Nonleptonic decays of chiral solitons and a possible resolution of the S-wave/P-wave puzzle

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We calculate the nonleptonic weak decays of hyperons in the Skyrmion model of baryons. The weak-interaction chiral Lagrangian is that known from kaon decay, providing a zero-parameter description of the decay amplitudes. In the S waves the PCAC (partially conserved axial-vector current) relation is verified, but the magnitude emerges as a factor of 3 too small, probably indicating a slow convergence of the chiral expansion. In the P waves, new and important terms are found which are not included in standard analyses. These modify the PCAC treatment of hyperon decay and can provide the resolution of the long-standing S-wave/P-wave puzzle.

I. INTRODUCTION

The theory of weak nonleptonic decay remains one of the least understood aspects of the low-energy weak interactions. While the weak currents are believed to be accurately given by the standard Weinberg-Salam model, there remains the difficulty of obtaining hadronic matrix elements of the weak Hamiltonian. Some progress has been made through use of the quark model,[1] which yields fairly reasonable predictions for hyperon S-wave amplitudes, but which fails for kaon decays and has difficulty explaining hyperon P-wave matrix elements. The quark-model approach, however, does not possess the chiral-symmetric properties of real matrix elements involving pions.[2] We are therefore limited to the extent with which we can probe nonleptonic decays.

The recent development of a chiral-soliton model for baryons (i.e., Skyrmions[3,4]) provides a new "theoretical laboratory" with which to explore nonleptonic decays. In this paper we examine the weak decays of the chiral solitons and find that they do in fact provide new information previously unavailable in the quark model.

The program for the study of Skyrmions involves the use of effective chiral Lagrangians. It is the strong-interaction portion of the Lagrangian which dictates the structure of the soliton itself. Within the context of the present paper what is more important, as we review below, is that we know to a certain extent the chiral Lagrangian which leads to $\Delta S=1$ kaon decays.[5] In Skyrmion models it is this *same* Lagrangian which can also yield hyperon decay amplitudes. This presents us with the exciting possibility of predicting hyperon decay solely in terms of kaon decay. Unfortunately, at the simplest level of approximation, we find that this program does not work very well.

When applied to Skyrmions, the weak chiral Lagrangian will produce both S-wave and P-wave amplitudes. One aspect of this topic, the "D/F ratio" in the S-wave amplitudes, has previously been discussed by Bijnens, Sonoda, and Wise.[6] We look more closely at the magnitude of the S waves, including quartic Lagrangians, and extend the analysis to the P waves. It is the P-wave amplitudes which contain new terms which are most often neglected in standard treatments of the subject, and which we feel have important implications (see Sec. III).

The low-energy behavior of the *strong* interactions can be described by an effective Lagrangian involving the pseudoscalar fields $\phi^i=\pi,K,\eta$,

$$L_{\rm st}=\frac{F_\pi{}^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial^\mu\Sigma^+)+\tfrac{1}{2}\,\mathrm{Tr}[(a+b\lambda_8)(\Sigma+\Sigma^+)]+\cdots\,, \tag{1}$$

with

$$\Sigma=\exp(i\lambda\cdot\boldsymbol{\phi}/F_\pi),\quad F_\pi\simeq 94\ \mathrm{MeV}\,, \tag{2}$$

where we have not exhibited terms with higher numbers of derivatives. The precise nature of $L_{\rm st}$ will not be very important in our study.

The dominant *weak* interactions are those with a $\Delta I=\frac{1}{2}$, SU(3) octet behavior under left-handed chiral transformations, while being a singlet under right-handed transformations, i.e., $(8_L,1_R)$. We shall neglect the small $\Delta I=\frac{3}{2}$ $(27_L,1_R)$ effects in this paper. Chiral Lagrangians can also be written to describe the $(8_L,1_R)$ interactions

$$L=g\,\mathrm{Tr}(\lambda_6\partial_\mu\Sigma\partial^\mu\Sigma^+)+g'\mathrm{Tr}(\lambda_6\partial_\mu\Sigma\partial^\mu\Sigma^+\partial_\nu\Sigma\partial^\nu\Sigma^+)+g''\mathrm{Tr}(\lambda_6\partial_\mu\Sigma\partial_\nu\Sigma^+\partial^\mu\Sigma\partial^\nu\Sigma^+)+\cdots\,. \tag{3}$$

Here a discussion of the terms which we have not written is very important. The leading term with two derivatives is the usual "Cronin" Lagrangian which by itself does a good ($\simeq$25%) job of describing kaon decays.[7] There is clear evidence in $K\to 3\pi$ data (the so-called quadratic terms in the Dalitz plot) for quartic Lagrangians, i.e., those with four derivatives. An excellent ($\approx$10%) fit to the overall magnitude, slopes, and quadratic terms of $K\to 3\pi$, and the magnitude of $K\to 2\pi$ can be obtained with[5]

$$g = 3.6\times 10^{-8} m_\pi{}^2 ,$$
$$g' = g/(0.51\ \text{GeV})^2 , \tag{4}$$
$$g'' = g/(0.76\ \text{GeV})^2 .$$

In this paper we will use these values to explore the weak decays of Skyrmions. However it must be pointed out that this parametrization is not unique. At the level of four derivatives there are many more Lagrangians in addition to the two which we have listed. Other linear combinations of other Lagrangians can provide an equally good fit to kaon physics, and one does not have enough information to uniquely determine the coefficients of all quartic Lagrangians. See Ref. 5 for a more detailed discussion of this effect. Nevertheless we will adopt Eq. (3) as a trial description of the weak interactions. This will certainly be sufficient to justify our rather negative conclusion in Sec. III about the numerical reliability of the predictions of this approach. Our analysis is presented in Sec. II, to which we now turn.

II. ANALYSIS

The Skyrme model of the baryons involves a soliton solution

$$\Sigma_0 = \begin{bmatrix} e^{iF(r)\tau\cdot\hat{x}} & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

to the strong-interaction Lagrangian. The functional form of $F(r)$ depends on the details of the quartic terms in the Lagrangian. For the ansatz (5), the static Lagrangian is the same for both the SU(2) and SU(3) cases (only the part depending on collective coordinates changes). Thus the soliton radial profile is the same for each, so we shall employ $F(r)$ as determined in Ref. 4. The baryonic collective coordinates are found by allowing slow rotations of the soliton

$$\Sigma = A(t)\Sigma_0 A^{-1}(t) \tag{6}$$

with A being an arbitrary time-dependent SU(3) matrix. The wave functions for the $J=\frac{1}{2}$ baryons have been given by Guadagnini.[8] Using the conventional SU(3) labels $|Y,I,I_3\rangle$ the wave functions are given by matrix elements of the SU(3) octet rotation matrices defined by

$$|Y,I,I_3\rangle = \sum_{Y',I',I_3'} \langle Y',I',I_3' | D^{(8)}(A) | Y,I,I_3\rangle \, | Y',I',I_3'\rangle \tag{7}$$

for a rotation by an SU(3) matrix A. A state with quantum numbers Y,I,I_3, and z component of spin S_z has the wave function

$$\psi_{Y,I,I_3 S_z}(A) = N\langle Y,I,I_3 | D^8(A) | 1,\tfrac{1}{2},-S_z\rangle \tag{8}$$

which we will abbreviate, following Ref. 6, as $D_{ab}(A)$ with $a=Y,I,I_3$ and $b=1,\frac{1}{2},S_z$. Schnitzer has shown how to add pions and kaons to this system in a manner consistent with the soft-pion theorems.[9] This leads to the final form for the chiral matrix,

$$\Sigma = U_\pi A(t)\Sigma_0 A^{-1}(t) U_\pi , \tag{9}$$

where

$$U_\pi = \exp(i\boldsymbol{\lambda}\cdot\boldsymbol{\phi}/2F_\pi) . \tag{10}$$

It is this form which is used to explore the weak interactions.

Before considering S-wave and P-wave transitions separately, we should mention the "semiclassical" approximation which is associated with the fact that the solitons are slowly rotating. In this approach, the lowest order consists of neglecting all time derivatives acting on the field Σ. In the next order one time derivative appears, and so on. There is no convergence scheme, however, associated with the number of spatial derivatives. Any truncation thereof is essentially a dynamical assumption. Whether or not such a truncation provides a reasonable approximation depends ultimately on the success a description like the Skyrme model attains in reproducing various weak and electromagnetic properties of the baryons.

At this point we introduce the A,B amplitude of nonleptonic S-wave, P-wave decays, respectively, in $B(\mathbf{p},\lambda)\to B'(\mathbf{p}',\lambda')+\pi(\mathbf{q})$,

$$\bar{u}(\mathbf{p}',\lambda')(A+B\gamma_5)u(\mathbf{p},\lambda) . \tag{11}$$

In the limit of small momentum, Eq. (11) reduces to

$$A\chi^\dagger(\lambda')\chi(\lambda) - \frac{B}{2\overline{M}}\chi^\dagger(\lambda')\boldsymbol{\sigma}\cdot\mathbf{q}\chi(\lambda) , \tag{12}$$

where $\overline{M}$ is the average baryon mass, $\overline{M}=(M+M')/2$ and terms of order $p(M'-M)/\overline{M}$, p^2/M^2, etc., are neglected. Equation (12) provides us with the prescription necessary for extracting the A,B amplitudes in our calculation of soliton nonleptonic transitions.

A. S-wave decays

We insert the ansatz Eq. (9) for Σ into the weak Lagrangian of Eq. (3). In view of the semiclassical approximation, only spatial derivatives are taken. For S-wave emission of a pion, no derivatives of pion fields can appear in the Lagrangian, i.e.,

$$\partial_i\Sigma \underset{S\ \text{wave}}{\longrightarrow} U_\pi A(t)\partial_i\Sigma_0 A^{-1}(t) U_\pi . \tag{13}$$

It is of interest to exhibit terms in the weak Lagrangian which contain zero or one pion field. Accordingly one need expand U_π to no further than first order.

As an example, upon performing these operations on the first (i.e., "quadratic") term of Eq. (3), we obtain

$$g\,\text{Tr}(\lambda_6\partial_\mu\Sigma\partial^\mu\Sigma^+) \tag{14}$$

becoming

$$-g\,\text{Tr}(\lambda_6\partial_i\Sigma\partial_i\Sigma^+) \tag{15}$$

and employing Eq. (9),

$$-g\,\text{Tr}(U_\pi^\dagger\lambda_6 U_\pi A\partial_i\Sigma_0\partial_i\Sigma_0^\dagger A^\dagger) \tag{16}$$

which after some algebra (see the Appendix) yields for π^0 emission,

$$-\frac{2g}{\sqrt{3}}\left[F'^2+2\frac{\sin^2 F}{r^2}\right]\left[D_{68}(A)-D_{78}(A)\frac{\pi^0}{2F_\pi}+\cdots\right]. \tag{17}$$

In Eq. (17), the SU(3) representation matrices $D(A)$ are given by [see also Eqs. (7) and (8)]

$$D_{ab}(A)=\tfrac{1}{2}\,\mathrm{Tr}\lambda_a A\lambda_b A^{-1} . \tag{18}$$

Explicit calculation demonstrates that the relation between D_{68} and D_{78} is such as to put the terms containing zero and one pion field in the correspondence expected in the soft-pion limit. This is of course precisely what we would expect in a chirally invariant description such as the one used here.

Concentrating on the terms containing one pion field, we find for the angular integral of Eq. (3) the form

$$\int d\Omega\, L_{S\text{ wave}}=\frac{8\pi}{\sqrt{3}}\frac{\pi^0}{2F_\pi}D_{78}(A)I(r) , \tag{19}$$

where

$$\begin{aligned} I(r)&=g\left[F'^2+\frac{2\sin^2F}{r^2}\right]-g'\left[F'^2+\frac{2\sin^2F}{r^2}\right]^2\\ &\quad-g''\left[F'^4-\frac{4F'^2\sin^2F}{r^2}\right]\\ &\equiv I+I'+I'' . \end{aligned} \tag{20}$$

To determine S-wave amplitudes for individual transitions, all that need be done is to determine the relevant baryon-to-baryon matrix elements of $D_{78}(A)$ and also perform the radial integral of the quantity $I(r)$,

$$\chi^\dagger(\lambda')\chi(\lambda)A(\alpha\to\beta\pi^0) =\frac{4\pi}{\sqrt{3}}\int_0^\infty dr\, r^2 I(r)\frac{\langle\beta\,|\,D_{78}(A)\,|\,\alpha\rangle}{F_\pi} . \tag{21}$$

The former is considered in the Appendix. For the latter, note first that convergence is not a problem. As $r\to\infty$, it is simple to show $F(r)\sim r^{-2}$, so that the asymptotic behavior of $r^2I(r)$ for the three terms in Eq. (20) is, respectively, r^{-4}, r^{-10}, and r^{-12}.

Results of the model turn out to be decidedly mixed. We verify the result of Ref. 6 that the SU(3) structure of the model amplitudes is $f/d=-\frac{5}{3}$. The reader should realize that the analysis of Ref. 6, where this result first appears, holds for all orders in the number of spatial derivatives. The model described here is a subcase of this, being based on a linear combination of two and four spatial derivatives. The prediction $f/d=-\frac{5}{3}$ is of course in reasonable accord with experiment; the empirical amplitudes are well fit by $f/d\simeq-2$ (Ref. 1). However the magnitude of the predicted S-wave amplitudes turns out to be small by roughly a factor of 3:

$$\begin{aligned} f(\text{soliton})&=-3.2\times10^{-8} ,\\ f(\text{expt})&=-9.2\times10^{-8} ,\\ d(\text{soliton})&=1.9\times10^{-8} ,\\ d(\text{expt})&=3.8\times10^{-8} . \end{aligned} \tag{22}$$

This comes about not because the individual terms I,I',I'' are each too small but rather because there is considerable cancellation between I and I' in Eq. (20). That is, it is the relative phase of g and g' [see Eqs. (3) and (4)] which is crucial to this conclusion. How reliably is this relative phase known? Recall that it is deduced from $K\to3\pi$ data in Ref. 5. We have restudied the analysis of this paper and stand by its conclusions.

B. P-wave decays

A full treatment of the P-wave amplitudes must include baryon and kaon pole terms. As usual, these are proportional to single-particle matrix elements of the parity-conserving weak Hamiltonian. Self-consistency of our calculation would demand that we determine these from the meson and soliton sectors of the chiral model defined by Eq. (3). However, in a phenomenological analysis of the P-wave amplitudes, it is known that the baryon poles dominate the kaon poles.[1] Since the soliton-to-soliton matrix elements of H_w^{pc} are a factor 3 too small, we expect that the P waves cannot satisfactorily be explained either.

But this is not the whole story in the soliton model. It turns out there exists yet another contribution to P-wave decays arising from a "contact" term. Thus the full P-wave amplitude is as depicted in Fig. 1. To see how the contact term arises, we proceed analogously to the work just described in Sec. II A, except now focusing attention on terms having one derivative acting on a pion field. Such interaction terms would naturally vanish in the soft-pion limit which perhaps explains why they have not been heretofore (at least to our knowledge) considered.

Upon substituting the ansatz (9) into Eq. (3) and integrating over solid angles, we obtain after a tedious computation

$$\int d\Omega\, L_{P\text{ wave}}=\frac{8\pi}{3}\frac{\partial_a\pi^0}{2F_\pi}J_a(r) , \tag{23}$$

where we consider π^0 emission only and

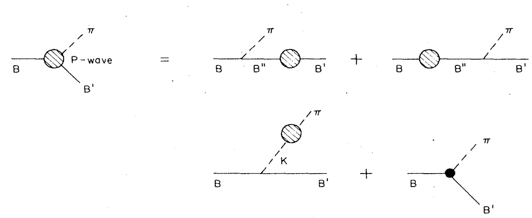

FIG. 1. The diagrams which contribute to P-wave hyperon decay. The last diagram is the contact interaction described in the text.

$$J_a(r)=g\left[D_{6a}(A)\left[F'+\frac{2\sin F}{r}\right](1+\cos F)+\frac{2}{\sqrt{3}}[D_{6a}(A)D_{38}(A)+D_{3a}(A)D_{68}(A)]\left[-F'+\frac{2\sin F}{r}\right](1-\cos F)\right]$$
$$+ig'\left[F'^2+2\left[\frac{\sin F}{r}\right]^2\right]\left[D_{7a}(A)\left[(1+\cos F)F'+\frac{\sin 2F+2\sin F}{r}\right]\right.$$
$$\left.+\epsilon_{ajk}D_{6j}(A)D_{3k}(A)\left[(1+2\cos 2F-\cos F)2F'+\frac{6\sin 2F-4\sin F}{r}\right]\right]$$
$$-ig''[D_{7a}(A)(C-\tfrac{2}{3}E)-2\epsilon_{ajk}D_{6j}(A)D_{3k}(A)E]\ , \tag{24}$$

where

$$C=-\tfrac{1}{3}(4\cos 2F+\cos F+5)F'^3+\tfrac{2}{3}\sin F(1+5\cos F)\frac{F'^2}{r}+2\sin^2F(2\cos 2F+\cos F+3)\frac{F'}{r^2}-\frac{8\sin^3F\cos F}{3r^2} \tag{25a}$$

and

$$E=(1-\cos F+2\cos 2F)F'^3+2\sin F(1-\cos F)\frac{F'^2}{r}+2\sin^2F\cos F\cos 2F\frac{F'}{r^2}+\frac{4\sin^3F\cos F}{r^3}\ . \tag{25b}$$

The contact contributions to the B amplitudes are found by doing the radial integral for $r^2J(r)$, taking its $B\to B'\pi^0$ matrix elements, and finally referring to the correspondence in Eq. (12). A simplified form for the contact contribution to the B amplitudes corresponding to π^0 emission along the 3 axis is given by

$$B_{\text{contact}}=\frac{8\pi}{3}\frac{\overline{M}}{F_\pi}gR_0{}^2\{0.34D_{63}(A)-2.94i\epsilon_{3jk}D_{6j}(A)D_{3k}(A)+1.31[D_{63}(A)D_{38}(A)+D_{33}(A)D_{68}(A)]\}\ , \tag{26}$$

where the dimensional quantity $R_0=1$ fm arises from the radial integrals, and the baryon-to-baryon matrix elements of the $D_{ij}(A)$ representation matrices remain to be evaluated. Upon doing so, we obtain (in units of 10^{-7})

	Contact	Experiment
$B(\Lambda_0^0)$	-4.1	-15.8
$B(\Sigma_0^+)$	2.3	26.6
$B(\Xi_0^0)$	0.6	-12.3

(27)

for several representative cases.

Observe in Eq. (26) the presence of both antisymmetric and symmetric combinations of D matrices. These imply that B_{contact} contains terms transforming as an antidecuplet and 27-plet, respectively, in addition to the usual octet contributions. Unfortunately the complexity in structure of the P-wave Lagrangians makes it impossible to reach a conclusion good to all orders in spatial derivatives as was obtained for S-waves in Ref. 6. Of course, the spin structure of the P-wave amplitudes is correctly reproduced. For example, the sign of the amplitude changes for pion emission along the 3 axis when we reverse the direction of the soliton 3 component of spin. Finally, what about the magnitude of these contact contributions? Given the difficulty of the model in describing the S-wave amplitudes, it is hard to be definitive about this. However, it would appear to be safe in concluding that they are nonnegligible. For example, a modification of the relative phases among the three g,g',g'' contributions could make the $\Lambda\to N\pi^0$ twice as large as the already important contribution displayed in Eq. (27). That is, a soliton model based on a different choice of Lagrangian than the one given in Eq. (3) could generate even larger contact contributions than the ones found here.

III. CONCLUSIONS

Our analysis of the nonleptonic decays in a model of baryons as solitons has been instructive in several regards.

First, it reflects on the potential of the simplest soliton models for being phenomenologically useful descriptions. Recall that the analyses of Refs. 4 and 5 regarding the strong-interaction sector were generally positive. In Ref. 5, the meson and baryon channels were found to be simultaneously describable in an acceptable manner, albeit within the rather large error bars occurring in existing $\pi\pi$ data. Reference 4 considered a number of nucleon observables and found qualitative agreement with experiment in most cases. However the axial-vector coupling was found to be too small by a factor of 2. The prediction $f/d=-\frac{5}{3}$ in Ref. 6 for the SU(3) structure of S-wave amplitudes to all orders of spatial derivatives (and zeroth order in time derivatives) lent further credence to the soliton approach. The calculation described here must be regarded as a failure of the soliton picture, but it is important to interpret this finding in a careful manner.

Specifically we are referring to the Lagrangian of Eq. (3) which contains three terms, one quadratic in the number of derivatives and two which are quartic. As we have mentioned, this truncation in the number of spatial derivatives is a dynamical assumption necessary to make the model mathematically tractable. Given the energy range explored for physical baryons (masses as large as 1.3 GeV), there can be no justification based solely upon chiral symmetry for this approach. Moreover, as we have also mentioned, even at this level of truncation the model is not unique. It is quite possible to employ two different quartic Lagrangians, whose coefficients could be fixed from $K\to 3\pi$ data, and which could in principle render a superior fit to the hyperon decay amplitudes. Unfortunately this would not be very assuring for it would imply an overly sensitive dependence on the choice of "basic Lagrangians" at the quartic level.

The reader should not confuse our negative conclusion regarding the soliton model based on Eq. (3) with the convergence of the "semiclassical approximation" mentioned in Sec. I. The latter refers to the number of time derivatives whereas the truncation assumption mentioned above pertains solely to spatial derivatives. Indeed, based on the successful prediction in Ref. 6 of the f/d structure for the *S*-wave amplitudes, it seems likely that inclusion of more and more Lagrangians with larger numbers of spatial derivatives would ultimately yield a successful model of hyperon nonleptonic physics. Evidently, such a series in the number of spatial derivatives converges more slowly than one might have hoped.

On the positive side, the model contains a hint regarding the resolution of the *S*-wave/*P*-wave puzzle in nonleptonic hyperon decay. Recall[1] that the standard PCAC (partially conserved axial-vector current) analysis of weak hyperon decay results in both the *S*-wave and *P*-wave amplitudes being described by the *same* set of baryon-to-baryon amplitudes. The analysis is based on the soft-pion theorem which states that $B\to B'\pi$ amplitudes are related to those of $B\to B'$ when the pion's four-momentum vanishes, a statement that is a consequence of PCAC. For the *P* waves the $B\to B'$ amplitude vanishes [in the SU(3) limit] and this leaves only the baryon poles to be included in the standard analysis. Thus PCAC predicts

$$\lim_{q_\pi\to 0}\langle B'\pi^0|H_w^{\rm PV}|B\rangle=\frac{-i}{2F_\pi}\langle B'|H_w^{\rm PC}|B\rangle\ ,$$
$$\lim_{q_\pi\to 0}\langle B'\pi^0|H_w^{\rm PC}|B\rangle=\sum_{\rm poles}\frac{g_{\pi B'B''}\langle B''|H_w^{\rm PC}|B\rangle}{m_B-m_{B''}}+(B\leftrightarrow B')\ . \quad (28)$$

The appearance of the same set of parity-conserving $B\to B'$ amplitudes in both channels leads to predictions of the magnitude of *P*-wave amplitudes. Despite the firm foundation of PCAC, such a treatment fails by about a factor of 2 when applied to the experimental amplitudes. This is the *S*-wave/*P*-wave puzzle, which has been around since the 1960s.

In order to accommodate the data one must either explain why SU(2)×SU(2) PCAC is broken by such a large factor or find the flaw in the standard analysis. The Skyrme model has PCAC being respected, and contains a new ingredient which allows PCAC to be consistent with experiment. This is the new nonpole "contact" contribution found in the *P* waves. The contact term is one which is explicitly proportional to q_μ; hence it vanishes in the soft-pion limit. Nevertheless in the Skyrme model it is a quite large contribution to the physical amplitude. The correct analysis of the *P* waves would be

$$\langle B'\pi^0|H_W^{\rm PC}|B\rangle=A_{\rm contact}(q)+\sum_{\rm poles}\frac{g_{\pi B'B''}\langle B''|H_W^{\rm PC}|B\rangle}{m_B-m_{B''}}+(B\leftrightarrow B')\ . \quad (29)$$

This is consistent with the soft-pion theorem, Eq. (28),

$$\lim_{q\to 0}A_{\rm contact}(q)=0\ , \quad (30)$$

but nevertheless is a major modification of the standard analysis if $A_{\rm contact}$ is large on shell. Viewed in this light, the flaw of the standard analysis lies not with PCAC, but with the assumption that terms which vanish as $q_\pi\to 0$ are not important.

A proposal which is similar in spirit has been made in Ref. 10. The suggestion of these authors, however, differs from ours in that they place the momentum variation in the *S*-wave amplitudes, while the *P*-waves retain only the pole contributions. The Skyrme model in the semiclassical approximation has no momentum variation in the *S* wave, although it is possible that some is generated if one goes beyond the semiclassical limit.

Actually, Georgi and Manohar[11] have suggested from a phenomenological viewpoint that a solution such as appears in the Skyrme model must occur. Their reasoning is based on an analysis of (non-soliton)-baryonic chiral Lagrangians. Chiral Lagrangians without derivatives would yield predictions equivalent to the standard analysis. However there are four additional contributions to baryonic chiral Lagrangians which involve a derivative on the pion field. These would yield a momentum dependence analogous to our contact term. Although these chiral Lagrangians have no predictive power (aside from isospin relations) for *P*-wave transitions, they do anticipate the presence of nonpole terms such as those found in our work. If PCAC is to be valid in hyperon decays, these new features must be the resolution of the *S*-wave/*P*-wave puzzle. It remains to be seen whether further thought and study can extend our findings into a successful quantitative description of hyperon decay.

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APPENDIX: DETAILS OF THE CALCULATION

In the following we describe two numerical aspects of our calculation, viz., evaluation of radial integrals and determination of matrix elements involving the time-dependent SU(3) matrices $A(t)$.

As explained earlier the *S*-wave and *P*-wave amplitudes describing nonleptonic transitions of solitons are given in terms of certain radial integrals. The integrands typically involve functions of the soliton radial profile $F(r)$ [see

Eq. (5)] along with its radial derivative $F'(r)$. An analytic form of $F(r)$ does not exist; rather, $F(r)$ is determined numerically as the solution to a nonlinear differential equation. It is known that $F(0)=\pi$ and $F(r)\sim Ar^{-2}$ as $r\to\infty$. A plot of $F(r)$ for $0\le r\le 3$ fm appears in Ref. 4. We found an excellent fit to this plot was given by

$$F(r)=\begin{cases}\pi-3.65r+1.285r^2 & (r<1),\\ 0.62/r^2 & (r>1),\end{cases} \tag{A1}$$

where r is in units of fm. The representation of Eq. (A1) formed the basis of our numerical work.

Another important component in extracting specific predictions from the soliton model involves trace theorems associated with the collective coordinates $A(t)$. The simplest such relation, given in Eq. (18), is all that is needed in computing S-wave amplitudes. More complicated expressions are encountered in the P-wave sector. In particular we found ($i=1,2,3$)

$$\mathrm{Tr}[\lambda_6(A\lambda_i A^\dagger\lambda_3 A\lambda_8 A^\dagger - A\lambda_8 A^\dagger\lambda_3 A\lambda_i A^\dagger)] = 4\sqrt{3}i[\tfrac{1}{3}D_{7i}(A)-\epsilon_{jki}D_{6j}(A)D_{3k}(A)] \tag{A2}$$

and

$$\mathrm{Tr}[\lambda_6(A\lambda_i A^\dagger\lambda_3 A\lambda_8 A^\dagger + A\lambda_8 A^\dagger\lambda_3 A\lambda_i A^\dagger)] = 4\sqrt{3}\left[\tfrac{1}{3}D_{6i}(A)+\frac{1}{\sqrt{3}}[D_{6i}(A)D_{38}(A) + D_{68}(A)D_{3i}(A)]\right] . \tag{A3}$$

The proof of Eqs. (A2) and (A3) rests upon several ancillary results. The reader should have no difficulty in demonstrating

$$D_{6i}(A)D_{aj}(A)d_{ijb}=d_{6aj}D_{jb}(A) \tag{A4}$$

and

$$D_{6i}(A)D_{aj}(A)f_{ijb}=f_{6aj}D_{jb}(A) . \tag{A5}$$

Less obvious are ($a=1,2,3$)

$$f_{8kl}f_{laj}=\frac{\sqrt{3}}{2}\left[d_{ajk}-\frac{1}{\sqrt{3}}(\delta_{ja}\delta_{k8}+\delta_{ka}\delta_{j8})\right] \tag{A6}$$

and

$$f_{8kl}d_{jla}=\frac{\sqrt{3}}{2}(f_{kja}-\epsilon_{kja}) . \tag{A7}$$

Equations (A6) and (A7) follow from a numerical analysis of the SU(3) f,d coefficients.

Finally there is the matter of taking matrix elements of the D matrices between baryon states of specified spin alignment and flavor. This is an entirely straightforward procedure. Products of two D matrices are reduced by means of the corresponding Clebsch-Gordan series and the group integration utilizes the orthogonality property of the D matrices. For example, consider the matrix element $\langle N\uparrow|D_{63}(A)|\Lambda\uparrow\rangle$. As a shorthand for the sequence Y,I,I_3 of SU(3) labels, we employ $K^0\equiv(1,\frac{1}{2},-\frac{1}{2})$, $\eta\equiv(0,0,0)$, and $\pi^0=(0,1,0)$. Then we find

$$\begin{aligned}\langle N\uparrow|D_{63}(A)|\Lambda\uparrow\rangle &= \frac{1}{\sqrt{2}}\int dA\, D^{(8)*}_{K^0K^0}(A)D^{(8)}_{K^0\pi^0}(A)D^{(8)}_{\eta K^0}(A)\\ &=\sqrt{1/2}\sum_k C\begin{bmatrix}8 & 8 & 8\\ K^0 & \eta & K^0\end{bmatrix} C\begin{bmatrix}8 & 8 & 8n\\ \pi^0 & K^0 & K^0\end{bmatrix}\\ &=\tfrac{2}{15}\sqrt{3/2} .\end{aligned} \tag{A8}$$

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