_\lambda|^2\ , \\ P_L^N&=\frac{1}{\Gamma}\sum_\lambda\lambda_N|T_\lambda|^2\ , \\ P^N&=\frac{2}{3}\frac{1}{\Gamma}\sum_\lambda\{\sqrt{2}\,\text{Re}[T(\lambda_N+1,\lambda_\nu)T^*(\lambda_N,\lambda_\nu)] \\ &\quad+\lambda_N(\lambda_N-\lambda_\nu)|T_\lambda|^2\}\ . \end{aligned} \tag{14}$$

The quantities Γ, P_L^N, and P^N are the ones already measured experimentally.

III. THE $V+A$ PIECE

Apart from the absolute value of F_A determined from the capture rate, experimental values of P_L^N, P^N and α allow the extraction of F_M/F_A,

$$\left[\frac{m_\mu E^{(\nu)}}{m_\pi^2}\frac{F_P}{F_A}-\frac{E^{(\nu)}}{2m_p}\frac{F_E}{F_A}\right],$$

and ϵ. The absence of information on α for muon capture forces us to use form factor values as obtained from other sources. The value of F_M/F_A is known from the β decay of ^{12}B(g.s.). This value and the experimental results for P_L^N and P^N will constitute our input for the analysis.

The weak magnetism $F_M(q^2)$ shows up in a deviation of the β spectrum from the allowed shape (F_M/F_A has been also extracted[8] from a combination of β-decay and μ-capture data), so that, using the well-supported relation[9]

$$\frac{F_M(q^2)}{F_M(0)}=\frac{F_A(q^2)}{F_A(0)} \tag{15}$$

we have at our disposal the experimental value[8]

$$\left.\frac{F_M}{F_A}\right|_{\text{expt}}=3.87\pm0.44 \ . \tag{16}$$

Nevertheless, this value is extracted from a pure $V-A$ analysis, that is, imposing that in β decay the parameters in the hadronic sector are $\lambda_V=\lambda_A=1$. If the leptonic currents for β decay were $V-A$, it would be evident that the result (16) would correspond to

$$\frac{\lambda_V F_M}{\lambda_A F_A}=\left.\frac{F_M}{F_A}\right|_{\text{expt}} \tag{17}$$

but the presence of a $(V+A)$ piece in the leptonic sector will give contributions depending on ϵ to the observables of the β-decay process. As we will see later, the essential contribution to the bounds of ϵ will come from the error of P_L^N so that we will go on with the present analysis assuming the identification (17). At this point we must stress that if we had used the general Lagrangian of the left-right-symmetric model ($M_{W_1}/M_{W_2}\neq0$) it would be impossible to use Eq. (17) as an input in the anslysis, because the structure of the new Lagrangian would not be of the current-current form.

The ratio $R=P^N/P_L^N$ is less affected by the systematics of P_L^N and P^N and by the corrections coming from the excited states contribution,[10] so we will use the experimental values of R and P_L^N, together with Eq. (17).

Let us define

$$Y\equiv\epsilon^2,\ \ X\equiv\frac{G_A-G_P}{G_A}\ ,$$
$$\beta_A\equiv\frac{G_A}{G_A-2G_V}\ , \tag{18}$$

where β_A depends on $\lambda_V F_M/\lambda_A F_A$ only, so it is an experimental input. Using Eqs. (12) and (14) for P_L^N and P^N, it is straightforward to get the following constraints for X and Y

$$F_1(X)=A_3X^3+A_2X^2+A_1X+A_0=0\ ,$$
$$F_2(Y)=B_3Y^3+B_2Y^2+B_1Y+B_0=0\ , \tag{19}$$

where the coefficients A_i, B_j are well-defined functions of R, P_L^N, and β_A.

The experimental values of R, P_L^N, and β_A are independent, so that in order to get the errors of X and Y, σ_X and σ_Y, we must combine quadratically σ_R, σ_L, and σ_A, the errors of R, P_L^N, and β_A.

Using the values $R=-0.509\pm0.041$ from Ref. 10 and a value of $\beta_A=1.462\pm0.066$ from relation (16) we get the results of Table I. Using Eqs. (19) we have made two evaluations of X and Y for two different values of P_L^N. In the first row we have used the value $P_L^N=-1.03\pm0.11$ given by Roesch *et al.*,[10] in the second row we cut down the allowed interval of P_L^N to the region compatible with conservation of angular momentum ($|P_L^N|\leq1$) taking a value -0.96 ± 0.04.

In the second column we present the values for ϵ^2; the result in the second row is closer to the $V-A$ value $\epsilon=0$, nevertheless we get in both rows the same bound for $|\epsilon|<0.26$, because in our analysis $Y=\epsilon^2>0$, so that the larger error in the first row is compensated with the larger deviation from the $\epsilon=0$ value in the negative direction. We conclude that, independently of how we use the P_L^N values, we get at present a bound on ϵ such that the mixing angle is bounded by the values $0\leq|\xi|\leq0.25$. This has to be compared with 0.05 and 0.12 extracted essentially from muon and β decay.[11] By making model-dependent assumptions, much stronger limits (at the $|\xi|<0.005$ level) have been obtained[12] using information on nonleptonic decays, primarily K decay, and semileptonic decay data, with the recently measured B lifetime playing an essential role. Although the bound from muon capture is not still competitive, it represents the first estimate coming from a different semileptonic process. Coming back to Table I, we have analyzed the contribution of the different errors. The dominant contribution to the error of Y comes from σ_L and the contribution of σ_A to σ_Y is negligible. This is so whenever $\sigma_L\geq0.02$. Therefore, at the present level of accuracy, the ϵ^2 value is not sensitive to the identification (17).

In the fourth column of Table I we have the values for X. Again these extracted values of X are not sensitive to the relation (17). To obtain the induced pseudoscalar form factor F_p, we need a value of F_E and its q^2 dependence. Giffon *et al.*[9] have proved that the relation

$$\frac{F_A(q^2)}{F_A(0)}=\frac{F_E(q^2)}{F_E(0)} \tag{20}$$

is valid, at the 10% level, for the q^2 value of muon cap-

TABLE I. Values for ϵ and X from different P_L^N values.

P_L^N	$Y=\epsilon^2$	$\lvert\epsilon\rvert$	$X=\frac{G_A-G_P}{G_A}$	$f_P=\frac{F_P}{F_A}$
-1.03 ± 0.11	-0.09 ± 0.16	<0.26	0.34 ± 0.16	-0.82 ± 0.44
-0.96 ± 0.04	0.009 ± 0.059	<0.26	0.26 ± 0.08	-1.02 ± 0.37

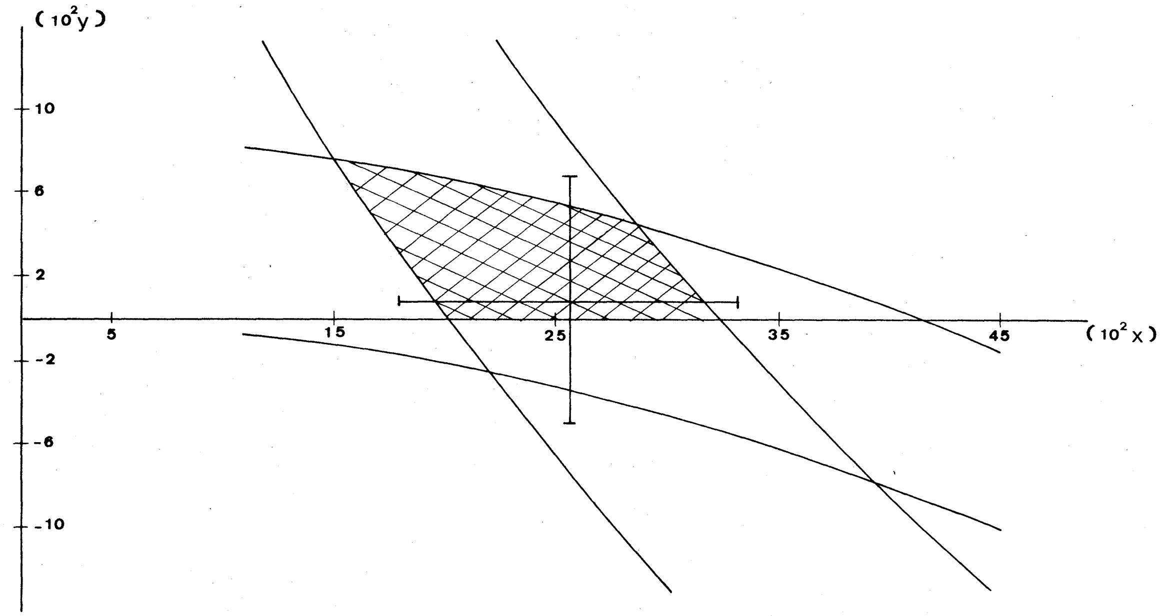

FIG. 1. Allowed region of values for $Y=\epsilon^2$ and X from the measurements of the ratio R and $-0.92 \geq P_L^N \geq -1$. The point shown is the result given in Table I, where the errors were added quadratically.

ture, although it is modified by the introduction of mesonic degrees of freedom. Using the β-decay value[8] $F_E(0)/F_A(0)=3.81\pm0.44$ and Eq. (20), we have evaluated in the fifth column of Table I the values for F_p. Both values of f_p are consistent with PCAC, which predicts[13] $f_P \simeq -1$.

One notices that the values extracted in Table I for X and Y, and their errors, are obtained (from P_L^N and R) without any information of one variable to the other. In general, the values of X and Y allowed from the experiment are correlated. We present in Fig. 1 two strips coming from $Y=Y(R,X)$ due to the experimental R and from $Y=Y(P_L^N,X)$ due to $-1 \leq P_L^N \leq -0.92$. β_A has been fixed because the contribution of the error σ_A is negligible. From Fig. 1, one sees the dominance of P_L^N in providing an upper bound for Y, as long as $X<0.29$. On the contrary, it is a better measurement of R which can constrain the value of f_p. In the weak-interaction framework used here to analyze the parameters, we have $Y=\epsilon^2 \geq 0$, so only the shadowed region shown in Fig. 1 is the one with full significance. Overimposed on the allowed region one finds the point extracted in Table I for $P_L^N=-0.96\pm0.04$, where the errors from R and P_L^N were added quadratically. One sees that the fixing of f_P to the theoretically expected value $f_P=-1$ does not improve much the bound on ϵ, going down to $|\epsilon|<0.23$.

IV. CONCLUSION AND OUTLOOK

In this paper we have shown in a quantitative way that the process $^{12}C(\mu^-,\nu_\mu)^{12}B$ can be used to probe the space-time structure of the weak charged currents. Using the experimental data at our disposal we obtain for the mixing angle of the left-right-symmetric model the bound $|\xi|<0.25$. This upper bound could be improved with higher precision for the measured value of P_L^N. In particular, it is the lower limit of $|P_L^N|$ which is of relevance: if one moved from $P_L^N<-0.92$ to $P_L^N<-0.96$, the mixing angle would be bounded by $|\xi|<0.12$. Independently of the value of ξ, we have extracted from the data the induced pseudoscalar coupling f_p, presented in Table I and consistent with the expectation from PCAC. For the standard value $\xi=0$, one gets $f_P=-1.02\pm0.30$.

Although our results are not sensitive to the identification (17), the β decay of ^{12}B should be analyzed within the same left-right-symmetric model if the precision in muon-capture observables increases in the future. Furthermore, the measurements of P_L^N and P^N themselves use the β-decay analysis for ^{12}B(g.s.). Complete consistency demands therefore the reanalysis of $^{12}B \rightarrow {}^{12}C\, e^- \bar{\nu}_e$ in the same scheme.

Let us finally mention the interest in measuring the asymmetry α in the angular distribution, in order to analyze the $V-A$ structure using information from muon capture only. In this case, the use of Γ, P_L^N, P^N, and α makes it possible to extend the present analysis to the full left-right-symmetric model in order to set bounds on the allowed region of the plane $(\xi, {M_{W_1}}^2/{M_{W_2}}^2)$.

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